subgrade stress than did the concrete-tie section. The larger ballast strain could have resulted from the development of a gap between the tie and the ballast because the upper part of the ballast strain gauge was fixed to the tie.

The ballast and subballast dynamic strains were greater for the tangent concrete track than for the curved concrete track, possibly because the tangent track had a thinner ballast layer.

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Study of Analytical Models for Track Support Systems

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Data on the dynamic responses of ballast, subballast, and subgrade of track sections at the Facility for Accelerated Service Testing track in Pueblo, Colorado, are compared with predictions from three available analytical models for track support systems. The response data include ballast strain, subballast strain, subgrade deflection, and subgrade stress. The analytical solutions are provided by (a) a model that combines Burmister's three-dimensional elasticity solution with a structural analysis model that solves for the tie-ballast reaction (MULTA), (b) a finite-element, three-dimensional model that has prismatic elements combined with a structural analysis model (PSA), and (c) a quasi-three-dimensional, finite-element model, in which a longitudinal two-dimensional analysis is followed by a transverse two-dimensional analysis (ILLI-TRACK). The results show that all three models can reasonably predict the behavior of the track system, provided that values for the material properties and model parameters are correctly specified. Each model has advantages and limitations compared with the others. ILLI-TRACK is the only model that can vary properties in the vertical, longitudinal, and transverse directions and also the only one having a nonlinear stress-strain representation. However, the accuracy of ILLI-TRACK predictions is less certain because it depends on two empirical parameters, the effective tie-bearing length and the angle of distribution. The PSA model permits property variation in the transverse and vertical directions, but its computer costs are an order of magnitude greater than those for the other two models. The MULTA model is restricted to homogeneous layers of ballast and underlying materials, but it combines the features of both three-dimensionality and economy.

To provide a foundation for the prediction of track performance, which is a prerequisite for rational track design and maintenance-life prediction, it is necessary to have an analytical model that realistically represents the actual behavior of a track system subjected to various vehicle-loading conditions. One of the requirements for such a model is that it adequately characterize the three-dimensional aspects of the problem. Another is that it must distinguish the various soil and ballast layers and give them independent properties.

Several models that use the beam-on-elastic-foundation approach (1-3) have been employed to provide a basis for track design procedures in the past (4, 5). Although this approach has been extended to include a nonuniform foundation modulus (6) and a nonuniform finite-beam section (3, 7) to represent more closely the rail-tie system, its significant limitations are that it does not adequately model the ballast and subgrade system and that the interaction between the soil and the track structure is not properly represented.

To interrelate the components of the track structure to properly represent its complex interactions in determining the net effect of traffic loads on the stresses, strains, and deformations developed, several morecomprehensive models are available. However, some of these models involve a plane-strain assumption, which does not represent the three-dimensionality condition (8), and others use three-dimensional finite elements, which are too expensive and not feasible for practical purposes (9). Considering all alternatives currently available, three models, which do account for the three-dimensionality condition, include soil-structure interaction and proper representation of the soil layers, and are reasonably economical to use, were chosen for study. They are

1. MULTA: a model that combines Burmister's three-dimensional, multilayer elastic solution with a structural analysis model that solves for the tie-ballast reaction;
2. PSA: a three-dimensional finite-element model that uses prismatic elements together with a similar structural analysis model; and
3. ILLI-TRACK: a quasi-three-dimensional finite-element model, in which a longitudinal two-dimensional analysis is followed by a transverse two-dimensional analysis.

In this paper, a brief description is given of the basic assumptions, important features, and limitations of the three models. Then, the general trends of the track behavior predicted by using them are presented and compared with the results of field measurements made at the Facility for Accelerated Service Testing (FAST) track in Pueblo, Colorado. Finally, the models are evaluated in relation to the suitability of their predictions and their cost-effectiveness.

MULTA MODEL

The MULTA model is a combination of two computer codes: BURMISTER and LOADS AND COMBINATIONS (LAC).

The BURMISTER code uses Burmister's multilayer

Figure 1. Schematic representation of BURMISTER code.

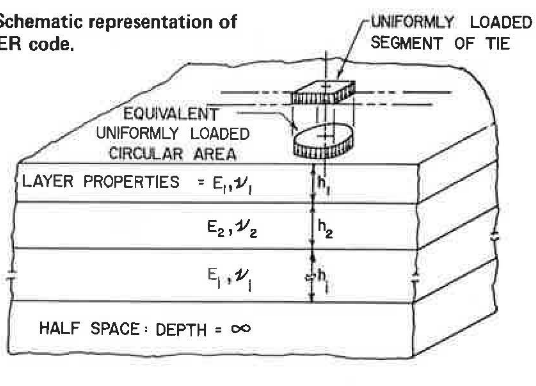
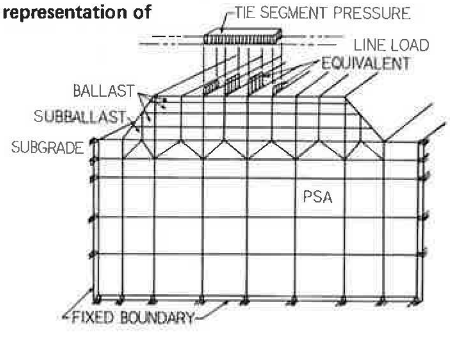


Figure 2. Schematic representation of PSA model.



elastic theory to represent the ballast and the soil layers. The tie-bearing area is divided into segments of approximately square dimensions, and then the area of each segment is converted to a circular area of uniform pressure (see Figure 1) that represents the same vertical force. These uniformly loaded circular areas are used to generate influence coefficients for stresses and displacements for the multilayer linear-elastic model.

