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Method of Allocating Airport Runway Slots

KENNETH E. GEISINGER

Each operation (takeoff or landing) at an airport takes some period of time, referred to as a "slot." Federal Aviation Administration (FAA) regulations set quotas on the number of operations per hour at each of four major U.S. air-carrier airports: Washington National, New York LaGuardia, Chicago O'Hare International, and New York Kennedy International. The runway slots designated for scheduled air carriers are assigned to the various carriers in advance, and airline schedules are built around them. How many slots each airline gets each hour at each airport is determined by mutual agreement among the airlines through airline scheduling committees. These committees have served since the quotas were put into effect in 1969. With the advent of the Airline Deregulation Act, these committees have been questioned as being anticompetitive. If the committees are abolished, their function might have to be performed by FAA. In view of this possibility, FAA is considering several possible approaches. Among them are auctioning of slots, peak-hour pricing, and direct assignment of slots. There are many ways to effect any of these approaches. This paper presents one approach to slot assignment, which was designed to be implementable with as few changes to the current system as possible. The decision criteria consider the current airline requests and constraints (the historic share of the slots) and airline service to the local public in determining which airline gets a contested slot.

Federal Aviation Administration (FAA) regulations designate an upper limit to the number of operations (takeoffs or landings) per hour at four major U.S. airports: Washington National (DCA), New York LaGuardia (LGA), New York Kennedy International (JFK), and Chicago O'Hare International (ORD). The quotas apply only to instrument operations. During good visibility, operations (particularly nonscheduled ones) can exceed the quota. The quota rules (from Code of Federal Regulations, Title 14, Part 93, Subpart K) are shown in Table 1.

The use of the runways for one operation is referred to as a "slot." The runway slots designated for scheduled air carriers are assigned to the various carriers in advance, and airline schedules are built around them. How many slots each airline gets for each hour at each airport is determined through mutual agreement among the airlines through airline scheduling committees. These committees, which consist of representatives of the airlines that serve a particular airport, have served since the quotas were put into effect in 1969. They were granted a special exemption to the antitrust regulations by the Civil Aeronautics Board (CAB).

With the advent of airline deregulation, the possibility that the committees inhibit airline competition has been suggested, and CAB is currently questioning whether these exemptions should be continued. If the committees are abolished, their functions might have to be performed by some governmental authority such as FAA. The administration's proposed 1979 Airport and Airway Improvement Act (S. 1582) would give the Secretary of Transportation the authority to establish allocation procedures.

In view of this possibility, FAA's Office of Aviation Policy is considering several possible approaches. Among them are the auctioning of slots, peak-hour pricing, and administrative assignment of slots. There are many possible ways to do each.

Actions and pricing methods would involve a financial burden on the air carriers, which would be passed on (perhaps inequitably) to the airline passengers. These methods favor airlines that can best afford the slots, which are not necessarily those that would make the best use of them. On the other hand, the assignment of slots opens up the danger of political pressure. Thus, assignment rules must be complete and adhered to firmly.

This paper presents an administrative assignment technique to maximize both passenger service and consideration of the airlines' constraints and requirements. It is hoped that this procedure will take the place of the current procedure with as little disruption as possible and that it will result in an improvement in passenger service.

CURRENT PROCEDURES

The following discussion relates to the assignment of slots to certificated air carriers. Slots are currently assigned to air taxi and commuter carriers by separate procedures, which are less sophisticated. The following is a brief summary of the current procedures:

1. Airline scheduling committees meet twice a year--in July to assign slots for the winter schedule and in January to assign slots for the summer schedule. A separate committee meets for each airport. Additional meetings are called when needed.
2. The airlines submit a request for the number of slots desired each hour of each day of the week at each airport to the reservation center about one month prior to the meeting. The reservation center handles all the bookkeeping involved in the process, both during and between meetings.
3. At the meetings, these requests are whittled down by voluntary concessions from the participating airlines until the quota levels are reached. The committee concentrates on one particular day (when requests are maximum). Other days (different days of the week or different weeks) then generally fall into place. The first step is to reduce the requests to a total number of slots that does not exceed the total available. The second step is to get the airlines to slide their submission so that the number requested does not exceed the quota for any hour. Some airports are easy to resolve (e.g., JFK) and some are difficult (e.g., DCA).

SUGGESTED PROCEDURE

The procedure suggested in this paper also handles one given day for a given airport at a time. It requires as input the number of slots requested in each hour by each airline. It also consists of two steps: (a) allocating a total for the day to each airline and (b) assigning slots by hour to each airline. A schematic diagram of the proposed procedure is shown in Figure 1. The number of slots currently allocated to an airline is recognized by almost every other airline as a valid consideration. It represents an investment made by the airline and a vital interest not to be drastically altered.

Passenger service can be defined in many ways, but the measure suggested here is the average number of passengers enplaned (for departures) or deplaned (for arrivals) per operation. This indicates how many passengers are served for a given slot. Some advantages of this definition are as follows:

1. Data for this measure are available (CAB Form 536);

2. It is based on demonstrated passenger preference;

3. The operations with the highest service tend to be more profitable to the airlines and to the airport operator, except that stage length increases airline profitability but not passenger service;

4. It favors larger aircraft (more service per slot); and

5. It fosters airline competition (more business means more slots).

This procedure also has provision for special exemptions to permit slots to be allocated based on government policy. Examples of this would be honoring international agreements made with foreign carriers and flights that provide essential service to small communities. These must be considered on a case-by-case basis, but the total should be restricted to a small percentage of the total slots.

Increasing passenger service and respecting historic shares of runway slots are somewhat contradictory goals. This procedure allows the balance between the two to be set by a control variable called the reallocation factor. Setting this variable will require some experience and experimentation and an executive decision. It might appear reasonable to disregard current allotments altogether. But this could result in a 10-slot airline getting 100 slots and a 100-slot airline getting 10 slots--a change neither airline could absorb. Even if the airlines could absorb the change, overrewarding one or two airlines would eliminate competition on the next round rather than foster it. Also, the statistics used are based on current schedules and are not valid for gross variations from them.

A realistic but hypothetical example will show how the procedure works. It is based on actual requests for slots at DCA submitted for August 1979. For practical purposes, there are about 620 usable slots during the day: 40 slots/h between 6:00 a.m. and 10:00 p.m. and 20 slots/h at 10:00 p.m. (because of low airline demand for this hour and a voluntary jet curfew after 10:00 p.m.).

The hypothetical allocation is shown in Table 2.

Table 1. Quota rules.

	Instrument-Flight Operations per Hour			
Class of User	DCA	LGA	JFK	ORD
Certificated air carrier	40	48	70-80[a]	115
Scheduled air taxi and commuter	8	6	5	10
Other	12	6	5	10
Total	60[b,c]	60[c]	80-90	135[c]

Note: Hours in force: DCA, all day; LGA, all day; JFK, 3:00-8:00 p.m.; ORD, 3:00-8:00 p.m.

[a] 70/h between 3:00 and 5:00 p.m.; 80/h between 5:00 and 8:00 p.m.

[b] Does not include charter flights or other nonscheduled flights of scheduled or supplemental air carriers.

[c] Does not include extra sections of scheduled air-carrier flights.

Figure 1. Schematic diagram of proposed slot assignment procedure.

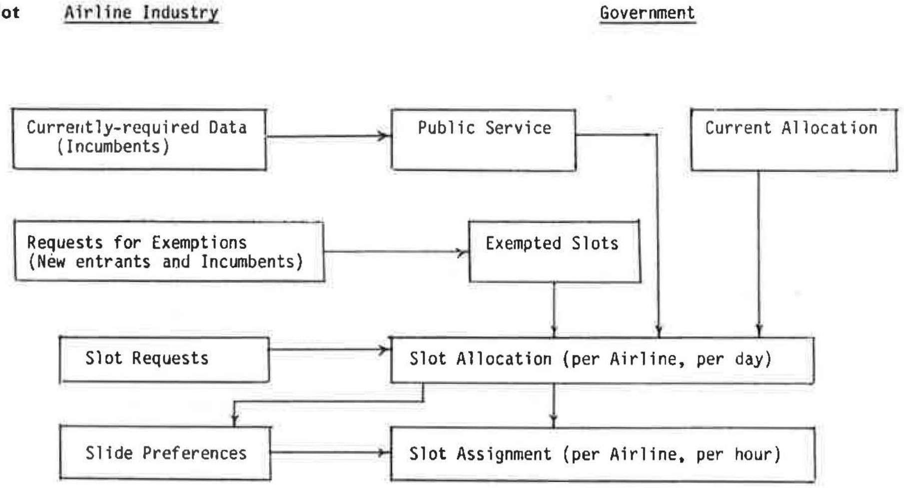

Table 2. Hypothetical slot allocation for DCA on weekdays, August 1979.

