Park-and-Ride Planning for Energy Conservation: An Optimization Methodology

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A linear programming approach is used to allocate the location and size of park-and-ride facilities to minimize energy use. The basic objective is to minimize the use of energy by estimating the minimum vehicle kilometers traveled for specific corridor or areawide park-and-ride programs. Parameters such as vehicle occupancy, travel costs, constraints that include transference of core-area parking to fringe areas, and community-imposed limits on parking in suburban areas are considered. Thus, the model is made responsive to areawide energy conservation, program costs, transportation system management and transportation control planning actions, and community policy concerns. This normative approach to park-and-ride planning is performance oriented, the results of other demand models are used as inputs to the linear programming process. It can assist also in contingency planning for energy savings by defining the allocation of park-and-ride facilities to meet specific levels of energy use subject to related system capabilities and constraints. The use of standardized linear programming routines permits rapid and relatively inexpensive evaluation of alternative scenarios by planners and decision makers. The paper describes the modeling techniques used and the areas of approximation in the techniques and provides a hypothetical example to illustrate the type of results available. It concludes with a brief discussion of areas that warrant further investigations to assist in developing the techniques discussed.

Extensive interest is currently focused on park-and-ride facilities because of their beneficial impacts such as fuel savings, reduced air pollution and downtown traffic congestion, and more limited use of the private automobile in general.

Also, specific policy actions such as transportation system management (TSM) and transportation control planning (TCP) measures have emphasized public transportation and, therefore, the need for park-and-ride facilities. These factors, in addition to federal, state, and local involvement in the planning process, require a rational, coordinated planning effort.

Accordingly, this paper describes investigations and initial results of a transportation analysis approach, by using mathematical programming techniques to minimize the fuel needs of park-and-ride users consistent with TSM and other appropriate public policy concerns.

ENERGY SAVINGS AND PARK-AND-RIDE LOCATION

If the locations and number of spaces at specific park-and-ride facilities can be arranged to minimize the vehicle kilometers traveled by commuters who drive to main-line transit facilities, the maximum amount of fuel savings will result from this category of users. At present, however, many commuting motorists do not drive to the nearest (or any) park-and-ride facility. Their choice is affected by mode selection factors, which may include the following park-and-ride-related concerns:

1. Access problems from adjacent arterials and freeways;
2. Certain lots being full before others;
3. Cost of using specific lots;
4. Level of attractiveness (fare, travel time, or convenience) of associated main-line transit;
5. Restrictions by specific municipalities on the use of lots within their jurisdiction (often due to environmental considerations) and community concern about traffic impacts;
6. Comparative advantage of using automobile versus main-line transit for specific route segments; and
7. Security conditions at specific parking lots.

An areawide or corridor plan that details the locations and sizes of park-and-ride facilities to minimize vehicle kilometers of travel consistent with the above policy and operational concerns can provide a guide for planners faced with coordinating transit (rail and bus) and related programs. The plan becomes all the more useful if changes in inputs (such as costs, vehicle occupancy, and development constraints) can be readily accommodated in the analysis process.

PLANNING METHODOLOGIES

Current Approaches

Various methods of mode-of-access analysis have been proposed to rationally estimate the demand for park-and-ride spaces for motorists who use public transportation for the major portion of their commuting trip. Mode of access means the mode of transportation used between home and the change-of-mode (in this case park-and-ride) facility. These methods may be categorized as descriptive (deterministic or probabilistic) demand models or normative (performance-oriented) models.

Examples of recent deterministic models are those of Abdus-Samad and Grecco (1, 2), which use linear regression analysis based on experience at existing facilities. A similar mathematical basis is provided by Kegg and Liou (3), primarily based on motorists' travel times and costs. Probabilistic methods of estimating demand include those used for Altrincham, England (4), and a probit analysis used for facilities in Washington, D.C. (5).

