Estimation of Trip Tables from Observed Link Volumes

Mark Turnquist*, School of Civil and Environmental Engineering,
Cornell University, Ithaca, New York
Yehuda Gur, Urban Systems, Inc., Chicago

Traffic assignment studies, which are instrumental in transportation plan evaluation, require an origin-destination trip table as input. A method of estimating the trip table from observed link volumes within the framework of small-area assignment is described. The technique is based on solution of an optimization problem and is compatible with equilibrium traffic assignment assumptions. It thus differs significantly from previous methods based on statistical estimation. Because it avoids expensive home-interview surveys and time-consuming work with trip generation and distribution models, the method provides an attractive and cost-effective alternative analysis procedure for small and medium-sized communities. It is likely to be particularly effective in the evaluation of short-term, low-capital improvements in the transportation network.

This paper describes a model for estimating a trip table, or origin-destination (O-D) matrix, based mainly on the knowledge of link volumes and possibly turning movements. This matrix can be used in a number of traffic assignments in which alternative network improvements are examined. A major intended application of the model is as an element in a process of developing and evaluating short-range, subregional traffic improvement plans. As such, this tool should prove very useful to transportation planners in small and medium-sized communities.

The proposed model provides a cost-effective alternative to available methods for constructing trip tables, methods that are dependent on expensive, time-consuming surveys and a chain of trip generation and distribution models. Such methods often require planning resources in excess of what is available in many smaller communities and metropolitan areas. The proposed model, on the other hand, is based mainly on count data, which are easily obtainable. It will be most effective in analyzing short-range problems in which no significant changes in land use are expected.

This paper discusses the general nature of the problem and summarizes previous attempts to solve it. It then describes the proposed model and an example of its application.

NATURE OF THE PROBLEM

Any solution procedure for determining an O-D matrix from observed link volumes must address itself to two major problems. The first of these is that the set of link volumes may be internally inconsistent; that is, there may be no trip table that, when assigned, exactly duplicates the observed flows. There are several possible reasons for this, some of which are described below. It is important that the solution procedure be capable of finding an approximate solution to the problem in this case.

A second major problem is that, if link flows are consistent, exactly the same set of flows might be generated by many, quite different trip tables. Because there are typically far fewer links (pieces of information) in a network than O-D interchanges (unknowns), the problem is underspecified. Thus, there will be a set of feasible trip tables (perhaps many), all of which produce the same link volumes. A major element in the solution procedure must be a mechanism for identifying a desired trip table from among this set.

Inconsistency of Input Data

There are a number of reasons why there may be no trip table that exactly reproduces the observed link flows. First, a coded network is an abstraction of the road network being represented. Vehicle counts from the real network must be converted into volumes on more abstract links of the coded network, and this process can give rise to internal inconsistencies. This is particularly the case for the ends of trips, since the coded network describes trips as beginning or ending in a limited number of "load nodes" whereas, in reality, these trips originate and terminate at many locations on the network.

A second source of inconsistency is that the logic of traffic assignment is always based on the assumption (expressed in various forms) that travel is made along minimum impedance paths between an origin and a destination. This assumption, although plausible, is not a completely accurate description of the real world. Thus, one should expect certain discrepancies between the real world and its description in the model.

Finally, inconsistencies may arise from errors in measurement and definition. Traffic counts will likely contain some errors, and the counts on various links in the network will often be from different days and times. The travel impedance that is minimized by travelers is likely to be incompletely defined, and its relation to measurable attributes, such as travel time and distance, is not completely understood. These problems, among others, are likely to result in a set of input link volumes that cannot be reproduced by assigning any trip table.

The approach taken in this research is to "smooth out" these inconsistencies so as to find a set of link volumes that are approximately the same as the observed volumes but that could be produced by assigning a trip table to the coded network. Once such a set of link volumes is found, the problem of underspecification can be addressed.

