BENEFIT/COST RATIOS

Table 6 outlines the 1.7 benefit/cost (B/C) ratio estimated for the Philadelphia project. The relationship between annualized and capital costs is based on the capital recovery factor of 8.33 percent, which is equivalent to 6 percent interest on a 22-year amortization or 7 percent on a 27-year period. The B/C ratio would increase to 1.9 if a 35-year project life were assumed at 6.5 percent interest. Subways have a much longer life than this. Commuter rail cars usually operate effectively for 35 years before replacement. The value of time saved has not been taken as a cash saving.

Thorough independent analyses by the Transportation Systems Center of the U.S. Department of Transportation and by the Delaware Valley Regional Planning Commission technical staff have also determined B/C ratios of 1.7 or better.

SUMMARY

With costs and highway congestion inexorably increasing, and with both cities and the energy supply declining, major projects that will reduce the cost of travel while improving mobility are becoming essential to maintain convenient access and travel efficiency in metropolitan areas. Construction alone may not provide the solution. Care in design and skill in implementation will be required to achieve the projected results.

The Center City Commuter Railroad Connection may not equal the impact of the Hudson River tunnels of 1910, but it will certainly tend in that direction. The favorable impact is already apparent in Philadelphia's Center City.

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Abridgment

Rapid Transit Time and Energy Requirements

W.H.T. HOLDEN

The results of an analysis to compare the trade-off between time and energy in the propulsion of a rapid transit train are discussed. Faster schedules consume more energy but reduce other operating costs and are an asset in attracting riders. Methods of reducing energy consumption, mainly by recovery of all or part of the kinetic energy used, are also described.

In planning and operating a rapid transit system today, the energy required for operation is a major consideration because of the rapid increase in energy costs. Faster schedule speeds are desirable because they increase patronage and reduce operating costs for train attendants and the quantity of equipment required. It is the purpose of this paper to determine the energy increase attributable to higher speed and the corresponding reduction in time to operate a train for a number of interstation distances.

RAPID TRANSIT TRAIN

For this analysis, a theoretical train has been assumed, the properties of which are based on those of the New York R-46 rapid transit car. The quantities that are significant for this purpose are (a) car dimensions (length of 23 m and area of cross section of 10 m²), (b) weight with one-half maximum load (60 000 kg), (c) train consist (8 cars), and (d) rotational inertia, which is taken as 10 percent of empty car weight, so that inertial mass for the train is 525 600 kg.

The following ranges of speeds and accelerations are considered: maximum speeds of 20, 25, 30, and 35 m/s and initial accelerations of 0.5, 1.0, and 1.5 m/s².

SPEED-TIME AND DISTANCE-TIME RELATIONS

It is necessary to express speed-time and distance-time relations in terms of a mathematical formula to permit the necessary integrations for energy determination during acceleration. The following exponential approximation has been adopted for this purpose:

$$V_t = V_o[1 - \exp(-t/T_o)] \tag{1}$$

where

V_t = speed t seconds after a start at t = 0,
V_o = maximum speed, and
T_o = maximum speed divided by initial acceleration, or V_o/A_o.

By integrating Equation 1 from t = 0 to t = t, it is found that

$$D_t = V_o t - V_t T_o \tag{2}$$

where D_t is speed at time t starting at t = 0.

When $t = 3T_o$, Equation 1 indicates that $V_t = 0.9502V_o$. The 5 percent difference from $V_t = V_o$ for this value of t is neglected and, for $t = 3T_o$ or greater, it is assumed that $V_t = V_o$. This leads to $D_t = 2V_oT_o$ when $t = 3T_o$.

Braking is assumed to be at the same rate as initial acceleration. Then the braking distance (D_b) is $(1/2)V_t^2/A_o$; if $V_t = V_o$, this becomes $D_b = (1/2)V_oT_o$. It follows that the minimum length of run, start to stop, in which V_o is attained is $2.5V_oT_o$. Braking time (t_b) is V_t/A_o, and equals T_o if $V_t = V_o$.

