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Planning Slope Stabilization Programs by Using Decision Analysis

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Maintenance funds are rarely sufficient for all needs, and this requires that decisions be made as to the most effective allocation of these funds. In the case of slope stabilization, these decisions will be based on the frequency and location of failures, the consequences of failures (i.e., the cost of accidents), and the cost of stabilization. Decision analysis is a simple but useful tool to determine the most cost-effective stabilization program. The expected costs of slope failures are calculated for different stabilization programs, and these costs are added to the costs of the stabilization work to determine the expected total cost. The program that has the minimum total cost is likely to be the most cost effective. An example of the use of decision analysis is given that shows the variation in expected total cost for rockfalls along a section of highway for no stabilization work, a limited scaling program, and a more-comprehensive ditching, scaling, and bolting program. It is shown that the frequency of rockfalls must be substantially reduced before there is any significant reduction in the cost of accidents and that this requires an extensive stabilization program. The example also illustrates how the probability values used in the decision analysis can be related to the design of the stabilization measures.

It is often necessary in transportation engineering to determine the optimum allocation of the limited funds available to maintain slopes in acceptably safe condition. These decisions are rarely straightforward because the likely types of failure are varied, the consequences are diverse, and their occurrences are difficult to predict. This paper describes the use of decision analysis, a simple but effective tool, for the analysis of the impact of different stabilization programs on the expected cost of slope failures.

Recent applications of decision analysis in engineering include the selection of safe routes for the transportation of hazardous materials (1) and surveys carried out to assess the safety of dams (2). In this paper, the focus is on the optimization of a maintenance program for a series of highway or railway rock cuts that have a history of rockfalls, some of which have interrupted traffic and caused accidents. The costs of these events and of different stabilization programs are estimated, and these costs and the probabilities of rockfalls occurring are used to calculate the expected costs of rockfalls under alternative maintenance-program scenarios. This information shows which stabilization program is more cost effective. Probability analysis can then be used to ensure that the probability of failure of the stabilized slope is consistent with the probability used in the decision analysis.

PRINCIPLES OF DECISION ANALYSIS

Decision analysis is a technique in which the consequences of all of the events that might occur in a particular situation are evaluated. Probabilities are assigned to events that occur by chance, and the costs of those events are determined. This information is then used to calculate the expected costs of different courses of action, which can be used as a guideline in making decisions.

The first task in decision analysis is to draw a decision tree that shows all possible events. In this paper, rockfalls from highway cuts are considered, although the same approach can be used on railroad cuts. On the tree, events that occur as a result of a decision are distinguished from events that occur by chance. The decision point in this analysis is whether or not to carry out a stabilization program. Once this decision has been made, regardless of what has been decided, a chance event will occur; that is, the slope will be either stable or unstable. Probabilities can be assigned to each of these events and, because they are mutually exclusive, the sum of probabilities at each chance point is 1.0.

Establishment of realistic probabilities for different events requires both experience and sound judgment. This is particularly true for rare events; experimental evidence shows that people tend to overestimate the likelihood of their occurrence (3). The best method for establishing probabilities is to study existing records and modify them where necessary to suit local conditions.

The next task is to assign the total costs to society (e.g., maintenance, injury, business losses, traffic delays) of each of the events at the tips of the decision tree and to determine the costs of stabilization at appropriate decision points. If the cost of an event cannot be expressed in terms of a single value, it can be expressed as a probability distribution in which all the costs within the range are given probabilities of occurrence. Summation of the area under the probability distribution curve will give the expected cost of the event. The determination of costs usually involves the cooperation of the owner, who is also likely to provide useful input on the structure of the decision tree and the assignment of probabilities.

The final task in the analysis is that known as averaging out and folding back (4) each branch of the tree. The product of cost and probability, summed over all events at a particular chance point, gives the expected cost. This procedure is started at the tips of the branches and worked back to the decision point. If the objective of the analysis is to determine the least costly option, the path that has the least expected cost is selected.