				Computation of Slot Allocation					
Airline	Current Slots	Requested Slots	E+D per Operation	Base	FM	Δ	Raw	Share	New
A	74	63	70.3	37	2 601	37	74	0.119	64
B	82	78	50.0	41	2 050	29	70	0.113	70
C	22	28	51.5	11	5 665	8	19	0.031	20
D	34	34	88.5	17	1 504	21	38	0.061	34
E	142	144	64.5	71	4 580	64	135	0.218	140
F	48	46	58.5	24	1 404	20	44	0.071	46
G	42	42	70.8	21	1 487	21	42	0.068	42
H	68	72	46.8	34	1 591	22	56	0.090	58
I	40	44	81.6	20	1 632	23	43	0.069	44
J	68	70	68.6	34	2 332	33	67	0.108	70
K	0	6	NA	6	0	0	6	0.010	6
L	0	6	NA	6	0	0	6	0.010	6
M	0	4	NA	4	0	0	4	0.006	4
N	0	8	NA	8	0	0	8	0.013	8
O	0	4	NA	4	0	0	4	0.006	4
P	0	4	NA	4	0	0	4	0.006	4
Total	620	653		342	19 748	278	620	0.999	620

Column 1 shows the current allocation and column 2 shows the number of slots requested. Column 3 shows the enplanements and deplanements (E + D) per operation for each incumbent carrier (average for all DCA operations). The new entrants have no passenger service history and are given a special exemption for a duration (assuming that their requests are reasonable). In general, incumbent carriers could qualify for both regular and exempted slots.

Computations begin with column 4, in which a base number of slots is allocated to each carrier by multiplying the current share by the reallocation factor (0.50 was used in this example). Exempted slots are added directly to the base. The product of columns 3 and 4 forms a figure of merit (FM), shown in column 5. The number of base slots is 342, which leaves 278 of the 620 slots to be allocated. This is accomplished by taking the fraction of FM to the total of FM times 278. This yields an increment (Δ) to the base (column 6). The base plus the increment form the raw allocation to each carrier (column 7). The fractional fair share of the total slots is obtained by dividing by 620 and is shown in column 8. The final allocation is shown in column 9. Here, slots allocated in excess of requests are redistributed proportionally to other carriers. For DCA the final allocation is rounded to an even number, since the quota period covers the whole day and every flight in is matched to a flight out.

Several observations can be made for this procedure:

1. It is perfectly general and can apply to any airport.
2. Slots are assigned based on a balance of historic share and local passenger preference. Since this process is iterated every six months, carriers would be encouraged to adjust their schedules, fleet mix, or both to improve passenger service or face a possible loss of slots on the next round.
3. An airline cannot gain more slots simply on the basis of asking for more nor can an airline retain all of its slots simply because it has had them.
4. If no more slots are requested than are available, this procedure will allocate to each airline the number requested.
5. Even if an airline has a low public service rating compared with others, it will not lose as many slots as its fair share would indicate because of limited requests by higher-rated carriers.

ASSIGNING SLOTS BY HOUR

Once the allocation of total slots for the quota period has been determined, the next step is to assign a given number of slots to each airline for each hour.

Table 3 shows the DCA slots requested at a special meeting in April 1979 for August 1979 by each airline for each hour at DCA. Note that for some hours the number requested is greater than the quota and for some hours the number is less than the quota. Even when the total allocation is reduced to 620, it will be necessary to slide some of the requested slots from one hour to another.

One of the best features of the current procedure is that the negotiations are conducted by airline officials with the ability and the authority to adapt their tentative schedules to accommodate slides. In many cases this can be done with little inconvenience to the airline (e.g., when an operation is planned near the beginning or end of an hour). In other cases this requires a considerable sacrifice and a slide is offered only when competing airlines have made suitable concessions. One disadvantage of the current system is that, since the decisions must be unanimous, stubbornness is rewarded by both the quantity and the placement of the slots obtained.

Some of the participants come to the meeting with preplanned slides to offer, and others consider revisions to their schedules during the course of the meeting. Under this procedure it will be necessary to have all airline slide offerings available simultaneously so that they can all be considered together and an assignment can be selected that maximizes the total benefit.

Each airline will be required to submit a package of proposed slides. The form of this submission has yet to be determined, but the net result would be similar to the hypothetical submission by one airline shown in Table 4, assuming that it is given 28 slots for the day (as it was by the committee). Each row in the table corresponds to the distribution of slots that could result from one or more slides.

The total number of slots must not exceed the quantity allocated. The airline assigns a value from 1 to 100 to each row to indicate the relative desirability of that distribution compared with the others. Row 1 is the distribution originally requested; therefore its value is presumed to be 100. As the row number gets higher, less-desirable slides are listed. Row 16 corresponds to slots actually used; this shows a rather large deviation from the original request, which evidently would have also been acceptable. It is quite possible to have two or more distributions given the same value, which indicates alternatives that are equally desirable.

The first step is to examine the slide preferences to see whether a feasible solution exists. A feasible solution would consist of a set of slots (one set from each airline) distributed so that the total slots in each hour did not exceed the quota. If one or more feasible solutions exist, the solution is chosen that maximizes the sum $\Sigma_i(F_i/A_i)(S_i/Z_i)V_i(k_i)$ in which i is the airline index, A_i is the number of slots allocated, F_i is the fair share, S_i is the number of slot-distribution choices provided, Z_i is the fair number of distribution choices, and $V_i(k_i)$ is the airline's evaluation of slot distribution k_i. The selected slot distributions indicated by k_i form the solution set.

The objective function shown above accomplishes three objectives:

1. It considers the slot-distribution preferences of the airlines.
2. If a decision has to be made between airlines for a preferred choice, an airline that received fewer slots than its fair share relative to other airlines will be given an advantage.
3. The airlines that have offered more slot-distribution choices relative to the number of choices they should have provided will be given an advantage.

The fair number of slot-distribution choices should be based on the number of slots allocated. A reasonable value is given by $Z_i = 2\sqrt{A_i}$. To prevent an airline from submitting a large number of slot distributions that differ from each other only in the off-peak hours and offer no help in the problem hours, some criteria could be adopted to determine whether a suggested distribution should be counted in the S_i.

Table 3. Actual slot submission for DCA, August 1979.

	Hour[a]																	
Airline	06[b]	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22[b]	Total
A	0	7	3	4	3	4	5	4	2	5	4	4	3	6	3	4	2	63
B	0	3	9	8	2	3	4	6	7	2	4	6	5	4	6	5	3	78
C	0	1	1	2	0	2	2	3	2	2	3	2	2	2	2	2	0	28
D	0	3	2	2	2	2	2	2	2	2	2	4	2	2	3	0	2	34
E	0	11	12	7	10	9	9	7	10	8	9	7	10	9	9	11	6	144
F	0	1	1	4	5	6	1	2	5	5	2	4	3	2	3	2	0	46
G	0	3	2	2	2	1	4	2	2	4	3	2	2	3	3	3	4	42
H	1	3	3	5	5	4	6	4	4	4	5	5	6	3	4	5	5	72
I	0	2	2	3	3	2	3	2	3	2	3	2	4	2	3	5	3	44
J	0	4	4	4	7	8	1	7	4	3	2	4	6	6	3	3	4	70
K	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	1	6
L	0	0	1	1	0	0	1	1	0	0	0	1	1	0	0	0	0	6
M	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	4
N	0	0	2	0	0	0	2	0	0	0	2	0	0	2	0	0	0	8
O	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	4
P	0	1	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	4
Total	1	40	43	43	40	42	42	40	41	38	40	42	45	43	41	42	30	653

[a]Example: 15 = 3:00-3:59 p.m. [b]No turbo-jet operations before 7:00 a.m. or after 10:00 p.m.

Table 4. Hypothetical slot slide submission, Airline C.

		Hour																
No.	Value	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	Total
1	100	0	1	1	2	0	2	2	3	2	2	3	2	2	2	2	2	0	28
2	99	0	0	2	2	0	2	2	3	2	2	3	2	2	2	2	2	0	28
3	85	0	0	2	1	1	2	2	3	2	2	3	2	2	2	2	2	0	28
4	85	0	1	1	2	0	2	2	3	3	2	2	2	2	2	2	2	0	28
5	70	0	1	1	2	0	2	2	3	4	2	2	2	1	2	2	2	0	28
6	70	0	0	2	2	0	2	2	3	4	2	2	2	1	2	2	2	0	28
7	60	0	1	1	2	0	2	2	3	3	3	2	2	1	2	2	2	0	28
8	55	0	0	2	2												2	0	28
9	50	0	0	1	2	·	·	·	·	·	·	·	·	·	·	·	2	0	28
10	50	0	0	1	2	·	·	·	·	·	·	·	·	·	·	·	2	0	28
11	50	0	0	1	2	·	·	·	·	·	·	·	·	·	·	·	2	0	28
12	45	0	1	1	2												2	0	28
13	45	0	1	1	2	0	2	2	4	3	2	2	3	1	1	2	2	0	28
14	45	0	1	1	2	0	2	2	4	3	2	3	2	1	1	2	2	0	28
15	40	0	1	1	2	0	2	2	3	4	2	3	2	1	1	2	2	0	28
16	40	0	1	1	2	0	2	2	3	4	3	2	2	1	1	2	2	0	28

Note: No. 1 was actually requested; nos. 2 through 15 are hypothetical; no. 16 was actually flown, August 10, 1979.

If no feasible solutions exist, the airlines could be offered another chance to provide more flexibility in their slot distributions. If this fails, slots could be administratively deleted based on criteria similar to the airline preference criteria already presented.

After this process has been completed, some slots may remain unused. Applications for these slots can be entertained, and preference can be given to airlines who have received less than their fair share compared with other airlines.