Regarding normative approaches to park-and-ride planning that specify or define the performance of a given plan for the allocation of parking spaces in terms of criteria such as energy saving and increased transit ridership, relatively few methodologies have evolved. One example, proposed by Schneider and others (6) for use in Seattle, uses interactive computer graphics and a worth score of travel characteristics to estimate, iteratively, a preferred plan. A linear programming approach to the allocation of parking spaces for minimum vehicle kilometers of travel and minimum cost objectives subject to community and other constraints on the number of spaces and associated TSM and TCP measures has been proposed by Schoon and others (7, 8).

Mathematical Programming Approach for Park-and-Ride Planning

Development of normative park-and-ride planning models can assist in transportation planning by providing es-
estimates of systems performance in terms of vehicle kilometers of travel and cost, subject to transportation operations and management programs and to public policy constraints.

The analysis outlined in this paper, therefore, is the result of ongoing efforts at Northeastern University to formulate a methodology for developing park-and-ride plans, specifically in terms of energy savings and monetary costs, that are also responsive to community concerns and transportation operations efforts. The methodology is being designed to be as direct as possible in its inputs, analysis processes, and outputs to enable planners and decision makers to explore a full range of scenarios and policies with maximum flexibility.

Some of the more significant capabilities of the mathematical programming approaches to park-and-ride planning are as follows:

1. Determination of the absolute minimum total vehicle kilometers of travel and the allocation of lots and spaces can be made, consistent with any given set of parameters and constraints. This may also be done manually for a limited number of park-and-ride facilities; however, it becomes tedious or impractical when a large number of facilities are planned.

2. Determination of the allocation of lots and the number of parking spaces within each lot to provide the absolute minimum cost for any given set of parameters and constraints can be determined.

3. TSM actions can be tested to see how effective they may be in terms of energy use on an areawide or corridor basis. This can be done by varying the input parameters such as facility locations, vehicle occupancy, and facility costs.

4. Effects of community-related requirements can be explored by varying input constraints such as the maximum number of available spaces at a given location. For instance, if the likelihood of obtaining parking spaces at one lot is very low, the effects on areawide vehicle kilometers of travel due to a potential redistribution of park-and-ride facilities can be determined.

5. Under contingency circumstances, it may be necessary to induce motorists to park at specific park-and-ride facilities in order to limit fuel consumption to predetermined levels. A mathematical programming approach can determine which park-and-ride location is preferable for each motorist in order to attain this objective.

Thus, the mathematical programming approach can be considered as an extension of demand analysis. By using this concept, demands that result from uncontrolled or controlled variables can function as input parameters and constraints that affect the attainment of objectives by means of a specific park-and-ride plan.

PRINCIPAL FEATURES OF THE METHODOLOGY

Overall Process

Overall concepts associated with formulation of an integrated, areawide or corridor plan for park-and-ride facilities by use of mathematical programming techniques can be illustrated as shown in Figure 1. This diagram summarizes the technical analysis and public policy factors that lead to a final presentation of alternative strategies or plans for the location and size of park-and-ride facilities in fringe or suburban areas. The process is divided into two principal stages: (a) the initial establishment of travel variables such as modal split and vehicle occupancy and (b) the analysis process that leads...
to initial results, which can be used as guidelines for the park-and-ride plan and as inputs to a number of iterations for progressively refining the plan and incorporating potential operational and policy options. The major features of this two-stage process are described below.

Establish Travel Variables

The essential steps in the identification of zone-specific variables are as follows. First, identify line-haul public transportation facilities within the area, including service frequency, capacity, level of service, usage determinants, and a detailed investigation of park-and-ride and other facilities associated with public transportation. The investigation includes an inventory of facilities, user origin patterns, fares, and usage levels. Next, identify existing and potential park-and-ride locations and capacities. Those at shopping centers, highway interchanges, and other locations can be identified at this time, as well as the more usual rail-transit-related park-and-ride lots.

Based on analysis of the extent of detail required, determine the extent and boundaries of line-haul transit station influence areas or analysis zones. Then, formulate station influence area or zone-specific constraints, including those for community concerns, modal split, mode of access, and parking-lot capacities. Formulation of the appropriate limits for use in the constraints will result from factors such as carpooling, extent of likely feeder bus, kiss-and-ride, dial-a-ride service, and other mode-of-access determinants.

