Using Computers in the Development of Statistically Based Acceptance Plans

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This paper presents a number of ways in which a computer can be used to assist in the development of statistically based acceptance plans. These techniques include statistical analysis of preliminary data to determine the appropriate parameters to use in the development of the acceptance plan. Data collected from 13 bituminous airport pavement construction projects were used to develop a proposed acceptance plan for asphaltic concrete density. A computer algorithm was used to calculate the probabilities necessary to determine the operating characteristics (OC) curves for the plan. Another computer program was used to estimate the integral necessary to determine the expected payment curve associated with a continuous price adjustment schedule. The continuous price adjustment schedule eliminates the problem of having relatively large differences in payment associated with small differences in quality. This situation is encountered frequently in the discrete price adjustment schedules typically used in pavement construction. A computer simulation program used to determine the OC curves for the proposed density acceptance plan is presented. The OC curves from the simulation agree very closely with OC curves that were calculated by theoretical means. The simulation program provides an advantage over the theoretical solution in that it provides OC curves in terms of means and standard deviations that can be related easily to the contractor's process capabilities. The simulation program presented can be used to develop OCs for properties that require either one-sided or two-sided specification limits.

In 1978, for the first time, the Federal Aviation Administration (FAA) Eastern Region incorporated statistically based concepts into its bituminous surface course specification (Item P-401). This specification included price adjustment factors for mat density. In order to expand the scope of its statistical specification to include additional acceptance characteristics and price adjustment factors, FAA contracted with Pennsylvania State University through the Pennsylvania Transportation Institute to investigate the Eastern Region's P-401 specification.

Data for the study were collected by FAA on 13 projects from the 1978 construction season. These projects represented over 180 000 Mg (200 000 tons) of asphaltic concrete. These data were analyzed by using standard statistical computer packages (1, 2) to determine the statistical parameters representative of existing acceptable construction. These parameters were then used in the development of proposed acceptance plans for asphaltic concrete.

The objective of this paper is to present the original mat-density acceptance plan and the modifications recommended for it as a result of the research study. The acceptance plan originally developed by FAA will first be presented and discussed. Recommended modifications to this plan will then be presented. The role of the computer, which made possible in seconds tedious and laborious calculations that would have required many hours if done by hand, will be discussed in detail.

ORIGINAL ACCEPTANCE PLAN

The original acceptance plan based acceptance for mat density on a lot-by-lot basis, with a lot defined as one day's production. The method of acceptance was based on the percentage of the material in the lot that was above the lower specification limit of 96.7 percent, as estimated from the test results for the lot. The percentage of material within limits (PWL) was estimated by the range method discussed below. The acceptance plan allowed material to be accepted at full payment, to be accepted at an adjusted price, or to be rejected and replaced.

The price adjustment schedule from the FAA plan is presented in the following table:

Percentage Above Lower Tolerance Limit	Percentage of Contract Price to Be Paid
90-100	100
85-89	98
80-84	95
75-79	90
70-74	80
65-69	70

It can be seen that 90 percent of the material must be within the specification limits (referred to as 90 PWL) for the material to be accepted at full payment. For material that is less than 90 PWL, the material is accepted but at a reduced payment. The incremental payment reductions increase as the level of quality, as measured by PWL, decreases. Because the price reductions become more severe as the quality of the material decreases, the contractor is encouraged to produce acceptable material rather than to produce inferior material at a reduced price.

The operating characteristics (OC) curves for the FAA plan are shown in Figure 1. The OC curves graphically represent the relation between the actual quality of a lot and its probability of acceptance. For the FAA plan, a set of OC curves is required in order to show the probability of a lot's being accepted at any of the possible payment levels. The curves shown in Figure 1 are for a sample size of four.

Another useful relation for examining the reasonableness of an acceptance plan is quality of the material, as measured by PWL, versus expected payment. This relation is plotted in Figure 2 for the FAA plan. Expected payment can be thought of as the average payment over the long run. To arrive at the curve shown in Figure 2, it is necessary to make certain assumptions concerning the probability of receiving 50 percent payment when the material has an estimated PWL less than 65. When the material has less than 65 PWL, it can either be removed and replaced or it can be accepted as it is at a payment level of 50 percent. In arriving at the expected payment curve, the following assumptions were made in the case when the estimated PWL is below 65:

1. When the actual PWL is between 50 and 80, the material is accepted at 50 percent payment 75 percent of the time.
2. When the actual PWL is equal to 80, the material is accepted at 50 percent payment 90 percent of the time.
3. When the actual PWL is above 80, the material is accepted at 50 percent payment 100 percent of the time.