Underspecification of the Problem

Link volumes alone (with or without turn volumes) do not provide enough information to construct a unique trip table; the same link volumes might be generated by highly different trip tables. A simple example will serve to demonstrate this problem.

Consider the simple network shown in Figure 1, with travel times and volumes as indicated. Nodes 1 and 2 serve as origins, and nodes 3 and 4 are destinations. If Wardrop's first principle—i.e., that all paths used between a given origin and destination have equal impedance and no path that is not used has an impedance less than that of paths that are used (1)—is assumed to determine network equilibrium, the observed flows could have arisen from either of the O-D matrices described in Figure 2.

There is no way to determine from link volumes alone which of the alternative O-D matrices is the appropriate one. However, these two O-D matrices might produce substantially different flows as a result of a change in the characteristics of a network link. Since the objective of this method is to provide trip tables that can be used to evaluate such changes, attention must be given to the problem of distinguishing among the alternative solutions.

Previous Attempts to Solve the Problem

The principal previous work on determination of O-D matrices from observed link volumes has been done by Robillard (2, 3), Nguyen (4, 5), and Gur and others (6). Robillard's work represents the first major effort in this area. He studied only the class of proportional traffic assignment algorithms, namely algorithms that do not take link capacity or congestion into account, either explicitly or implicitly.

Robillard's method of determining the trip table is to minimize a measure of the difference between observed link flows and assigned flow. For the case of proportional assignment algorithms, the assigned flows can be written explicitly in terms of the O-D interchange volumes, since the proportion of each O-D volume that uses a given link is constant. Robillard's approach is to solve a linear regression problem to determine total originating and terminating trips for each zone. A generalized gravity model is then used to determine the trip table.

Robillard's work offers some useful insights into the problem. First, it focuses attention on the need to obtain estimates of total origins and total destinations at each node. Second, the need to bring to bear additional information on the nature of trip distribution is recognized. This is an important element in the construction of a solution. However, Robillard's proposed solution technique is quite crude.

Nguyen (4, 5) has presented an approach to estimating an O-D matrix based on the assumption that observed link flows represent network equilibrium in the sense of satisfying Waldrop's first principle. Nguyen's work is valuable because it demonstrates a formulation that is compatible with nonproportional assignment techniques. This is a significant generalization of Robillard's earlier work, which was limited to proportional assignment methods. Nguyen's formulation of the problem is described in more detail later in this paper.

Figure 1. Simple network with indicated link flows and travel times.

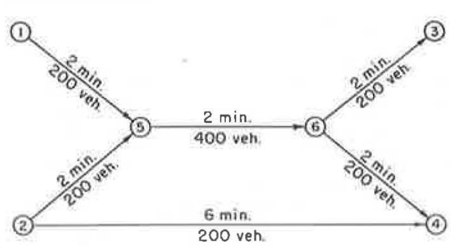


Figure 2. Possible trip tables for observed flows shown in Figure 1.

(a)

From \ To	3	4
1	0	200
2	200	200

(b)

From \ To	3	4
1	100	100
2	100	300

The approach Nguyen proposes for solving the problem suffers from a number of major deficiencies:

1. He does not address the problem of how to deal with the large number of potential solutions that reproduce the same link volumes.
2. He assumes that the observed flows are in equilibrium and does not address the problem of cases in which the observed volumes are inconsistent.
3. This proposed solution procedure, although theoretically correct, is extremely inefficient in application.

A third approach for solving the problem was formulated as a part of the present project (6). This approach used a linear programming formulation that specifically addressed the problems of volume inconsistencies and multiple solutions. The approach was formulated primarily for proportional assignment procedures. Although it was conceptually promising, it was not developed to its fullest extent. The major reasons for this were the need (as a part of project objectives) to address primarily nonproportional assignments and the need to invest heavily in software development before the approach could be fully developed and tested.

After the nature of the problem and possible solution procedures were studied, the direction chosen in this research was to use Nguyen's basic approach as a point of departure for developing an operational procedure.