Progressive Optimization and Evaluation

This stage of the process is concerned with the optimal capacity of the park-and-ride lot and the evaluation, iteration, possible modification, and assessment of the implications of each scheme. The optimization process for each of the strategies mentioned previously is conducted, with the necessary iterations, as follows:

1. Conduct initial optimization for each alternative by using the measure of effectiveness defined from an initial assessment of vehicle kilometers of travel and costs by using the constraints defined earlier;
2. From the initial allocation of park-and-ride facilities, reassess vehicle kilometers of travel for each zone and conduct the first iteration optimization of each strategy;
3. Conduct further iterations by modifying vehicle kilometers of travel inputs until the final optimization is achieved;
4. Conduct sensitivity analyses to establish implications of varying levels of investment and other determinants;
5. Present findings and implications of each of the strategies under the defined constraints; and
6. Modify public policy emphasis, funding levels, or other constraints if implications of the initial master plan are unacceptable or if policy options require modification.

Although linear programming is the specific optimization method described here, potential may also exist for other forms of analysis such as goal programming or dynamic programming.

Linear Programming Applications

Linear programming is often used as a tool for selecting a course of action given a quantitatively defined objective and associated constraints. Land use and transportation planning applications of linear programming to determine the optimum location of land uses related to transportation facilities approaches have been described by Herbert and Stevens (9), Harris (10), and Blunden (11). Recent investigations in network planning, which involved linear programming techniques (12-14), have indicated a potential for its use, although effective applications have often been hampered by a lack of truly quantitative data and difficulties in controlling levels of the factors involved.

A formal mathematical statement of the general linear programming problem may be stated as follows: Find $x_1, x_2, \ldots, x_n$, which maximizes (or minimizes) the linear function

$$z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

subject to the restrictions

$$a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq b_1$$

$$a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq b_2$$

$$a_{m1} x_1 + a_{m2} x_2 + \ldots + a_{mn} x_n \leq b_m$$

where $x_i \geq 0, x_2 \geq 0, \ldots, x_n \geq 0$ and $a_{ii}, b_i$, and $c_i$ are given constants.

The above model, interpreted in terms of park-and-ride planning, states that, given $n$ competing activities, the decision variables $x_1, x_2, \ldots, x_n$ represent the levels of these activities (the number of parking spaces in each of $n$ park-and-ride lots). If each activity is the formation of units of the $j$th product, $c_j$ is the increase in the overall measure of effectiveness (vehicle kilometers of travel or cost) that results from production of each unit of a corresponding product. The number of relevant scarce resources is $m$, and each of the $m$ linear inequalities expresses a restriction (constraint such as available land for park-and-ride spaces) on one of these resources. Each $b_i$ is the amount of resource $i$ (such as total program vehicle kilometers of travel or cost) available to the $n$ activities, and $a_{ij}$ is the amount of resource $i$ consumed by each unit of activity $j$. The total usage of the respective resources is given by the left side of these inequalities. The nonnegativity restrictions ($x_i, 0$) express the fact that a negative quantity of an activity cannot exist.

**EXAMPLE: PARK-AND-RIDE FACILITIES IN A RADIAL CORRIDOR**

**Problem**

A frequent situation is the problem of allocating park-and-ride spaces in lots throughout a specific corridor served by various forms of main-line transit for the major portion of the commuting trip between home and the city core area. The hypothetical, simplified example presented here illustrates the main features of a linear programming approach.

The following features are assumed in the corridor analysis:

1. Five stations (referred to as Q, R, S, T, and U) constitute the possible park-and-ride stations in a corridor (see Figure 2);
2. The average vehicle kilometers of travel per vehicle associated with each station is such that the distances between users' homes and park-and-ride facilities (mode-of-access distance) are greater farther the station is from the core area;
3. Average cost per park-and-ride space associated with each station tends to decrease the farther the station is from the core area;
4. The constraints on the number of parking spaces (modal split, mode of access, street access, and community concerns) have been consolidated to provide one upper and one lower level of parking space constraints for each station;

5. If the number of available parking spaces at a given park-and-ride facility is reduced, it is assumed that park-and-ride users will divert to the next park-and-ride location nearest the core area; and

6. For purposes of simplifying the example and for clarifying the essential relationships, the effects of kiss-and-ride and feeder bus users have not been included.