CONCLUSION

The procedure described here is preliminary and is subject to change. It has been programmed in FORTRAN, and experiments are being conducted to see how the process would react to realistic slot requests at different airports and for different values of the reallocation factor. Procedures for handling exemption requests and slide submissions are being developed. This procedure was designed for certificated air carriers. It could not be applied to unscheduled operations. It might be applicable (perhaps with some modifications) to commuter services, the present system for which is based on a waiting list and not on a periodic reassignment. That system, which is probably less fair than the present certificated air-carrier system, rests on the approval of FAA and not CAB. It should be emphasized that the method proposed in this paper is only one solution to the problem (if, indeed, it even becomes FAA's problem).

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I wish to acknowledge the guidance and direction of Scott Sutton, Acting Director of the FAA Office of Aviation Policy, who initiated this study, and to other individuals who offered helpful suggestions--Harvey Safeer, Walter Faison, John Wesler, and John Rodgers. I also wish to thank Walter Coleman of the Airline Reservation Center for his courtesy and for the information he provided. The proposal presented here is only one approach being considered and does not represent an official FAA position.

Discussion

John R.G. Brander

A basic problem in the provision of transportation infrastructure is the temporal variation in demand coupled with a capacity that is fixed in the short

Table 5. Results of two methods of slot allocation among incumbent carriers.

	Raw Allocations		Final Allocations	
Airline	Geisinger	Brander	Geisinger	Brander
A	74	67	64	64
B	70	63	70	78
C	19	33	20	28
D	38	55	34	34
E	135	99	140	114
F	44	49	46	46
G	42	51	42	42
H	56	54	58	68
I	43	55	44	44
J	67	63	70	70

run. Given this peak-demand problem, there arises the question of how the available capacity can best be allocated among users. It would be generally agreed that there exist three demand-management techniques that might be applied in these circumstances. There are peak-hour pricing, the auctioning of available capacity, and the development of artificial allocation mechanisms. In his paper, Geisinger focuses on the third of these mechanisms; he dismisses the first two on the grounds that they would have undesirable side effects on the existing carriers that serve a given airport. It is my intent here to discuss these issues.

By way of introduction, it may be instructive to inquire what a particular airline is demanding when it asks for a particular slot at a particular airport. Implicitly, Geisinger suggests that it is seeking to carry out a single operation in isolation. There is, however, much more involved. This is a true case of joint demand in the economic sense. The demand for a takeoff slot at one airport leads automatically to the demand for a runway landing slot at some other and for space on the airways that link the two together. This fact must be incorporated into the analysis. The initial criticism of the Geisinger approach, therefore, is that it is too narrow and does not meet the needs of the air transportation system as it now exists.

ALLOCATING SLOTS TO AIRLINES

The Geisinger approach involves a two-stage allocation procedure. The initial step allocates the total daily slots among the airlines. The second step adjusts for peak-hour problems. Both need to be discussed. However, the whole issue cannot be divorced from the question of the deregulation of air transportation. Deregulation is predicated on the assumption that market forces are better able to allocate resources in air transportation than is some government agency. The suggested allocative mechanism appears to substitute one form of government regulation for another. More specifically, as will become clear below, the Geisinger approach to slot allocation is overly protective of the existing carriers and discriminates against the entry of new competitors.

With respect to the initial step in the process, the major problem is that Geisinger's attempt to balance historic shares with local passenger preference results in a serious bias in the allocations. More specifically, there is a bias in favor of those airlines that have a large number of current slots but only a moderate enplanement plus deplanement (E + D) per operation. In contrast, airlines that have a higher E + D per operation but a much smaller current allocation suffer. An examination of Geisinger's Table 2 makes this clear (specifically, compare Airline D with Airline E). This bias is created by giving double weight to the current slots held--the first time when computing the base allocation, the second time when computing the change.

Changing the factor of merit, the means by which nonbase slots are allocated, significantly alters the raw allocation of runway slots. In order to test this hypothesis, the nonbase slots were reallocated among the existing airlines by using only the E + D per operation. A χ^2-test was then applied to the raw allocations shown in Table 5. The calculated χ^2 was 32.90 with 9 df. At the 0.005 level of significance, the critical value of χ^2 is 23.59. The conclusion is that the two raw allocations differ significantly. The difference is caused by the double use of the current slot allocations.

The final allocations determined by using both approaches are also set out in Table 5. As one might suspect, these two distributions are not significantly different, largely because of the upper limits placed on allocations to individual airlines by their total requests.

The second part of the allocation mechanism involves slot slides to ensure that hourly runway capacities are not exceeded. Geisinger suggests that a reasonable number of slides would be twice the square root of the number of slots allocated to each airline. However, this procedure introduces another bias in favor of those carriers that currently possess a large number of slots. The problem is that as the number of slots held increases, the proportion of required slides to be offered to slots held declines. By using Geisinger's approach, Airline E receives 140 slots and must offer 24 slides, or 17 percent of its slots. However, Airline C, which receives only 20 slots, must offer 9 slides, or 45 percent. Because of the small number of slots received, the new entrants must offer still higher percentages of their allocations as slides. Airlines M, O, and P have a slide/slot ratio of 1.0.

Finally, there are intertemporal problems with which the approach in its present form cannot come to grips. Implicit in the allocation mechanism is the assumption that the E + D per operation is determined by factors exogenous to the model. This, however, is only partly true. It must be admitted that a major determining factor here is the route density of the average route flown by a given airline. However, at least one endogenous factor is involved as well. There is ample evidence (which need not be reviewed here) that, other things being equal, load factors--and therefore E + D per operation--are sensitive to time of day. This fact coupled with the biases noted above in favor of larger operations increases the probability that such airlines will receive relatively larger proportions of the preferred slots and must be viewed as essentially anticompetitive.

All these shortcomings could be dealt with, at least in part. Both components of the allocation mechanism could be modified to meet these criticisms. It seems preferable, however, to explore the mechanisms that were rejected by Geisinger.

OTHER ALLOCATION MECHANISMS

Peak-Hour Pricing

The first of the rejected approaches to be considered is peak-hour pricing. The theory of peak-hour pricing has received considerable attention in the literature and involves setting the price equal to the marginal social cost in order to ration available capacity. So viewed, the price at peak periods is composed of two elements. The first is a basic user fee charged at all times, the purpose of which is to finance the infrastructure involved. The second is a congestion tax, which is set at zero during periods of low use and rises as the volume/capacity ratio increases.

Peak-hour pricing suffers from one of the major deficiencies of the mechanism discussed above: It typically considers a single facility in isolation from the remainder of the system. It could be applied to links in a transportation system by a process of aggregation. In such a scheme, the prices of each component in a given link would be separately estimated and the results added together. However, such a practice would be both cumbersome and costly, assuming that the marginal social cost involved could be estimated at all. As a consequence, this approach must be rejected as well.

Auctioning of Slots

The final approach to be considered here is auctioning of slots. The first step in considering it is the introduction of the Marshallian concept of quasi-rent (1, p. 74). Stigler (2, p. 95) argues that

> the theory of quasi-rents is essentially the explanation of the return on what is called fixed (overhead) investments. Once capital has been invested, it will remain invested until it can be depreciated through use and its salvage value, and throughout its service life, it will continue in that use regardless of its return. . . . The earnings of the fixed investment are price-determined in the short run and thus partake of the nature of rent. In the long run, however, they must be covered or capital will leave the industry.

The relationship between quasi-rents and airport slots is straightforward. The larger the number of such slots is and the more desirable the slots held are, the larger will be the quasi-rents earned. At the same time, it must be noted that the airlines possess all the requisite information to allow them to compute these quasi-rents on a link-specific basis. It need hardly be noted that this calculation can also be done for specific aircraft types (3). [I have recently given an example of the relationship between highways and site rent and a justification of the approach (4).] Geisinger suggests that airlines can place values on different slot combinations and, in so doing, they would presumably use precisely these data.

With an auction system for slot allocation, each airline would submit a bid for each slot in which it was interested. These bids, in turn, would presumably be based on the quasi-rent that could be earned. A given slot would be awarded to the airline that submitted the highest bid. In order to offer revenue security to the airport operators, an upset price could be attached to each of the slots. Any slot not allocated in the initial round could be auctioned a second time, and the process could continue either until all airlines were satisfied or until all slots were allocated.

Such a system of slot allocation seems to offer a number of advantages. First, it reflects the nature of the joint demand for facilities and the nature of the industry, since it is based on the concept of links in a system rather than operations at a single airport. Second, it does not protect the existing carriers at the expense of new entrants into a market. Third, it avoids the necessity of developing an artificial slide mechanism, for slots are allocated uniquely except for the possibility of identical bids for a given slot. In this case, a retendering would take place. Fourth, it, like the approach Geisinger attempts to develop, takes account of passenger preference (at least as far as these are reflected in the quasi-rents). Finally, and this is important in a period when deregulation is being considered, it places primary reliance on the forces of the market. A major concern with the Geisinger approach must be that it substitutes one form of regulation for another. One is driven to the conclusion that the auction approach to runway-slot allocation offers overwhelming advantages.

A postscript is necessary. Geisinger opposes the auction approach on the grounds that it does not guarantee the best use of slots and that it might cause inequity. The former is not true. The latter is irrelevant.

