CONCLUSION

We have shown through a hypothetical problem of investment in rural roads that multicriteria optimization methods may be used to assist the decision maker in evaluating transport projects. The methods considered—the generating technique known as the constraint method and the yes-no, iterative preference-incorporation method of Zionts and Wallenius—differ substantially in the types of information required by the decision maker and the degree of interaction between the decision maker and the analyst. The constraint and weighting methods, prototypical of the methods usually classified as generating methods, strive to approximate the noninferior (or efficient) set, under the implicit assumption that knowledge of this set will allow the decision maker to select a best compromise solution. The iterative techniques of Zionts and Wallenius and Geoffrion and others are preference-incorporation techniques and they seek to identify the best compromise solution without generating the entire noninferior set by soliciting preference information from the decision maker. The Zionts and Wallenius method does not lead to a definitive statement of the best compromise solution unless the overall utility measure is assumed to be a linear additive function of the multiple objectives. The method of Geoffrion and others by contrast always leads to a definitive best compromise solution. This precision is at the expense of requiring more sophisticated preference information from the decision maker.

An ideal multicriteria optimization algorithm would accommodate yes-no preference information and be able to treat discrete alternatives (i.e., integer variables). This last capability can only be introduced in the methods discussed here with a severe loss of computational efficiency.

REFERENCES


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Abridgment:

Goal-Programming Approach to Multiobjective Highway Network Design Model

Jossef Perl

A new approach to the highway network design model is presented that allows comparisons of networks on the basis of multiple incommensurable objectives with different degrees of importance. The goal-programming approach is not only capable of solving the multiobjective network design problem in a relatively efficient manner, but it can also be used to generate the multidimensional trade-off curve that provides additional important information to that provided in the two-dimensional curve derived by using the linear programming model. An example illustrates the application of the linear goal-programming model with four objectives.

Decision problems in general, and particularly those associated with transportation systems, are made in the context of multiple conflicting objectives. Decisions about transportation systems should be weighted against the social, economic, environmental, and aesthetic needs of the community. Among the most important decisions in the transportation planning process are those regarding the structure of transportation networks and the level of service to be offered on them.

In recent years there has been interest in the application of advanced analytic techniques to the search for good alternative transportation networks. A class of models known as network design models has been developed to solve the following problem: Given an existing network, a list of improvement options for various links, and projected increase in demand between various origin and destination pairs, select the optimal set of links to be improved or added to the existing network. The models perform two tasks simultaneously: (a) they choose the optimal set of links to be improved or added and (b) they assign the projected traffic to the new network.

This paper deals with the extension of a continuous Highway Network Design Model (HNDM) developed by Agarwal (1-3). The approach adopted by Agarwal follows that used by Hay and others (4) in an urban context and Morlok and others (5) for the Northeast Corridor Transportation Project. The HNDM is a linear programming model developed as a sketch planning tool.

There are two ways to incorporate multiple objectives in linear programming—as elements of the objective function or as constraints. In the first method, various objectives are collapsed into a single objective by using
Table 1. Link characteristics for example network.

<table>
<thead>
<tr>
<th>Link Segment</th>
<th>Average Daily Travel Time, Cij (h)</th>
<th>Average Daily Capacities, kij (veh/lanes/day)</th>
<th>Total Cost, Bij ($1000/vehicle/day)</th>
<th>Link Length, Vij (miles)</th>
<th>Dwelling Units to Be Relocated per Capacity Unit (000s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ij</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.021 43</td>
<td>15 000</td>
<td>600 400</td>
<td>2.0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0.805 80</td>
<td>5 000</td>
<td>252 800</td>
<td>1.0</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>0.040 0</td>
<td>40 500</td>
<td>252 800</td>
<td>1.0</td>
<td>3</td>
</tr>
<tr>
<td>23</td>
<td>0.246 73</td>
<td>12 500</td>
<td>376 200</td>
<td>1.4</td>
<td>4</td>
</tr>
<tr>
<td>34</td>
<td>0.057 14</td>
<td>15 000</td>
<td>347 600</td>
<td>1.4</td>
<td>4</td>
</tr>
<tr>
<td>45</td>
<td>0.100 0</td>
<td>15 000</td>
<td>126 400</td>
<td>1.7</td>
<td>10</td>
</tr>
<tr>
<td>35</td>
<td>0.021 43</td>
<td>15 000</td>
<td>505 600</td>
<td>0.6</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Demand matrix for example network.

