Equilibration of Supply and Demand in Designing Bus Routes for Small Urban Areas

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A methodology that models the interactive relations between bus-system supply and demand and results in an optimal or near-optimal bus-route structure is described. On the supply side, the route structure is developed by using a heuristic algorithm called SWEEP, written in FORTRAN language. The algorithm partitions the total bus stops in the urban area into sectors and uses a three-optimum traveling-salesman algorithm or Hamiltonian-path algorithm to link these stops. The objective function of the algorithm is to minimize the total distance traveled by all buses, subject to the capacity and distance constraints on each bus. On the demand side, the program uses the already developed bus network to determine the percentage of total community travel that requires bus service. This is carried out by using a disaggregate mode-choice model that is based on the total time and cost difference between travel by automobile and travel by bus for each individual user. Costs of bus operations are calculated from a four-variable unit-cost model. An iterative, interactive feedback process is used to achieve the equilibrium state of the transportation market. Equilibrium is reached when the bus share of the transportation market cannot be increased by improving the bus network, under certain resource conditions and financial constraints. The program is tested in developing feeder bus routes to the proposed Glebe Metro station in Arlington County, Virginia.

The problems of bus transit planning in small urban areas can be boiled down to two major aspects—supply and demand. On the supply side, the development of route structure, the frequency of buses on each route, and the estimation of system operating costs and required subsidies are all bus functions that require improvement in the existing transit planning process. The system-wide configuration of bus routes, which is the working skeleton of the transit system and the medium of contact between the users and the bus company, is still developed by hand. Bus routes and frequencies are developed on a qualitative basis by using a number of routing and operating criteria to judge the route network (1, 2). This procedure limits the number of alternative configurations to be considered, does not have a defined objective function, and does not make use of the interactive relations between supply and demand. Expected operating costs and required subsidies cannot be treated independently of developed route configurations and equilibrium demand functions. On the other hand, demand is a function of the attractiveness of the supply system, which includes such factors as the characteristics of the bus system and its performance under specific physical and financial constraints. In short, a computerized methodology that will equilibrate the supply and demand functions where the supply functions are endogenous to the model is not available for bus-system planning in small urban areas.

Few mathematical models have been formulated to determine the equilibrium condition of the supply and demand functions in a transportation network. Equilibrium conditions in highway networks in which demand is elastic have been investigated by Florian and Nguyen (3), Martin and Manheim (4), Wilkie and Stefanek (5), and Wigan (6). A survey of the literature and possible approaches to the problem are presented by Ruiter (7). Kulash (8) has developed two simulation models for analyzing fixed-route bus systems. These models evaluate the quality of service that results from various operating policies and are used to predict the impacts of various operating decisions that are needed to improve route and schedule designs. Yet the route structure in Kulash's models is still developed manually, and no demand interaction is considered. Similarly, Rapp and Gehner (9) developed an interactive graphic computer system known as the Urban Transit Analysis System (UTRANS). UTRANS is used to evaluate different route and schedule policies based on quality of service. Frequency and route structure, again, are used as input data.

The methodology presented in this paper develops bus-route structure and computes equilibrium flows in a network in which demand is elastic.

METHODOLOGY

Equilibrating the supply and demand functions of the bus system requires the following models: a collection of supply models, a demand model, and an integrating model. The demand model represents the demand side of the system. It predicts the passenger demand for each bus stop in the system based on economic and population forecasts and on characteristics of the various transportation modes that serve the city. The supply models include the set of activities that represent the flow of passengers on bus routes. Two models represent the supply activities: the network development model and the cost model. The integrating model is a program that processes the supply and demand functions to determine an equilibrium of supply and demand quantities. The structure of the supply-demand equilibrating framework is shown in Figure 1.

It is useful to discuss the methodology in three sections: (a) supply functions, (b) the demand model, and (c) the integrating model.