The LOADS-AND-COMBINATIONS code is a matrix structural-analysis model that solves for the tie-ballast reactions by using the method of consistent deformations. Wheel loads are applied on the opposite rails to represent an axle load. Each rail is assumed to be a beam of finite length that is supported by 11 ties, which are also represented as beams having multiple supports (one for each segment of the tie-ballast contact area, as indicated in Figure 1).

In each division, the uniformly distributed pressure that is converted into a resultant tie-support force is assumed to be unknown. The force is represented by the influence coefficients from the BURMISTER program (10). The reaction between rails and ties and the displacement at the intersection of rail and ties are also unknowns. This indeterminate structural problem is then solved by imposing compatibility and equilibrium equations to form a set of simultaneous equations containing the unknowns.

After the magnitude of the tie-ballast pressures is determined for each division of each of the ties, these pressures are superimposed on the roadbed system for all ties, by using BURMISTER, to obtain the displacements and stresses within the multilayer soil system.

Some limitations of this model include the following:

1. There is no relative displacement between tie and ballast. In addition, no separation of tie and ballast is allowed and tension may be developed between tie and ballast (which is not realistic).
2. The reactions between rail and tie and between tie and ballast are in the vertical direction only; shear forces are neglected.
3. The material properties for each roadbed layer are linear elastic, and they are constant throughout the layer. Thus, each layer is assumed to be composed of a homogeneous, isotropic, linear elastic solid.

PSA MODEL

This model is similar to MULTA in that it also considers the foundation representing the ballast and subgrade layers separately from the track structure for developing stress and displacement influence coefficients and then imposes the compatibility of displacements and stresses between the bottom of the structure and the top of the foundation to effect an overall solution to the system.

The PSA code generates foundation stress and displacement influence coefficients based on an analysis of periodically loaded prismatic solids (11-16), as shown in Figure 2. A prismatic solid is defined as a body that (a) is infinite in extent in the longitudinal direction (i.e., z), (b) has a cross-section (which may be arbitrary in shape) that is identical for all values of z, and (c) has material properties that do not vary in the z-direction. The analysis is restricted to those problems in which the spatial dependence of the loading can be approximated as periodic in the z-direction. The period of the loading, however, can be made sufficiently large so that the effects of isolated single loads or groups of loads can be effectively considered.

In the currently used PSA code, materials are considered to be linear elastic, but different elements in the vertical plane perpendicular to the rails (i.e., the x-y plane) may have different elastic constants. The three-dimensional solution is approximated as a Fourier series in the direction parallel to the rails (z). The coefficients in the series are obtained from two-dimensional finite-element analyses (one for each term in the series) that produce displacement series coefficients of all the nodes as a function of the x and y coordinates. Summation of those series terms gives the final displacements from which the strains and stresses at any point in a prismatic solid can be obtained. The input to the program consists of a finite-element representation of the cross-section of the body and the Fourier coefficients of the body forces, temperature terms, and boundary conditions. The output consists of displacements, strains, and stresses for any desired point in the body.

Figure 3. Typical representation of the two ILLI-TRACK two-dimensional finite-element meshes.

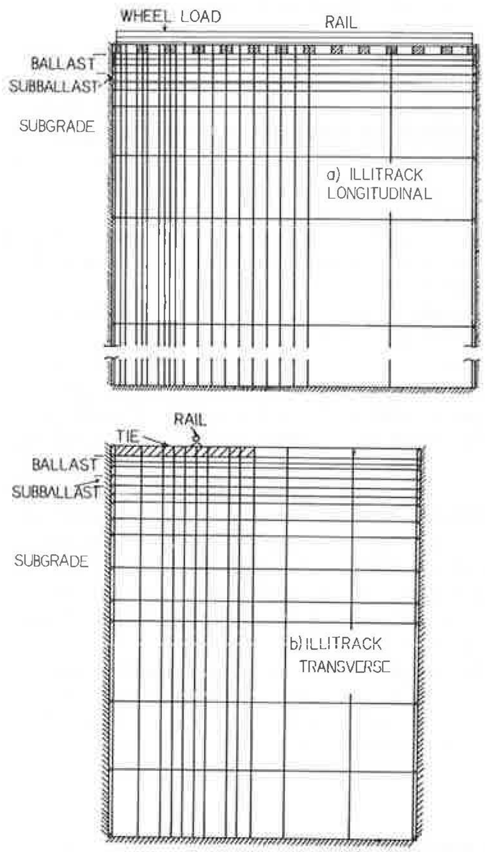
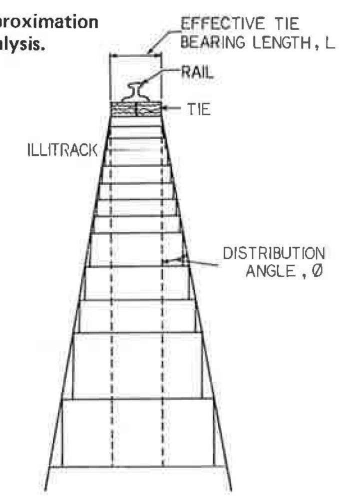


Figure 4. Pseudo-plane-strain approximation for ILLI-TRACK longitudinal analysis.