Table 1. Dimensionless values for speed-time and distance-time relations.

t/T_o	V_t/V_o	D_t/V_oT_o	D_1/V_oT_o	t_1/T_o
0.1	0.0952	0.0048	0.0053	0.0103
0.2	0.1813	0.0187	0.0351	0.3813
0.3	0.2592	0.0408	0.0858	0.5592
0.4	0.3297	0.0703	0.1247	0.7297
0.5	0.3935	0.1065	0.1345	0.8935
0.6	0.4517	0.1483	0.2264	1.0317
0.7	0.5034	0.1966	0.3233	1.2034
0.8	0.5507	0.2493	0.4009	1.3507
0.9	0.5934	0.3066	0.4827	1.4934
1.0	0.6321	0.3679	0.5677	1.6321
1.1	0.6671	0.4329	0.6554	1.7671
1.2	0.6988	0.5012	0.7454	1.8988
1.3	0.7275	0.5725	0.8371	2.0275
1.4	0.7534	0.6466	0.9304	2.1534
1.5	0.7769	0.7231	1.0249	2.2769
1.6	0.7981	0.8019	1.1166	2.3981
1.7	0.8173	0.8827	1.2167	2.5173
1.8	0.8347	0.9653	1.3137	2.6347
1.9	0.8504	1.0500	1.4116	2.7504
2.0	0.8647	1.1357	1.5091	2.8647
2.1	0.8775	1.2225	1.6075	2.9775
2.2	0.8892	1.3108	1.7061	3.0892
2.3	0.8997	1.4003	1.8050	3.1997
2.4	0.9093	1.4907	1.9041	3.3093
2.5	0.9179	1.5821	2.0034	3.4179
2.6	0.9257	1.6743	2.1028	3.5257
2.7	0.9328	1.7672	2.2637	3.6328
2.8	0.9392	1.8608	2.3018	3.7392
2.9	0.9450	1.9550	2.4015	3.8450
3.0	0.9502[a]	2.0498[b]	2.5012[c]	3.9502[d]

[a]Use 1.0000.
[b]Use 2.00.
[c]Use 2.50.
[d]Use 4.00.

RUN LENGTHS

The run lengths, or interstation intervals, considered here are 800, 1600, and 3200 m. If D_r is run length, then $D_r - 2.5V_oT_o$ must be positive if V_o is attained during the run. The distance ($D_f = D_r - 2.5V_oT_o$) is run in a time D_f/V_o in this case, and drag forces are constant at the V_o value during this time t_f. If $D_r - 2.5V_oT_o$ is negative, then it is necessary to determine V_t and t and also D_b and t_b. This must be done by trial and error, since it is not possible to solve the equations directly for t because it occurs as both an exponential and an algebraic term. Graphical methods may be used.

DIMENSIONLESS FORM OF ABOVE RELATIONS

If we divide Equation 1 by V_o, we have

$$V_t/V_o = 1 - \exp(-t/T_o) \quad (3)$$

where V_t/V_o has a maximum value of unity and states V_t as a fraction of V_o. Similarly, Equation 2 can be divided by V_oT_o, which results in

$$D_t/V_oT_o = t/T_o - V_t/V_o \quad (4)$$

In addition, $D_b/V_oT_o = (1/2)(V_t/V_o)^2$ and, if $V_t/V_o = 1$, $D_b/V_oT_o = 1/2$.

A quantity D_1 is used in this determination of energy and time. It is the distance run in attaining maximum speed in a run plus the distance required to brake to a halt from that speed. At $t = 3.0T_o$, $D_1 = 2.500V_oT_o$. There is also a time t_1, which is the time it takes to run the distance D_1. At $t = 3.0T_o$, $t_1 = 4T_o$.

