The suggested vehicle kilometers of travel procedures are practical and are based on sampling theory. The random selection of days and locations to count obviates the need to apply specific adjustment factors. It is hoped that cities and states will apply these procedures in the development of their initial sampling plan. As information on the reliability of vehicle kilometers of travel estimates are assembled, it will be possible to refine methods and pinpoint parameters.

Further research is desirable to obtain information on the variance of the distribution of freeway, arterial, and local street links. This information will make it possible to directly estimate the vehicle kilometers per link, thereby allowing greater clarity in sampling frames and procedures.

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Abridgment

Empirical Comparison of Various Forms of Economic Travel Demand Models

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Transportation planners are interested in assessing future conditions of intercity travel demands and in knowing the passenger's response to a fare hike. Planners need to be able to forecast correctly how a reduction in air fare would affect the passenger demand for airlines and other competing modes such as rails and buses.

To answer these questions, we introduce several different forms of demand equations, which were developed by many economists (1, 2). To be consistent with the theory of consumer behavior, all demand equations should satisfy three basic properties: homogeneity, summability, and symmetry. Traditional demand analysis in intercity travel demand (3-5) has never explicitly introduced the three properties in the formulation of demand equations. In many cases, the final form of reduced equations becomes a double-log form. The own and cross elasticities are a popular tool for the evaluation of the passenger's response to the price hike. When the signs of the estimated parameters are inconsistent with their experiences, an inequality-constrained double-log equation is often introduced to impose correct signs (4, 6). The market cross elasticities fail to correctly measure the substitutability among alternative travel modes because those elasticities include the income effects.

We adopt five popular demand models:

1. A double-log demand model,
2. An inequality-constrained double-log demand model,
3. A weighted Stone model (7),
4. The Rotterdam system of demand equations (2), and
5. A homogeneous translog demand model (8).

The last three models have firm foundations in the theory of consumer behaviors, and the parameter estimation of the models has been done by imposing three basic properties (i.e., homogeneity, summability, and symmetry). The first two models have very loose ties with the theory of consumer behavior.

One feature of our analysis is a comparative study that answers the following questions: (a) Does the choice of functional form matter in predicting substitutability among intercity travel demand? (b) Is it necessary to tie the model to the theory of consumer behavior to get a reliable result? and (c) Are market cross elasticities proper indicators of substitutability?

Another interesting feature of our model is the use of a compensated demand concept. Conventional intercity travel demand models (1, 3-5, 9, 10) fail to introduce this theoretically important and useful concept. The compensated demand concept can be used to correctly measure both consumer surplus and substitutability.

Our demand analysis differs from conventional intercity modal-split models in one important aspect. The conventional models employ trips as the variable of interest whereas our model employs the distance of travel. Use of travel distance instead of trip simplifies conceptual understanding of intercity travel-demand behavior by excluding trip-related variables, such as trip origin, destination, and length. Furthermore, travel distance, which is a continuous variable, directly ties with many policy-related variables, such as energy consumption in transportation, accident frequency rates, and pollution control measures.

THE MODELS

We assume that a consumer has an additively separable utility function in terms of several group commodities such as food, intercity travel, clothing, energy, and leisure. The consumer maximizes his or her utility in terms of these group commodities, which have money and time constraints. From the first-stage maximization, the consumer decides how much money (M) is required for the intercity travels and how much time (T) he or she can allocate for the intercity travels.

At the second stage, the consumer allocates the intercity travel budget (M) and travel time (T) on various travel modes so as to maximize his or her utility. The usual Lagrangian solutions provide the derived demand equations, which are expressed in terms of unit costs (p_i, $i = 1, \ldots, n$), speed (t_i, $i = 1, \ldots, n$), time (T), and money (M) budgets for the intercity travel demands:

$$x_i = x_i\,(p_1, \ldots p_n, t_1, \ldots t_n, M, T) \tag{1}$$

where $i = 1, \ldots, n$. Equation 1 is the usual starting point of empirical demand equations. We have selected four popular forms of demand equations.

Double-Log Form

The double-log demand equation provides various market elasticities directly from the estimated coefficients. However, there is no guarantee that all estimated coefficients will have a right sign. To avoid such difficulty, we introduce the inequality-constrained double-log demand equation.