<table>
<thead>
<tr>
<th>From Node</th>
<th>To Node</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7685</td>
<td>15 000</td>
<td>4516</td>
<td>5 222</td>
<td>7000</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5517</td>
<td>3 239</td>
<td>508</td>
<td>711</td>
<td>2217</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6234</td>
<td>3 439</td>
<td>1704</td>
<td>858</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1333</td>
<td>2 129</td>
<td>3209</td>
<td>5 817</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The purpose of this paper is to demonstrate an approach to the multiobjective HNDM. The proposed goal-programming approach has significant advantages over the interactive programming techniques for solving multiobjective problems previously applied by Agarwal (5). The goal-programming approach can generate trade-off curves in their full dimensionality. The interactive approaches are very time consuming because they require a continuous interaction with the decision maker. Goal programming can solve multiobjective problems while still employing the simplex algorithm (on a modified basis). This allows exploration of a much larger number of alternatives in a given time period than does an interactive programming technique.

In the process of using the HNDM, information that describes the trade-offs between the achievement of various objectives is perhaps more valuable than the actual point solution. This information is presented by a trade-off curve. A serious issue in the multiobjective trade-off analysis is the display of the multidimensional trade-off curve in two-dimensional space. This paper will demonstrate an approach to the presentation of multidimensional trade-off curves.

GOAL-PROGRAMMING MODEL

A discussion of linear goal programming is beyond the scope of this paper and can be found in Ignizio (7). Linear goal programming can be defined as (7) "a systematic methodology for solving linear, multiple objective problems wherein preemptive priorities and weights are associated with the objectives."

The linear goal-programming model presented here includes the following objectives:

- G1 = flow conservation objective,
- G2 = flow definition objective,
- G3 = link capacity objective,
- G4 = budget objective,
- G5 = level-of-service objective,
- G6 = household relocation objective, and
- G7 = vehicle-miles-of-travel objective.

Since G1, G2, and G3 can be legitimately viewed as absolute objectives (objectives that must be satisfied), they are assigned a top priority. In the following formulation of the linear goal-programming model, the household relocation objective is assigned to priority level two, the level of service objective to priority three, the budget objective to priority four, and the vehicle-miles-of-travel objective to priority five. The linear goal program for a network with N nodes, a set of L undirected links, and a set of S origins and destination nodes can be written as follows:

Find: xij, xi, xij, kij, so as to minimize

\[ s = \{ (n_1 + p_1 + n_2 + p_2 + \ldots + n_H + p_H + \ldots + n_Q + p_Q + p_{Q+1} + \ldots + P + \ldots + P_{L}, (P_i), (P_j), (P_k)) \} \] (1)

such that for G1,

\[ \sum_{k \in N} x_{kij} - \sum_{k \in N} \sum_{h \in H} x_{hi} = \{ D_i \} \forall i, j \in N \forall h = 1, \ldots, H \] (2)

For G2,

\[ \sum_{m=1}^{M} x_{mij} - \sum_{m=1}^{M} (x_{mj} + x_{ij}) + N_{ij} - P_{ij} = O_{ij} \text{ for } s \in S, q = H + 1, \ldots, Q \] (3)

For G3,

\[ x_{ij} - P_{ij} + n_{ij} - p_{ij} = K_{ij} \text{ or } x_{ij} + n_{ij} - p_{ij} = K_{ij} \forall i, j \in N \] (4)

For G4,

\[ \sum_{q=1}^{Q} N_{qij} + n_{ij} - p_{ij} = W \forall i, j \in N \] (5)

For G5,

\[ \sum_{i \in L} \sum_{m=1}^{M} c_{ij} x_{mij} + n_{ij} - p_{ij} = T \] (6)
Figure 1. Multiobjective trade-off curve.