The main advantage of the prismatic-solid analysis is that it can provide solutions to three-dimensional elasticity problems at a relatively low cost when compared with an equivalent general three-dimensional finite-element analysis (that uses three-dimensional brick elements), which usually requires inordinately high and often impractical costs. The PSA model has the same limitations as MULTA. Its advantage over MULTA is the ability to vary the material parameters across the track section along the length of the tie.

ILLI-TRACK MODEL

The ILLI-TRACK model (17-19) represents an attempt to incorporate a realistic representation of the nonlinear and stress-dependent behavior of roadbed materials.

Recognizing the three-dimensional nature of the geometry and loading conditions of a track system and the complexity and inordinate cost associated with actual three-dimensional finite-element formulation, Robnett and others (17) have attempted to simulate the track system by using two two-dimensional, pseudo-plane-strain finite-element analyses. A longitudinal two-dimensional analysis (see Figure 3a) is performed, and this is followed by a transverse two-dimensional analysis (see Figure 3b) that uses as input the results of the longitudinal analysis. Rectangular plane-strain elements are used to represent the ballast, subballast, and subgrade, and beam-spring elements are used to represent the rail-tie subsystem as a continuous beam supported on tie springs.

In standard two-dimensional, plane-strain finite-element formulations, the thickness of the elements (t) is maintained constant in all the elements. Thus, in the plane-strain state, the load is distributed in two directions only. Three-dimensional load dissipation is simulated by allowing the finite-element thickness to increase with depth. It is assumed that the rate of increase of element thickness with depth is constant. This is denoted by a parameter called the angle of distribution (ϕ) as shown, for example, in Figure 4 for the longitudinal analysis.

Also for the longitudinal analysis, it is assumed that the initial thickness of the element at the surface is equal to an effective tie-bearing length (L). This length is assumed to be the region of effective load transfer between the tie and the ballast.

Material nonlinearity is accounted for in the ILLI-TRACK model through the use of a resilient modulus, which is defined as the repeated deviator stress divided by the elastic or recoverable strain in a triaxial test, as established in pavement research (20, 21). For granular materials, such as ballast and subballast, the resilient modulus (E_r) has been found to increase with increasing bulk stress (Θ), as given by Equation 1.

$$E_r = K_1 \Theta^{K_2} \qquad (1)$$

where

Θ = sum of the principal stress = $\sigma_1 + \sigma_2 + \sigma_3 = \sigma_1 + 2\sigma_3$ in a triaxial test, and

K_1 and K_2 = constants determined from laboratory tests.

Table 1. Rail, tie, and roadbed properties assumed for FAST section 18B.

Material	Cross-Sectional Area (cm^2)	E (MPa)	Moment of Inertia About Major Axis (mm^4)	Layer Thickness (cm)	
Rail[a]	86.13	207 000	39.5×10^6		
Tie[b]	406	10.3	107.2×10^6		
Roadbed					
Ballast[c]		207		38	0.37
Subballast		13.3		15	0.37
Subgrade		3.3			0.33

Notes: 1 cm^2 = 0.155 in^2; 1 MPa = 145 lbf/in^2; 1 mm^4 = 2.40×10^{-6} in^4; 1 cm = 0.39 in.
The properties of Section 18 A are identical to those given above except that the ballast layer thickness is 53 cm (21 in).
[a] 68 kg/m (136 lb/yd) jointed.
[b] 17.8 x 22.9-cm x 2.59-m (7x9-in x 8.5-ft) hardwood on 49.5-cm (19.5-in) center-to-center spacing.
[c] Granite.

Figure 5. Comparison of calculated rail-seat load and deflection profiles under single-axle load: MULTA, PSA, and ILLI-TRACK.