These assumptions were made by the FAA Eastern Region (3) and have been used by the researchers be-

Figure 1. Set of operating characteristics curves for FAA density acceptance.

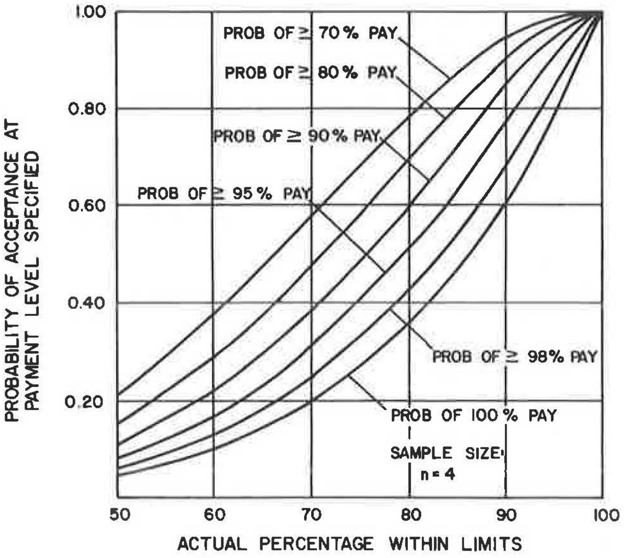
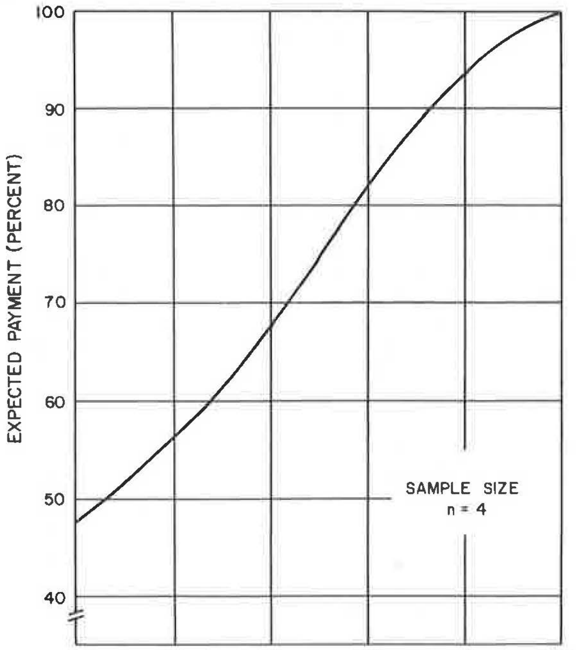


Figure 2. Expected payment curve for FAA density acceptance plan.



cause they appeared to be reasonable. It seems likely that in many cases the option of acceptance at 50 percent payment will be chosen because it is the easiest course of action.

DEVELOPMENT OF PROPOSED ACCEPTANCE PLAN

It was decided to base the development of the acceptance plan and price adjustments on the reasonableness of its OC and expected payment curves. This approach is one of four described by Willenbrock and Kopac (4). Because this approach requires some subjective decisions concerning the reasonableness of the acceptance plan and price adjustments, the starting point used in the development of the proposed acceptance plan was the FAA Eastern Region's density plan.

Range Method Versus Standard Deviation Method

The FAA acceptance plan for density, as outlined in the Eastern Region P-401 specification, is based on the range method for estimating PWL. In this method a quality index Q is calculated for each set of samples. For the case of density, which has only a lower specification limit L, the value of Q_L is calculated from

$$Q_L = (\bar{X}_n - L)/R_n \tag{1}$$

where

Q_L = quality index for lower specification limit,
$\bar{X}_n$ = mean value of n samples in the lot,
n = number of samples in each lot,
R_n = range (difference between largest and smallest) of n sample in the lot, and
L = lower specification limit.

Once the value of Q_L has been calculated, the estimate of PWL can be determined from tables that relate Q values to estimated PWL.