Inequality-Constrained Double-Log Form

We may impose positive signs on all cross elasticities and negative signs on all own elasticities. However, they may not satisfy the basic properties of the theory of consumer behaviors.

Weighted Stone Model

We impose the summability, homogeneity, and symmetry conditions on the Stone model (7). This can be done by multiplying the budget share to Stone's demand equation and properly restricting the values of parameters:

$$s_i \log x_i = \sum_{j\epsilon C} (b_{ji} \log p_j) + b_{mi} \log M^* + b_{ti} t + b_{si} \log SR + b_{oi} \tag{2}$$

with the restrictions

$$\sum_{i\epsilon C} b_{mi} = 1 \text{ (summability)} \tag{3}$$

$$b_{ji} = b_{ij} \text{ (symmetry)} \tag{4}$$

$$\sum_{i\epsilon C} b_{ji} = 0 \text{ (homogeneity)} \tag{5}$$

where

$$\begin{aligned} b_{mi} &= s_i\, a_{mi}, \\ b_{ji} &= s_i\, a_{ji}, \\ b_{ti} &= s_i\, a_{ti}, \\ b_{si} &= s_i\, a_{si}, \\ b_{oi} &= s_i\, a_{oi}, \\ \log M^* &= \log M - \sum_{j\epsilon C} (s_j) \log p_j, \\ C &= \text{air, bus, or rail mode,} \\ s_j &= \text{the money budget share of } j\text{th mode,} \\ SR &= \text{the ratio of airline speed to the bus-rail speed, and} \\ t &= \text{time trend.} \end{aligned}$$

a_{ji}, a_{mi}, a_{ti}, a_{si}, and a_{oi} are the parameters of Stone's demand equation and a_{ji} for j, $i \epsilon C$ are compensated elasticities.

Rotterdam System of Demand Equations

$$s_i d \log x_i = \sum_{j\epsilon C} (b_{ji} d \log p_j) + b_{mi} d \log M^* + b_{ti} + b_{si} d \log SR \tag{6}$$

(with the same restrictions as in Equations 3-5)

Homogeneous Translog Demand Equations

Alternatively, we may begin with a specific form of indirect utility function. We assume that the consumer has a homogeneous translog indirect utility function. By using Roy's identity, we have the following budget share equations:

$$s_i = -\sum_{j\epsilon C} [b_{ji} \log (p_j/M)] + a_{si} \log (SR) + a_{ti} t + a_{oi} \tag{7}$$

The symmetry and homogeneity in Equations 4 and 5 and normalization ($\Sigma\, a_j = -1$) are imposed.

DESCRIPTIONS OF DATA AND EMPIRICAL RESULTS

Intercity passenger kilometers, prices per passenger kilometer, and number of passengers by each mode (airline, bus, and railroad) for 1947-1974 were collected from Transportation Facts and Trends (11). The average annual speed of the airline service is obtained from the Handbook of Airline Statistics (12). The average speed of bus and rail is gathered from Federal Highway Administration (FHWA) and Amtrak, respectively. The speed data of the rail mode include not only intercity trains but also suburban trains, including waiting time, whereas the speeds of airline and bus are the average maximum speed of the trip, not including waiting time. Because of this factor, the difference in speed between the bus and rail modes is not great. Hence the speed of the airline mode versus the speed of the bus-rail mode is considered.

An ordinary least-squares estimation was used to estimate the parameters of the double-log model. Some of the estimated coefficients in the double-log model turn out to have wrong signs. For example, several market cross elasticities turn out to be negative.

We impose correct signs on the parameters of the double-log demand model and estimate the parameters. We use an inequality-constrained least-squares estimation method (13, 14).

These two models fail to satisfy the homogeneity, symmetry, and summability conditions. We impose the three conditions on Stone's model, the Rotterdam system of demand equations, and the homogeneous translog demand models. Parameters of these models are estimated by the nonlinear maximum-likelihood estimation method. Table 1 gives the results of parameter estimations.

The latter three models—weighted Stone (WS), Rotterdam (RD), and the homogeneous translog (HTD)—provide fairly consistent empirical results, particularly in compensated cross elasticities, income elasticities, and own price elasticities. The first two models—double-log (DL) model and inequality-constrained double-log (ICDL) model—provide rather inconsistent empirical results. The market cross elasticities, which include the income effects, substantially differ, depending on the choice of the models. The results of demand elasticities are given in Table 2.

