Abridgment

Comparison of Multicriteria Optimization Methods in Transport Project Evaluation

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The evaluation of transport projects frequently requires consideration of multiple criteria other than or in addition to economic efficiency. Nonetheless, few of the important methodological advances of recent years in the areas of multicriteria decision making and multicriteria optimization have been discussed in the literature about transport project evaluation. This paper compares certain key multicriteria optimization methods and illustrates their relative strengths and weaknesses through application to a hypothetical transport project in a developing country. In particular, the results of an earlier paper that compares the weighting method suggested by Zadeh and Marglin to the iterative preference-incorporation method of Geoffrion, Dyer, and Feinberg are combined with new analyses based on the constraint method suggested by Marglin and the yes-no iterative algorithm of Zionts and Wallenius. Conclusions are drawn concerning the relative attractiveness of these solution methods and the characteristics of an ideal multicriteria optimization algorithm for the evaluation of transport projects.

That the evaluation of transport projects involves the consideration of multiple criteria or objectives that are noncommensurable is increasingly recognized by planners and engineers. Although the importance of multiple criteria has been recognized, agreement as to an appropriate methodological approach for handling noncommensurable criteria in transport project evaluation has not been reached.

A useful vehicle for examining the role of multiple criteria in transport project evaluation is provided by rural roads in developing countries. The problem of investment in rural roads may be used to illustrate the application of standard multicriteria evaluation tools and to compare the attractiveness of solution methods.

Multicriteria evaluation problems are frequently most naturally articulated as vector mathematical programming problems, as the rural roads example of this paper will illustrate. Methods for the solution of vector mathematical programming problems can be divided into two categories [Cohon (1)]:

1. Generating methods that identify all efficient solutions and
2. Preference-incorporation methods that use decision-maker preferences to examine only a subset of all efficient solutions.

We will give a detailed illustration in subsequent sections of the characteristics of the so-called constraint method, one of the most widely known generating techniques, which is generally attributed to Marglin (2), and the iterative preference-incorporation technique of Zionts and Wallenius (3). The constraint method offers certain computational advantages over another standardly employed generating technique, the so-called weighting method, which was also suggested by Marglin (2). Similarly, the Zionts and Wallenius technique provides, at least theoretically, certain advantages over the preference incorporation of Geoffrion and others (4), which is sometimes considered the prototypical preference-incorporation method. For comparison we will also review the results of applying the weighting method and the method of Geoffrion and others.

IDENTIFICATION OF THE RURAL ROAD PROBLEM

The lack of adequate transportation facilities has been a major determining factor of rural underdevelopment in developing countries, particularly through its constraining effect on the agricultural sector. In order to analyze alternative rural transport project evaluation techniques, we will work with a simplified model based on the following assumptions:

1. Transport cost savings caused by a road investment project are fully transferred to agricultural producers in the form of correspondingly higher prices,
2. Total cultivable land area is fixed,
3. Agricultural producers are price takers,
4. Agricultural producers are profit maximizers, and
5. The total amount of the homogeneous agricultural product of concern is marketed only after transport over the road system.

Let us further suppose that we have two agricultural regions, 1 and 2, that follow the assumptions above and that are connected through roads R_1 and R_2, respectively, to market A where their output is sold. Both regions draw on common fixed resources. Improvement to road R_i will decrease the market transport cost and will thus increase the product price perceived by the producer. As a result, agricultural production in region i will expand. On the other hand, a production increase in one region will lead to a production decrease of smaller magnitude in the other region since the common resource's share to the latter will diminish as a consequence of the production expansion in the former. A transport investment program is being considered that will improve both roads R_1 and R_2.

Before the implementation of the R_1 improvement the perceived price is p_0, and total output is determined by $p_0 = MC_0$ (price equals marginal cost of production) at q_0, which is the profit-maximizing output. After the implementation of the R_1 improvement, the product demand curve (as perceived by the farmers) shifts upward to $p = p_1$ due to transporation cost reduction. If the improvement of R_2 is also implemented, production in region 2 will increase and, as a result, the marginal cost curve of region 1 will shift upward from MC_0 to MC_1. Final equilibrium output in region 1 will be q_2, which is larger than q_0 but less than q_1, the level that could have been achieved had no R_2 investment been made.