It has been pointed out (5) that the range method, which is used in the FAA plan, provides a biased estimate of PWL. Willenbrock and Kopac (5) recommend the use of the standard deviation method for estimating the percentage within limits because "it provides unbiased estimates of the percentage within limits.... The standard deviation method also requires smaller sample sizes than the range method in order to provide a given degree of protection." In light of the better estimate afforded by the standard-deviation method, it was recommended that the standard-deviation method be adopted in lieu of the range method. The advantage attributed to the range method is that range is a more easily understood concept. With the advent of inexpensive pocket calculators that provide the capability of quickly determining mean and standard deviation, we consider the use of the range method to be no longer warranted.

The method for estimating PWL in the proposed new acceptance plan is based on the calculation of a quality index for the lot by using the standard-deviation method. The quality index Q_L for density can be calculated from

$$Q_L = (\bar{X}_n - L)/S_n \tag{2}$$

in which Q_L, $\bar{X}_n$, n, and L are defined as noted earlier and S_n refers to standard deviation of n samples in the lot. S_n can be calculated from

$$S_n = \sqrt{\sum_{i=1}^{n} (X_i - \bar{X}_n)^2/(n-1)} \tag{3}$$

where X_i, i = 1, 2, 3, ..., n = individual sample results, and n = number of samples in each lot.

As in the case of the range method, once Q_L has been calculated by the standard-deviation method, the estimated value of PWL can be determined from tabled values of Q. Appropriate tables for the standard-deviation method have been developed by Willenbrock and Kopac (5).

Continuous Versus Discrete Price Adjustments

The price adjustment schedule from the FAA acceptance plan was noted earlier in this paper. This discrete schedule works quite well in the long run, as shown by the expected payment curve in Figure 2. However, this type of schedule can present some problems in the short run and, in particular, on smaller projects. There are two potential problems with discrete price adjustment schedules.

The first problem deals with the uncertain areas between different pay levels. Material that is 90-100 percent within limits is to receive 100 percent payment and material that is 85-89 percent within limits is to be paid for at 98 percent of the contract price. This could present problems of interpretation when the estimated PWL is between 89 and 90. These potential areas of uncertainty occur at the boundary of each price adjustment level.

Another area of concern is the large incremental differences in the price adjustment levels. A situation in which, for example, 75.1 PWL is worth 90 percent payment but 74.9 PWL is worth only 80 percent payment is an undesirable one, particularly in light of the uncertain areas between price adjustment levels. It can be argued that in the long run these situations (i.e., 74.9 versus 75.1 PWL) will balance out, as shown by the expected payment curve in Figure 2. However, on a small project or in the case of a contractor who works infrequently under the acceptance plan, there is no long run in which this balancing effect can take place. It is suggested that a continuous rather than discrete price adjustment schedule would eliminate potential problems when the estimated PWL is near the boundary between price adjustment levels. Willenbrock and Kopac (5) have recommended that a specifying agency "seriously consider the use of a continuous price adjustment schedule which can be presented in a graphical fashion or as a series of straight-line equations between the various price adjustment levels. This approach would eliminate a lot of potential field problems of interpretation which could develop." For these reasons, it was decided to use a continuous price adjustment schedule in the proposed new acceptance plan.

Table 1. Continuous price adjustment schedules considered for mat density.

Schedule	Estimated Percentage of Material Above the Specification Limit (PWL)	Percentage of Contract Price to Be Paid
1	90-100	100
	80-90	$\frac{1}{2}$ PWL + 55
	65-80	$\frac{5}{3}$ PWL - $38\frac{1}{3}$
	Below 65	-[a]
2	90-100	100
	80-90	$\frac{1}{2}$ PWL + 55
	65-80	2 PWL - 65
	Below 65	-[a]
3	90-100	100
	85-90	$\frac{2}{5}$ PWL + 64
	80-85	$\frac{3}{5}$ PWL + 47
	75-80	PWL + 15
	65-75	2 PWL + 60
	Below 65	-[a]
4	90-100	100
	65-90	$-606.45 - 6.56\,(\mathrm{PWL}) + 13.7\,(\mathrm{PWL})^{1/2}$
	Below 65	-[a]
5	90-100	100
	65-90	$-1153.40 - 13.8\,(\mathrm{PWL}) + 263\,(\mathrm{PWL})^{1/2}$
	Below 65	-[a]

Note: PWL = percentage of material within limits.