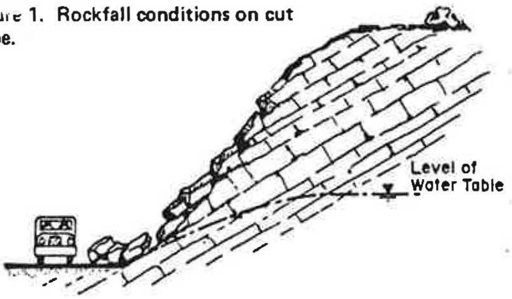
DATA COLLECTION FOR DECISION ANALYSIS

The purpose of decision analysis is to assist in predicting the outcome of future events, and the reliability of the prediction will be greater if reliable data on past events are available and if the mechanism and causes of failure are thoroughly understood.

In the cause of rockfalls from slopes adjacent to a highway or a railway (see Figure 1), three types of information are required:

1. If a record of rockfalls exists, the locations, frequencies, and consequences of these falls should be summarized and these data used to estimate the probabilities with which such events occur. It is unlikely, however, that there will be sufficient records to establish the complete rockfall population from which to calculate true probabilities. Therefore, it may be necessary to make appropriate modifications based on judgment and experience to the calculated probabilities. For instance, the records may have been collected during a period when the winters were more severe than usual and frost action produced an unusually large number of rockfalls. In such a case, the probability should be adjusted downward.
2. The impact of rockfalls on traffic should be studied to determine the average costs of different classes of events. For example, the costs of a delay caused by a major rockfall would be due to the interruption to traffic, removal of the rock, and repairs to the slope and the pavement. In the case of an impact, costs will result from damage to the car, injury to or death of its occupant(s), and damage to the pavement. Even when there is neither a delay nor an impact, it may still be necessary to remove the rockfall and perform repairs. In addition, there are indirect costs such as the lost wages of those injured, engineering studies of stability conditions, and legal fees in the event of a court case.
3. The physical and geological characteristics of the slopes should be studied to determine the causes of failure and whether further falls are likely. One possibility is to evaluate the stability conditions for each slope on a numerical point rating from very high to very low probability that a rockfall will occur. The detailed information should include the length and spacing of the natural fractures in the rock, their strength characteristics and orientation with respect to the slope face, groundwater pressures, and whether heavy blasting has caused damage to the rock behind the face (5).

Figure 1. Rockfall conditions on cut slope.



APPLICATION OF DECISION ANALYSIS

To illustrate the use of decision analysis, consider the case where a number of unstable slopes above a major highway have experienced frequent rockslides. A decision is required on whether a preventive stabilization program to reduce the likelihood of future slides is economically justified and, if so, how much money should be spent.

An examination of the length of highway where the rockfalls have been occurring shows that, on a 1.5-km-long section of essentially constant geological characteristics, there are a number of potentially unstable slopes. Rockfalls have been occurring because the slopes were cut at 45°, which undercuts the bedding planes that dip at about 30° toward the highway. In addition, groundwater pressures exist within the slope (Figure 1).

The first step in the decision analysis is to draw a decision tree to show the range of conditions expected (see Figure 2). The first point in the tree is the decision point for the three alternative courses of action. These are

1. No stabilization,
2. Option 1—expenditure of $6000/0.1-km segment

Figure 2. Decision analysis: existing stability condition.