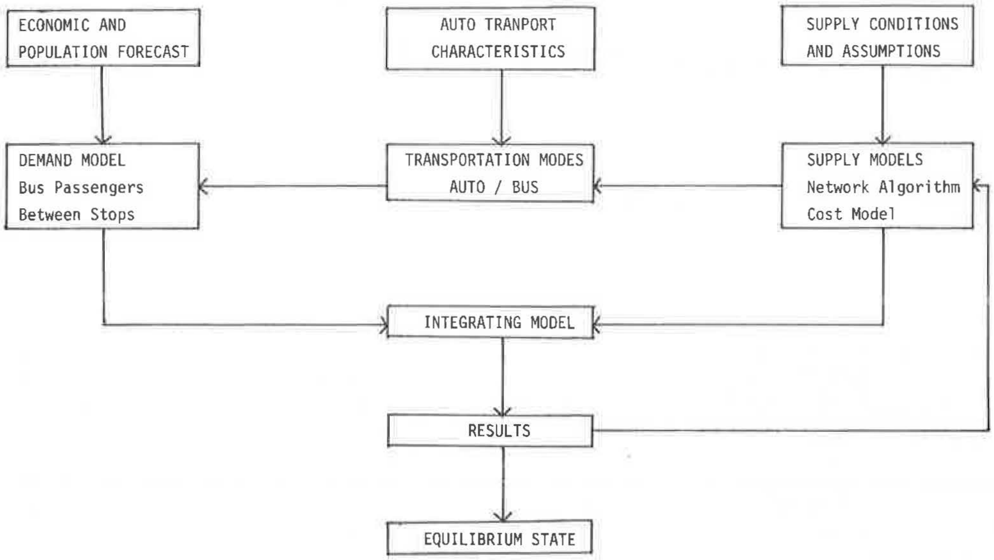
Supply Functions

Network Development Model

The model used to develop route structure, referred to as the SWEEP algorithm, was originally proposed by Gillet and Miller (10) as a solution to the general problem of vehicle dispatch. This algorithm was modified and applied to bus network design (11).

Optimal computer algorithms for development of bus routes had been investigated and applied in the past and found to be ineffective and to require an extensive amount of computer time. But, in light of the new generation of heuristic algorithms now available in the literature for solving vehicle-dispatching problems (12-19), these limitations are no longer valid. Near-optimal solutions to problems of large bus networks could be accomplished

Figure 1. Supply-demand equilibrating framework.



in a reasonable amount of computer time.

The objective of the SWEEP algorithm is to minimize the total distance or time traveled by all buses in satisfying demand at all bus stops, subject to the load and distance constraints on each bus. The problem is to determine the number of routes and the paths in each route according to the above objective. The coordinates of each bus stop and bus-stop demand are inputs to this algorithm. The following notation will aid in its explanation:

N = number of bus stops, including the transfer station (the transfer station is bus stop 1 and the dispatch point of all buses);
Q(I) = demand at bus stop I (I = 2, 3, ..., N);
X(i), Y(I) = rectangular coordinates of I bus stop (I = 1, 2, 3, ..., N);
C = capacity of each bus;
D = maximum distance each bus can travel;
A(I, J) = distance between stops I and J;
An(I) = polar coordinate angle of the Ith stop (I = 2, 3, ..., N); and
R(I) = radius from transfer station to stop (I).

The constraints on the problem are as follows:

$$Q(I) \leq C \quad (1)$$

for all I;

$$A(I, J) > 0 \quad (2)$$

for all I $\neq$ J;

$$A(I, I) = W \quad (3)$$

where W is a constant that denotes extra distance or time per stop; and

$$A(I, 1) + A(1, I) \leq D \quad (4)$$

for all I.

The stops are renumbered according to the size of their polar-coordinate angle, and the transfer station is stop 1. The stops are partitioned into routes beginning with the stop that has the smallest angles—namely, stop 2. The first route, then, consists of stops 2, 3, ..., J, where J is the last stop that can be added without exceeding the vehicle capacity or distance constraint. The second route contains stops (J + 1, J + 2, ..., L), where L is the last stop that can be added to the second route without exceeding the constraints. The remaining routes are formed in the same manner.

The total distance or time traveled is just the sum of the distances for each route. An iterative procedure is then used to improve the total distance traveled by replacing one stop in route K with one or more stops in route (K + 1) for (K = 1, 2, ..., m - 1), where m is the number of routes formed.

The process of adding one or more stops to route K and deleting another stop continues until no improvements are found. The X and Y axes are then rotated counterclockwise so the first location becomes the last, the second becomes the first, and so forth.