[a] The lot shall be removed and replaced to meet specification requirements as ordered by the engineer. In lieu of this, the contractor and the engineer may agree in writing that, for practical purposes, the deficient lot shall not be removed and will be paid for at 50 percent of the contract price.

Development of Price Adjustments

Because the development of acceptance plans by the OC curve approach requires some subjective analysis and engineering judgment, it was decided to base the continuous price adjustment on the discrete schedule developed by FAA. This was done for two reasons. First, FAA considered it to be a reasonable schedule and, second, the schedule had gone through one construction season with no major complaints from contractors about its fairness coming to our attention.

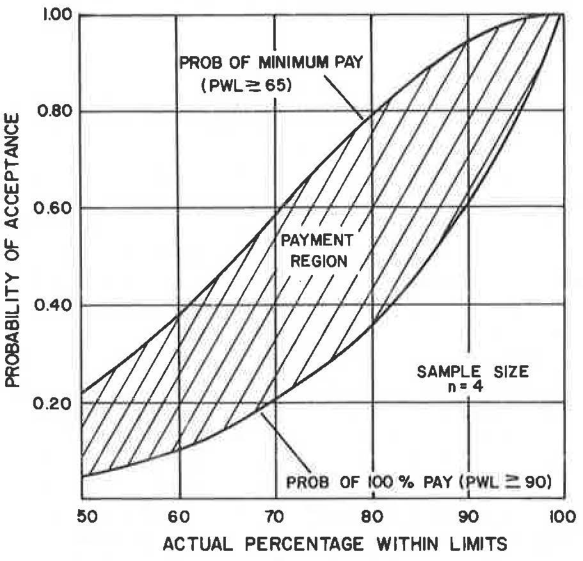
Five different continuous price adjustment schedules, all based on the FAA schedule, were considered. The five price adjustment schedules (labeled 1, 2, 3, 4, and 5) are presented in Table 1. The first three schedules were based on the use of several straight-line equations to relate payment level and estimated PWL. The last two schedules attempted to fit one curved line to the FAA schedule to eliminate the need for more than one equation. These curves were fitted to the FAA plan by using multiple regression analysis and Minitab 2 (1), a general purpose statistical computer system. To choose among these five schedules, it was decided to compare their OC curves and expected payment curves with those of the FAA price adjustment schedule.

It has been shown by Resnikoff and Lieberman (6, 7) that the noncentral t-distribution is appropriate for the estimate of the proportion of a normal population that lies above a given limit. This is the case for the density acceptance plan in which acceptance is based on an estimate of the proportion (or percentage) of the population that falls above the specification limit. A detailed discussion of the use of the noncentral t-distribution to estimate PWL for the case of asphaltic concrete density is presented by Willenbrock and Kopac (8).

The OC curves for the FAA price adjustment schedule and the proposed new schedules were determined by use of a computer program that calculates the area beneath the noncentral t-distribution. This program was subroutine MDTN, obtained from the International Mathematical and Statistical Library (IMSL) of Subprograms (9). This routine, which is available only in single precision in IMSL, was modified for double precision by Terry L. King, a graduate student in the Department of Statistics at Pennsylvania State University.

The set of OC curves for the FAA acceptance plan is shown in Figure 1. For the case of the discrete FAA price adjustment schedule, an OC curve can be developed for each price adjustment level. For the case of a continuous price adjustment schedule, there are an infinite number of OC curves possible because there are an infinite number of potential payment levels. The operating characteristics for the case of a continuous schedule can therefore be indicated by a region that has as its bound a curve corresponding to the probability of receiving 100 percent payment and another curve corresponding to the probability of receiving the minimum possible payment. For each of the five proposed schedules, 100 percent payment occurs at an estimated PWL value ($\widehat{\mathrm{PWL}}$) of 90 or more, and the minimum payment level occurs at a $\widehat{\mathrm{PWL}}$ of 65. The operating characteristics of the proposed schedules can be represented, then, by a region bounded by one curve corresponding to the probability of $\widehat{\mathrm{PWL}}$ equal to or greater than 90 for each actual value of PWL and by another curve corresponding to the probability of $\widehat{\mathrm{PWL}}$ equal to or greater than 65 for each actual value of PWL. The region within

Figure 3. Payment region for proposed density price adjustment schedule.



these curves corresponds to the probability of receiving some payment.