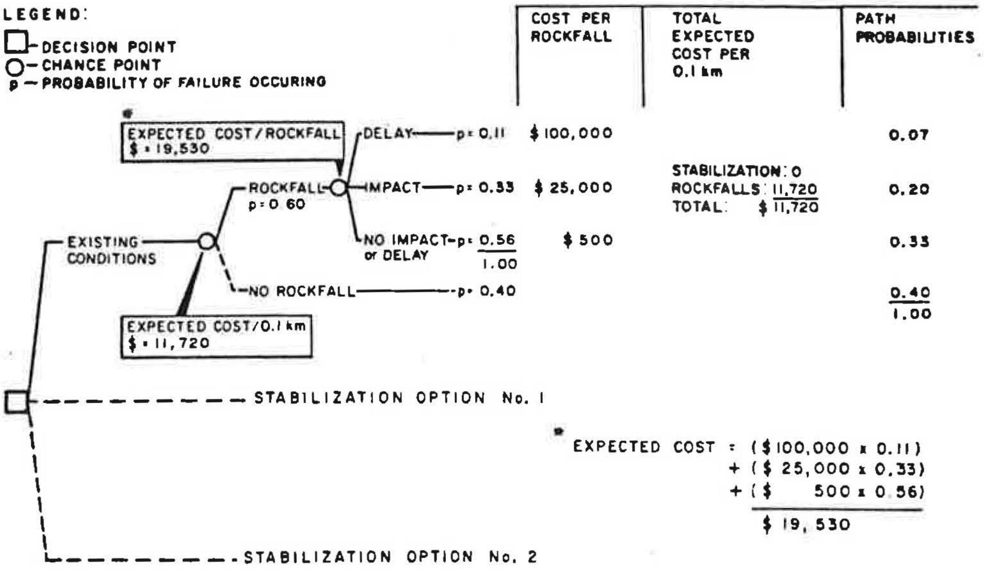
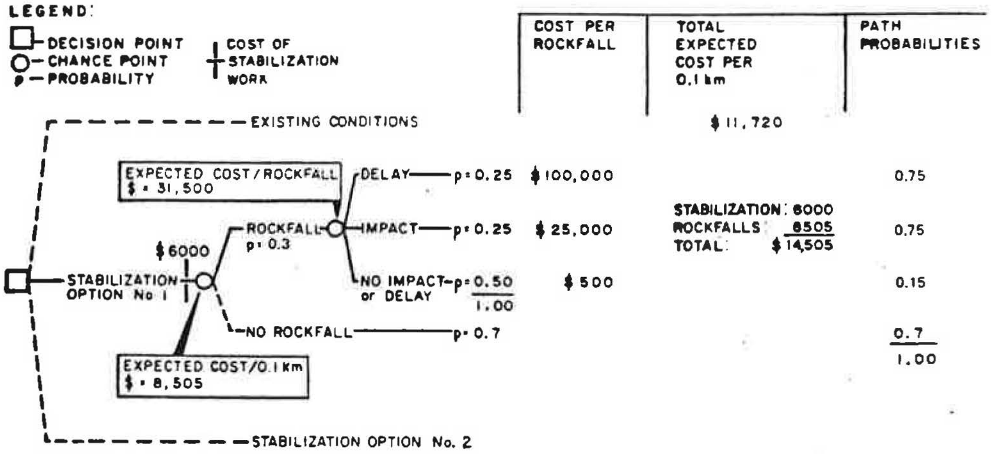


Figure 3. Decision analysis: stabilization option 1.



of highway for removal of loose rock from the face, and

3. Option 2—expenditure of $10 000/0.1-km segment of highway to install tensioned rock anchors and to excavate a ditch and construct a gabion wall along the toe of the slope.

Whichever course of action is taken, the same events can take place, although the probabilities of their occurrence will differ if the stabilization program is effective. Thus, the structures of the trees are identical for each of the three options. The events that can occur at the first chance point are either

1. A rockfall takes place or
2. The slope is stable and no rockfall takes place.

If a rockfall does occur, then one of three types of events can take place at the second chance point:

1. There is a delay,
2. There is an impact, or
3. There is neither an impact nor a delay, but there may be some damage to the highway.

The probabilities of these events occurring per 0.1-km segment of highway can be estimated by dividing the expected number of rockfalls by 15, i.e., the number of 0.1-km segments in 1.5 km of highway. For example, if nine rockfalls have occurred on this 1.5-km section, then the probability of a rockfall occurring on a given 0.1-km segment is 9/15 or 0.6, and the probability of no rockfall occurring on that segment is (1 - 0.6) or 0.4. This probability unit can then be used to compare the expected rockfalls over other sections of highway that have the same geology.