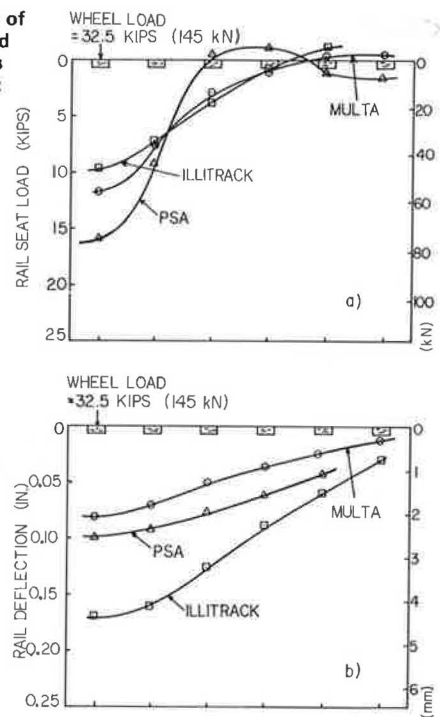
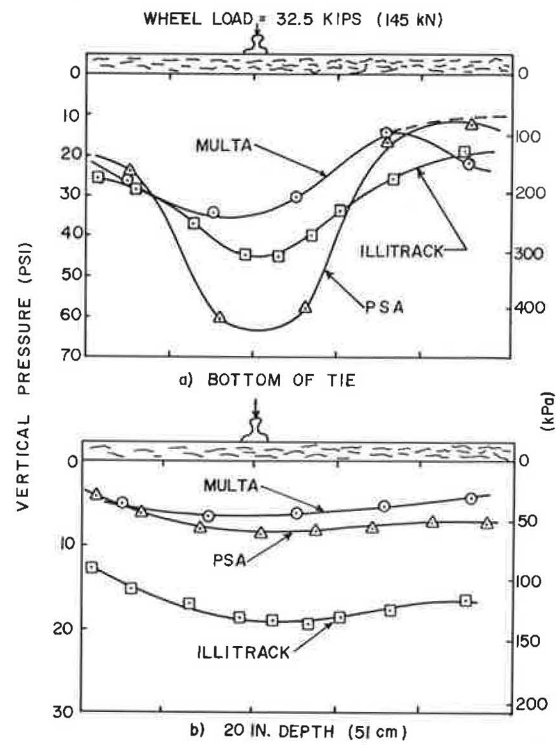


Figure 6. Comparison of vertical pressure distribution across tie under single-axle load: MULTA, PSA, and ILLI-TRACK.



The E_r of fine-grained soils has been found to decrease with increases in the deviatoric stress (17, 20, 22). At higher values of deviatoric stress, the E_r is almost constant, resulting in a bilinear relationship.

The basic limitation of the ILLI-TRACK model is the pseudo-three-dimensional assumption. The accuracy of the model predictions depends critically on assumed model parameters, such as effective tie-bearing length and angle of distribution. However, the criteria for choosing these parameters have not been well defined.

FIELD MEASUREMENTS OF INSTRUMENTED SECTIONS

As described by Yoo and Selig in the preceding paper in this Record, an extensive instrumentation program has been undertaken at the FAST track to monitor the performance of ballast, subballast, and subgrade layers under repeated traffic loading. Sensors were installed in the ballast and subballast layers under the rails to determine the vertical strains in these layers. Vertical extensometers were used to measure the settlement at the subgrade surface relative to that at a depth of 3.05 m (10 ft) below the top of the subgrade. Soil stress gauges were installed at the subballast-subgrade interface to measure the vertical stress on the surface of the subgrade. The measurements of instantaneous response during traffic loading obtained were then compared with the values predicted by using the three analytical models, MULTA, PSA, and ILLI-TRACK.

The single axle load for the test cars was 289 kN [65 000 lbf (65 kips)], assumed to be equally distributed to each of the two wheels. The distance between the axles on a truck was 178 cm (70 in).

COMPARISON OF THE THREE MODELS

The track response was predicted by using each of the three models and the same geometry, soil and track properties, and loading conditions. These predictions were then compared to develop an understanding of the variations and trends of predicted results.

Because the present forms of the MULTA and PSA models are limited to a linear elastic assumption, constant modulus values (Es) and Poisson's ratios (νs) for each layer were selected from the range for track and highway roadbed materials available in the literature (14-18, 22, 23-25). These values and the

Figure 7. Comparison of distribution of vertical pressure with depth under single-axle load: MULTA, PSA, and ILLI-TRACK.

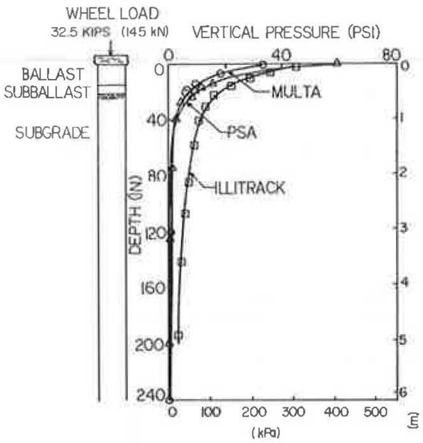
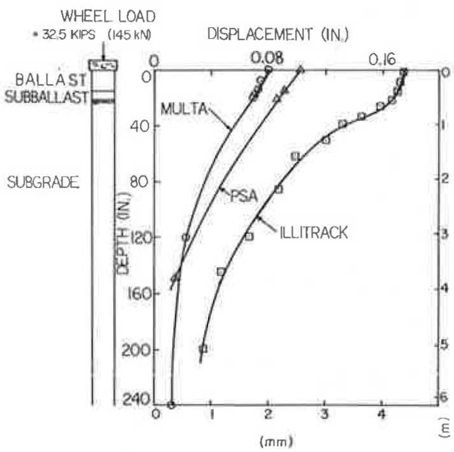


Figure 8. Comparison of distribution of vertical displacement with depth under single-axle load: MULTA, PSA, and ILLI-TRACK.



measured values for the FAST materials are summarized in Table 1.