We assume that the objective of the investment program is to maximize the vector of regional agricultural

productions and we make the following definitions:

$Z = [Z_1, Z_2]$,
Z_i = agricultural production level of region i,
X_i = amount of investment in road R_i ($000 000s),
$\alpha, \beta, \gamma, \delta > 0$,
$\beta, \gamma < 1$,
b_1 = total national (both regions) budget devoted to rural road improvements ($000 000s),
b_2 = a maximal allowable excess of region 2's transport investment share over region 1's (a nonnegative number) ($000 000s),
b_3 = rural road improvement budget of region 1 ($000 000s), and
b_4 = rural road improvement budget of region 2 ($000 000s).

Then our problem becomes

$$\text{MAX } Z = [Z_1, Z_2] \quad Z_1 = \alpha X_1 - \beta Z_2 \quad Z_2 = -\gamma Z_1 + \delta X_2 \tag{1}$$

subject to:

$$X_1 + X_2 \le b_1 \quad -X_1 + X_2 \le b_2$$
$$X_1 \le b_3 \quad X_2 \le b_4 \quad X_1, X_2 \ge 0.$$

The constraints are purposely written to allow the implicit bounding $X_1 - X_2 \le b_3$; this simplifies the exposition. The more general constraint $|X_2 - X_1| \le b_2$ may be introduced if desired with relatively simple modifications of the numerical examples discussed subsequently. In addition, we will assume that a bicriterion welfare function is defined and given by

$$U = U[Z_1(X), Z_2(X)] \tag{2}$$

where X denotes the vector of decision variables (X_1, X_2). The function $U(\cdot)$ is a monotonically increasing function of the objectives with convex isoquants in objective space.

Solution of an Example Problem

In this section, we show that the rural road problem can be reformulated as a (linear) multiobjective programming problem. We then assign some numerical values to the coefficients and solve the example problem by a standard generating method, the constraint method [Marglin (2)], and by the iterative preference-incorporation technique of Zionts and Wallenius (3). These results are then compared to those obtained from the weighting method [Marglin (2)] and the iterative procedure of Geoffrion and others (4). Friesz and others (5) show that

$$Z_1 = aX_1 - bX_2 \tag{3a}$$

and

$$Z_2 = cX_1 + dX_2 \tag{3b}$$

where the coefficients are all positive. To illustrate the solution process, we assign some arbitrary positive integers as the coefficients of expressions 3a and 3b and of the constraint set stated in problem 1. The example problem with two objectives and two decision variables is thus as follows:

$$\text{Maximize } Z(X_1, X_2) = [Z_1(X_1, X_2), Z_2(X_1, X_2)] \tag{4}$$

where

$$Z_1(X_1, X_2) = 6X_1 - 1X_2 \quad Z_2(X_1, X_2) = -1X_1 + 4X_2$$

subject to:

$$-X_1 + X_2 \le 3 \quad X_1 + X_2 \le 8 \quad X_1 \le 6 \quad X_2 \le 4 \quad X_1, X_2 \ge 0.$$

We now discuss techniques for solving the example problem 4. Throughout the discussion that follows, the general solution of a multicriteria or vector optimization problem is taken to be the set of all efficient or noninferior alternatives. A noninferior alternative is a feasible alternative such that it is impossible to improve one objective (criterion) without causing a degradation in at least one other objective. The noninferior alternative that maximizes some aggregate social welfare measure (in our two-objective case, the welfare function, Equation 2) is termed the best compromise solution.

Application of the Constraint Method

We can apply the constraint method [see Cohon (1)] to generate the noninferior set for Equation 4 in the following steps.

1. Step 1. Construction of a payoff table; solve individual maximization problems to find the optimal solution for each of the objectives. If there is more than one optimum for any of these problems, then choose the noninferior solution from among the alternative optima.
2. Step 2. Construction of a single objective problem; transform the multiobjective optimization problem into a single objective-constrained problem.
3. Step 3. Define lower bounds for the objectives placed in the constraint set during step 2.
4. Step 4. Solve the constrained problem of step 2 for the lower bounds determined in step 3.

The results of the application of the constraint method to the example problem with n = 4 are summarized below.

Bound (L_2)	Optimal Value of Objective Function (Z_1^*)	Optimal Solution: In Decision Space (X_1^*, X_2^*)	Optimal Solution: In Objective Space (Z_1^*, Z_2^*)
-6	36	(6, 0)	(36, -6)
1	34.25	(6, 1.75)	(34.25, 1)
8	25.6	(4.8, 3.2)	(25.6, 8)
15	2	(1, 4)	(2, 15)

Application of the Zionts and Wallenius Method

Zionts and Wallenius (3) developed an iterative preference-incorporation procedure by which the decision maker is requested to provide yes-no answers to questions regarding certain trade-offs. The procedure leads to an approximation of the best compromise solution. It is assumed in this method that all relevant criteria or objective functions are concave functions to be maximized and that the constraint set is convex. The overall utility function is assumed to be unknown to the decision maker, but it is implicitly assumed to be a linear function or, more generally, a concave function of the individual scalar objective functions.