APPROACH TO SOLVING THE PROBLEM

In general, the problem of finding an O-D matrix that, when assigned, would replicate observed link volumes can be stated formally as follows:

$$\min F = \sum_a \left[\int_0^{f_a} t_a(x)\,dx \right] - \sum_j \hat{u}_j T_j \qquad (1)$$

subject to

$$T_j - \sum_k h_j^k = 0 \quad \text{for each O-D pair } j \qquad (2)$$

$$\sum_j \sum_r d_{ja}^k h_j^k = f_a \quad \text{for each link } a \qquad (3)$$

and

$$f_a, T_j, h_j^k > 0 \qquad (4)$$

where

f_a = observed flow on link a,
$t_a(x)$ = impedance function for link a,
$\hat{u}_j$ = observed O-D impedance for interchange j,
T_j = trips for interchange j,
h_j^k = number of trips from interchange j using path k, and
$d_{ja}^k = \begin{cases} 1 \text{ if link a is in path k for interchange j} \\ 0 \text{ otherwise.} \end{cases}$

This is a nonlinear programming problem. The general form reflects the situation in which the observed link flows represent network equilibrium. In the special case in which no congestion effects are present, $t_a(x)$ is a constant and the first term of the objective function becomes simply $\Sigma_a t_a f_a$. This results in a linear programming problem.

In either event, the decision variables (or unknowns) are the T_j's and the h_j^k's, which represent trips on each interchange and allocation of these trips among paths, respectively. Note, however, that the h_j^k's are internal to the problem and can be computed indirectly. Nguyen (4) proved that the problem has a unique solution in link volumes (the observed link volumes); thus, the set of decision variables T_j (the output trip table) is a solution to the problem.

A solution to this mathematical programming problem is one trip table that could have resulted in the observed link flows. In light of the previous discussion, however, there are likely to be a number of alternative optima to this problem (i.e., alternative trip tables), each of which would produce the same link flows. The solution procedure, then, must be sensitive to this aspect of the problem and be capable of converging to a desired solution from among this set.

Since the optimization problem described above is very similar to the equilibrium traffic assignment problem, it is not surprising that solution algorithms for the two problems are quite similar. An iterative solution algorithm for this problem has been developed that requires the following basic steps:

1. Specify an initial trip table T^1 and a volume-delay (impedance) function for each link.
2. Find the $\hat{u}_j$ (skim trees) by using observed link impedances.
3. Assign T^1 to the unloaded network by using free-flow impedances to obtain a set of link volumes f^1. Denote this current solution as a vector (f^1, T^1).
4. Let $i = 1$.
5. Determine link impedances at the current volume f^i and again build minimum impedance trees. Denote the resulting values u^i.
6. Given T^i, $\hat{u}$, and u^i, find a correction trip table V that is closer to a solution.
7. Assign V^i to the trees built in step 5 to obtain correction link volumes s_i.
8. Find a weight r^i such that $0 \leq r^i \leq 1$ and the solution $[(f^{i+1}, T^{i+1}) = r^i(s^i, V^i) + (1 - r^i) \cdot (f^i, T^i)]$ minimizes the objective function F.
9. Check the convergence criterion. If it is met, stop; otherwise, set $i = i + 1$ and go to step 5.

Within the basic framework of this algorithm, there are a number of opportunities for variation. Specifically, one would expect the results to be sensitive to (a) choice of the initial trip table, (b) choice of link impedance functions, and (c) choice of procedure for computing V^i. The initial trip table is important because the problem generally has multiple optima. The algorithm will have a tendency to converge to a solution that is "nearest" to the initial solution. Link impedance functions are important because they play a central role in determining the order in which alternative paths for a given interchange are selected and thus the way in which equilibrium is approached. Obviously, the procedure for computing the correction trip table is a crucial element in the algorithm, since it determines the efficiency of the technique as well as the nature of the final solution.

A substantial battery of tests has been performed on this algorithm to determine its sensitivity to the choices indicated above under various network situations. The major findings are summarized here.