The payment region for the five proposed schedules for a sample size of four is shown in Figure 3. The upper and lower bounds, which correspond to the probability of receiving some payment, are the same for all five plans. Even though the boundaries of the payment region are the same for all five schedules, a particular payment level may occur at a different location within the region. In order to differentiate the five schedules, it was therefore necessary to determine their expected payment curves.

For the discrete FAA price adjustment schedule, the expected payment curve shown in Figure 2 can be calculated from the relation:

$$\text{Expected payment} = \sum_{\text{all } i} X_i P(X_i) \qquad (4)$$

where

X_i = payment level i and
$P(X_i)$ = probability of receiving payment level i.

The values of $P(X_i)$ are determined by the probabilities that $\hat{\text{PWL}}$ will fall within the limits of the various payment levels (e.g., $\hat{\text{PWL}} \geq 90$ is necessary for 100 percent payment). These probabilities were determined from the noncentral t-distribution by using subroutine MDTN from IMSL.

For the case of a continuous price adjustment system, an integration procedure must be used because the number of possible price adjustment levels is infinite. The expected payment in this case can be calculated from

$$\text{Expected payment} = \int_{-\infty}^{\infty} G(X)\, P(X)\, dX \qquad (5)$$

where

X = estimated PWL value ($\hat{\text{PWL}}$),
G(X) = payoff function relating the estimated PWL value ($\hat{\text{PWL}}$) to the payment level, and
P(X) = probability density function (noncentral t) of the estimated PWL values ($\hat{\text{PWL}}$).

This integral is not convenient to use for computational purposes, so a computer program was developed to approximate the integral for the purposes of estimating expected payment. As a convenient approximation to this integral, the potential $\hat{\text{PWL}}$ values were partitioned into small intervals. The probability of $\hat{\text{PWL}}$ falling within each interval was calculated by using subroutine MDTN to determine the appropriate area under the noncentral t-distribution. The probability of $\hat{\text{PWL}}$ falling within an interval was then multiplied by the average payment associated with that interval. These products were then summed for all of the intervals to achieve a good estimate of the expected value. This procedure was performed for six different actual PWL values to identify the expected payment curve.

The area corresponding to a $\hat{\text{PWL}}$ greater than 65 but less than 90 is the probability of receiving some reduced payment; the area falling above a $\hat{\text{PWL}}$ of 90 is the probability of receiving 100 percent payment; the area falling below a $\hat{\text{PWL}}$ of 65 is the probability that the material will have to be either removed and replaced, or accepted as it is at a payment of 50 percent. In determining the expected payment curves for the proposed schedules, the same assumptions concerning the 50 percent payment level were made as were made in the calculation of the expected payment curve for the FAA plan.

To determine the expected payments associated with the partial payment levels, two cases were considered for price adjustment schedule 1. The region between $\hat{\text{PWL}}$ equal to 65 and $\hat{\text{PWL}}$ equal to 90 was partitioned into 25 and 50 intervals for the two cases. The expected values were then calculated for both cases and compared. At all actual PWL values tested for the two cases, the expected values calculated by using 25 intervals and 50 intervals were identical to four significant figures. It was therefore decided that the results obtained by using 25 intervals were sufficiently accurate. The expected payment curves for the five proposed schedules for sample sizes of four, five, and seven were then calculated by using 25 intervals. Schedules 2 and 4 have expected payment curves that are very similar to those of the FAA plan. These schedules deviate less from the FAA schedule than do the other three schedules, with schedule 2 providing the closest match. All three schedules are very similar, and it is difficult to discriminate among them.

Because the expected payment curves for these three schedules are so similar, the choice between schedule 2 and schedule 4 was based on their ease of application in the field. Schedule 4 has the advantage of having only one equation, but this equation [$\text{Percent pay} = -606.45 - 6.56\,(\text{PWL}) + 136.7\,(\text{PWL})^{1/2}$] is somewhat more complicated than the straight-line equations for schedule 2. For this reason, it was decided to adopt price adjustment schedule 2 for the proposed new density acceptance plan. This price adjustment schedule along with the original FAA schedule is illustrated in Figure 4. The expected payment curves for sample sizes of four, five, and seven are shown in Figure 5.