In order to directly compare the models, rather than using the nonlinear version of ILLI-TRACK, the same constant moduli and Poisson's ratios were used in that also.

Figure 5a shows the distribution of the rail-seat load when a single-wheel load is supported over 11 ties. The distribution, in general, agrees with the trend observed by Talbot (1) over 7-9 ties from a single wheel load. The prediction obtained by using the PSA model indicates that a substantial part of the single-wheel load is distributed to only 3 ties, i.e., the loaded tie plus one tie on either side. The distributions obtained by using the MULTA and ILLI-TRACK models indicate that the load is distributed to about 5 ties. MULTA predicts a higher rail-seat load for the tie directly beneath the wheel load than does ILLI-TRACK.

Except for MULTA, the predicted pressures at the tie-ballast interface are highest under the rail and lowest at the center (see Figure 6a), which is considered to be typical of flexible wooden ties. The subgrade surface pressure along the tie under the wheel load is relatively smooth and close to being uniform (Figure 6b). ILLI-TRACK predicts much higher values of subgrade stress than do MULTA and PSA. This discrepancy may be caused by the value of ϕ [10° (19)] used for the pseudo-plane-strain analysis. A larger value of ϕ is required to match the predicted subgrade pressures of ILLI-TRACK with those of MULTA and PSA.

Vertical pressures under the wheel load are shown as a function of depth in Figure 7. The pressures predicted by PSA in ballast and subballast and at the top of the subgrade are higher than those predicted by MULTA, because, in the PSA model, more load is transmitted to the tie under the wheel than to the adjacent ties. ILLI-TRACK predicts much higher vertical pressure with depth than do PSA and MULTA. The low dissipation rate of stress with depth is caused by the small angle of distribution. The high vertical stress predicted by ILLI-TRACK is also reflected in the high vertical displacements, as shown in Figure 8. For the same reason, PSA predicts greater vertical deformation than does MULTA.

The rail-deflection profile is shown in Figure 5b. The deflections of the rail under the wheel load are 2.0, 2.5, and 4.3 mm (0.08, 0.10, and 0.17 in), respectively, for MULTA, PSA, and ILLI-TRACK. It can be seen that rail deflection is still significant up to the fifth tie away from the loaded tie. Superposition is therefore necessary to represent the effect of the adjacent axle load.

ILLI-TRACK predicts higher deflection than do other models, perhaps due to the value [46 cm (18 in) (19)] used for the tie-bearing length under each rail in the longitudinal analysis.

In general, except at the tie-ballast interface, PSA and MULTA predict similar vertical pressures and deflections, while ILLI-TRACK predicts values in the order of 100 percent higher.

The PSA model predicts the highest values of tie-ballast pressure and rail-seat load directly under the load and lowest values away from the load. The reason for this greater stress concentration is not known.

EFFECT OF ILLI-TRACK PARAMETERS

Tie-bearing length is one of the parameters used to simulate the three-dimensional effect of ties in the longitudinal analysis of the ILLI-TRACK model. The wheel load transmitted through this tie-bearing area into the roadbed system, and therefore the stiffness of the rail-tie system, depends very much on the specified value of this parameter. Angle of distribution is another parameter used in the ILLI-TRACK model to further simulate the three-dimensional problem. This parameter allows the thicknesses of the elements to increase with depth for both transverse and longitudinal analysis. The stiffness of the roadbed system is also a function of the angle of distribution.

The currently used values of the tie-bearing length and the angle of distribution are 46 cm and 10°, respectively. The 10° value for the angle of distribution was selected to give the best agreement between the ILLI-TRACK solution and a closed-form elastic solution of stress distribution with depth under a strip footing (25). This value however, does not appear to be suitable for the FAST track structure. Calculations also show that the vertical pressure distribution with depth given by the longitudinal analysis is not the same as that given by the transverse analysis when this angle of distribution is used. Thus, various values of the tie-bearing length and the angle of distribution were used, and the results were compared with solutions given by MULTA.

The tie-ballast and subgrade pressures were shown to be affected significantly by the tie-bearing length.

Table 2. Comparison of MULTA, PSA, and ILLI-TRACK predictions with measured response at FAST sections 18B and 18A.