Table 1 gives dimensionless values for these relations, where

Table 2. Time and energy comparisons for various runs and performances.

Acceleration (m/s^2)	Time or Energy	800-m Run				1600-m Run				3200-m Run			
0.5	V_o (m/s)	20	25	30	35	20	25	30	35	20	25	30	35
	T_o(m/s)	40	50	60	70	40	50	60	70	40	50	60	70
	V_oT_o (m/s)	800	1250	1800	2450	800	1250	1800	2450	800	1250	1800	2450
	Max V (m/s)	15.4	16.3	17.2	18.9	18.4	20.7	22.3	23.2	20	25	24.8	29.2
	U_k (MJ)	62.5	70.3	77.6	94.1	88.8	109.8	130.5	141	105	164	190	223
	U_a (MJ)	14.4	43.9	53.3	80.7	43.4	42.7	91	89.2	38	76	140	278
	U_f (MJ)	0	0	0	0	0	0	0	0	28	2	0	0
	U_r (MJ)	76.9	114.2	131	175	132	153	222	230	171	242	330	501
	t_r (s)	90	86	86	92	137	130	126	123	220	203	190	186
1.0	V_o (m/s)	20	25	30	35	20	25	30	35	20	25	30	35
	T_o (m/s)	20	25	30	35	20	25	30	35	20	25	30	35
	V_oT_o (m/s)	400	625	900	1225	400	625	900	1225	400	625	900	1225
	Max V (m/s)	17.3	18.2	21.8	22.6	20	25	26.7	29.2	20	25	30	35
	U_k (MJ)	78.6	113	151	143	105	164	187	223	105	164	237	322
	U_a (MJ)	15.4	16.5	35.7	47.2	19	38	54	60.4	19	38	69	115
	U_f (MJ)	0	0	0	0	1.4	1	0	0	52	49	27	6
	U_r (MJ)	111	129	187	190	125	203	241	287	176	251	333	443
	t_r (s)	57	65	72	62	100	89	95	92	190	166	153	144
1.5	V_o (m/s)	20	25	30	35	20	25	30	35	20	25	30	35
	T_o (m/s)	13.33	16.67	20	23.33	13.33	16.67	20	23.33	13.33	16.67	20	23.33
	V_oT_o (m/s)	267	414	600	817	267	414	600	817	267	414	600	817
	Max V (m/s)	20	22.8	25.1	26.8	20	25	30	32	20	25	30	35
	U_k (MJ)	105	136	166	188	105	164	237	269	105	164	237	322
	U_a (MJ)	32	24	11	42	13	16	45	63	13	16	45	77
	U_f (MJ)	3	0	0	0	22	17	4	0	56	65	65	55
	U_r (MJ)	140	160	177	230	140	197	286	332	174	245	347	454
	t_r (s)	60	56	53	52	100	89	83	86	180	153	137	127

Table 3. Input energy per unit energies and schedule speeds.

Run Length (m)	Initial Acceleration (m/s^2)	Balancing Speed (m/s)	Input Energy (MJ)	Energy per Kilometer (MJ)	Energy per Car Kilometer (MJ)	Power per Car Kilometer (MJ)	Schedule Speed (m/s)
800	0.5	20	96	120	15	6.8	7.3
		25	143	179	22	9.9	7.6
		30	164	205	26	11.7	7.6
		35	219	274	34	15.3	7.14
	1.0	20	139	173	20.6	20.6	13
		25	108	136	17	17	11.8
		30	108	136	17	17	10.9
		35	115	144	18	18	12.2
	1.5	20	175	219	27.4	27.4	10
		25	200	250	31.3	31.4	10.5
		30	221	276	34.5	34.6	11
		35	288	360	45	45	11.1
1600	0.5	20	165	103	13	5.9	10.2
		25	191	119	15	6.8	10.7
		30	278	174	22	9.9	11.0
		35	288	180	23	10.4	11.2
	1.0	20	156	98	12	12	13.3
		25	254	159	20	9	14.7
		30	301	188	24	24	13.9
		35	359	224	28	13.7	13.9
	1.5	20	175	109	13.6	13.7	13
		25	246	154	19.3	19.5	14.7
		30	358	224	28	28	15.5
		35	336	210	26.3	26.3	15.1
3200	0.5	20	214	67	8	3.6	13.3
		25	254	79	10	4.5	14.4
		30	413	129	16	16	14.4
		35	626	195	29.5	24.5	15.5
	1.0	20	220	69	9	9	15.2
		25	314	99	12	12.6	17.2
		30	416	130	16	16	18.5
		35	554	173	21	20.6	19.5
	1.5	20	218	68	8.5	8.6	18
		25	306	96	12	12	18.5
		30	434	136	17	17	20.4
		35	403	126	16	16	21.8