To calculate the expected cost of rockfalls in the future, probabilities of future rockfalls and their consequences are calculated from the existing rockfall conditions by assuming that the instability problem will be similar in the future to what it has been in the past (although some allowance might be made for increases in traffic). If, of the nine rockfalls that have occurred, one caused a delay (p = 1/9 or 0.11), three caused an impact (p = 3/9 or 0.33), and five caused neither a delay nor an impact (p = 5/9 or 0.56), then the probabilities can be assigned as shown in Figure 2. Path probabilities are then calculated by multiplying the probability along each path on the tree. This gives the overall probability of an event occurring if a previous event has occurred with a certain probability (4).

Average costs for the three types of events for the case of a heavily used highway that has a high proportion of commercial traffic are estimated (6) to be as follows:

Type of Event	Cost ($)
Delay	100 000
Impact	25 000
Damage to highway only	500

Finally, these probabilities and costs are averaged out and folded back to determine the expected cost of rockfalls per 0.1-km segment. For no stabilization, this cost is calculated to be $11 720. The objective of the stabilization work is thus to reduce the probability of failure so that the expected cost of rockfalls plus the stabilization cost is less than $11 720.

The first stabilization option consists of removing loose rock from the slopes. This option is estimated to cost $6000/0.1-km segment; from experience, this will approximately halve the number of rockfalls. The probabilities are calculated by assuming that four rockfalls will occur in the same time interval as in the no-stabilization option of which one will be a delay, one an impact, and two will cause no delay (see Figure 3). (It should be noted that, because probabilities of occurrences have been rounded to whole numbers, small differences in path probabilities will have no significance.)

Calculation of the probabilities of these events shows that, although the probabilities of the impact and no-delay events have been considerably reduced from existing conditions, the path probability of a delay occurring is essentially unchanged. This is reasonable because the stabilization work has done nothing to improve the stability of the overall slope and rockfalls can still be expected to occur. Calculation of the expected costs by using these probabilities and the same costs for each type of event as before shows that the expected cost of rockfalls per 0.1-km segment of highway is $8505. This plus the stabilization cost of $6000/0.1-km segment gives a total expected cost of $14 505. This cost is greater than the existing cost of rockfalls, which means that a scaling program is not economically justified.

The second stabilization option consists of excavating at the toes of the unstable slopes to form a ditch, constructing a gabion wall to catch small rockfalls, and installing tensioned rock anchors where necessary to prevent large rockfalls (see Figure 4). It is estimated that this option will cost $10 000/0.1-km segment of highway. The ditch, however, is designed to prevent small rockfalls from reaching the highway so that the probability of impact and no-impact events will be very low.

Figure 4. Illustration of stabilization program.

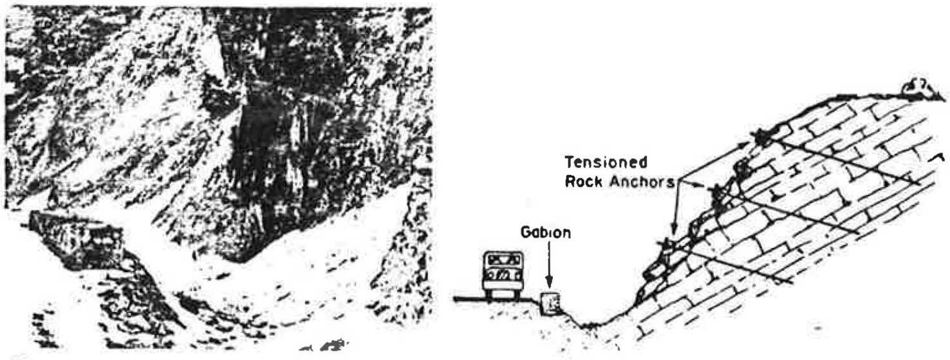


Figure 5. Decision analysis: stabilization option 2.