REFERENCES

1. A. Marshall. Principles of Economics, 8th ed. Macmillan, New York, 1920.
2. G.J. Stigler. Production and Distribution Theories. Macmillan, New York, 1941.
3. N.K. Taneja. The Commercial Airlines Industry. Heath, Lexington, MA, 1976.
4. R.G. Brander. Highway Finance: Some Vicissitudes. In Proc., International Conference on Transportation Research (Bruges, Belgium, June 1973), Transportation Research Forum, Chicago, 1974, pp. 897-903.

Author's Closure

Brander raises some interesting points that deserve some further discussion. First, it would be well to review some developments that have occurred since my paper was written.

The proposed administrative method has been revised as follows:

1. The measure of passenger service has been modified to include locations served as well as passengers served. The locations served per slot is defined as the number of different airports served by the airline by direct flight to or from the airport in question divided by the number of slots. Both factors are combined into a single measure through the use of weighting factors. This balances the original measure's preference toward large aircraft that serve dense markets with a preference toward airlines that serve many small communities with few slots.
2. Some guidelines on acceptable distribution choices were developed; these included a limit on the number of slots an airline could request in any hour (based on the airline's share of slots for the day).

In February 1980, FAA conducted a test of the revised administrative allocation and an auction method that handles all quota airports simultaneously (somewhat similar to Brander's proposal). The test involved airline schedules and a computerized airline-management game. The airline schedulers ran a set of five simulated airlines that served 16 simulated airports. No conclusive findings were reached about which method was superior; both proved workable and some valuable lessons were learned.

FAA is preparing a draft Notice of Proposed Rule Making (NPRM), which will present both the administrative method and the auction method for comment. This should have been released by the time that this article is published.

Brander's points will now be discussed in the order in which they were raised.

1. Slots used at one airport are linked to slots used at other airports. Therefore, the suggested procedure is too narrow and does not meet the needs of the air transportation system as it now exists.

It is true that every scheduled operation at a quota airport is linked to a scheduled operation at some other airport. But only four airports have quotas, and for two of those the quota applies only during 5 h of the day. Thus, the majority of flights that require a slot at one end do not require a slot at the other end.

However, the problem of getting slots at both ends of some flights does exist. The existing scheduling committees (which have met the needs of the industry for over 10 years) solve the problem in this way. The quota airports are handled sequentially, beginning with the hardest to resolve (DCA) and ending with the easiest to resolve (JFK). Usually, the schedule at DCA is not completely resolved in the time allotted. In that case, the DCA committee resumes negotiations after the other airports' schedules have been resolved. In any case, there is provision for turning in unusable slots and obtaining unused slots after negotiations are closed. I would handle this problem in just that way.

2. The Geisinger approach to slot allocation is overly protective of the existing carriers and discriminates against new entrants.

New entrants do not compete with incumbents but are given a set of slots by exemption. The current thinking is that four slots would be a reasonable number. It could be 8, 16, or 100. The process itself does not discriminate.

3. There is a bias in favor of airlines that have a large number of current slots.

Yes, there is such a bias. In fact, if we neglected exempted slots and if all airlines had an equal measure of passenger service, they would all get their current allocations. The current allocation is the starting point and deviations are made only as passenger service differs and then only in modest amounts. The airlines with more slots do risk losing a larger number of slots in the process, however.

The reasons for this bias are as follows: (a) The current allocation is recognized as an investment that an airline has made in developing markets and providing service capacity; (b) the measure of service is an average made over the current schedule and is not valid for gross variations (many more or many fewer slots) from that schedule; and (c) turbulence caused by sudden and drastic changes in allocations would be harmful to everyone.

Nevertheless, if service differentials persist, significant changes in allocations could occur after repeated applications of the procedure (every six months).

4. Airlines that have a large number of slots have to propose fewer alternative slot plans proportionately than do airlines that have a small number of slots.

This is true. Moreover, the number of variations mathematically possible increases much faster than linear proportion to the number of slots. But the problem is that preparing alternative slot plans is a great burden to the airline schedulers, and the airlines that have many slots are faced with serious real-life constraints that counteract their supposed flexibility. FAA tests revealed the need to ask for as few alternatives as possible.

5. Airlines that have many slots will receive a disproportionate share of slots during the prime hours and get an advantage in increasing their service measure.

A limit is now placed on the number of slot requests that each airline can make in any hour. This limit is proportional to the total number of slots allocated to the airline. FAA tests revealed that this limit should be applied only for the problem hours.

6. A slot auction offers overwhelming advantages.

The objective of this paper was not to debate the relative merits of alternative allocation methodologies but rather to set forth one of many alternatives and stimulate public discussion thereof.

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Method for Forecasting General Aviation Activity

FRANK R. WILSON AND HAROLD M. KOHN

This paper describes a study of the method used to develop demand-estimation models for itinerant and local movements of general aviation aircraft. The study area consisted of seven airports in the Maritime Provinces of Canada (New Brunswick, Nova Scotia, and Prince Edward Island). Confidential data on aircraft movements were made available from the Aviation Statistics Centre for this study. Econometric models were developed for each airport separately, and one system model was developed for all traffic that flows on the 49 links between the seven airports. The approach used generation-distribution-type models in contrast to the pure generation models attempted by others and found to be only marginally successful. Cross-section demo-

graphic, economic, and system data for the base year 1975 were used. The adequacy of these models is analyzed in a series of statistical and intuitive tests. The model calibrated on link flows produced marginally acceptable results. Although the model is not recommended for detailed planning, it represents the first known attempt in Canada to calibrate, by using actual data, a model to forecast general aviation activity. In this context the work can be considered a departure point for the development of general aviation forecasting techniques for the Canadian transportation environment.

The movement and storage of aircraft, their servicing, and the handling of passengers and cargo associated with them are assumed to be the principal functions of an airport. Other land uses are complementary or supplementary. Operations in the airspace adjacent to and at airports that have a large passenger volume cater to the large aircraft that operate scheduled air services. However, a significant portion of the aviation system service demands associated with these airports is composed of other types of aircraft activity.

Considerable attention has been directed to the forecasting of the service demands that arise from commercial airlines operations, which is understandable because this activity produces the major economic impact of civil aviation. Although they do not produce the reliable results often desired, current techniques and estimates of future passenger demand are nevertheless the aspect of airport activity forecasting that appears so far to be accepted most readily by planners. These techniques also give some indication of the price elasticity of passenger traffic.

Forecasts of the demand for goods movements by air are generally not considered as reliable as those for passenger demand because of the close coupling of service supply to the passenger demand. It has been well established that air-cargo service availability depends on surplus aircraft space on passenger flights. Pricing practices tend to stimulate the market for air cargo in areas in which such surplus space exists. Therefore, the independent predictions of air-cargo service are generally not so reliable as are forecasts in which all cargo operations can be identified.

The most-difficult (and apparently the least-reliable) aircraft activity forecasts are those related to general aviation. Despite rising fuel prices and the general economic slowdown in recent years, the general aviation industry in North America has continued to grow and prosper. In 1978, U.S. manufacturers delivered about 18 000 general aviation aircraft that had a value of $1.78 billion. This represented 19.2 percent more billings and 5.3 percent more unit output than in 1977 (1). Canada imported slightly more than 500 general aviation aircraft in 1978. Canadian ownership of general aviation aircraft has grown phenomenally: It has risen at a rate of slightly more than 12 percent per year from approximately 2150 in 1960 to 15 000 in 1977.

Many firms and institutions in Canada own and operate general aviation equipment. It is estimated that approximately 100 corporations own jets and that there are hundreds of corporately owned piston- and turbo-powered propeller craft. There are literally thousands of other privately owned jets and propeller-driven aircraft (2).

There are, of course, a large number of smaller aircraft owned and operated purely for the joy of flying. Expense does not appear to be a major consideration. The joy of flying has contributed to the sales of aircraft to individuals and to flying clubs; flying is as much a sport and hobby as is stamp collecting and photography.

However, the economic gains and growth of the general aviation industry have also produced additional pressures on air-traffic control, safety, and airport congestion. More than half the civil aviation activity in Canada consists of general aviation. At Malton and Dorval International Airports, for example, general aviation accounts for roughly one-third of all itinerant movements. Several major Canadian and U.S. airports have become congested with both air carriers and general aviation to the level that a series of satellite airports have been built exclusively for use by general aviation. Buttonville Airport serves this purpose in the Toronto area, Pitt Meadows in the Vancouver area, and White Plains in the New York City area. At other Canadian airports, general aviation accounts for the majority of itinerant aircraft movements.

Increasing pressures on the existing and future aviation infrastructure dictate that reliable forecasts be made available to planners. Not only is it important to provide adequate airport and airside capacity, it is also vital that adequate safety standards be maintained in congested areas in which there exists a mix of large commercial aircraft and the smaller, slower, and increasingly greater numbers of general aviation aircraft.

This discussion has so far centered on forecasting itinerant aircraft movements. Local movements around an airport are normally even larger than itinerant movements. If an airport has a flying school or club, a large number of aircraft movements take place locally. These, together with the itinerant aircraft movements, make general aviation the largest segment of civil aviation in terms of volume. Table 1 contains data on local movements for various years at selected airports in the Atlantic Provinces of Canada compared with itinerant movements for base year 1975.