The major strategies to be examined will be called strategy 1, minimization of vehicle kilometers of travel, and strategy 2, minimization of system costs. These two strategies represent the extreme points of the relationship of cost and vehicle kilometers of travel for the specific constraints and, as such, assist in defining the cost and vehicle kilometers of travel domain within which possible variations in the master plan can be formulated.

Formulation of the model, indicating the strategy and alternative breakdowns and the associated constraint levels, is shown in Figure 3. This summarizes the essential components described above and provides the inputs for the linear programming analysis. Key parameters, constraints, and other features of the problem formulation are also shown in this figure.

The major outputs of the analysis for strategies 1 and 2, by using a standard computer linear programming package, are summarized in Table 1. The principal points of note in this table concerning vehicle kilometers of travel and total costs are that the minimum attainable vehicle kilometers of travel for the commuters in the example corridor for the specified parameters and constraints is 32,536 km. The corresponding cost for implementation of this plan is $52,621. When the minimum cost of implementing a plan consistent with the specified parameters and constraints is the objective, the total cost can be reduced to $45,208. However, under this plan the corresponding vehicle kilometers of travel increases to 54,628 km.

Differences that correspond to the above can be seen in terms of average vehicle kilometers of travel and cost per park-and-ride space. A check on the allocated park-and-ride spaces at each of the stations indicates that all of the specified constraints are met.

The basic output data shown in Table 1 also provide overall assessment of each strategy and establish relationships between key variables that can be adjusted to investigate sensitivity and general relationships.

Sensitivity Analysis

The sensitivity of the cost versus vehicle kilometers of travel relationship to changes in vehicle occupancy, unit costs, and reductions in core parking is shown in Figure 4. This illustrates, for specified changes in these parameters, how the cost and vehicle kilometers of travel...
Table 1. Park-and-ride optimization plan—summary of results.

<table>
<thead>
<tr>
<th>Station</th>
<th>Park-and-Ride Spaces</th>
<th>Vehicle Occupancy</th>
<th>Persons</th>
<th>Distance (km)</th>
<th>Vehicle Kilometers Traveled (km)</th>
<th>Unit Cost ($)</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>2000</td>
<td>1.4</td>
<td>2800</td>
<td>11</td>
<td>14000</td>
<td>17</td>
<td>34000</td>
</tr>
<tr>
<td>R</td>
<td>617</td>
<td>1.3</td>
<td>740</td>
<td>13</td>
<td>4928</td>
<td>13</td>
<td>8021</td>
</tr>
<tr>
<td>S</td>
<td>800</td>
<td>1.1</td>
<td>860</td>
<td>15</td>
<td>7200</td>
<td>9</td>
<td>7200</td>
</tr>
<tr>
<td>T</td>
<td>200</td>
<td>1.1</td>
<td>220</td>
<td>16</td>
<td>2000</td>
<td>7</td>
<td>1400</td>
</tr>
<tr>
<td>U</td>
<td>400</td>
<td>1.1</td>
<td>440</td>
<td>16</td>
<td>4400</td>
<td>5</td>
<td>2000</td>
</tr>
<tr>
<td>Total</td>
<td>4017</td>
<td></td>
<td>5080</td>
<td></td>
<td>32536</td>
<td></td>
<td>52021</td>
</tr>
</tbody>
</table>

| Strategy 2 |
| Q       | 400                 | 1.4               | 560     | 11           | 2800                             | 17            | 5800    |
| R       | 616                 | 1.2               | 760     | 13           | 4928                             | 13            | 8008    |
| S       | 800                 | 1.1               | 880     | 15           | 7200                             | 9             | 7200    |
| T       | 2100                | 1.1               | 2310    | 16           | 21000                            | 7             | 14000   |
| U       | 1700                | 1.1               | 1870    | 18           | 18700                            | 5             | 4500    |
| Total   | 5616                |                   | 6360    |              | 54628                            |               | 45208   |

Notes: 1 km = 0.62 mile.