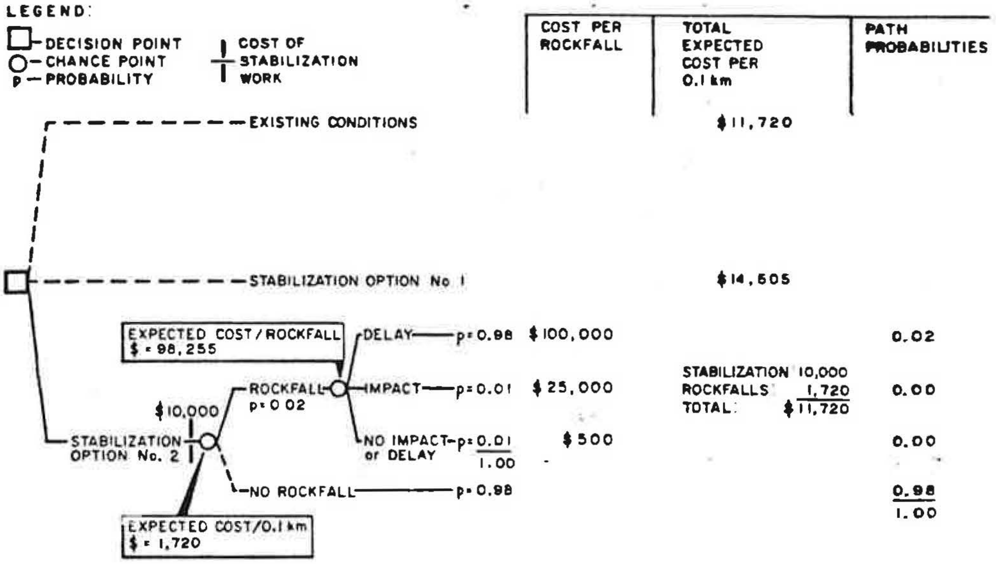
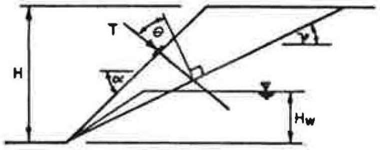


Figure 6. Definition of parameters used in stability analysis.



The installation of rock anchors will reduce the probability of a large rockfall occurring that would cause a delay. For the stabilization option to be economically justified, the expected cost of stabilization and rockfalls must be less than the existing cost of \$11 720/0.1-km segment of highway. As shown in Figure 5, this expected cost will be achieved if the probability of a delay is less than 0.0175 (approximately 0.02). The required probability is calculated from the required expected cost by working from left to right through the tree.

The design of the rock-bolting program to achieve this level of probability of failure can be carried out by using probability analysis in conjunction with standard factor-of-safety (FOS) analysis. In this way, it is possible to relate the consequences of failure to the amount of stabilization work carried out.

PROBABILITY ANALYSIS

A probability analysis can be used as a guideline in the objective selection of an appropriate FOS. This analysis takes account of the variability and lack of definition in the parameters used. Furthermore, this probability of failure can then be used in the decision analysis to examine the consequences of failure. If the consequences are unacceptable, then the decision can be made to take action to reduce the probability of failure to a level that has an acceptable consequence.

Alternatively, the design of stabilization measures can be carried out on the basis of a selected FOS. This is a somewhat subjective selection and may not be consistent from case to case. Usually, a sensitivity analysis is also carried out to determine which factors have the greatest effect on the FOS. If the definition of the more-sensitive factors is uncertain, then a further subjective decision is made to increase the FOS.

The following example shows how a probability analysis, in conjunction with the decision analysis, can be used to select an appropriate FOS for the stabilization work. One method of calculating the probability of failure of a rock slope is as follows.