Note: Input energy at 80 percent efficiency conversion and distribution, based on 20-s station delay or dwell time.

$V_tV_o = 1 - \exp(-t/T_o)$,
$D_t = t/T_o - V_t/V_o$,
$D_1 = D_t + (1/2)(V_t/V_o)^2$, and
$t_1/T_o = t/T_o + V_t/V_o$.

DRAG FORCES OR TRAIN RESISTANCE

Any moving vehicle encounters frictional forces that oppose motion. These are not readily determined analytically, and empirical formulas derived from tests are used to determine these forces. Davis and Dover have derived such relations. The Dover formula for drag force attributable to train resistance (F_d, in newtons) is

$$F_d = mg(0.001\,832 + 0.000\,054\,8V) + AV^2(0.6702 + 0.0095nL) \tag{5}$$

where

m = mass of train (kg),
g = acceleration of gravity (9.81 m/s^2),
V = speed (m/s),
A = area of cross section (m^2), and
n = number of cars L meters in length in the train.

When one introduces the numbers from the train data cited earlier, this becomes

$$F_d = 8627 + 258V + 24.18V^2 \tag{6}$$

KINETIC ENERGY

The kinetic energy in a mass m in motion at a speed V is $(1/2)mV^2 = U_k$, where U_k is kinetic energy (J), m is inertial mass (kg), and V is speed (m/s). For the train here considered, the inertial mass weight is 65 700 kg. This is the largest component of the energy to propel and accelerate. Part of it is recoverable by regenerative braking systems.

ENERGY TO OVERCOME DRAG FORCES DURING "POWER-ON" PERIOD

The product F_dV is the power required to overcome drag forces at speed V. Integrating the expression for this, which can be derived from Equation 6, results in

$$U_a = 8627\int_0^t V_t\,dt + 258\int_0^t V_t^2\,dt + 24.18\int_0^t V_t^3\,dt \tag{7}$$

where U_a is the energy expended in acceleration to overcome train resistance (MJ). The expression for V_t is that of Equation 1, and thus

$$U_a = 8627V_o\int_0^t [1 - \exp(-t/T_o)]\,dt + 258V_o^2\int_0^t [1 - 2\exp(-t/T_o) + \exp(-2t/T_o)]\,dt + 24.18V_o^3\int_0^t [1 - 3\exp(-t/T_o) + 3\exp(-2t/T_o) - \exp(-3t/T_o)]\,dt \tag{8}$$

The first term above is $8672(V_ot - V_tT_o)$ or

$8672D_t$, where D_t is distance at $t = t$. The second term results in

$$258V_o[D_t + (1/2)V_{2t}T_o] \quad (9)$$

and the third becomes

$$24.18V_o^2[V_ot - 3V_tT_o + (3/2)V_{2t}T_o + (1/3)V_{3t}T_o] \quad (10)$$

Thus, for times at which full speed is not attained, we have

$$U_a = 8672D_t + 258V_o[D_t + (1/2)V_{2t}T_o] + 24.18V_o^2[V_ot - 3V_tT_o + (3/2)V_{2t}T_o + (1/3)V_{3t}T_o] \quad (11)$$

Note that V_{2t} is speed attained in a time 2t and V_{3t} is that attained in a time 3t.