	Section 18 B					Section 18 A		
		Predicted					Predicted	
				ILLI-TRACK				
Response	Measured[a]	MULTA	PSA	Constant Moduli	Nonlinear[b]	Measured[c]	MULTA	ILLI-TRACK (constant moduli)
Ballast strain (mm/mm)	0.004	0.0007	0.0006	0.0008	0.0005	0.006	0.0006	0.001
Subballast strain (mm/mm)	0.0005	0.0005	0.0008	0.0009	0.0006	0.0004	0.0003	0.001
Subgrade surface deflection (mm)		2.46	3.02	6.17	2.03		1.04	1.88
Subgrade deflection at extensometer bottom-anchor location (mm)		1.07	0.64	1.85			1.22	3.71
Subgrade surface deflection relative to extensometer bottom anchor (mm)	0.79	1.40	2.39	4.32				
Subgrade surface vertical stress (kPa)	45.5	48.3	68.9	186	139		38.6	153

Note: 1 mm/mm = 1 in/in; 1 mm = 0.039 in; 1 kPa = 0.145 lbf/in².
[a]Average for ties 18 B - 0375, 18 B - 0383, 18 B - 0391, 18 B - 0399, 18 B - 0417, and 18 B - 0425.
[b]Taken from results of Tayabji and Thompson (26).
[c]Average for ties 18 A - 0319 and 18 A - 0147.

Table 3. Types of roadbed stiffness.

Type of Roadbed		Young's Modulus (MPa)		
Ballast	Foundation	Ballast	Subballast	Subgrade
Stiff	Stiff	2067	138	138
Stiff	Soft	2067	138	34.5
Soft	Stiff	207	138	138
Soft	Soft	207	138	34.5

Note: 1 MPa = 145 lbf/in².

Similarly, the magnitude of the vertical pressure distribution with depth is greatly affected by the angle of distribution. Better agreement between the vertical stress distributions given by MULTA and ILLI-TRACK is achieved by using $\phi = 30°$ and a tie-bearing length of 61 cm (24 in).

The values of the angle of distribution and the tie-bearing length are expected to be different for different tie spacings, rail and tie stiffnesses, roadbed moduli, and loading conditions for any roadbed-track-structure-interaction problem. Further studies on the appropriate values to use for these two parameters are necessary to use the ILLI-TRACK model effectively.

COMPARISON BETWEEN PREDICTED AND MEASURED BEHAVIOR

A preliminary analysis showed that the two axles on a truck are close enough so that the peak response measured directly under one axle is affected by the load from the other axle. Thus, to compare the predictions with the field measurements, the computations were done by using superposition of two axles. The results are given in Table 2. For comparison, the results given by the nonlinear version of ILLI-TRACK (26) are also shown in Table 2.

To study the effect of roadbed moduli on the predictions, moduli for four types of roadbed systems were assumed and analyzed by using MULTA. The assigned moduli for ballast and foundations ranged from stiff to soft and are shown in Table 3. The results are given in Table 4 compared with the average and range of measured values.

As can be seen from these results, all three models predict ballast strains that are significantly lower than the values measured at FAST. A possible explanation for this discrepancy is that the measured displacements at the tie-ballast interface actually include the closure of the small gap that may exist between the tie and the ballast before the application of the train loads, i.e., tie-seating effects. This can significantly affect the measured ballast strains, because the upper coils are attached to the tie.

The MULTA predictions for various roadbed moduli give values of subballast strains, subgrade deflections, and subgrade pressures that are in the range of the measured data at FAST Section 18B. It is believed that reasonable predictions can be made by using MULTA if appropriate moduli values for roadbed layers are chosen.

In general, MULTA, which is a three-dimensional elasticity solution, and PSA, which is a three-dimensional finite-element solution, predict values that are nearly the same at the subgrade level. Although the tie-ballast pressures predicted by PSA are significantly higher than those predicted by MULTA (Figure 6), the rate of pressure dissipation with depth is nearly the same.

The ILLI-TRACK model appears to be the most complete model in the sense that it is a nonlinear, stress-dependent model that incorporates failure criteria for roadbed materials. It is, however, suspected that the two-stage pseudo-plane-strain analyses do not truly represent the three-dimensional state of the track system. Thus, it is possible that no advantage will accrue from sacrificing the three-dimensionality of the track system by using these detailed nonlinear formulations. It should be pointed out, however, that it may be possible to obtain realistic predictions from the ILLI-TRACK model by using a systematic variation of the angle of distribution and an effective tie-bearing length. However, because these are empirical parameters whose values may vary from problem to problem, the accuracy of predictions obtained by using this model is uncertain.

In terms of computer cost and input-data preparation effort, the ILLI-TRACK model is the least expensive.

Table 4. Comparison between measured and predicted responses (MULTA) for different types of roadbed stiffness.

Response	Predicted				Measured	
	Stiff Ballast and Stiff Foundation	Stiff Ballast and Soft Foundation	Soft Ballast and Soft Foundation	Soft Ballast and Stiff Foundation	Average	Range
Ballast strain (mm/mm)	0.000 13	0.0007	0.000 25	0.0007	0.004	0.001-0.005
Subballast strain (mm/mm)	0.000 58	0.000 48	0.000 35	0.0005	0.0005	0.0003-0.0007
Subgrade surface deflection relative to extensometer bottom anchor (mm)	0.38	1.06	0.46	1.40	0.79	0.20-0.89
Subgrade surface vertical stress (kPa)	56.5	31.7	71.0	48.2	45.5	30.3-60.6

Note: 1 mm/mm = 1 in/in; 1 mm = 0.039 in; 1 kPa = 0.145 lbf/in².