The stability of a slope is dependent on the relative magnitudes of two forces—a displacing force (D) that acts to cause failure and a strength (or resisting) force (R) that acts in the opposite direction. The difference between the two forces (R - D) is the margin of safety and is positive when the slope is stable and negative when the slope is unstable. The ratio of the two forces (R/D) is the FOS and is greater than 1.0 when the slope is nominally stable. In the case of a planar type, the

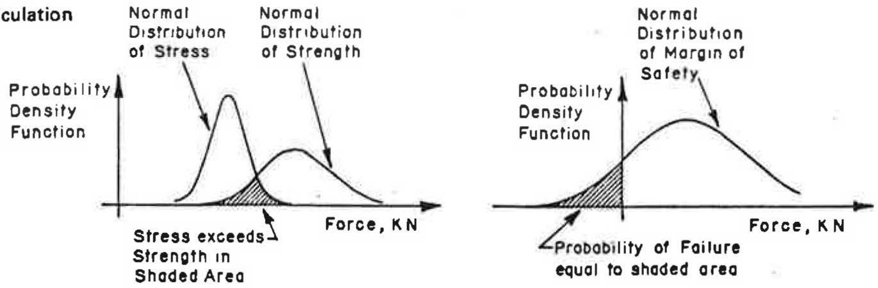


Figure 7. Normal distributions used in calculation of probability of failure.

two forces are calculated by using Equations 1 and 2.

$$R = (W \cos \psi + T \cos \theta - U) \tan \phi \quad (1)$$

$$D = (W \sin \psi - T \sin \theta) \quad (2)$$

where W = weight of sliding block, U = water pressure action on failure plane, and α, H_w, ψ, T, and θ are defined in Figure 6 and have the values given below.

Parameter	Value	Estimated SD	Comments
Friction angle (α) (°)	41	5	Determined from rock texture and surface roughness, cohesion = 0
Height of water table (H_w) (m)	3.3	1.5	Variation in peak spring water levels (determined by piezometer measurements)
Dip of bedding planes (ψ) (°)	30	2.5	Determined by dip measurements made during surface geological mapping
Bolt tension per linear meter of slope (T) (kN)	120	10	Actual load (which is less than design load due to anchor relaxation)
Bolt angle (ϕ) (°)	16	3	Variation due to changes in rock surface

Because of the variable properties of rock, it is rarely possible when calculating these forces to define the magnitudes of parameters used in the analysis precisely, and it is more realistic to express their magnitudes in terms of ranges of values. One of the most convenient expressions for variability is the normal distribution. This is a bell-shaped curve that is symmetrical about the mean value and has a width that is defined by the standard deviation of the sample. An important property of the normal distribution is that the area under the curve between any two values on the horizontal axis represents the probability of a sample occurring within that range (7).

If all parameters used in the calculation of the resisting and displacing forces are independent and can be expressed as normal distributions, these can be combined by appropriate methods (8) to obtain the normal distributions of the two forces. If the two curves are plotted on the same figure and intersect at some point (as shown in the left-hand side of Figure 7), then D > R and the probability of failure of the slope is equal to the shaded area shown on the right-hand side of Figure 7. Alternatively, Monte Carlo techniques can be used to combine different types of distributions (8).

The probability of failure can be calculated by subtracting the two distributions to obtain the distribution of the margin of safety, i.e., the area under the curve to the left of the vertical axis. These calculations can be performed on a programmable pocket calculator.

If the parameters used in the analysis have little variation, then the distribution curve for the margin of safety will be narrow and only a slight increase in the strength will be required to produce a significant decrease in the probability of failure.

To illustrate the application of probability analysis in the design of stabilization measures, consider the slope discussed in the decision analysis above. Here, for the stabilization program to be economically justified, it is necessary to reduce the probability of failure by approximately 70 percent, i.e., from 0.07 to 0.02. The first step is to calculate the probability of failure and the factor of safety of the existing slope. Rock anchors are then added progressively, and the probability of failure is calculated until it is reduced by about 70 percent. For example, consider a 15-m-high slope cut at 45° and having the parameters shown above.