The theoretical constraints on the problem leave considerable latitude in selecting link impedance functions. Nguyen (5) has proved that the problem solution will replicate observed link flows as long as the impedance functions satisfy two simple criteria: (a) They must be increasing functions of volume, and (b) they must take on the value of observed impedance at the observed link volume.

Several forms of impedance functions have been tested, and the best results have been obtained by using a piecewise-linear form, as shown in Figure 3, in which t_0 is the observed impedance and f_0 is the observed volume. This function has provided superior results in terms of both the speed of convergence and the quality of the final solution.

A number of procedures for computing the correction trip table V^i are also possible. In his initial work on this problem, Nguyen (4) suggested a procedure in which a single interchange would be updated at each iteration. Although the convergence properties of such a technique can be established, it represents a very inefficient method for problems of realistic size. As a result, considerable effort has been devoted to developing more efficient procedures. To date, the procedures developed must be regarded as heuristics; i.e., there is no proof available that they will always converge to a solution. However, a number of these procedures have performed very well in empirical tests. The best correction technique identified thus far is as follows:

$$V_j^i = \begin{cases} T_j^i * 1 + 2 * [(\hat{u}_j - u_j^i)/(u_j^i - u_j^o)] & \text{if } \hat{u}_j > u_j^i \\ 0 & \text{if } \hat{u}_j \leq u_j^i \end{cases} \quad (5)$$

where u_j^o is the impedance for interchange j on the unloaded network (free flow). This correction is based on projecting the history of past corrections and has performed well in empirical tests. It must be emphasized, however, that it is a heuristic method.

As an example of the performance of the algorithm, Figure 4 shows how a solution is approached for a given network from three different starting points. The value on the ordinate is total volume error, computed as

$$\text{error} = \sum_a |\hat{f}_a - f_a| \quad (6)$$

where $\hat{f}_a$ is the observed volume on link a and f_a is the estimated volume from the algorithm. The network in question is shown in Figure 5, and the three initial trip tables are shown in Figure 6. Table 1 gives the observed volume and impedance attributes assumed for the network.

Trip table A represents essentially a "no-prior-information" starting point. All feasible interchanges are assigned equal numbers of trips, and the only information represented is that the matrix is scaled to match total observed vehicle hours traveled on the network. Trip table B represents a better starting solution but one that still contains major errors; i.e., this trip table, when assigned, would not produce the observed link volumes. Finally, trip table C represents a starting point that is quite close to an acceptable solution.

Figure 4 shows two important points about the algorithm. First, at least in terms of total volume error, the better the initial solution is, the better the computed solution will be after a given number of iterations. This is to be expected. However, the second point is that the procedure is capable of movement to a reasonable solution even when the starting point is far from a solution, as in the case of trip table A. This tends to increase our confidence in the convergence properties of the heuristic method.

It is also interesting to examine the estimated trip tables produced from the three different starting points. Figure 7 shows the solutions after 15 iterations. Note that the three solutions differ considerably from one another. This indicates the presence of alternative solutions, as discussed previously. Note also, however, that each solution has retained several important properties of its starting solution. These properties include the number of nonzero interchanges and variability in the relative size of interchange volumes.

This is very important, for it indicates the tendency of the algorithm to identify a solution that is close to the starting solution. This provides a mechanism for distinguishing among the alternative optimal solutions. By using as a starting point a "target" trip table that represents a number of properties we would like to see retained in the final trip table, we can generate a final solution that is close to this target, subject to the restriction that it (approximately) replicates observed link volumes.

Figure 3. Pseudo volume-delay function used in experiments.

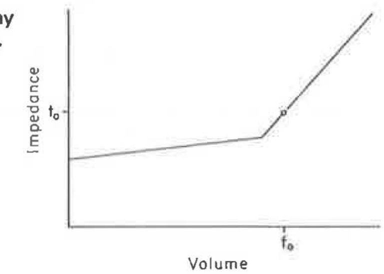


Figure 4. Total volume error in test network for three initial trip tables.