This procedure of partitioning routes and interchanging locations between routes is then repeated until all possibilities have been exhausted and the minimum total distances are calculated. The smallest of these minimums provides a good heuristic solution for a bus-route network.

Each time a set of bus stops is considered for a given route, a "traveling-salesman" algorithm is solved to determine the minimum path to service each of the stops in the route. As a consequence, a loop route is formed. To modify the algorithm to form linear bus routes instead of loop routes, a Hamiltonian-path algorithm is developed to replace the traveling-salesman algorithm.

The solution to the well-known traveling-salesman problem, in which the origin and destination points coincide, represents a "Hamiltonian circuit". A Hamiltonian path represents a route in which the origin and destination vertices are at two distinct points. The linear bus route, on which the bus goes to the end of the route and returns along the same route, is mathematically equivalent to the determination of the shortest Hamiltonian path.

In the algorithm for determining the shortest Hamiltonian path, which is given below, the following notation is used: Edges are incident to or incident from a vertex; $v_1, v_2, \ldots, v_n$ = the set of vertices; and L_{ij} = distance or time for the edge incident from v_i and incident to v_j.

Figure 2. Sample of output of SWEEP algorithm.

```
Best solution is:
Route 1 has load 46.00 with distance 184.54 is
102  49  50  1  44  45  103  107  106  104  105
78  79  80  82  75
Route 2 has load 46.00 with distance 255.44 is
68  91  92  66  94  111  114  110  109  81  108
69  76  1  48  72  77  113  73  112  90
Route 3 has load 44.00 with distance 281.23 is
2  1  70  74  83  95  64  65  67  63  62  84  85
61  58  57  86  87  71
Route 4 has load 48.00 with distance 235.73 is
100  43  34  33  1  46  47  42  41  54  59  55
56  60  101
Route 5 has load 45.00 with distance 218.14 is
96  97  99  98  52  53  37  36  35  1  32  89
Route 6 has load 47.00 with distance 205.15 is
1  6  24  88  38  39  40  31  18  10
Route 7 has load 47.00 with distance 192.33 is
1  6  24  88  38  39  40  31  18  10
Route 8 has load 43.00 with distance 182.97 is
5  1  11  9  14  29  28  27  26  25  30  17  3
Route 8 has load 43.00 with distance 182.97 is
4  23  20  19  15  1  8  16  13  22  21
Route 9 has load 19.00 with distance 54.49 is
1  12  7  51
Total Distance Is. . . . .1810.01685  (Distances are in 100 feet)
```

The steps in the Hamiltonian path are as follows:

1. Define a T set of edges to be included in the Hamiltonian path. Initially, T is empty.
2. Determine the shortest L_{ij}. Include the edge $(v_i - v_j)$ in T.
3. Merge the vertices v_i and v_j to form a single vertex. This would imply the revision of distance L_{ik} and L_{pj}, where v_k and v_p are typical vertices other than v_i and v_j.
4. Check as to whether the set T has (n - 1) edges. If yes, go to step 5; if no, go to step 2.
5. The set T defines the shortest Hamiltonian path.

The routes developed by the algorithm, whether one uses the traveling-salesman path or the Hamiltonian path, conform to the general guidelines for bus-route development (1) and yet minimize the total distance or time traveled by all buses. The computer output under each variation shows the number of routes, the path of each route, the distance traveled by each bus, and the total number of passengers carried on each route. A sample of the output of the SWEEP algorithm is shown in Figure 2.

The algorithm described above is version 1 of the SWEEP algorithm, which determines a bus network in which there is only one central transfer point in the service area. To overcome this weakness, version 2—development of which is under way—deals with multi-terminal network design. The basic concept of version 2 is the same as that of version 1. Its modifications can be briefly described as follows.

Assume M to be the number of assigned terminal stations, which usually are the major traffic attractors in the city. The algorithm first assigns each bus stop to its most appropriate terminal by a ratio scheme and then partitions the primary problem into M smaller subproblems. These individual subproblems are considered as problems of single-terminal network design and can be solved by using version 1 of the SWEEP algorithm. The so-called ratio scheme involves the following steps:

1. Initially, all bus stops are unassigned. For each bus stop i, find the closest terminal j1 and the second closest terminal j2 and compute the ratio

$$r(i) = d(i, j1)/d(i, j2) \tag{5}$$

where d(i, j) is the distance between nodes i and j.