All five models predict the own market price elastic-

Table 1. Parameter estimation.

	Double Log		Inequality-Constrained Double Log		Weighted Stone		Rotterdam		Homogeneous Translog	
Equation	Parameter	t-Value	Parameter	t-Value	Parameter	t-Value	Parameter	t-Value	Parameter	t-Value
Air										
Air fare	-1.08	-20.4	-1.04	-7.53	-0.139	70.9	-0.120	-16.0	-0.045 7	-2.87
Bus fare	0.084 1	1.36	0.212	1.32	0.039 0	16.3	0.031 2	5.05	0.012 7	1.94
Rail fare	-0.063 2	-2.42	0.010 0	-[a]	0.099 8	75.0	0.089 3	11.6	0.033 0	2.53
Money budget	1.15	13.3	1.02	5.12	0.836	141.0	0.857	55.3		
Time trend	-0.006 77	-5.71	-0.008 45	-2.23	-0.003 96	-5.86	-0.006 09	-2.88	0.003 35	4.97
Speed ratio	0.153	5.14	0.186	2.19	0.120	5.35	0.130	2.75	-0.136	6.20
Intercept	0.005 93	0.035 0	0.397	0.892	-0.277	-10.7			-0.642	-20.2
R^2	0.998		0.997		0.996		0.859		0.607	
D-W statistic	1.34		1.48		0.646		1.34		0.784	
Bus										
Air fare	0.446	1.39	0.338	1.13	0.039 0	16.3	0.031 2	5.05	0.012 7	1.94
Bus fare	-0.859	-2.29	-0.965	-1.93	-0.044 7	-6.44	-0.032 3	-4.02	-0.005 54	-0.786
Rail fare	0.326	2.06	0.298	1.49	0.005 73	1.09	0.001 10	0.157	-0.007 19	-1.93
Money budget	-0.207	-0.394	0.010 0	-[a]	0.046 7	2.27	0.031 8	2.30		
Time trend	0.002 91	0.403	0.003 75	0.311	0.000 059 5	0.141	0.000 553	1.36	0.000 195	0.833
Speed ratio	0.508	2.81	0.459	1.84	-0.032 8	-2.81	0.001 66	0.194	-0.015 7	-2.79
Intercept	-5.01	-4.87	-4.98	-3.00	-0.082 1	-4.62			-0.027 5	-2.71
R^2	0.712		0.711		0.914		0.041 7			
D-W statistic	1.16		1.13		0.845		1.68			
Rail										
Air fare	0.537	1.59	0.463	1.54	0.099 8	75.0	0.089 3	11.6	0.033 0	2.53
Bus fare	-0.650	-1.64	0.010 0	-[a]	0.005 73	1.09	0.001 10	0.157	0.007 19	-1.93
Rail fare	-0.666	-4.00	-0.585	-2.23	-0.106	-25.3	-0.090 4	-7.90	-0.025 8	-2.26
Money budget	0.271	0.491	0.010 0	-[a]	0.117	7.26	0.111	5.33		
Time trend	0.048 8	6.45	0.003 76	4.01	-0.003 79	-5.82	0.003 05	2.76	0.003 54	-6.17
Speed ratio	-1.26	-6.61	-1.13	-3.65	0.136	6.56	-0.075 8	-3.12	0.152	7.96
Intercept	-2.84	-2.63	-1.20	-0.980	-0.415	-16.3			-0.330	-12.2
R^2	0.869		0.848		0.803		0.386		0.706	
D-W statistic	1.53		1.20		0.784		1.62		0.979	
Log of likelihood function					401.0		388.8		248.3	

[a]Denotes the bounded value.

Table 2. Demand elasticities.