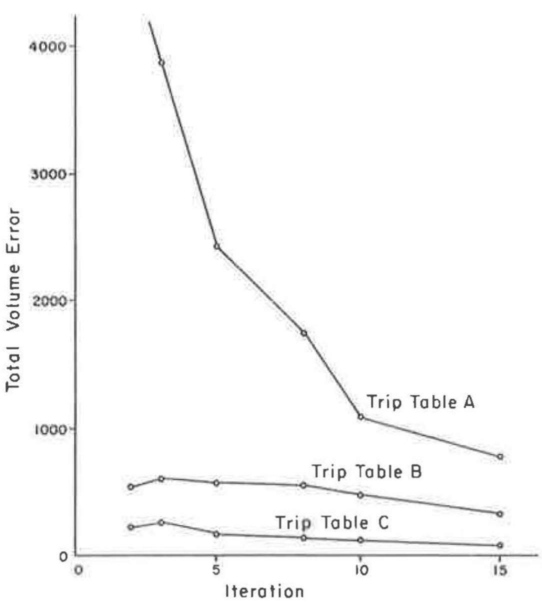
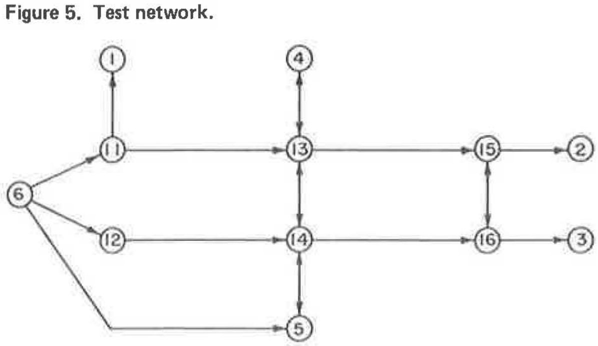


Figure 5. Test network.



This property of the algorithm has led us to devote substantial attention to the problem of constructing a reasonable target trip table. This was necessary because our major concern is small-area analysis and available trip distribution models deal primarily with large regions. A special trip distribution model—SMALD—has been designed for this purpose and is described in detail elsewhere (7).

If the solution trip table is judged by the analyst to be too far from the target table to be acceptable, postprocessing procedures are available to formally identify the trip table that will reproduce (approximately) observed link volumes and that lies closest to the target table. This procedure uses the basic information produced by the iterative algorithm in a second-stage mathematical programming problem. The technique is described by Gur and others (6). Experience to date has indicated, however, that this second-stage procedure is seldom necessary.

APPLICATION TO A REALISTIC NETWORK

To demonstrate the feasibility of the model in a realistic setting, it has been applied to a network that represents an area of about 15 km^2 (6 $miles^2$) in Hudson County, New

Figure 6. Initial trip tables for test network.

(A)

From \ To	1	2	3	4	5
4	-	983	983	-	983
5	-	983	983	983	-
6	983	983	983	983	983

(B)

From \ To	1	2	3	4	5
4		800	500	-	1100
5	-	1500	500	0	-
6	500	2000	0	2000	600

(C)

From \ To	1	2	3	4	5
4	-	600	300	-	1500
5	-	2000	3000	1500	-
6	500	4000	400	500	200

Table 1. Attributes of test network.

Link Origin Node	Link Destination Node	Loaded Impedance	Zero Volume Impedance	Observed Volume
4	13	10	7	2400
5	14	10	7	2000
6	5	40	30	100
6	11	10	7	5000
6	12	10	7	500
11	1	10	7	500
11	13	20	15	4500
12	14	20	15	500
13	4	10	7	2000
13	14	10	7	1500
13	15	20	15	4900
14	5	10	7	1600
14	13	10	7	1500
14	16	20	15	900
15	2	20	15	4800
15	16	10	7	300
16	3	20	15	1000
16	15	10	7	200

Jersey. The area includes 34 "boundary" load nodes and 24 internal load nodes. The coded network includes a total of 132 nodes and 369 links. The observed link volumes are based on adjusted ground counts collected from 1973 to 1975.