2. Assign all bus stops i that have r(i) greater than 2 to their closest terminal.
3. For each unassigned bus stop i, find j*, k* such that

$$\text{Min}\,\{d(i,j) + d(i,k) + d(j,k)\} = d(i,j^*) + d(i,k^*) + d(j^*,k^*) \tag{6}$$

for all j, k where j and k are bus stops already assigned to the same terminal, either j1 or j2. Assign stop i to the terminal to which j* and k* are assigned.

In this procedure, if r(i) is large it means that stop i is relatively close to one terminal. All such stops are immediately assigned to the closest terminal. Stops that are more or less midway between two terminals are assigned more carefully. The minimization implies including stop i between j* and k* as linearly as possible.

Cost Model

The task of explaining total operational costs as a function of the output and characteristics of the system has proved in many cases to be very difficult. But this is not the case for the operational costs of bus systems.

Several different approaches have been taken to developing cost models for bus systems, but they are all basically single-equation expressions of cost as a function of the output of the system. These models can be primarily categorized into three types: the four-variable unit-cost model, the four-variable regression model, and the slowness function model. A review and comparison of these models can be found elsewhere (20). Hurley and Siegel (21) suggest that "the unit-cost method of determining parameters appears to be an accurate method when used to predict future costs for the same system" and "the four-variable model is equal to, and usually superior to, the slowness function." Therefore, the computer program uses the unit-cost model, under some reasonable assumptions, to generate operational cost estimates.

The four-variable unit model has the following general form (since the models presented in this paper were formulated in U.S. customary units of measurement, no SI equivalents are given):

$$OC = a*VM + b*VH + c*PV + d*RP \tag{7}$$

where

OC = annual operational costs,
VM = annual vehicle miles,
VH = annual vehicle hours,
PV = number of peak-hour vehicles,
RP = annual revenue passengers, and
a, b, c, d = unit costs for their corresponding variables.

According to data collected in 1970, the national averages for these unit-cost coefficients for public bus operations are as follows:

$$OC = 0.277\,VM + 5.700\,VH + 6527.480\,PV + 0.038\,RP \quad (8)$$

These costs are based on the 1970 dollar value and are multiplied by the inflation factor to estimate design-year operational costs. In this case, the average inflation factor is considered to be 7 percent for each year since 1970.

In the absence of particular specifications, the magnitude of these four variables is calculated by using the following relations:

VM = total bus-route miles * service frequency in vehicles per hour * operating hours per day * operating days per year,
VH = vehicle miles/bus average speed in miles per hour,
PV = number of vehicles in peak-hour operation, and
RP = total number of passengers on all routes for each bus trip during peak hours (capacity) * service frequency in vehicles per hour * [peak-hour operation per day + (operation per day - peak hours per day)/2] * operating days per year.

Demand Model

Travel demand is divided into captive and choice riders. The captive riders are further subdivided into automobile captives and bus captives. The division between choice and captive riders is assumed to be known a priori, through survey or other data.

The demand model for choice riders used in this research is an individual mode-choice model based on logit functions. The mathematical expression of this model is

$$P_{i(b)} = \exp(B_i) \Big/ \sum_{j=1}^{n} \exp(J_i) \quad (9)$$

This equation states that the probability of a passenger i taking the bus travel mode (b) is the exponential of the bus model utility (B_i) divided by the sum of the total exponentials of all modal utilities $\sum_{j=1}^{n} \exp(J_i)$ in the market, where n is the number of total available modes. In a small urban area in which there are only two major modes available, the equation is

$$P_{i(b)} = \exp(B_i)/\exp(A_i) + \exp(B_i) \quad (10)$$

where A_i and B_i are the utilities of automobile mode and bus mode for an individual traveler i.

The development and use of these types of models, also referred to as disaggregate travel behavior models, are fully described and discussed elsewhere (22-26).

The above equation is simplified by dividing both the numerator and the denominator by $\exp(B_i)$. The following equation is obtained:

$$P_{i(b)} = 1/[1 + \exp(Z_i)] \quad (11)$$

where Z_i is the difference in utility function between bus and automobile.