We are interested in the underlying problem of maximizing a multiattribute utility or welfare function defined over K objectives, which can be denoted symbolically as

$$\text{Maximize } U[Z_1(X), Z_2(X), \ldots, Z_K(X)] \tag{5}$$

subject to

$$X \in F_d \tag{6}$$

where X is a vector of decision variables and F_d is the feasible region of the problem. The basic structure of the Zionts and Wallenius algorithm that addresses this problem is as follows:

1. Step 1. Initialization. Determine a set of initial weights λ_i^0 for each objective $i = 1, \ldots, K$. Let $\lambda_i = \lambda_i^0$ for all i.
2. Step 2. Consider the linear program

$$\text{Maximize } U = \sum_{i=1}^{K} \lambda_i Z_i(X) \tag{7}$$

subject to

$$X \in F_d \tag{8}$$

where F_d is the feasible region and X is the vector of decision variables. Solve this linear program to obtain solution X^0 and the reduced cost w_{ij} of each nonbasic variable X_j with respect to objective i.

3. Step 3. Partition the set of nonbasic variables into two subsets: those that when introduced into the basis lead to efficient adjacent extreme points in objective space and those that do not. Call these, respectively, efficient and inefficient variables.

To check if a nonbasic variable X_ℓ is efficient, consider the following linear program:

$$\text{Minimize } Z = \sum_{i=1}^{K} w_{i\ell} \lambda_i \tag{9}$$

subject to

$$\sum_{i=1}^{K} w_{ij} \lambda_i \geq 0 \qquad j \neq \ell \text{ and } X_j \text{ nonbasic} \tag{10}$$

$$\sum_{i=1}^{K} \lambda_i = 1 \tag{11}$$

$$\lambda_i \geq 0 \qquad i = 1, \ldots, K \tag{12}$$

Solve this linear program where the w_{ij}'s are reduced costs obtained in step 2. If Z is negative, X_ℓ is efficient; otherwise X_ℓ is inefficient. If no efficient variables are found, stop. The best compromise solution has been reached. Otherwise go to the next step.

4. Step 4. For each efficient variable X_j, the decision maker is asked to respond yes or no to a potential trade-off among objectives. These trade-offs are described by the reduced cost w_{ij}. (There will be at least one negative and at least one positive w_{ij} for each efficient variable X_j.) For each yes response construct an inequality of the form:

$$\sum_{i=1}^{K} w_{ij} \lambda_i \leq -\epsilon \tag{13}$$

where ϵ is a sufficiently small positive number. For each no response, construct an inequality of the form:

$$\sum_{i=1}^{K} w_{ij} \lambda_i \geq \epsilon \tag{14}$$

5. Step 5. If this is the first iteration, go to step 6a. For the second and successive iterations, if the overall utility function is assumed to be a linear additive function, go to step 6a; if the overall utility function is assumed to be a general concave function of objectives that are, in turn, linear functions of the decision variables, go to step 6b.
6. Step 6. (a) Put the constraints that are generated in step 4 together with all previously constructed constraints, including Equations 11 and 12. Find a feasible solution λ_i^*, $i = 1, \ldots, K$ that satisfies all constraints of the form of Equations 11-14. Let $\lambda_i = \lambda_i^*$ for all i and go to step 2.

Next, (b) query the decision maker to determine whether the new solution is preferred to the old. If so, all old responses are purged. This implies that only the constraints generated in step 4 of the current iteration together with constraints 11 and 12 will be used to generate new feasible weights λ_i. Find a feasible solution λ_i^*, $i = 1, \ldots, K$ that satisfies all these constraints. Let $\lambda_i = \lambda_i^*$.

If the old solution is preferred to the new solution or if the decision maker is indifferent, the analyst presents the decision maker with all the efficient solutions adjacent to the old solution in objective space. If any of the adjacent efficient solutions are preferred to the old solution, the procedure is continued from one of these preferred solutions. That is, find the efficient (nonbasic) variables associated with the preferred adjacent solution and then go to step 4. If none of these adjacent solutions is preferred to the old solution, stop. The optimal solution lies in a neighborhood of the old solution comprised of that portion of the noninferior set defined by the old solution and its adjacent efficient solutions in objective space.

