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Abridgment

Formation and Dissipation of Traffic Queues: Some Macroscopic Considerations

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Queue lengths at signalized intersections are state variables that are frequently used for optimal control of traffic signals particularly at high-volume intersections. In the absence of a reliable macroscopic model that describes queue lengths as a function of the demands, intersection capacity, and the control decisions, existing control schemes are using effective queue size rather than queue length for optimal control. Effective queue size is defined as the actual number of automobiles waiting for service on a particular approach to the intersection at an instant. Queue length, on the other hand, is the distance immediately behind the stopline within which traffic conditions are on the right side of the flow-versus-concentration curve (i.e., they range from congested to capacity).

Anyone familiar with traffic signal control problems will recognize that queue length rather than effective queue size is the parameter that should be used to describe the state of the intersection. This is because an efficient signal control policy should prevent upstream intersection blockage; it should effectively control queue lengths rather than queue sizes. This criterion for optimal operations is somewhat relaxed when traffic demands are relatively low and queue lengths do not pose any immediate threat to adjacent intersections.

In this paper, a rigorous mathematical model shows the evolution of queue length in time at any approach to the intersection as a function of the demands, the intersection capacity, and the signal control policy. Due to space limitations, the results of the simplest possible model are given here. More detailed (therefore, more realistic) models, along with stability analysis and numerical examples, can be found in Michalopoulous and Stephanopoulos (1). The mathematics of queue dynamics discussed here can be used for optimal control of traffic signals. It is believed that, in light of these results, the traffic signal control problem can be placed on a new, more realistic, and rigorous framework of analysis.

BACKGROUND

Consider distance L behind the stopline of a particular approach to the intersection without entrances or exits. Further, assume that L is long enough so that queues never extend beyond this section. Within L, the following equation of continuity applies (2):

$$(\partial K/\partial t) + (\partial q/\partial x) = 0 \qquad (1)$$

where

K = density,
q = flow,
x = space, and
t = time.

Assuming that flow is a function of density, that is, $q = f(K)$, it can be seen that Equation 1 is a first-order partial differential equation in which K is the dependent variable and x and t are independent variables. Solution of this equation allows the estimation of density at any point in the time-space domain. Although space limitations preclude a detailed presentation here, the solution of Equation 1 (also known as the continuity equation) leads to these conclusions.

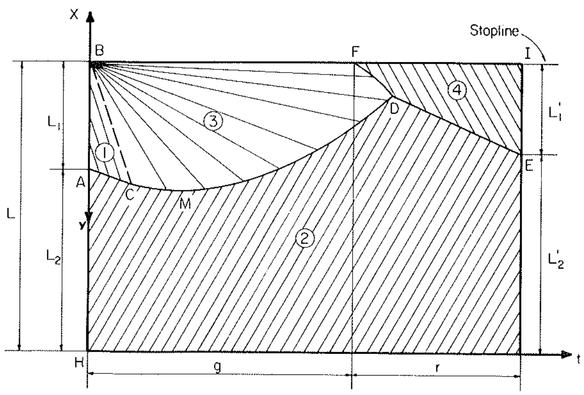
1. Density K is constant along a family of curves called characteristic lines or characteristics.
2. The characteristics are straight lines emanating from the boundaries—$x = L$ (stopline), $x = 0$ (end of the section), and $t = 0$—that have a slope tangent to the flow-concentration curve:

$$(dx/dt) \equiv h(k) = (dq/dK) \qquad (2)$$

3. The characteristics carry the value of density at the point from which they emanate.

These findings suggest that density at any point that has the coordinates x and t is found by drawing the appropriate characteristic line emanating from one of the boundaries and passing through the point. The value of density at the boundary is carried through the characteristic line (i.e., it is maintained constant), and it corresponds to the density of the point of interest. If k_1 is

Figure 1. Queue length developments behind the stopline during a saturated cycle.



ZONE	DENSITY
1 & 4	$K = K_j$
2	$K = K_a$
3	$K_j \geq K \geq K_m$

the boundary value of the density of the characteristic passing through points x and t, then density at this point is also k_1 and the slope of the characteristic line is (dq/dK) evaluated at point k_1.