There are many exogenous variables that can be considered in calibrating the difference utility function Z_i. Because of unavailable data, a model calibrated for Schaumburg/Hoffman Estates Transit demand prediction (23) is used here. The difference utility function is

$$Z_i = -1.37 + 0.0544(T_a - T_b) + 0.0021(C_a - C_b) \quad (12)$$

where

T_a = total travel time by automobile,
T_b = total travel time by bus,
C_a = total travel cost by automobile, and
C_b = total travel cost by bus.

Integrating Model

The integrating model is a simulation model that uses the route network and the operating costs developed by the supply functions and the demand quantities from the demand model to determine the equilibrium state of bus-system supply and demand, under specific operational and financial management policies such as bus fare, the boundaries of the service area, and bus headways and capacity. The input data for the integrating model include

1. The bus network and its associated operating cost developed from the supply model;
2. The O-D matrix for the system service area;
3. The split for automobile-captive riders, choice riders, and transit-captive riders; and
4. All values of travel time and cost parameters used in the demand model.

If the split for automobile captives, choice riders, and transit captives is not available for the service area, it is assumed that 75 percent of total trip makers are choice riders and 25 percent are captives. Among the captives, 25 percent are assumed to be transit captives and 75 percent automobile captives. In other words, 75 percent of total trips are sensitive to the level of service of the bus system and applicable to the demand model.

The simulator is based on the Monte Carlo technique. It first calculates the probability of a trip origin for each bus route and each bus stop. The individual route probability is the initial total demand on the route divided by the total bus demand over the whole service area. The nodal probability is the amount of initial demand at the particular bus stop divided by the total amount of initial demand along the route. Uniformly distributed random numbers between zero and one are generated and fed into the foregoing cumulative probabilities for the routes and stops, respectively. Comparisons with the calculated route and nodal probabilities determine the location of the trip origin. The location of the trip destination is determined by repeating the same process based on the number of passengers getting off at each bus stop.

The travel times for this trip are calculated separately for the two travel modes, private automobile and public bus. Direct travel times are determined based on the available street network and the first developed bus network. Automobile access time and bus walking time, waiting time, boarding and departing time, and transfer time are all determined from appropriate distributions and appropriate boundary values (8, 27, 28) that correspond to the system under consideration. The total travel times for each individual trip, plus other utility variables determined or obtained from supply functions and operational policies (such as automobile travel cost, bus fare, and automobile parking fare), are fed into the demand model to obtain the individual probability of choice rider i traveling by bus, referred to here as the bus system's attractiveness to choice rider i.

This simulation process is repeated for a large sample of individual trips. The sample size is chosen to be statistically acceptable. Then statistics on all of these individual probabilities (bus attractiveness) are collected according to classes of trip length. The mean of total probabilities—the bus share of choice riders in the transportation market—is assigned the variable name PROB2, which implies the "current" attractiveness of the bus system, in contrast to PROB1, the "previous" attractiveness of the bus system.

PROB1 is among the initial input data, which are determined by either demand survey or analogy. PROB1 can be calculated as follows:

$$\text{PROB 1} = (\text{total bus riders} - \text{bus captives})/[\text{total travelers} - (\text{bus captives} + \text{automobile captives})] \quad (13)$$

If the numbers of automobile captives and bus captives are not available, the model determines PROB1 by the previously assumed division between captive and choice riders:

$$\begin{aligned}\text{PROB 1} &= (\text{total bus riders} - \tfrac{1}{16}\ \text{total travelers}) \\ &\div (\text{total travelers} - \tfrac{1}{4}\ \text{total travelers}) \\ &= (\text{bus choice riders}/\text{total choice travelers}) \quad (14)\end{aligned}$$

The values of PROB1 and PROB2 are compared. If they are different, the market is in an unstable condition and demand is subject to change. Demand at each bus stop (one of the initial input data) is modified by the following relation:

$$Q(I) = Q(I)\{[1 - (\text{bus captives}/\text{total choice riders})]\ (\text{PROB 2} \div \text{PROB 1}) + (\text{bus captives}/\text{total choice riders})\} \quad (15)$$

or, for the assumed division,

$$Q(I) = Q(I)[(11/12) \times (\text{PROB 2}/\text{PROB 1}) + (1/12)] \quad (16)$$

where bus captives/total choice riders = (1/16)/(3/4) = 1/12. The previous system attractiveness is then dropped and the current attractiveness is substituted; i.e., PROB1 = PROB2.