PREVIOUS WORK

There has been a limited amount of work done on developing general aviation forecasting models. These have been completed primarily in the United States by the Federal Aviation Administration (FAA) and there have been several works published in the literature. The usual approach has been to develop time-series trip-generation models on an individual-site basis. In Canada, most general aviation forecasts have been no more sophisticated than time-trend analysis.

Two studies that were carried out by FAA represented the most extensive econometric modeling exercise that pertained to general aviation at that time. These two studies were entitled Forecasting General Aviation Activity at Federal Aviation Administration Facilities: An Econometric and Time Series Analysis, by A.M. Schwartz, and A Recursive Forecasting Model of General Aviation Activity Levels with Policy Implications for Alternative Cost (Fuel) Scenarios, by J.E. Tom and S.G. Vahovich (these reports are not available to the public). These models employ a complex multiequation, simultaneous-regression, two- and three-stage, least-squares econometric model to forecast general aviation. This technique permits more explanatory variables to enter the model and equations, each of which may describe a certain behavioral aspect. These can then react with each other in a cause-and-effect relationship.

The Schwartz model was used as a basis for the development of the model described in this paper. Unfortunately, the Schwartz model cannot readily be calibrated in Canada due to the difficulty of collecting data for many of the variables.

Table 1. Number of local aircraft movements for various years versus itinerant movements for 1975.

Airport	Itinerant Movements, 1975	Local Movements				
		1975	1974	1973	1972	1971
Charlottetown	11 559	18 652	4 008	5 647	10 782	7 134
Fredericton	20 763	31 059	23 957	19 711	15 466	20 168
Halifax	15 657	35 947	39 428	32 777	30 360	51 725
Moncton	22 599	81 632	79 871	70 788	57 574	57 480
Saint John	16 356	27 603	27 518	15 555	14 771	10 608
Sydney	6 913	9 196	9 646	9 096	7 296	2 276
Yarmouth	5 187	–	672	1 090	39	96
Bathurst	375	3 863	753	1 199	9 475	189
Charlo	1 447	4 760	3 077	3 281	2 770	2 205
Deer Lake	3 139	41	102	–	2	16
Edmunston	559	1 257	94	702	2 292	2 443
Stephenville	3 472	363	3	4	62	2
St. John's	8 461	11 716	7 364	14 496	15 895	19 467
Gander	14 169	8 634	10 735	13 139	15 809	36 768

DEFINITION AND DATA BASE

In both the industry and the literature, there is considerable confusion and controversy about the definition of general aviation. There is no standard or universally accepted definition for the term. The difficulty in arriving at a mutually acceptable definition is often the result of different study objectives.

For planning purposes, general aviation must be defined as all civil aviation other than scheduled and charter operations. Scheduled-aviation space requirements are readily determined. One normally excludes charter from general aviation for two main reasons: (a) the majority of this traffic is, in effect, scheduled and (b) it is not thought of conceptually as typical general aviation. Military aviation is excluded since this sector of aviation generally frequents military bases and uses military navigational facilities. Of course, there are several airports (e.g., Fredericton in New Brunswick) that do experience military activity because of their proximity to major Canadian Air Force bases. However, the military's use of civilian facilities is limited, so studies on general aviation have excluded this component.

Despite the fact that movements of aircraft are being forecast, it must be remembered that, in essence, it is movements of people that should be forecast. Load-factor data are readily available for translating scheduled and charter passenger forecasts into aircraft movements, but these data are not available for general aviation.

Confidential data were made available from the Aviation Statistics Centre (ASC) in a form consistent with the use of the preferred trip-generation-distribution approach. The data in this paper are an aggregate of the raw data, and it is not possible to trace any particular aircraft movement to a specific time, place, or owner. Therefore the confidentiality of the data has been maintained.

The raw data made available from ASC were computer outputs in which each airport in Canada that has a control tower is included. Each airport report consists of two parts--one that lists aircraft trips to the airport and one that lists trips from the airport. It is based on a last-stop, next-stop system. These are not entirely true origin-destination (O-D) data, but they were the only data available with respect to where the aircraft were operating. Nonetheless, the reliability or workability of the model is not jeopardized, because the system is being described in terms of aircraft flow between airports. Furthermore, a large percentage of general aviation trips are short, because the aircraft are generally small and have limited range. Therefore the last-stop, next-stop data are significantly close to true aircraft O-D data.

The data base for each airport is extensive. For this reason, it was decided to limit the itinerant-model development work in the study to seven Maritime Province airports--Charlottetown, Fredericton, Halifax, Moncton, Saint John, Sydney, and Yarmouth--for the base year 1975.

Aircraft movements to and from airports were broken down by aircraft type and further disaggregated into 21 classifications. Each aircraft movement was classified by sector and category.

In choosing the data to be used in the study, scheduled, charter, and military movements were eliminated. Approximately 20 000 data entries were used. For the aircraft that leave from the seven test airports, the movements were summed for each node pair by aircraft type and by travel sector within each classification. The process was repeated for the aircraft that arrive at the seven airports, and the two totals were added. This produced a two-way trip table of flows for each node pair.

The most-disaggregate level of data collection produced 375 nodes scattered throughout North America and Europe. By using the Financial Post magazine's survey of markets, 70 catchment areas were defined. The catchment area of an airport was defined as the major geographical area from which the airport attracted business. O-D trip tables for each of the 70 areas were produced, and the data were used for model calibration.

In the development of the model, forecasts were attempted for two types of aircraft movements--itinerant and local. Within the itinerant class, a model was built for each of the seven airports as well as one model that incorporated all airports. The latter model was an attempt at forecasting on a systems basis (i.e., on a node-to-node basis).

Since data on economic variables were available on a metropolitan basis for major Canadian cities and their respective airports as well as for the catchment areas, two sets of calibration data were collected. One set used catchment-area data for each node, whereas the other replaced the catchment-area data with metropolitan-area data for those nodes at which these data were available. The latter data set therefore used data from both catchment and metropolitan areas.

Several of these catchment areas contained major metropolitan areas that influence an airport, and there could exist other airports within the greater area. For example, British Columbia as a provincial catchment area remained unchanged in both data sets. The Moncton catchment-area population was greater than that of only the metropolitan area, and therefore it consisted of the geographic area that made up its catchment area. In the combined data set, the Moncton metropolitan-area population replaced the catchment-area population.

MODELS

By using the FAA model as a starting point, several formulations of the model were tested based on the availability of data for the independent variables. The initial formulation of the models tested is presented below in Equations 1, 2, 3, and 4. The final models selected are presented in Equations 5 and 6 (the variables used are listed in Table 2).

Equations 1 and 2 present the initial models

Table 2. Variables used in models.

Variable	Definition	Variable	Definition
LCL	Local general aviation aircraft movements	USD	Dummy variable for United States
ITN	Itinerant general aviation aircraft movements (two-way total flow for each node pair)	FD	Dummy variable for foreign areas
i,j	Origin (i)-destination (j)	APD	Dummy variable for Atlantic provinces
a,b	Coefficients	AD	Dummy variable for seven maritime airports
BA	Based aircraft	DIST	Distance
W	Weather index	CPOP	Population of metropolitan and catchment areas
FSD	Dummy for flying schools	CINC	Per-capita income of metropolitan and catchment areas
POP	Population of catchment area	CIMR	Market-rating index of metropolitan and catchment areas
INC	Per-capita income of catchment area	CRS	Per-capita retail sales of metropolitan and catchment areas
RS	Per-capita retail sales of catchment area	CII	Income-rating index of metropolitan and catchment areas
IRI	Income rating index of catchment area		
MRI	Market-rating index of catchment area		
TOAD	Dummy variable for type of airport		

developed based on catchment areas. The model for local general aviation aircraft movements on a node basis is as follows:

$$\mathrm{LCLi} = a_1 + b_1\mathrm{BAi} + b_2\mathrm{Wi} + b_3\mathrm{FSD} + b_4\mathrm{POPi} + b_5\mathrm{INCi} + b_6\mathrm{RSi} + b_7\mathrm{IRIi} + b_8\mathrm{MRIi} \tag{1}$$

The model for itinerant general aviation aircraft movements on a link basis is as follows:

$$\mathrm{ITNij} = a_2 + b_9\mathrm{POPi} + b_{10}\mathrm{POPj} + b_{11}\mathrm{INCi} + b_{12}\mathrm{INCj} + b_{13}\mathrm{RSi} + b_{14}\mathrm{RSj} + b_{15}\mathrm{IRIi} + b_{16}\mathrm{IRIj} + b_{17}\mathrm{MRIi} + b_{18}\mathrm{MRIj} + b_{19}\mathrm{TOAD} + b_{20}\mathrm{USD} + b_{21}\mathrm{FD} + b_{22}\mathrm{APD} + b_{23}\mathrm{AD} + b_{24}\mathrm{DISTij} \tag{2}$$

Equations 3 and 4 present the initial models developed based on metropolitan areas. The model for local general aviation aircraft movements on a node basis is as follows:

$$\mathrm{LCLi} = a_3 + b_{25}\mathrm{BAi} + b_{26}\mathrm{Wi} + b_{27}\mathrm{FSDi} + b_{28}\mathrm{CPOPi} + b_{29}\mathrm{CINCi} + B_{30}\mathrm{CIMRi} + b_{31}\mathrm{CRSi} + b_{32}\mathrm{CIIi} \tag{3}$$

The model for itinerant general aviation aircraft movements on a link basis is as follows:

$$\mathrm{ITNi} = a_4 + b_{33}\mathrm{BAi} + b_{34}\mathrm{CPOPj} + b_{35}\mathrm{CINCi} = b_{36}\mathrm{CINCj} + b_{37}\mathrm{CRSi} + b_{38}\mathrm{CRSj} + b_{39}\mathrm{CIMRi} + b_{40}\mathrm{CIMRj} + b_{41}\mathrm{CRSi} + b_{42}\mathrm{CRSj} + b_{43}\mathrm{TOAD} + b_{44}\mathrm{USD} + b_{45}\mathrm{FD} + b_{46}\mathrm{APD} + b_{47}\mathrm{AD} + b_{48}\mathrm{DISTij} \tag{4}$$

The above equations (and operational signs) present only the conceptual model and not individual independent-variable hypotheses. In fact, some variables were run in various multiple combinations.