Elasticities	Double-Log	Inequality Constrained	Weighted Stone	Rotterdam	Homogeneous Translog
Own price					
Air	-1.08	-1.04	-1.00	-1.01	-0.945
Bus	-0.859	-0.965	-1.15	-0.830	-0.863
Rail	-0.666	-0.585	-0.979	-0.846	-0.790
Income					
Air	1.15	1.02	1.00	1.03	1.00
Bus	-0.207	0.010 0	1.15	0.785	1.00
Rail	0.271	0.010 0	0.951	0.902	1.00
Speed					
Air	0.153	0.186	0.144	0.156	0.163
Bus	0.508	0.459	-0.810	0.041 0	0.390
Rail	-1.26	-1.13	1.11	-0.616	-1.24
Cross elasticity					
Air-bus fare	0.084 1	0.212	0.006 2	-0.004 40	-0.015 2
Air-rail fare	-0.063 2	0.010 0	-0.004 0	-0.020 0	-0.039 4
Bus-air fare	0.446	0.338	0.002 0	0.114	-0.315
Bus-rail fare	0.326	0.298	0.000 45	-0.069 4	0.178
Rail-air fare	0.537	0.463	0.016 0	-0.028 0	-0.268
Rail-bus fare	-0.650	0.010 0	0.008 1	-0.027 6	0.058 5
Compensated cross elasticities					
Air-bus fare	0.131	0.253	0.046 7	0.037 3	0.025 2
Air-rail fare	0.077 8	0.135	0.119	0.107	0.083 5
Bus-air fare	0.273	0.346	0.963	0.770	0.522
Bus-rail fare	0.301	0.299	0.141	0.027 2	0.301
Rail-air fare	0.764	0.471	0.811	0.726	0.568
Rail-bus fare	-0.639	0.010 4	0.046 6	0.008 94	0.098 9

ity of airline demand to be near 1: -1.08 for DL model, -1.04 for ICDL model, -1.00 for WS model, -1.01 for RD model, and -0.945 for HTD model. The bus and rail own price elasticities vary, depending on which model we choose. WS predicts as high as -1.15 for bus and -0.979 for rail services, whereas ICDL predict as low as -0.585 for rail service. RD predicts -0.830 for bus. All five models correctly predict the sign of the own price elasticities of all three modes. The income elasticities of intercity airline demand range from 1.00 to 1.15. WS, RD, and HTD predict the income elasticities of railroad demand to be very close to 1.0. DL and ICDL appear to predict very low income elasticity of rail. The income elasticity of bus demand varies from 1.15 by WS to 0.785 by RD.

A change in price of a transportation mode affects not only the demand for that mode but also the demand for the alternative modes. The latter is measured by the cross elasticities. However, the market cross elasticities do not correctly measure the substitutability among alternative modes since the elasticities include the income effects. We notice that the empirical results of the market cross elasticities become substantially different depending on the choice of the model. For example, the cross elasticity of airline price with respect to railroad demand varies from 0.537 by DL model to -0.268 by HTD model. The unstable nature of the market cross elasticities seems to originate from the unstable income effects. As we take income effects out and constrain the homogeneity, summability, and symmetry conditions, the compensated cross elasticities (predicted by different models) become relatively close values.

The compensated cross elasticities that exclude the income effect correctly measure the degree of substitution among alternative transportation modes. The RD model has all negative cross elasticities except one case. However, when we take the income effect out, all compensated cross elasticities become positive, which implies that airline, bus, and rail intercity demands have a substitutional relationship. Similar results are obtained in the WS and HTD models.

Since the market cross elasticities do not correctly provide the degree of substitutability, our discussion concentrates on the compensated cross elasticities. WS, RD, and HTD predict that a change in air fare causes a significant change in demand for bus and rail. The models predict that a reduction in air fare could severely reduce the demand for intercity rail and bus travel. For example, WS model predicts that a 1 percent decrease in air fare could cause a 0.963 percent decrease in bus travel demand and 0.811 percent decrease in rail travel demand. RD model predicts that the decreases will be 0.770 percent for bus travel demand and 0.726 percent for rail travel demand when the air fare drops by 1 percent. The HTD model predicts slightly lower percentages, 0.522 for bus demand and 0.568 for rail demand.

Neither the change in bus fare nor the change in rail fare significantly affects the intercity travel demand for airline services. Passengers are more sensitive to air fare change than to bus fare or rail fare change.