New demands [Q(I)] at each bus stop are fed back to the SWEEP algorithm, and the whole process is performed again. Current system attractiveness (PROB2) is once again generated from the integrating model. PROB1 and PROB2 are compared. If they are different, the whole process is repeated. The termination of the iterative process occurs only if PROB2 falls within a 95 percent confidence interval of PROB1. At this stage, it is assumed that the equilibrium state is reached—i.e., that the supply and demand sides are in stable condition. The equilibrating strategy is shown in Figure 3.

As Figure 3 shows, the equilibrium state or condition changes with system management policies. Different sets of policies will produce different levels of bus-system attractiveness. Under a specific set of policies, maximum local attractiveness is obtained at the equilibrium state. When different policies are evaluated by sensitivity analysis, the one that generates global maximum attractiveness is the most preferable set of policies if maximization of attractiveness is the management objective.

APPLICATION

The methodology described above is now applied to the development of feeder bus routes to the proposed Glebe Station of the Washington, D.C., Metro rail rapid transit system. The study area is located in the midwestern section of Arlington County, Virginia. It surrounds the last Metro station on the Rosslyn-Ballston Corridor and acts as a catchment area for the station. The boundaries of the designated area, determined by using a conservative radius from the transit station, extend 3.2 km (2 miles) to the north, south, and west of the station. The area to the east is assumed to be serviced by the previous station on the Metro line.

The locations of bus stops are determined by land-use patterns, the major attractors, the road network, and passenger walking time. Candidate streets for bus routes are initially identified. These streets fall into the categories of major and minor arterials. Bus stops are then located on these streets based on land-use patterns, the major generating and attracting points of the study area, and the consideration that passenger walking distance to a bus stop should not exceed 0.4 km (0.25 mile). Because 70 percent of morning-peak work trips in the study area are bound for Washington, D.C., the Metro station can be considered as the major traffic attractor in the area and version 1 of the SWEEP algorithm appears to be applicable. The Metro station is considered to be bus stop 1 and the dispatch point of all buses. One hundred and thirteen bus stops, excluding the Metro station, were developed for the study area. The study area and its boundaries, as well as the location of bus stops, are shown in Figure 4.

Initial demand for each bus stop is calculated from the 1970 census-tract data for bus ridership in the study area.

Various computer runs were conducted by using different input parameters. On the supply side, bus capacity, maximum allowable travel distance for each bus, and the schedule of bus headways are all input variables. On the demand side, bus-fare structure, automobile parking fare, and automobile cost per mile are also input parameters that affect the measure of bus-system attractiveness. All of these variables are tested in order to investigate how alternative policies will affect the attractiveness of bus to the community.

The findings can be summarized as follows:

1. Capacity and distance constraints versus operational cost—Two types of buses were chosen for computer runs: buses with 32 seats and a maximum allowable travel distance of 9 km (5.68 miles) and buses with 50 seats and a maximum allowable travel distance of 13 km (7.97 miles). The results show that the system with the larger buses will operate at lower cost.
2. Capacity and distance constraints versus attractiveness—From intuitive judgment, a decrement in attractiveness should result if buses of greater capacity are used. The bus with greater capacity can serve longer routes, which will consume longer travel time. Not too surprisingly, however, bus capacity does not significantly affect system attractiveness in this case study. The variation ranged between 0.005 and 0.01. Such a small range of variation can be attributed to the structure of the behavioral model.
3. Fare structure and parking price versus attractiveness—Four levels of bus-fare policies were tested: \$0.25, \$0.30, and \$0.50/ride and free fare. Three levels of parking-price policies were tested: \$1, \$2, and \$3/trip. The results show that higher bus fares suppress ridership and higher parking prices increase ridership. Increasing parking price will be a more effective means of increasing ridership than decreasing bus fare.
4. Trip length versus attractiveness—The simulation output stratifies the data on trip length. The results show that the range of maximum distance for bus use is approximately 4.8-6.4 km (3-4 miles). When trip length is longer, the attractiveness of bus decreases rapidly.