The ILLI-TRACK model also has an attractive automatic mesh-generating feature that reduces the number of cards needed to describe the track system.

The PSA model is an order of magnitude more expensive than the MULTA model. The input data preparation for PSA requires a minimum of one day compared with about five hours for MULTA. In addition, a lot of time is needed to check the connectivity data, nodal-point coordinates, and Fourier coefficients needed by the PSA model. In the present form of the PSA model, the influence coefficients for each of the five tie divisions are generated in separate computer runs. This requires a large turnaround time in order to obtain all the influence coefficients.

The close agreement between the predictions obtained by using the MULTA and PSA models and the measured responses and the fact that these models are three-dimensional and incorporate most of the required components of the track system make them good potential candidates for use in track analysis. Their basic limitation is that, in their present form, they are linear elastic and do not account for the stress-state- and stress-path-dependent behavior of roadbed materials.

The PSA model is more advanced than MULTA in that it permits variation of the properties of the roadbed transverse to the rail. This capability is particularly attractive for the study of center-bound track conditions.

SUMMARY AND CONCLUSIONS

Three analytical models—MULTA, PSA, and ILLI-TRACK—have been studied and evaluated by comparing their predicted results with field measurements.

1. For a set of chosen roadbed properties, the predictions obtained by using MULTA and PSA show similar trends of behavior in comparison with field measurements. On the basis of a materials parametric study of MULTA and the similarity of PSA and MULTA in the mathematical representation of the three-dimensionality of the track system, it is believed that PSA and MULTA can reasonably predict the response of a track system.

2. It is not certain whether the pseudo-plane-strain assumption in ILLI-TRACK is actually representing the three-dimensionality of the track system as desired. The parameters involved in this assumption—angle of distribution and effective tie-bearing length—are both problem dependent and require experience in their specification. The usefulness of this model might be improved by a systematic study of these two parameters to provide a guideline for selecting the proper values.

3. Relative to cost-effectiveness, the PSA model is an order of magnitude more expensive than MULTA and ILLI-TRACK.

4. The linear elastic assumption currently used in MULTA and PSA is not considered adequate for representing the actual stress-dependent behavior of roadbed materials. Further studies of material characterization and the adaptation of these models to properly represent the nonlinear behavior of roadbed materials should be carried out.

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Discussion

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The development and justification of the ILLI-TRACK model is described elsewhere (17, 19). The emphasis in the development of this model was the proper and realistic simulation of the ballast, subballast, and subgrade materials in the track support system.

The repeated-load behavior [as characterized by the resilient modulus (E_R = repeated deviator stress/recoverable strain)] of granular materials and fine-grained subgrades is stress dependent (17). Because stress states vary throughout the support system (ballast-subballast-subgrade), adequate materials modeling cannot be achieved by assigning a constant modulus.

In the MULTA and PSA models, it is assumed that the materials are linearly elastic and no provision is made for failure. Stress-dependent resilient behavior and failure criteria, however, are considered in the original ILLI-TRACK program; there is no advantage to using this model for the analysis of linear-elastic systems in which stress-dependent resilient behavior and material failure criteria are not stipulated.

In the initial ILLI-TRACK model, material failure criteria were defined in terms of the maximum principal stress ratio (σ_1/σ_3) and the minimum-allowable minor principal stress (σ_3) (generally, $\sigma_3 = 0$; i.e., no tensile stress is permitted) for granular materials. A maximum-allowable shear stress $[(\sigma_1 - \sigma_3)/2]$ was designated for fine-grained soils. If an element failed during the ILLI-TRACK analysis, a failure modulus was assigned. A value of 27.6 MPa (4000 lbf/in^2) was recommended for the failure modulus of granular materials. The assigned failure modulus for a fine-grained soil was the resilient modulus corresponding to a repeated deviator stress equal to the shear strength of the soil. The effects of the assumed failure criteria on ILLI-TRACK-predicted responses are significant (18).