The initial trip table was estimated by using an early version of the SMALD model (7). This table contained a number of obvious imperfections, particularly total vehicle kilometers traveled, which was overstated by 14 percent. In the observed data, vehicle kilometers of travel = 1 482 304. The table below gives the results of the input trip table and the solution trip table for this measure (1 km = 0.62 mile):

Figure 7. Output trip tables resulting from various initial trip tables.

(A)

From \ To	1	2	3	4	5
4	0	884	483	0	1147
5	0	1248	313	415	0
6	478	2743	259	1636	520

(B)

From \ To	1	2	3	4	5
4	0	725	554	0	1164
5	0	1547	453	67	0
6	495	2540	5	1955	590

(C)

From \ To	1	2	3	4	5
4	0	597	708	0	1096
5	0	1700	291	4	0
6	500	2503	0	1996	600

Measure	Input Trip Table	Solution Trip Table
Vehicle kilometers of travel	1 729 567	1 482 169
Volume error (number of vehicles)		
With approach links	609 364	234 041
Without approach links	550 085	141 280
Root-mean-square error (%)		
With approach links	42.5	18.3
Without approach links	42.7	13.5

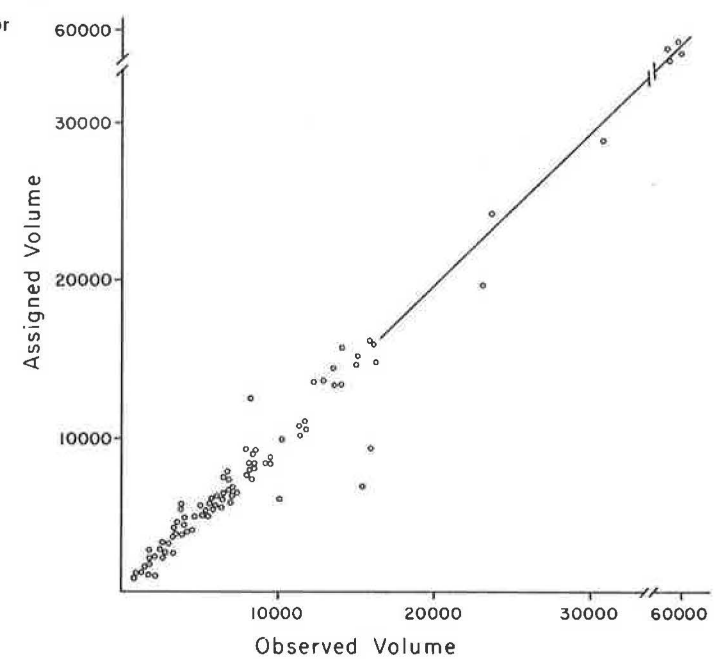
In assigning the inital trip table to the network by using five iterations of equilibrium assignment, a root-mean-square (RMS) error of 42.7 percent of the mean volume was found.

The model was applied to the problem for 35 iterations. The model changed the trip table substantially; the final trip table, when assigned, showed an RMS error of 13.5 percent, as given above, and underestimated the vehicle kilometers by about 2 percent. (Successful capacity-restrained assignments usually approximate observed volumes with an RMS error of 25-35 percent.) Figure 8 shows a plot of the observed versus assigned volumes that demonstrates close replication of the observed volumes over the entire volume range.

In a detailed analysis of the results, the following findings were made:

1. Many of the significant residual volume errors can be traced to inconsistencies in the input data (e.g., the total observed volume entering a network node is different from the total volume leaving that node). The algorithm distributed those errors in an acceptable way.
2. The extent to which total productions and attractions are preserved for internal zones can be controlled by the user, through the slope of the pseudo-delay function. Changes in production-attraction rates can be traded off against lower residual errors in the observed volumes. The trade-off should be based on the user's knowledge of the quality of the data.
3. The algorithm preserves the original trip table to the extent possible, subject to the objective of approximating the observed link volumes.