Computer Simulation

In addition to the theoretical development of the OC curves shown in Figure 3, additional OC curves were determined by use of computer simulation. The simulation program used was similar to that used by Willenbrock and Kopac (8) but was modified to use the standard-deviation method rather than the range method for esti-

Figure 4. Proposed density price adjustment, schedule 2.

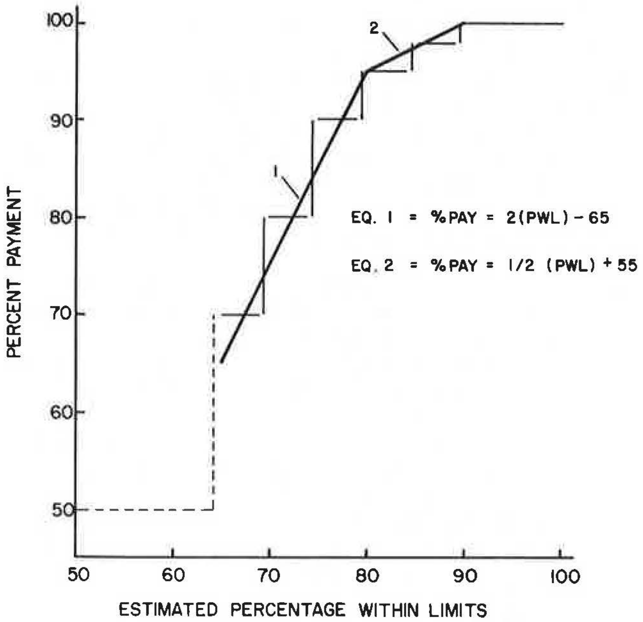
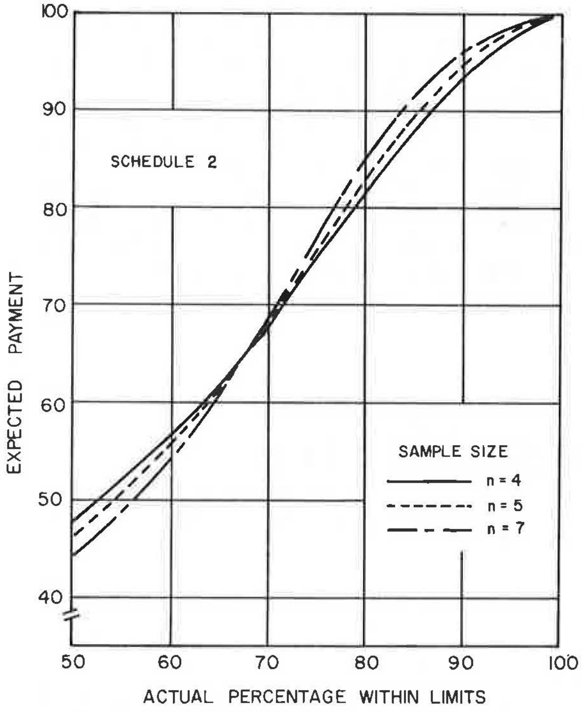


Figure 5. Expected payment curves for proposed density price adjustment, schedule 2.



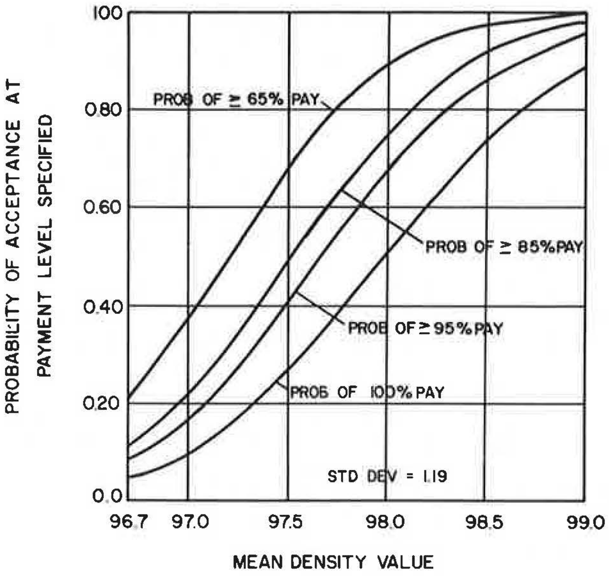
mating PWL. The program, which is similar in principal to those recommended by Weed (10), was originally written by Charles E. Antle of the Department of Statistics of Pennsylvania State University.