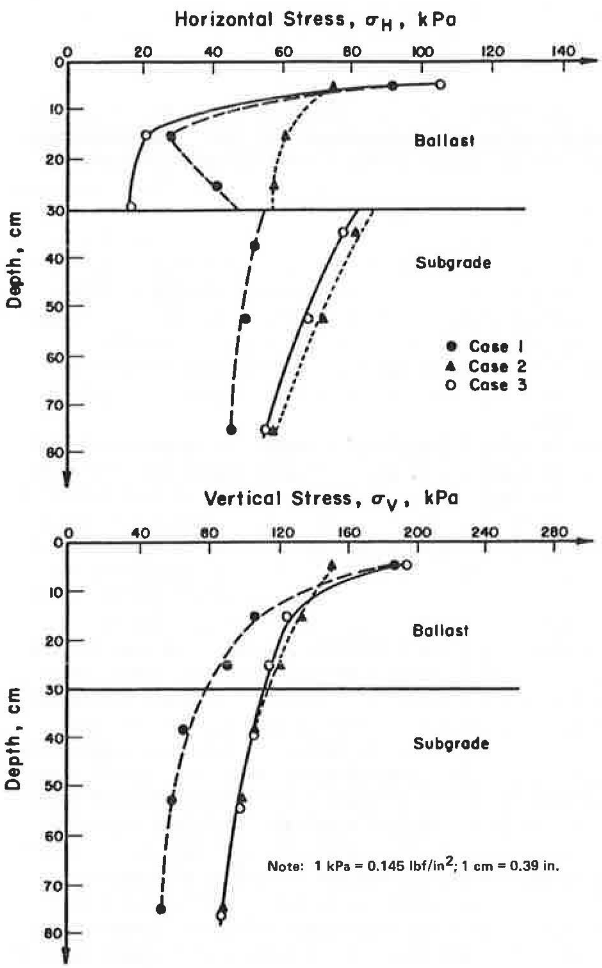
A new failure criterion for granular materials and subgrade soils under repeated states of stress has recently been developed by Raad and Figueroa (27) and incorporated into the original ILLI-TRACK model. In this modified version (ILLI-TRACK 2), the nonlinear properties of the granular material and subgrade layers are included by means of a successive iteration technique. The principal stresses are modified at the end of each iterative step so that they do not exceed the strength of the material as defined by the Mohr-Coulomb envelope. This is achieved by using the vertical stress (σ_v) in each element at the end of the iterative step to calculate limiting values for the major and minor principal stresses $(\sigma_1)_{max}$ and $(\sigma_3)_{min}$, respectively, in terms of cohesion

Table 5. Summary of response data.

Case	Failure Criteria: Ballast	Failure Criteria: Subgrade	Maximum Tie Reaction (kN)	Maximum Tie Deflection (mm)
1[a]	$\sigma_1/\sigma_3 = 10$, $(\sigma_3)_{min} = 0$	$\tau^b_{max} = 173$ kPa	36.1	1.30
2[c]	$\sigma_1/\sigma_3 = 5.8$, $(\sigma_3)_{min} = 0$	$\tau_{max} = 173$ kPa	32.7	3.30
3	$\phi = 45°$, $C = 0$	$\phi = 0$, $C = 173$ kPa	37.6	1.45

Notes: 1 kPa = 0.145 lbf/in²; 1 kN = 225 lbf; 1 mm = 0.039 in.
Details of loading and track system data are given by Tayabji and Thompson (18); 133.5-kN (30 000-lbf) wheel loads, wooden ties at 51-cm (20-in) spacing, and 68 kg/m (136 lb/yd) rail were used.
[a] Modulus of ballast at failure is assumed to be 27.6 MPa (4000 lbf/in²).
[b] τ_{max} = maximum-allowable shear stress.
[c] Modulus of subgrade at failure is assumed to be 690 kPa (100 lbf/in²).

Figure 9. Variation of vertical and horizontal stresses in ballast and subgrade.



(C) and angle of friction (ϕ), such that

$$(\sigma_1)_{max} = \sigma_v \tan^2 [45 + (\phi/2)] + 2C \tan [45 + (\phi/2)] \quad (1)$$

$$(\sigma_3)_{min} = \sigma_v \tan^2 [45 - (\phi/2)] - 2C \tan [45 - (\phi/2)] \quad (2)$$

If σ_3 and σ_1 are the minor and major principal stresses at the end of the iterative step, respectively, then σ_3 should not be smaller than $(\sigma_3)_{min}$ and σ_3 should not be larger than $(\sigma_1)_{max}$. However, σ_1 should not assume a value greater than σ_1', the major principal stress associated with σ_1 at failure, where

$$\sigma_1' = \sigma_3 \tan^2 [45 + (\phi/2)] + 2C \tan [45 + (\phi/2)] \quad (3)$$

The detailed procedure for the modification of σ_1 and σ_3 is described elsewhere (27). Elements that have modified stress states are in a plastic state and would exhibit large permanent deformation while maintaining a constant resilient response (defined by the specified nonlinear constitutive relationships of the subgrade and granular materials).

An example problem similar to that given by Tayabji and Thompson can be solved for the purpose of comparing the response obtained when the new failure model is used with that obtained when the original model is used. The results are given in Table 5 and Figure 9.

Although there is partial agreement between the responses predicted by the original model (cases 1 and 2) and those predicted by the new model (case 3), there are significant differences when predictions for total response are compared. For example, although vertical stresses for cases 2 and 3 seem to compare quite well (as shown in Figure 9), the resilient deformation for case 2 is twice that for case 3 (Table 5).

Stress state has a tremendous effect on the permanent deformation behavior of granular materials and subgrade soils subjected to repeated loadings (28). Adequate stress-state predictions are thus essential if a rational evaluation of a track support system (ballast-subballast-subgrade) is desired. For example, as shown in Figure 9, there are significant discrepancies in the predicted horizontal and vertical stresses. Linear-elastic theories such as MULTA and PSA frequently indicate the existence of significant tensile stresses at the bottom of the granular layer (even though the granular material has no tensile strength).

Therefore, we believe that ILLI-TRACK 2 (modified to incorporate the improved failure criteria) is the only currently available track-structure model capable of providing a realistic characterization of ballast, subballast, and subgrade response.

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