Research on local movements has been limited in past studies of forecasting general aviation movements at an airport. Most of the emphasis has been placed on itinerant movements, since these place a greater demand on sophisticated air-traffic-control systems and facilities. Most local movements are training flights, and many can be associated with flying schools and clubs. It has been stated many times that local movements not only are a function of flight training, but also depend on good flying weather, since many local movements are conducted under visual flight rules (VFR) and require suitable weather. With this in mind, a model was postulated that included a variable for weather and a dummy variable for flying schools. Other variables included were population, per-capita income, retail sales, income- and market-rating indices, and based aircraft for both catchment-area data and metropolitan-area data.

It is reasonable to expect that explanatory variables for itinerant and local movements may only be the same in specific circumstances. Itinerant movements can involve trips for specific purposes and thus general aviation could be considered a passenger mode. Business trips would fall in this category, as would most government trips. Some itinerant movements would be cross-country training, which might be more difficult to explain by means of the usual socioeconomic variables. Local movements are normally training and might have an explanatory variable similar to that for itinerant training. Unfortunately, there are almost no data on trip purpose for either itinerant or local trips.

The list of airports selected was restricted by the availability of based-aircraft data. These data, which are collected on an ongoing up-to-date basis, were obtained late in 1975 from the Atlantic Regional Office of Transport Canada.

Independent variables formulated in the model included based aircraft, a dummy variable for flying schools, and a weather index based on VFR flying-weather percentages. The airports used for the local-movement model were listed in Table 1.

ANALYSIS AND RESULTS

Itinerant-Movement Model

Figure 1 depicts the calibration procedure used in the development of the model for itinerant-aircraft movements. Both linear and log-linear transformations of the format data were used to test the model on the seven Maritime Province airports.

The log-transformation form for the model produced the best model from a statistical viewpoint. The values of the coefficient of determination were of the order of 0.6. Generally, little correlation between the independent variables was observed, and the signs associated with the coefficients were as expected. Even with R^2-values of the order of 0.6, very erratic residuals were produced. By using various dummy variables and combinations of variables, it was not possible to significantly reduce the magnitude of the residuals to a level acceptable for a forecasting model. Table 3 contains a comparison of values for observed and estimated itinerant-aircraft movements for the Moncton Airport by using the log-transformation form of the model and combined catchment- and metropolitan-area data. The model yielded an R^2-value of 0.63. The F- and t-statistics for the forecasting equation were significant at the 95 percent level. The node model for itinerant-aircraft movements did not produce highly reliable estimates when it was applied to all 70 catchment areas.

It should be emphasized that it was not the intention of the study to maximize the R^2 but rather to minimize the differences between the observed and the estimated trip values. The values of the coefficient of determination were useful as a guide to the expected reliability of the model; as the R^2-value rises, the difference becomes smaller. For forecasting purposes on a link basis, in which each observation is a link, it is extremely important to minimize the value of the residuals. Only in this fashion can the model be expected to perform in any meaningful way. Since the model did

Figure 1. Calibration procedure for itinerant-aircraft movements.

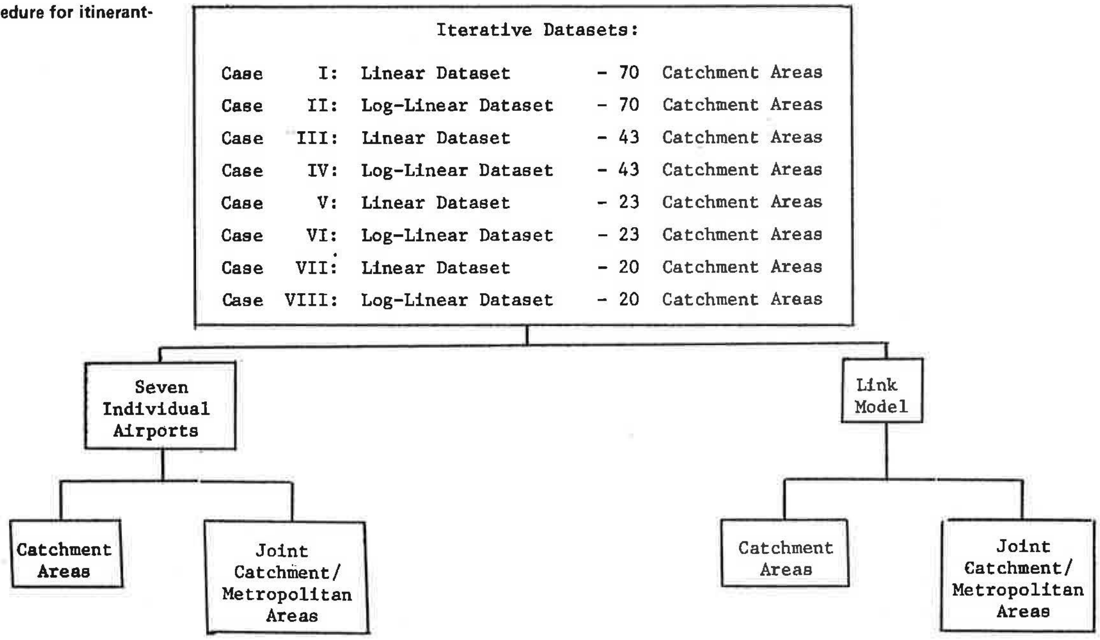

Table 3. Trip values for observed and estimated aircraft movements for node or airport model.

	Trip Values		
Node or Airport	Observed Movements	Estimated Movements	Percentage of Error
Deer Lake	37	25	-32
Stephenville	49	21	-57
Gander	84	23	-73
St. John's	71	28	-61
Halifax	2202	650	-70
Sydney	236	199	-16
Chatham	120	217	81
Charlo	231	69	-70
Fredericton	2081	696	-66
Saint John	1197	767	-36
Charlottetown	1354	718	-47
Massachusetts	25	27	8
New Jersey-New York	23	24	4
Maine	246	25	-89
Southeast United States	10	11	10
Canada (except areas specifically mentioned)	2841	49	-98
Metropolitan Toronto	50	53	-6
Eastern Ontario	111	16	-86
Metropolitan Montreal	210	103	51
Eastern townships	68	28	-59
Quebec	88	34	-61
Gaspesia	127	54	-57
House Harbour	96	33	-65
Greenwood	211	54	-74

not perform well with 70 catchment areas, a testing procedure was used that eliminated observations by removing those that made a relatively minor contribution to the trip values for total aircraft movements. The testing procedure indicated that by aggregating the catchment areas higher R^2-values (and thus lower residual values) could be obtained.

The original 70 catchment areas were first aggregated to 43, then to 23, and finally to 20. In each case, the linear and log-linear data base was tested for both the catchment area and the joint catchment and metropolitan conditions.

By using the 20 catchment areas, calibration on the basis of an individual airport or node did not produce acceptable results. This link model was tested by using the 20 catchment areas and produced marginally acceptable results when calibrated on the 49 links that serve the seven airports included in the study. The residuals shown in Table 4 indicate that, although this is the best model developed by using the most-recent link data available, the model could not be regarded as a highly reliable forecasting tool for planners.

The format for the 49-link model reduced from the general aviation models (Equations 2 and 4) is as follows:

$$ITNij^* = 89.08 - 0.64DIST + 4.31INC + 0.77RSI + 1.75MRI \qquad (5)$$

where ij* is itinerant trips outside the air-traffic-control zone that landed at point of origin, RSI is retail-sales index, and 89.08 is the constant. Related statistical data are $R^2 = 0.74$, SE = 0.89, F = 31.64, df = 4.44, and the coefficients and t-statistics listed below:

Coefficient	t-Statistic
0.64	0.69
4.31	0.43
0.77	0.21
1.75	0.13

Local-Movement Model

The local movements were calibrated for 14 individual Maritime Province airports by using linear and log-linear data for both the catchment area and the joint catchment and metropolitan areas. The final model selected from the initial models (Equations 1 and 3) was in the following form:

$$\text{Local trips} = 69\,545.25 + 29\,794FSD\ [4.23] + 874.07W\ [1.64] + 190.04BA\ [0.41] \qquad (6)$$

where the standard errors are in brackets.