When two characteristic lines of different slope intersect, then density should have two different values at the point of intersection that is physically unattainable. The discontinuity of density at the point of intersection is explained by the generation of a shock wave moving upstream or downstream of the highway with speed u_w as given in the well-known equation first proposed by Lighthill and Whitham (2):

$$u_w = [(k_2 u_2 - k_1 u_1)/(k_2 - k_1)] = [(q_2 - q_1)/(k_2 - k_1)] \tag{3}$$

where

k_1 = upstream section concentration,
k_2 = downstream section concentration,
u_1 = speed of the upstream section,
u_2 = speed of the downstream section,
q_1 = upstream flow, and
q_2 = downstream flow.

THEORETICAL RESULTS

Based on this preliminary discussion, Figure 1 was prepared to show the shock wave developments behind the stopline. This figure assumes that at the stopline the discharge rates are those as suggested by Webster (3). Briefly, Webster's model suggests that some time is lost due to the driver's response time, acceleration, and deceleration. During the remaining green interval, automobiles are discharged from the intersection at saturation flow as long as a queue exists, and they depart with no delay after the queue dissipates. Of course, during the red interval, no automobiles are discharged. This modeling leads to the conclusion that the entire cycle can be divided into two intervals called effective green, denoted by g, and effective red, denoted by r, in Figure 1.

In Figure 1, it should be noted that along the x axis, point B corresponds to the stopline and point A to the tail end of the queue at the beginning of the effective green interval. Thus, t = 0 corresponds to the start of the effective green. Within AB, jam density and zero flow conditions prevail. Upstream of A and in the remaining portion L_2 of section L, automobiles arrive at an average flow rate q_a. Thus, density within L_2 is K_a. Assuming an average arrival flow q_a and density K_a during the cycle, then flow and density at the beginning of section L (point H) are q_a and K_a during the period $g + r = c$, where c is the cycle length. Finally, assuming that the cycle is saturated, capacity flow and density conditions q_m and K_m prevail at the stopline during g (i.e., from point F to point I).

From this definition of initial and boundary conditions, the characteristic lines emanating from t = 0, x = 0, and x = L were drawn. These lines are tangent to the flow-versus-density curve evaluated at the flow and density conditions corresponding to the point of origin. For example, within AB, the slope of the characteristics is negative, and it is the same as the tangent at the point 0_1K_j of the flow-density curve. At point B, density changes instantaneously from K_j to K_m and, therefore, the characteristics at B fan out, i.e., they take all possible slopes from $(dq/dK)_0, K_j$ to zero.

The characteristic lines emanating from the boundaries divide the entire time-space domain $[0 \leq x \leq L, 0 \leq t \leq c]$ into four distinct zones of different flow and density conditions as shown in Figure 1. When the characteristics intersect, a shock wave is generated. The shock wave developments that result are shown in Figure 1. At the tail end of the queue, shock wave ACMDE is generated during the period of one cycle, and this line represents the trajectory of the tail end of the queue. Therefore, its vertical distance to the stopline represents queue length denoted as y(t). The slope of line ACMDE at any point represents the speed at which this shock wave (or, equivalently, the tail end of the queue) propagates upstream or downstream of the highway. At the end of the effective green, shock wave FD is generated and meets the tail end of the queue at point D. Finally, at the end of the cycle, the distance L_1' represents the final queue length of the existing cycle or, equivalently, the initial queue length of the next cycle.

It should be noted that, if the cycle is undersaturated, line ACMD intersects the stopline during green and point D falls on the stopline. After point D, queue length is zero. For the remainder of the green interval, automobiles depart without delay. At point F, the queue length starts increasing again linearly until the end of the cycle.

ANALYTICAL RESULTS

Each segment of line ACMDE and the coordinates of points C, M, D, and E can be described analytically. Analytical expressions are, of course, needed for the purpose of developing a control policy that restricts the queue lengths within predetermined upper bounds for each approach to the intersection. In order to obtain analytical results, however, one must assume a specific relation between flow and density or, equivalently, between speed and density. For simplicity, we assumed the linear speed-density model (4), but it should be noted that similar results can be obtained for any other model if the guidelines given here are followed. Because space limitations do not allow presentation of detailed proofs, only the final results are given. [See Michalopoulos and Stephanopoulos (1) for further details and for more elaborate models that take into account gradual transitions at point B (Figure 1) and capacities to the left or right of the theoretical capacity.]