For strategy 1, the average vehicle kilometers traveled to a park-and-ride space is 8.1 and the average cost is $13.1.

For strategy 2, the average vehicle kilometers traveled to a park-and-ride space is 9.7 and the average cost is $8.7.

Figure 4. Relationship between cost and vehicle kilometers of travel and example of selected changes in input parameters and constraints.

will change. It substantiates and quantifies the intuitive analysis that

1. Reductions in vehicle occupancy will simultaneously reduce costs and vehicle kilometers of travel and vice versa.

2. Increases in unit costs will increase total costs but will not increase vehicle kilometers of travel provided that no upper total cost is imposed on the program.

3. An increase in parking restrictions in the city core will require a greater provision of park-and-ride facilities. Hence, a greater increase in total cost and vehicle kilometers of travel associated with park-and-ride facilities will result. A decrease in the core parking requirements will have the opposite effect.

The sensitivity relationships illustrated in this example show changes in the basic condition when changes are made in only one parameter at a time. However, different combinations of parameters and constraints can be changed simultaneously. Also, the linear programming method would result in step functions rather than the generalized straight-line relationships shown in Figure 2.

Another important area of interest in sensitivity analysis is in exploring the effects of varying the acceptable number of parking spaces at a specific station. Consider, for example, the following four alternatives associated with station S:

Alternative 1—800 park-and-ride spaces must be provided at station S;

Alternative 2—upper limit of 400 park-and-ride spaces must be provided at station S;

Alternative 3—0 spaces allowed at station S; and

Alternative 3A—same as alternative 3, but with increased vehicle kilometers of travel for users from influence area of station S due to their diversion to station R.

Based on substitution of each revised constraint in the problem format and rerunning the program, Figure 5 indicates that for strategy 1 (minimization of vehicle kilometers of travel)

1. As the number of available park-and-ride spaces at station S is reduced (alternatives 2 and 3), the excess spaces are allocated to station R because allocation to
station R is compatible with the vehicle kilometers of travel minimization objective, and station R has a sufficiently high potential capacity to accommodate the excess park-and-ride users diverted from station S. Had there been insufficient capacity at station R to accommodate the diverted users, they would either continue to station Q or would be lost to the park-and-ride system (i.e., would select a different destination or would have traveled the entire distance between home and the downtown by automobile).

2. Vehicle kilometers of travel and costs reflect the allocation of spaces to each station.

3. The adjustment in vehicle kilometers of travel due to the larger influence zone of stations R plus S (alternative 3A) indicates a relatively small difference in the park-and-ride space, vehicle kilometers of travel, and cost apportionment.

For strategy 2 (minimization of costs), the apportionment of park-and-ride spaces for alternatives 2 and 3 emphasizes a reallocation primarily to stations T and R consistent with the cost-minimization strategy. Also, the park-and-ride spaces are assigned to station R when the upper constraints on station T are reached. As with strategy 1, additional average vehicle kilometers of travel due to combining influence areas for stations R and S is relatively minor.

The linear programming approach offers considerable opportunity to conduct further sensitivity analyses. A summary list of how these analyses can be approached in response to TSM concerns is as follows:

<table>
<thead>
<tr>
<th>TSM Element</th>
<th>Approach to Inclusion of TSM Element in Park-and-Ride Planning Methodology</th>
</tr>
</thead>
<tbody>
<tr>
<td>Improved vehicular flow</td>
<td>Affects access time between home and park-and-ride facilities; travel time and cost for line-haul transit vehicles include mode-of-access, modal-split, and cost parameters</td>
</tr>
<tr>
<td>Preferential treatment for high-occupancy vehicles</td>
<td>Improved travel times and routes of line-haul vehicles included in modal split, time, and cost estimates Included in estimates of access time and demand levels at park-and-ride facilities for specific time periods; also will affect transit costs</td>
</tr>
<tr>
<td>Reduced peak-period travel</td>
<td>Center-city parking restrictions will result in a transference of spaces to park-and-ride facilities; increased number of park-and-ride parking spaces must be allocated to provide minimum vehicle kilometers of travel</td>
</tr>
<tr>
<td>Parking management</td>
<td>Increased vehicular occupancy is included in basic linear programming format (see also example)</td>
</tr>
<tr>
<td>Promotion of high-occupancy and nonvehicular travel modes</td>
<td>Increased transit use will affect modal split and total passenger volumes on specific routes; paratransit between home and park-and-ride locations will lower the need for park-and-ride parking spaces</td>
</tr>
<tr>
<td>Transit and paratransit service improvements</td>
<td>Reductions in transit operating costs and improvements in operating efficiency will affect modal split and transportation system costs</td>
</tr>
</tbody>
</table>

CONCLUSIONS

Current provision of park-and-ride facilities attempts to achieve maximum use of line-haul public transportation from city centers. Park-and-ride planning described here can generate information on a range of options to form the basis for decision making.

Areas in which further data and research are desirable in this context include consideration of trip-making characteristics of park-and-ride users, estimation of park-and-ride-related travel costs, mode-of-access analysis, and variations in the mathematical programming approach.

Linear Approximation of Variables

The use of linear programming techniques implies that relationships used in the model, such as travel cost versus vehicle kilometers of travel, behave in a linear fashion; however, such relationships are rarely linear. Investigations should be made to determine the effects of such approximations on the accuracy of the results, within the typical limits likely to be obtained in practical situations.

Trip-Making Characteristics of Park-and-Ride Users

Although we can assume that most park-and-ride users will travel to the park-and-ride facilities nearest to their trip origin before they begin the line-haul portion of the trip (thus simplifying the estimation of average mode-of-access trip distances), investigations are warranted into the effects of imposing limits on certain park-and-ride facilities on the average trip length and the choice of park-and-ride facility.

Note that users from the same zone often have a different choice of park-and-ride facilities, depending on the time at which the journey is started, due to changes
in occupancy of facilities, temporary access deficiencies, and congestion points that fluctuate in extent and severity.

**Urban Travel Patterns and Modal-Split Analysis**

Probably the major effects of restrictions on private automobile use to and from city core areas and the associated park-and-ride facilities program will be the result of an imposition of a ceiling level on the use of a particular mode, which will thus distort the free demand level. This will also occur at certain of the park-and-ride facilities. In particular, two factors arise from the policy of imposing limits on parking space:

1. Economic discrimination between user categories if pricing policies are used as the mechanism for reducing parking needs (those users more able to meet increased costs will have a corresponding mobility advantage) and
2. Decreased advantage to some captive automobile users whose schedules may conflict with restricted availability of parking spaces.

Also, as mentioned earlier, the determiners of modal split frequently are not linear functions and each specific case would have to be assessed to determine the actual effects on modal split.

**Mathematical Analysis Variations**

The analysis approach described earlier to provide optimal master plans for park-and-ride facilities that have alternative objectives and constraints exhibits two distinct mathematical stages. First is the estimation stage in which the parameters such as average vehicle kilometers of travel per zone per vehicle and vehicle occupancy were estimated and also in which predictions about the range of modal split, mode of access, and other constraints were made. Second is the prescriptive stage in which the estimates of the first stage were assembled to provide boundary conditions within which levels of the variables could be determined in order to attain defined (or prescribed) objectives. In this second stage, a linear programming methodology was used and, as shown in the example, the linear approximations to the nonlinear functions were investigated.

In addition to the basic linear programming format, a number of refinements and variations could prove advantageous in more detailed studies. The most likely applications in this regard are the use of dynamic programming or of linear programming under uncertainty—involving either stochastic programming or chance-constrained programming. Also, goal programming, where each potential objective is ranked in terms of its priority, offers the potential for future applications.

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**REFERENCES**


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