To apply the Zionts and Wallenius algorithm to the example problem we first assume that the multivariate utility function can be expressed in product form $U[Z_1(X), Z_2(X)] = Z_1 Z_2$ where, of course, $X = (X_1, X_2)$. The algorithm terminates, having determined that the best compromise solution lies in a neighborhood $X^2 = (4, 4)$. This neighborhood is that portion of the noninferior set defined by line segments that join $X^0 = (6, 2)$, $X^1 = (4, 4)$ and $(1, 4)$ in decision space. This corresponds to that portion of the noninferior set defined by line segments that join $Z^0 = (20, 12)$, $Z^1 = (34, 2)$ and $(2, 15)$ in objective space.

Overview of Results Obtained

Friesz and others (5) applied both the weighting method and the method of Geoffrion and others to the sample problem. The table below summarizes the results for the former method.

Weight		Optimal Value of Objective Function	Optimal Solution	
			In Decision Space	In Objective Space
w_1	w_2	(Z^*)	(X_1^*, X_2^*)	(Z_1^*, Z_2^*)
1	0	36	(6, 0)	(36, -6)
1	1	36	(6, 2)	(34, 2)
1	2	44	(4, 4)	(20, 12)
1	3	56	(4, 4)	(20, 12)
0	1	15	(1, 4)	(-2, 15)

They also showed that, under a hypothetical decision environment, the method of Geoffrion and others correctly converges to the unambiguous best compromise solution $X^* = (4, 4)$ with $U^* = Z_1 Z_2 = (20)(12) = 240$ when applied to the example problem.

CONCLUSION

We have shown through a hypothetical problem of investment in rural roads that multicriteria optimization methods may be used to assist the decision maker in evaluating transport projects. The methods considered—the generating technique known as the constraint method and the yes-no, iterative preference-incorporation method of Zionts and Wallenius—differ substantially in the types of information required by the decision maker and the degree of interaction between the decision maker and the analyst. The constraint and weighting methods, prototypical of the methods usually classified as generating methods, strive to approximate the noninferior (or efficient) set, under the implicit assumption that knowledge of this set will allow the decision maker to select a best compromise solution. The iterative techniques of Zionts and Wallenius and Geoffrion and others are preference-incorporation techniques and they seek to identify the best compromise solution without generating the entire noninferior set by soliciting preference information from the decision maker. The Zionts and Wallenius method does not lead to a definitive statement of the best compromise solution unless the overall utility measure is assumed to be a linear additive function of the multiple objectives. The method of Geoffrion and others by contrast always leads to a definitive best compromise solution. This precision is at the expense of requiring more sophisticated preference information from the decision maker.

An ideal multicriteria optimization algorithm would accommodate yes-no preference information and be able to treat discrete alternatives (i.e., integer variables). This last capability can only be introduced in the methods discussed here with a severe loss of computational efficiency.

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Abridgment

Goal-Programming Approach to Multiobjective Highway Network Design Model

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A new approach to the highway network design model is presented that allows comparisons of networks on the basis of multiple incommensurable objectives with different degrees of importance. The goal-programming approach is not only capable of solving the multiobjective network design problem in a relatively efficient manner, but it can also be used to generate the multidimensional trade-off curve that provides additional important information to that provided in the two-dimensional curve derived by using the linear programming model. An example illustrates the application of the linear goal-programming model with four objectives.

Decision problems in general, and particularly those associated with transportation systems, are made in the context of multiple conflicting objectives. Decisions about transportation systems should be weighted against the social, economic, environmental, and aesthetic needs of the community. Among the most important decisions in the transportation planning process are those regarding the structure of transportation networks and the level of service to be offered on them.

In recent years there has been interest in the application of advanced analytic techniques to the search for good alternative transportation networks. A class of models known as network design models has been developed to solve the following problem: Given an existing network, a list of improvement options for various links, and projected increase in demand between various origin and destination pairs, select the optimal set of links to be improved or added to the existing network. The models perform two tasks simultaneously: (a) they choose the optimal set of links to be improved or added and (b) they assign the projected traffic to the new network.

This paper deals with the extension of a continuous Highway Network Design Model (HNDM) developed by Agarwal (1-3). The approach adopted by Agarwal follows that used by Hay and others (4) in an urban context and Morlok and others (5) for the Northeast Corridor Transportation Project. The HNDM is a linear programming model developed as a sketch planning tool.

There are two ways to incorporate multiple objectives in linear programming—as elements of the objective function or as constraints. In the first method, various objectives are collapsed into a single objective by using