Analytical expressions for the trajectory of the queue length (Figure 1) were obtained by using the following notation:

$y(0) = L_1$ = initial queue length at $t = 0$,
$y(c) = L_1'$ = final queue length at $t = c$,
$y(t)$ = queue length at any time point t,
g = effective green interval,
r = effective red interval,
$c = g + r$ = cycle length,
g_{min} = minimum green time required for undersaturation,
X_{IJ} = equation of any line IJ,
u_f = free-flow speed of the approach under consideration,
k_j = jam density of the approach under consideration,
q_a, k_a = arrival flow and density conditions,
x_i, t_i, y_i = coordinates of point i, and
y_{IJ} = equation of line IJ with respect to the y axis (Figure 1).

Thus, the following analytical expressions can be obtained by following the guidelines that are offered in Michalopoulos and Stephanopoulos (1):

$$X_{BC} = L - u_f t \quad (4)$$

$$X_{AC} = (L - L_1) - [(k_a u_f/k_j)]\, t \quad (5)$$

$$X_C = L - [k_j L_1/(k_j - k_a)] \quad (6)$$

$$t_C = [k_j L_1/u_f(k_j - k_a)] \quad (7)$$

$$y_C = [k_j L_1/(k_j - k_a)] \quad (8)$$

$$y_{CMD} = [u_f + h(k_a)]\,(t \cdot t_C)^{1/2} - h(k_a)t \quad (9)$$

where

$$h(k_a) = u_f\,[1 - (2k_a/k_j)] \quad (10)$$

$$t_M = [u_f + h(k_a)]^2\, t_C/4[h(k_a)]^2 \quad (11)$$

$$y_M = [u_f + h(k_a)]^2\, t_C/4[h(k_a)] \quad (12)$$

$$y_{FD} = u_f t - u_f(tg)^{1/2} \quad (13)$$

$$t_D = \{(t_C)^{1/2} + [u_f(g)^{1/2}/u_f + h(k_a)]\}^2 \quad (14)$$

$$y_D = u_f\{t_C - [u_f h(k_a)g]/[u_f + h(k_a)]^2 + [u_f - h(k_a)]/[u_f + h(k_a)]\,(gt_C)^{1/2}\} \quad (15)$$

$$y_{DE} = y_D + [u_f k_a(t - t_D)/k_j] \quad (16)$$

$$y_E = L'_1 = L_1 + [(k_a u_f c)/k_j] - [k_j u_f g]/4(k_j - k_a) \quad (17)$$

$$t_E = c \quad (18)$$

In an undersaturated cycle, the queue dissipates in time:

$$g_{min} = [(y_C/t_C) + h(k_a)]^2\, t_C/[h(k_a)]^2 \quad (19)$$

This is the minimum green time to dissolve the initial queue L_1. In such a cycle, the final queue length L_1' is independent of the initial L_1 and is given by

$$y_E = L_1' = (c - g)(k_a u_f)/k_j \quad (20)$$

QUEUE LENGTH STABILITY

The analytical relations between the initial and final queue developed in the preceding section of this paper can be used for stability analysis. Equation 17 can be rewritten as

$$L_1' = L_1 + b \quad (21)$$

where

$$b = (k_a u_f c/k_j) - [k_j u_f g/4(k_j - k_a)] \quad (22)$$

If c and g are given, b is constant, i.e., it is independent of the initial queue L_1. Thus, Equation 22 can be generalized for any cycle N and rewritten as

$$L_{N+1} = L_N + b \quad (23)$$

where L_N and L_{N+1} are the queues at the beginning of cycle N and N+1. Clearly, a steady state exists if $L_N = L_{N+1}$ or if $L_N = L_N + b$, i.e., if $b = 0$. Therefore, for steady state

$$(k_a u_f c/k_j) - [k_j u_f g/4(k_j - k_a)] = 0 \quad (24)$$

and to solve for g/c

$$g/c = [k_j g/4(k_j - k_a)] = \lambda \quad (25)$$

Since λ is positive, it is easily seen that if $g/c < \lambda$, the queue length at the end of the cycle will be growing for as long as this situation persists. Otherwise, if $b < 0$ or, equivalently, $g/c > \lambda$, the queue at the end of the cycle will decrease. It should be noted that Equations 23 and 25 are meaningful for saturated cycles, i.e., for green times less than the ones given by Equation 19. Otherwise, L_{N+1} is not related to L_N and it is given from Equation 20. A final note concerning the stability of the steady state is worthy of emphasis. As Equation 23 reveals, the steady state is metastable. If $b = 0$, a small variation of the demands will change the steady state to a nearby value that is also metastable. Therefore, the queue length at the beginning of each cycle will change according to the fluctuating values of b, which depend on the demands.