Other findings related to the supply side are that the

Figure 3. Flow diagram of equilibrating strategy.

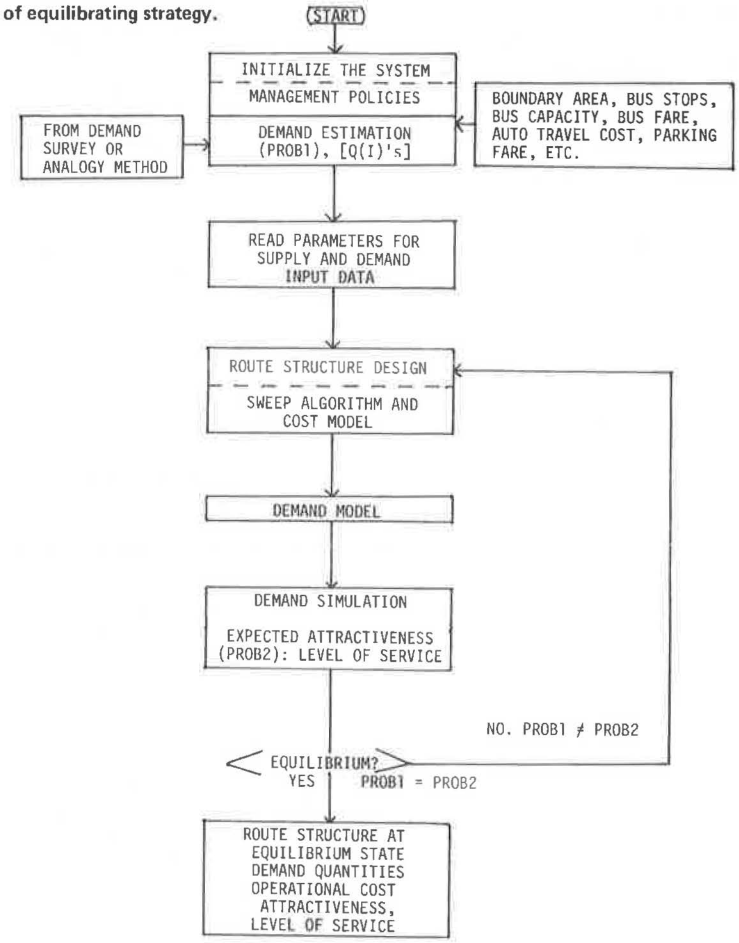


Figure 4. Selected bus-stop locations in the study area.

linear routes developed by the Hamiltonian-path algorithm, in comparison with the loop-routes developed by the traveling-salesman algorithm, saved 5-10 percent of the total distance traveled by all buses under the same conditions of bus capacity and travel-distance constraint. Although both algorithms are computationally efficient, the Hamiltonian-path algorithm is far superior for this specific problem. Each computer run for the Hamiltonian path took an average of 1 min of central processing unit (CPU) time versus 9.5 min for each run with the traveling-salesman algorithm. In both cases, computer time increased linearly with the total number of bus stops and quadratically with the number of stops per route.

Based on the sensitivity experiments, the following observations are made:

1. The larger the constraint is on the distance traveled by each bus, the fewer routes will be formed. This yields a lower distance traveled for all buses.
2. The greater bus capacity is, the fewer routes will be formed. This yields a lower total distance traveled.

CONCLUSIONS

The methodology described in this paper is a comprehensive computer model and an efficient tool for analyzing and evaluating bus systems in small to medium-sized urban areas. It determines the impacts of different policies and conditions on the bus system. It can be useful to small bus companies in evaluating and designing their systems and to local planning commissions in planning bus systems in their communities. The model is easy to use and provides quick answers to many of the decision maker's questions. In addition, it is a strong educational tool by means of which the user can easily learn the interactions among the different elements of the bus system.

However, additional improvements and modifications in bus-network development in the model are being sought, such as (a) inclusion of the actual street network and variable travel times on each link of the network and (b) a built-in capability in the algorithm to change the locations of bus stops and their spacing according to demand.

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