If $t = 3T_o$, it is assumed that V_t approaches V_o, and then Equation 11 becomes

$$U_a = 17\,344V_oT_o + 258V_o(2.5V_oT_o) + 24.18V_o^2[3V_oT_o - 3V_oT_o + (11/6)V_oT_o] \quad (12)$$

or

$$U_a = V_oT_o(17\,344 + 645V_o + 44.33V_o^2) \quad (13)$$

RESULTS IN SPECIFIC CASES

Equations 11 and 13 have been applied to specific cases of four balancing speeds, three acceleration speeds, and three run lengths. The results of these calculations are given in Table 2, where

U_a = energy expended in acceleration to overcome train resistance (MJ),
U_f = energy in full-speed portion of run where speed is constant (MJ),
U_r = total energy for the run (MJ), and
t_r = time required for the run.

Table 3 gives the results in a different form that is more convenient for evaluating the time-energy trade-offs under consideration.

ENERGY CONSERVATION

In view of the current need to conserve energy, it will be of interest to review the data presented here to determine what methods of conservation may be of value. As the energy required for propulsion is directly proportional to car weight, it is obvious that a reduction in weight per unit of capacity is desirable. There is another trade-off here, since weight reduction may involve excessive costs or reduced life of equipment. In addition, since it is essential that buffing strength be adequate to prevent danger in collisions or car damage in coupling, there is also a safety factor.

But, since the largest term in the energy account is kinetic energy (U_k), it will be seen that recovery of some fraction of this term by regenerative braking is an effective method of energy conservation. The problem is then what to do with this energy. With direct-current power distribution, the power supply network may be unable to accept such reverse energy flow because of a lack of load from other trains in the power-on condition or excessive increase in line voltage at the regenerating train. This energy can be stored by diverting it into some type of energy-storage device. Two proven storage devices are available: the storage battery and the flywheel. There is also the question of whether these devices should be on board or on the wayside. The flywheel appears to have economic advantages over the battery, and can be on board or on the wayside. The development of this device to a state that permits wide commercial use should be expedited.

It can also be seen from the data in Tables 2 and 3 that the equipment used should be adapted to the station intervals. High-speed cars with short station spacings appear to waste energy. Long station intervals conserve energy, and it may be possible to adopt skip-stop operation where denser areas necessitate short intervals.

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Rationale for Selection of Light Rail Transit for Pittsburgh's South Hills

E.L. TENNYSON

A project to update the 70-year-old South Hills electric railway system in Allegheny County, Pennsylvania, was among the first such projects to be subjected to intense scrutiny as part of a federally mandated alternatives analysis. The rationale of the accepted solution is examined, and the technical process by which consensus was achieved is described. The data used derive from the alternatives-analysis work of the consultants, from regional planning projections, and from the author's observations and experience in the area. The alternatives analysis did not include a final solution for the downtown Pittsburgh traffic problem, but the subsequent review process, based on good data, led to the conception and acceptance of the Sixth Avenue subway.

For more than 70 years, street railway service has been provided to the southern portion of Allegheny County, Pennsylvania, an area known as the South Hills. As of 1980, most of this rail operation is on private right-of-way. This may account, in part, for its success and continued existence. However, 100-year-old bridges, 70-year-old way and power facilities, and 35-year-old cars cannot continue on indefinitely. The leaders of Allegheny County (of which Pittsburgh is the county seat) recognized this and made plans years ago for a more contemporary replacement facility.

Allegheny County currently has a population of 1.7 million; 458 000 reside in the city of Pittsburgh and 114 553 in the suburbs served by the street railway system. As the table below indicates, both Pittsburgh and Allegheny County are in a population decline but the areas served by the