CONCLUSION

The approach taken here to a new and rigorous mathematical model and analysis that show the formation and dissipation of traffic queues at signalized intersections is macroscopic in nature in the sense that automobiles are treated as platoons rather than as single units. The shock wave analysis is fairly realistic for moderate to high demands where the shock waves can be clearly realized. In fact, it is this volume range in which the traffic signal control problem is more pronounced. The analytical results given here have been used for the derivation of a real-time control policy that minimizes total intersection delays subject to queue length constraints at isolated critical intersections (1, 5). It is important to note that the traffic queue dynamics noted here are valid for isolated intersections at which the assumption of constant average flow rates is fairly realistic. If traffic arrivals are affected by an upstream signal, the analysis becomes more complex. Extension of the basic theory to a system of two or more intersections in succession is not trivial due to the side streets and sinks or sources between the intersections. The fact that the input flows to the downstream intersection is a function of the output of the upstream intersection for any pair of intersections further complicates the analysis. We have also studied the queue dynamics for a system of intersections as well as the related optimal traffic signal control problems (1).

It should be recognized that the analysis presented here does not include all possible combinations of shock wave developments that can occur at signalized intersections. Rather, the ones that are most likely to occur are noted. However, analysis for the cases not discussed can be easily performed by following similar guidelines. A number of other cases and numerical examples are treated in Michalopoulos and Stephanopoulos (1).

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Abridgment

Discomfort Glare: A Review of Some Research

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Extensive research on discomfort glare as applied to roadways has been done in Europe by De Boer (1) and Hopkinson (2). However, discomfort glare research in the United States, as has most lighting research, has focused on interior applications. In the past few years, discomfort glare research conducted at Kansas State University under the sponsorship of the Illuminating Engineering Research Institute has been aimed primarily at fixed-roadway lighting. This paper surveys this research and briefly discusses its applications.

SINGLE-SOURCE STUDY

An initial major study was conducted with a single glare source.

Method

Putnam and others (3-5) did what might be considered pilot studies for this experiment by selecting the variables and the range of variation and by running a few subjects. The study summarized here is described in detail by Bennett (6).

Glare source size, position, and background luminance were independent variables; glare source luminance at the borderline between comfort and discomfort (BCD) was the dependent variable.

Glare source size was varied in five equal steps from 10^{-6} to 10^{-3} steradian. At arm's length, these vary from pinhole size to that of a quarter and were selected to cover the range of practicable sizes of the luminous parts of roadway luminaires. Source position varied in five equal steps from 0° (along the horizon) up to 30° above the line of sight (above the occluding angle of windshield tops). Background luminance was varied in five equal steps from 0.0034 cd/m² (0.001 footlambert) to 34 cd/m² (10 footlamberts). According to Kaufman (7), the former represents an overcast horizon night sky with the moon and the latter, an horizon sky on a very dark day.

Observers adjusted the luminance of a glare source to the BCD, which has been the common North American criterion for discomfort glare for about 30 years. The long instructions say that somewhere between a dim comfortable light and a bright uncomfortable one is a point of change or threshold called BCD. They further state that this threshold is neither the one that distinguishes pleasantness and comfort nor the one that distinguishes tolerable and intolerable. Rather, at BCD, if the glare source was made just slightly brighter, it would be uncomfortable.

The 97 paid participants in this study—primarily college students—adjusted (with replication) the incandescent glare source to BCD for 23 of the 125 possible combinations of the three variables in a confounded design. The observer looked at the pole of a 0.6-m (2-ft) radius hemisphere sitting on edge. By using a combination of a transformer and several neutral density filters (to reduce the voltage range and, hence, the lamp color variation), the glare source was set to BCD.

Multiple regression analysis, which involved some trial and error on transformations of variables, was performed. However, this work was largely guided by previously published research.

Results and Discussion

The selected multiple-regression model is as follows:

$$BCD = 200\,(L_B)^{0.3} \times e^{0.05A}/S^{0.6} \qquad (1)$$

where