The program determines the operating characteristics of the proposed acceptance plan by simulating the random sampling of 10 000 lots of material. The specification limits are variables that are entered into the program. The program is designed for both an upper and lower specification limit, so it is very convenient for properties such as air voids or asphalt content. In fact, a simulation program such as this is the best way for determining the operating characteristics for an acceptance plan that has two specification limits because it is possible for material to be outside both limits simultaneously. This program was used to investigate acceptance plans for Marshall stability, flow, and air voids as well as for density during the course of the research efforts (11). A copy of the program, together with a discussion of its use, can be found in Burati and Willenbrock (11).

In order to test the simulation program, it was used to determine the OC curves for the density acceptance plan. Because the program is set up for both an upper and lower specification limit and density has only a lower specification limit, a very high value that was certain never to be exceeded was entered as the upper limit. The operating characteristics derived from the simulation were compared with those obtained from the theoretical solution by using the noncentral t-distribution. The results agreed quite closely, thus verifying the applicability of the program.

The use of the computer simulation has an advantage over the theoretical solution. The results of the theoretical solution are presented in terms of the actual PWL. It is difficult for a contractor to relate what is meant by 90 PWL to the construction process. The simulation program allows the operating characteristics to be easily related to the mean density values for a given value of standard deviation. The contractor can determine from past tests results what values of mean and standard deviation can be achieved with the construction process. The simulation program would then allow the contractor to determine OC curves in terms of the mean target value and standard deviation that are being achieved. To do this the contractor must know the price adjustment schedule. Figure 6 presents OC curves for a standard deviation value of 1.19. The value of 1.19 is the pooled standard deviation for all the projects included in the research study. The figure shows curves for the probability of receiving payment greater than or equal to 100 percent, the minimum payment of 65 percent, and intermediate payments of 95 and 85 percent.

Figure 6. Operating characteristics for proposed density acceptance plan for a standard deviation of 1.19.



CONCLUSION

This paper has presented many methods in which the computer can be used in the development of a statistically based acceptance plan. These methods include the following:

1. Preliminary data analysis to determine parameters necessary for the development of the plan;
2. The determination of areas under probability distributions by mathematical algorithms, thereby eliminating the need for tedious hand calculations and interpolations from tables;
3. A means for obtaining a good estimate of the expected payment curves for acceptance plans that incorporate continuous rather than discrete price adjustment schedules;
4. Computer simulation to develop OC curves in terms of means and standard deviations that can easily be related to the contractor's process capabilities; and
5. Computer simulation to develop OC curves for acceptance properties requiring both upper and lower specification limits that would not be practical if theoretical methods were used.

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Conversational Programming: Making Computer Technology More Accessible

Richard M. Weed

Although the most practical way to handle many of today's sophisticated engineering and statistical analyses is with high-speed electronic computers, engineers often lack the programming skills necessary to take full advantage of this approach. Conversational programming, which provides explicit instructions to the user at an interactive terminal, enables even less-skilled subordinates to perform complex tasks on the computer. Almost any quantitative analysis can be computerized, and the potential applications are virtually endless and are limited only by the imagination and inventiveness of the program writer. Benefits of conversational programming are discussed, and a reference is cited to aid in the development of programs of this type. An example is presented that deals with the testing of statistical acceptance procedures by computer simulation, an extremely powerful technique of great potential value to quality assurance engineers. By using another conversational program, a theoretical analysis is made and compared with the simulation results.

In a time that historians are beginning to call the computer age, many engineers still have little or no programming ability. Although computer terminals are readily accessible to engineers in most organizations and computer programming is now strongly emphasized in engineering colleges, many practicing engineers have never acquired the ability to take full advantage of these facilities. In some cases, this means that considerable time will be wasted in making complex calculations that could be performed much more efficiently by computer. In other cases, potentially useful analyses may never be attempted because of the impracticality of doing them by any means other than a large computer. Although many engineering functions have been computerized by software specialists in recent years, much still remains to be done.

As more and more engineering graduates enter the profession, this shortcoming will gradually disappear.