The speed ratio elasticity of the airline equation ranges from 0.144 by the WS model to 0.186 by the ICDL model. This implies that, as the speed of airline service increases 1 percent faster as compared with the bus-rail speed, it attracts more passengers to the airline industry by 0.144 percent, according to the WS model. As is expected, the rail industry loses its passengers. All models except the WS model forecast that the rail industry is the major victim of air speed increases. For example, a 1 percent speed increase in the airline industry decreases the rail passenger demand by 1.26 percent (DL model), by 1.13 percent (ICDL model), by 0.616 percent (RS model) and by 1.24 percent (HTD model), respectively. However, the air speed impact on bus becomes positive except for the WS model. The elasticities range from 0.508 by DL to 0.041 by RD. Possible sources of the wrong sign are due to the model specification errors or partly due to the errors from aggregation. Bus is mainly used for shorter trips. A data stratification by trip distance or by trip purpose or inclusion of automobile could improve the empirical results.

CONCLUSION

Our empirical results indicate that the family of demand equations that are imposed by the homogeneity, summability, and symmetry conditions provides more stable results on the compensated cross elasticities than those equations that do not have such conditions imposed. In general, the market cross elasticities are very unstable and they vary depending on the choice of functional forms, even when we impose the three basic conditions on their demand equations.

Our conclusion is that it is desirable to impose the homogeneity, summability, and symmetry conditions. The market cross elasticities are theoretically improper and empirically unstable in measuring the substitutability of intercity travel modes. Theoretically sound and empirically stable indicators of substitutability are the compensated cross elasticities. When the three conditions are imposed, the choice of functional forms yields minimal variation on the compensated cross elasticities.

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Abridgment

Estimation of Demand for Public Transportation

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This paper describes the mode-split model used by the Chicago Area Transportation Study (CATS). The model was formulated in 1972 and was first used in mid-1973 for the evaluation of the 1995 regional transportation plan (1, 2). Since then, the model has been used as an operational tool (3) and has been refined, recalibrated, and validated. The current operational version of the model is described in this paper.

CHARACTERISTICS OF THE MODEL AS A PLANNING TOOL

The mode-split model operates as an integral part of the CATS transportation planning process. The major products of the model are estimates of the number of trips by automobile and transit for each origin zone and trip purpose. If necessary, these estimates are further processed through the CATS planning models, including mode-specific trip-distribution models, to provide estimates of volumes on specific roads and transit lines. In other cases, where such detailed information is not required, estimated changes in transit and highway demand (in response to proposed policies) are directly applicable to the evaluation of those policies.

As with most mode-split models, the CATS model is sensitive to the levels of service provided by various transportation modes and to the socioeconomic attributes of travelers. However, the model is unique in its emphasis on the effect of access and egress service on the demand for transit. The model provides for an accurate description of the access and egress service, considering both the availability of various submodes and the variations in the level of service within zones due to spatial dispersion of trip ends. The model's capabilities make it possible to describe accurately a wide range of policies related to improvements in access and egress service and to estimate the effects of those policies on travel demand.

STRUCTURE AND OPERATION OF THE MODEL

The CATS mode-split model may be described as an application of Monte Carlo simulation principles to travel-demand analysis. It may also be described as an aggregation procedure, which facilitates the application of disaggregate mode-choice models (4, 5) by the use of aggregate data.

Straightforward applications of mode-choice models in planning are done in the following way. Data are collected on a sample of the population under analysis, including (for each trip) relevant socioeconomic characteristics, service attributes, and chosen mode. The sum of the individual choices, properly weighted, provides an unbiased estimate of the population's modal shares. The sample is used to estimate a mode-choice model. A policy to be analyzed is introduced into the sample as changes in the level of the attributes that are affected by the proposed policy, and the resulting changes in mode-choice probabilities are calculated. The changes in the sum of those probabilities are used as estimators of the expected changes in modal shares of the population.

Many successful applications along these lines have been documented (6-8); however, this method has a number of deficiencies that seriously limit its applicability. The most obvious are the cost and time required to collect the data and the inability to sample future populations. Other deficiencies include the difficulties of identifying the population affected by a given policy and of selecting an effective sample.

The CATS mode-split model uses the same conceptual approach; the difference is that a pseudosample rather than a real sample is used. The pseudosample is generated by sampling the frequency distribution of the attributes of the population under analysis. This approach permits full exploitation of the power of disaggregate models without a need for a real sample. The procedures for creating the sample are designed to operate not only within the limitations imposed by considerations of data availability and analysis costs but also with the provision of means for accurately describing a wide range of proposed policies.

Operation of the Model

The heart of the CATS mode-split model is a procedure that repeatedly generates individual samples and mode-