For $G_0$,  
\[
\sum_{i \in L_p} B_i k_{ij} + n_i - p_i = B 
\]

For $G_s$,  
\[
\sum_{i \in M_s} x_{ij}^m + n_i - p_i = V, \quad x_{ij}^m, k_{ij} = 0 \tag{8}
\]

where

\[ n_i = \text{a negative deviation from the aspiration level} \]

\[ p_i = \text{a positive deviation from the aspiration level} \]

\[ x_{ij}^m = \text{flow on arc } jk \text{ going from mode } j \text{ to destination } s \]

\[ D_i = \text{demand from origin } j \text{ to destination } s \]

\[ A_j = \text{the set of modes after } j \]

\[ B_j = \text{the set of modes before } j \]

\[ H = S(N-1) \]

\[ x_{ij}^m = \text{flow on the } m \text{th segment of the total travel-time curve of link } ij \]

\[ M_{ij} = \text{number of segments on the total travel-time curve of link } ij \]

\[ Q = S(N-1) + L_s \]

\[ k_{ij} = \text{capacity added to link } ij \]

\[ K_{ij} = \text{existing capacity on segment } m \text{ of the total travel-time curve of link } ij \]

\[ L_p = \text{the set of links that can be improved} \]

\[ L_c = \text{the set of links that cannot be improved} \]

\[ F_{ij} = \text{the portion of capacity added to link } ij \text{ assigned to the } m \text{th segment of its total travel-time curve} \]

\[ N_{ij} = \text{number of households to be relocated per capacity unit added to link } ij \]

\[ W = \text{a desirable limit on the total number of households relocated as a result of network improvements} \]

\[ c^*_{ij} = \text{average travel time on the } m \text{th segment of the total travel-time curve of link } ij \]

\[ T = \text{a target value on the total daily travel time in the network} \]

\[ B_{ij} = \text{cost per additional unit of capacity on link } ij \]

\[ B = \text{total budget available} \]

\[ V_{ij} = \text{length of link } ij \]

\[ V = \text{a desirable limit on the total daily vehicle miles of travel} \]

The data used for demonstrating the application of the model are presented in Tables 1 and 2. The multidimensional trade-off curve derived by using the model is shown in Figure 1. This curve shows the sensitivity of total travel time, investment, and total vehicle miles of travel to changes in the desirable limit on the number of households that can be relocated.

If we ignore for a moment the objectives in priority levels four and five, we obtain in Figure 1 the two-objective trade-off curve normally generated by a linear programming model. The two-objective trade-off curve shows the change in level of service as a result of a change in the limit on the number of relocated households. It is unlikely that the alternatives in region $O$ are desirable, for a small increase in the number of relocated households will bring about a large reduction in travel time if the decision maker is willing to move to region $P$. Similarly, the alternatives in region $Q$ are probably not attractive because a large increase in the number of relocated households to move from $P$ to $Q$ results in a relatively small reduction in total travel time. A good solution probably lies in region $P$. However, since within region $P$ the reduction in travel time that results from allowing relocation of additional households is constant, the decision maker may have difficulties in selecting the most desirable network. The information on the level of achievement of other objectives provided by the multiobjective trade-off curve can help in selecting the preferred network.

In the multiobjective curve, the level of achievement of the objectives at lower priority levels are displayed next to each efficient point. The achievement level of a more-important objective is closer to the curve. Looking at Figure 1, suppose the achievement of total travel time of 9912 vehicle-h with the displacement of 55 households, total travel time of 9104 vehicle-h with the displacement of 70 households, and total travel of 9024...
A new approach to the HNDM that allows comparison of network alternatives on the basis of multiple incommensurable objectives with different degrees of importance was presented. The goal-programming approach is capable of solving the multiobjective highway network design problem in a speedy and efficient manner. Furthermore, it could be used to generate multidimensional trade-off curves that provide additional important information to that provided by two-dimensional trade-off curves derived from the linear programming model.

The goal-programming approach was shown to overcome some serious limitations of linear programming. In linear programming, a solution that violates one or more of the constraints is termed infeasible. It is easy to realize that this type of conclusion provides no useful information and can often be considered misleading. For example, a basic assumption of the HNDM is that interzonal demands are given with certainty. In reality, predicted demands are subject to great uncertainty. Consequently, certain combinations of prediction errors can result in an infeasible solution and no further information is provided to the decision maker.

The budget objective is formulated in the linear programming model as a constraint. There are two serious problems with such a formulation. First, the decision maker does not have an a priori knowledge of the investment required to satisfy the predicted demands. In fact, he or she would probably expect to obtain this information from the model. If the budget is set too low, it may result in infeasibility. Setting the budget too high to avoid infeasibility would lead to overconstruction and an unrealistic flow pattern. Second, the budget is not independent of the level of service in the network. In fact, the budget is determined to achieve a desired level of service or certain levels of other impacts. The goal-programming approach avoids these problems because aspirations about the level of service and other impacts can be specified and we are allowed to consider the budget as a nonabsolute objective.