Figure 8. Observed versus assigned volume for Hudson County network.



4. Required computer resources are quite modest. The run required 65 s of central processing unit time and 120K bytes of core on an IBM 370/165.

5. Advance knowledge of total area vehicle kilometers of travel is not essential for a successful application of the model. Thus, the model can easily be enhanced to treat cases in which volumes are known for only part of the links.

The tests succeeded in demonstrating the applicability of the model to actual planning problems.

CONCLUSIONS

Traffic assignment is an important tool in the evaluation of potential changes in the transportation network. Such assignments require an O-D trip table. These tables have traditionally been estimated based on home-interview data and a complex chain of trip generation and distribution models. Careful analytic evaluation of transportation plans has thus often been beyond the financial capabilities of small and medium-sized communities. Even if the required data are available, in many cases they are outdated and inaccurate and thus result in assigned volumes that hardly resemble observed traffic. Such large discrepancies may cause the assignment analysis to be unreliable or even useless, especially for small-scale, short-range problems.

This paper documents a procedure by which an O-D trip table can be estimated based primarily on link-volume data. Most communities maintain an ongoing program for monitoring link volumes. Thus, they can easily provide a set of good ground counts to be used as a basis for constructing the trip table.

An important property of the problem in general is that there are usually alternative optimal solutions that correspond to different trip tables that produce the same link volumes when assigned. A characteristic of the algorithm developed is that it tends to converge to a solution that is close to the given starting point. This provides the motivation for creating a starting point (a target trip table) that represents desirable attributes. The algorithm will then modify this target table but only enough so as to result in approximate replication of observed link volumes.

Tests of the algorithm have indicated that it is relatively efficient from a computational standpoint and should thus represent a cost-effective tool for transportation planning in small and medium-sized communities. It should be particularly useful in evaluating short-term and/or low-capital improvements in the transportation network, which are not likely to result in significant changes in land-use or general trip-making patterns.

ACKNOWLEDGMENT

The work described in this paper was performed by John Hamburg and Associates under the sponsorship of the Office of Research, Federal Highway Administration, U.S. Department of Transportation. We wish to thank Steven Cohen, the contract manager, for his encouragement and helpful suggestions in all phases of the project. The advice and ideas of Morton Schneider and Larry Leblanc have also been of great value.

We are solely responsible for the findings and opinions expressed here.

REFERENCES

1. J. G. Wardrop. Some Theoretical Aspects of Road Traffic Research. Proc., Institute of Civil Engineers, Vol. 5, Part 2 (I), 1952, pp. 278-325.
2. P. Robillard. Estimating the O-D Matrix and Network Characteristics from Observed Link Volumes. Proc., 1st International Conference of Transportation Research, 1973, pp. 736-740.
3. P. Robillard. Estimating the O-D Matrix from Observed Link Volumes. Transportation Research, Vol. 9, 1975, pp. 123-128.
4. S. Nguyen. Estimating an O-D Matrix from Network Data: A Network Equilibrium Approach. Centre de Réchèrche sur les Transports, Univ. of Montreal, Rept. 60, Feb. 1977.
5. S. Nguyen. On the Estimation of an O-D Trip Table Matrix by Equilibrium Methods Using Pseudo Delay Functions. Centre de Réchèrche sur les Transports, Univ. of Montreal, draft paper, 1977.
6. Y. Gur, M. A. Turnquist, M. Schneider, and L. J. Leblanc. Determining an Origin-Destination Trip Table Based on Observed Volumes. Presented at Annual Meeting of Operations Research Society of America and the Institute of Management Science, New York, May 1978.
7. D. Kurth, M. Schneider, and Y. Gur. Small-Area Trip Distribution Model. TRB, Transportation Research Record 728, 1979, pp. 39-41.

**M. Turnquist was at Northwestern University when this research was performed.*