The probability of failure of this slope is 0.34 (FOS = 1.2) and must be reduced (by 70 percent) to 0.10. If two rock bolts are installed, the probability of failure will be 0.18 (FOS = 1.47), and if three rock bolts are installed the probability of failure becomes 0.12 (FOS = 1.63). Thus, the required improvement to the stability of the slope can only be achieved by adding three bolts rather than two (which is an insignificant additional cost).

CONCLUSIONS

Decision analysis can be used as a guideline in making rational decisions when there are several courses of action available. This approach offers the following advantages over subjectively made decisions:

1. Decision analysis encourages decision makers to scrutinize their problems as a whole as well as to evaluate the interactions among various facets of their problems.
2. The systematic approach helps communication. It allows each expert to give testimony about his or her area of expertise.
3. Systematic examination of the value of information in a decision context helps evaluation of what information is important.
4. Analysis distinguishes the decision maker's preference for consequences, including attitudes toward risky situations.
5. The methodology of decision analysis is useful as a mediating device in situations in which the advisors to a decision maker disagree about an appropriate course of action.

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Abridgment

Advantages of Founding Bridge Abutments on Approach Fills

D.H. Shields, J.H. Deschenes, J.D. Scott, and G.E. Bauer

A set of controlled experiments has been carried out in which the ultimate bearing capacity at various locations within a granular approach fill for a spill-through bridge abutment was measured. It was shown that existing design procedures for spread-footing-supported abutments in approach fills are unduly conservative, and it is recommended that the experimentally determined bearing-capacity values be used as the basis for design.

Footing foundations would be competitive in cost with piled foundations for spill-through bridge abutments if the design bearing pressure for footings near slopes could be increased. That is, if the allowable bearing pressures could be located closer to the end slope of the approach fill, the resulting bridge length would be comparable to that of a bridge having a pile foundation.

Current bearing-capacity limits are based on theoretical considerations. This paper describes a set of controlled experiments in which the ultimate bearing capacity at various locations within a granular approach fill was measured. It was found that the theoretical approach seriously underestimates the capacity of footings close to the crest of a slope. Present indications are that piles can be omitted from existing spill-through abutment design, and the abutments can be placed directly on select, well-compacted gravel at lower cost. A concomitant benefit is that a footing-supported abutment and fill will settle as a unit; this will eliminate the maintenance cost often associated with bridge approaches that settle while the bridge itself does not.

In 1978, an actual underpass structure was built to a new design based on the tests reported here. The behavior of the structure is being monitored, and its performance will be compared with that of the corresponding model.

DEFINITION OF THE PROBLEM

Generally, one distinguishes two basic types of abutments—the retaining and the spill-through. In a retaining abutment, the approach fill is contained within the vertical abutment wall and the wing walls, whereas in the spill-through abutment, the approach fill is self-supporting and the bridge appears to rest on the fill near the top of the end slope. In fact, in the majority of cases, the bridge does not rest on the fill but is, instead, supported on piles that extend down through the fill to the natural soil or rock.

Why Use Piles?

Economics plays a large role in the design of bridges, in particular in the design of fairly routine highway and railway bridges of the overpass type. Based on present design practices, the economic advantage is nearly always in favor of founding spill-through abutments on piles rather than on spread footings. Generally, the bridge on spread footings is longer than the bridge on piles and the spread-footing alternative requires a fairly large zone of more-expensive, compacted select fill.

To design a spread footing for a spill-through abutment, the designer must resolve the dilemma of determining the probable ultimate capacity and settlement of the footing. At present, there are at least eight bearing-capacity theories that engineers can use, and all eight purport to take into account the effects of the proximity of the sloping face of the approach fill. The problem is that all eight give different answers.

Most of the theories are applicable only to a footing located right at the crest of the slope; only two—those of Meyerhof (1) and Giroud (2)—treat the general problem of the capacity anywhere within a slope and also use acceptable analytical techniques. Because it is unlikely that a designer would locate an abutment footing right at the crest at the end slope of the approach fill, Meyerhof's and Giroud's theories are the most widely used for design. Even then, the difference between the two theories can be considerable—particularly in dense material within the region close to the crest of the slope.