Many of the airports show widely fluctuating counts. This is in part due to reporting difficulties. Many air-traffic controllers fill in the daily record sheet at the end of the shift by making an estimate of movements. This is especially true at the smaller airports and at those airports that are only radio controlled.

Unfortunately, although the exercise was quite interesting in terms of the variables that were

Table 4. Residual errors for the 49-link model.

Airport	Link	Trip Values		Percentage of Error
		Observed Movements	Estimated Movements	
Charlottetown	1 Halifax	707	609	-14
	2 Yarmouth	45	62	38
	3 Sydney	525	214	-59
	4 Fredericton	280	346	24
	5 Moncton	1258	512	-59
	6 Saint John	163	360	121
	7 Charlottetown[a]	5957	5 795	-3
Fredericton	8 Halifax	993	689	-31
	9 Yarmouth	82	128	56
	10 Sydney	69	206	199
	11 Fredericton[a]	8787	14 011	59
	12 Moncton	2169	709	-67
	13 Saint John	1852	1 133	-39
	14 Charlottetown	275	346	26
Halifax	15 Halifax[a]	3583	22 247	521
	16 Yarmouth	765	161	-79
	17 Sydney	409	357	-13
	18 Fredericton	899	690	-23
	19 Moncton	2030	824	-59
	20 Saint John	598	810	35
	21 Charlottetown	635	609	-4
Moncton	22 Halifax	2209	824	-63
	23 Yarmouth	352	105	-70
	24 Sydney	236	229	-3
	25 Fredericton	2081	709	-66
	26 Moncton[a]	9373	11 189	19
	27 Saint John	1198	741	-38
	28 Charlottetown	1354	512	-62
Saint John	29 Halifax	687	785	14
	30 Yarmouth	222	155	-30
	31 Sydney	55	211	284
	32 Fredericton	1780	1 133	-36
	33 Moncton	1245	741	-40
	34 Saint John[a]	6796	13 161	94
	35 Charlottetown	166	360	117
Sydney	36 Halifax	408	357	-13
	37 Yarmouth	21	43	105
	38 Sydney[a]	2473	4 619	87
	39 Fredericton	62	206	232
	40 Moncton	230	229	0
	41 Saint John	54	211	291
	42 Charlottetown	552	214	-61
Yarmouth	43 Halifax	436	161	-63
	44 Yarmouth[a]	3485	676	-81
	45 Sydney	11	43	291
	46 Fredericton	51	129	153
	47 Moncton	144	105	-27
	48 Saint John	102	155	32
	49 Charlottetown	13	62	377

[a] Itinerant trips outside air-traffic-control zone that landed at point of origin.

identified as being significant, the results were not particularly encouraging and the R^2-values were relatively low.

The dummy for flying schools and the weather index very often were the first two variables to enter. In the linear formulation, these two variables combined to produce an R^2 of 0.57. Numbers of aircraft based at the airports entered next, and the marginal increase in the R^2 was 0.02.

The metropolitan linear data produced results that had little improvement. The maximum R^2 produced was 0.61. Table 5 presents the residual-error analysis. These results were to be expected, since the observed trip data ranged from 41 to more than 80 000 trips.

In the log-linear regressions, the socioeconomic variables were entered with the more-interesting explanatory structural variables of based aircraft, population, and weather, and these combined to produce an R^2 of 0.62. The income variables then entered with the reverse sign from that which would be expected.

Table 5. Residual-error analysis of best model developed for local movements at the node or airport level.

Airport	Observed Trips	Predicted Trips	
		Linear	Log-Linear
Deer Lake	41	7 980	376
Stephenville	363	12 328	405
Gander	8 634	5 635	17 581
St. John's	11 716	23 217	30 570
Halifax	35 947	37 803	46 520
Yarmouth	672	-5 102	804
Sydney	9 196	1 268	2 120
Charlo	4 760	3 726	1 190
Fredericton	31 059	41 742	66 455
Moncton	81 632	45 676	16 407
Saint John	27 603	39 518	19 008
Edmundston	1 257	10 454	3 204
Bathurst	3 863	11 187	5 826
Charlottetown	18 652	-38	3 375

DISCUSSION OF RESULTS

This paper has presented the method and results from the first known attempt to calibrate a general forecasting model on both a node and a link basis by using Canadian data recorded at the airport level. Similar work has been carried out in the United States and, very recently, Transport Canada has begun similar work.

The premise was accepted at the beginning of this study that forecasting general aviation in the manner described here would not be as precise as developing forecasts of passenger flows and then converting these flows to aircraft movements as is done for commercial aviation. When the passenger-forecasting approach is used, it is hypothesized that passengers are subject to economic and behavioral forces that direct them to use air transport as a mode of travel. However, there do not exist in Canada (or for that matter in the United States) reliable data on the volume of movements by individuals on an O-D basis for general aviation. Furthermore, the data on a passenger basis do not exist at the station-activity level. Trip-purpose data are not available. Such data could be compiled through user and pilot surveys of general aviation, but even for a region of Canada such surveys would be extremely expensive.

There are a number of reasons why the results obtained were not so reliable as required for detailed planning functions. Some of these reasons include the fact that a considerable portion of general aviation is recreational flying, and it is very difficult to find a socioeconomic indicator for this type of travel. The use of seven Maritime Province test airports may have restricted the model in that the main general aviation traffic routes in Canada were not included (for example, those routes that connect and center around Montreal and Toronto). Even so, a model in which the central nodes were Montreal and Toronto (if it were successful) would not necessarily be reliable in the less-dense traffic areas such as the Maritime Provinces. It is reasonable to suggest that one model would not necessarily work for all parts of the country.

The type of model used in this study does not explain fully the complex relationships between particular links due to specialty uses of general aviation (for example, between a head office and its regional counterpart, business links, government links, etc.). Data on travel at this level are simply not available without expensive and extensive surveys. To go one step farther, travel-demand forecasting has not progressed to the stage at which the behavior of individuals can be rationalized. Forecasting and modeling is still a crude process. It has been said that a forecast model is a muddled set of assumptions on an abstract piece of behavior.

The link model calibrated on the 49 links from the seven Maritime Province airports can be considered to produce marginally acceptable results. The statistical parameters associated with the forecasting model were significant at the 95 percent level.

The model, although not recommended for use in a detailed planning function, can be considered an acceptable departure point for the development of general aviation forecasting techniques for the Canadian Air Transport environment. The data supplied by Statistics Canada should be made available to other researchers so that development in this area can continue. The procedures for estimating commercial aviation activity are reasonably well advanced, and similar planning tools must become available for general aviation to enable the total air-transport mode to be evaluated on an ongoing basis.

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REFERENCES

1. Contracts. Canadian Aviation, March 1979.
2. Canadian Civil Aircraft Register. Transport Canada, Ottawa, 1978.

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Air Traffic Control Network-Planning Model Based on Second-Order Markov Chains

NEIL W. POLHEMUS

A method designed to assess the impact of increased air traffic demand on flow rates in a network of en route air traffic control sectors is described. Given projected arrival and departure rates at airports within a given region, a second-order Markov-chain model is employed that has transition probabilities estimated from historical data. The technique is designed to serve as a planning tool and is demonstrated by using data from the New York Air Route Traffic Control Center.

The primary purpose of air traffic control (ATC) systems is to ensure the safe and efficient movement of air traffic. Given projected increases in traffic levels, it is important that a method be developed to predict the impact of additional demand on the system. In particular, the need to restructure existing sector boundaries depends on the distribution of flow in the current system.

As an example of the structure of ATC networks, the New York Air Route Traffic Control Center (ARTCC) consists of 32 sectors that cover the entire states of New Jersey and Delaware and parts of New York, Pennsylvania, Connecticut, and Maryland. The center controls en route traffic by dividing the low- and high-altitude airspace into sectors, each of which is handled by an individual controller who has an assigned communications frequency. Figure 1 shows the orientation of the low-altitude sectors. The high-altitude sectors are configured similarly and control traffic at or above 24 000 ft.

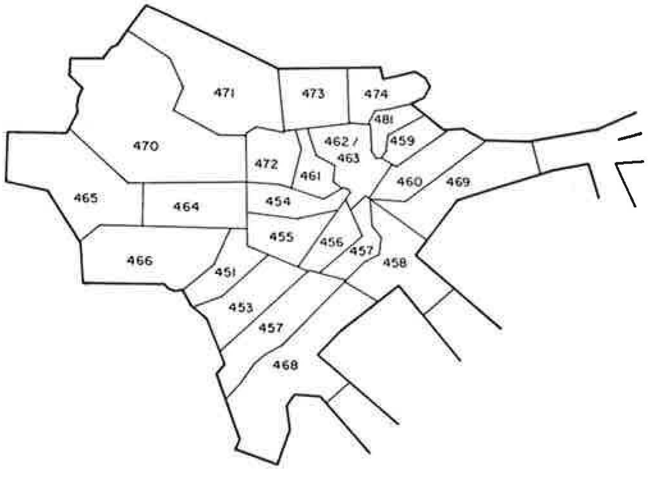
This paper describes a method designed to assess the impact of specified demand patterns on flow in the system. The approach is based on describing the sequences of sectors traversed by aircraft as second-order Markov chains. Although it is an approximation, the model provides a reasonable characterization of general system flow patterns with a simple-enough structure to allow for adequate parameter estimates. The need for a second-order Markov chain for terminal areas rather than a first-order chain as proposed earlier (1) is due to a lack of unidirectionality in the flow through many of the en route sectors.

The paper begins with a general formulation of the ATC system as a directed network and then considers characterizations of traffic generation and sector sequences. The use of the method in predicting system flows is discussed. Throughout, the techniques described are applied to the New York ARTCC.

NETWORK STRUCTURE

To represent an ATC system, let the sectors be rep-

Figure 1. New York ARTCC low-altitude sector control boundaries.



resented by the set of nodes $\{N_j, j = 1,2,\ldots, m\}$. Feasible movement among the sectors is characterized by a set of arcs (A) between adjacent sectors. If we adopt the notation used by Ford and Fulkerson (2), Potts and Oliver (3), and others, the system is defined as the network G = [N;A].

For a network containing m sectors, A could consist of as many as m(m - 1) arcs. However, for most ATC systems only a very small subset of the possible arcs ever exists, since most pairs of sectors are not physically adjacent. To specify which arcs are present in a network, a node-node incidence matrix D of dimension (m x m) may be defined with element

$$d_{ij} = \begin{cases} 1 & \text{if flow is possible from node i to node j} \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

The node-node incidence matrix for the 32-sector New York en route system (Figure 2) consists of 152 arcs.

To represent arrivals to and departures from the system, it is convenient to construct a supersource (s) and a supersink (t) for the point of entry and exit, respectively, for all traffic in the network. Arcs are then constructed from s to the sectors in node N and from N to t. The actual source of traffic, however, is one of the airports in the region or an en route sector in another center. To represent the actual sources, we may construct the set of sources $\{s_i, i = 1, 2, \ldots, m_s\}$ and, in a similar manner, a set of sinks for the termini $\{t, k = 1, 2, \ldots, m_t\}$. This formulation is illustrated in Figure 3.

In the New York center, traffic was observed departing from and arriving at 12 separate airports in the region. During a 2-h sample, there were (a) 253 aircraft departures from airports within the region covered by the New York en route sectors, (b) 238 aircraft arrivals at airports within the region, and (c) additional en route traffic that had both source and terminus outside the region. For this system, $m_s = m_t = 13$, and there is one source and one terminus for each airport and an additional

Figure 2. Node-node incidence matrix for New York en route network.

SECTOR	451	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
451	0	1	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
453	1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
454	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
455	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
456	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
457	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
458	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	1
459	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0
460	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
461	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0	0	0	0
462	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
463	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0
464	1	0	1	1	0	1	0	0	0	1	0	0	0	1	1	0	0	0	1	0	1	0	0	1	0	0	0	0	1	0	0	0
465	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
466	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
467	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
468	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
469	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
470	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
471	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	1	0	0	0
472	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0	1	0	0	0
473	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0
474	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0
475	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	1	1	1	0	0	0
476	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
477	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	1
478	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
479	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0
480	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	1	0	0
481	0	0	0	0	0	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1
482	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
483	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0

Figure 3. Schematic diagram of en route network traffic flow.

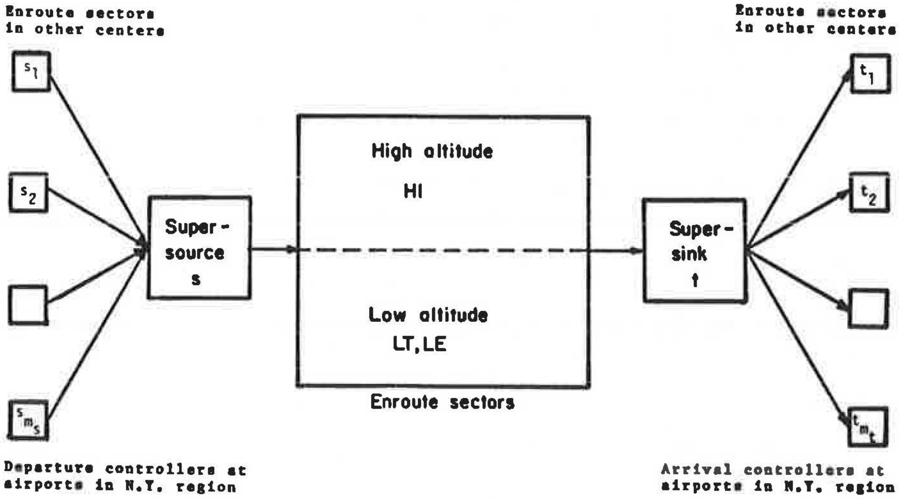


Table 1. Traffic to and from airports within New York region.

		Traffic Through En Route Network[a]	
Airport	Code	Departures	Arrivals
Newark	EWR	53	40
John F. Kennedy	JFK	52	64
LaGuardia	LGA	58	56
Philadelphia	PHL	36	40
Atlantic City	ACY	9	5
Wilmington	ILG	5	9
Wilkes-Barre	AVP	3	2
Binghamton	BGM	3	5
Harrisburg	HAR	12	0
Allentown	ABE	8	3
Elmira	ELM	7	3
Westchester	HPN	7	11
Total		253	238

[a] Aircraft per 2-h sample.

source and terminus for en route sectors outside the New York center. Table 1 is a summary of traffic to and from airports within the region.

To characterize flow through the network, we then need to determine the following:

1. The manner in which aircraft are generated at the various sources,
2. The arcs over which they enter the en route network,
3. The sequence of sectors through which they proceed, and
4. Their final termini.

Given a finite set of data, achievement of such a characterization in a meaningful and consistent fashion raises various problems. In particular, one must ensure that the flow-conservation equations are satisfied yet allow for manipulation of system input at a sufficiently macroscopic level to provide a usable tool for the decision maker. The technique described below is designed to generate meaningful predictions of system flows in a manner suitable for planning purposes.

TRAFFIC SOURCES

To characterize sources of traffic in the network, let λ_i be the rate of traffic generated at source s_i. If

$$p_j = \text{prob [aircraft enters network over arc } (s,N_j)] \quad (2)$$

where prob represents probability and if

$$p_{j|i} = \text{prob [aircraft enters network over arc } (s,N_j) \text{ given generation at source i]} \quad (3)$$

then

$$f(s,N_j) = \sum_{i=1}^{m_s} p_{j|i}\,\lambda_i \quad (4)$$

For specified flow rates λ_i, estimation of the entry-arc flow rates $f(s,N_j)$ requires estimation of the conditional entry probabilities $p_{j|i}$.

To estimate these probabilities from a finite set of data, define a source-node entry count matrix C of dimension (m_s x m) with elements

$$c_{ij} = \text{number of aircraft generated at source } s_i \text{ that entered sector } N_j \quad (5)$$

Then the total number of aircraft generated at source s_i is given by

$$c_{i.} = \sum_{j=1}^{m} c_{ij} \quad i = 1, 2, \ldots, m_s \quad (6)$$

and the number of aircraft that enter the network over arc (s,N_j) is given by

$$c_{.j} = \sum_{i=1}^{m_s} c_{ij} \quad j = 1, 2, \ldots, m \quad (7)$$

The top 13 rows of Figure 4 show part of a matrix determined from the New York sample period in which 000 indicates entry from en route sectors in another center. The totals $c_{i.}$ and $C_{.j}$ are given in the last column and row of the figure.

If the selection of entry sector N_j for the arrivals from source i is independent, the probabilities $p_{j|i}$ are parameters of a multinomial distribution. The maximum-likelihood estimates are given by

$$\hat{p}_{j|i} = c_{ij}/c_{i.} \quad (8)$$

and the estimated entry-arc flow rates by

$$\hat{f}(s,N_j) = \sum_{i=1}^{m_s} \hat{p}_{j|i}\,\lambda_i \quad (9)$$

To test the assumption of independence in selection of entry sector, the selections of consecutive departures from the four major airports in the region were examined. By using a technique described by Anderson and Goodman (4), χ^2-test statistics indicated significant violation of the assumption only at LGA, at which successive departures tended to alternate between sectors 461 and 462.

Entry of aircraft to the network is completely determined by the set $\{p_{j|i};\ i = 1, 2, \ldots, m_s,\ j = 1, 2, \ldots, m\}$. The movement of aircraft after they enter the initial sector is the subject of the next section.

CHARACTERIZING SECTOR SEQUENCES

As aircraft move through an ATC system, they pass from sector to sector (from node to node) in sequences affected by their origin and destination. In a network of many sectors, the number of possible sequences is enormous, which makes the specification of the relative frequencies of all such sequences prohibitive. In order to reduce the complexity of the problem and still maintain the general patterns of network flow, an approach based on Markov chains will be presented.

To state the problem formally, consider a Markov chain with $M = m_s + m + m_t$ states, where the states represent the m_s-sources, m-en route sectors, and m_t-sinks, numbered in that order. Further, let $\{s_n(h),\ h = 0, 1, 2, \ldots\}$ be the sequence of sectors through which the nth aircraft passes, in which

$$s_n(0) = i \quad \text{if n th aircraft is departure from i th source} \quad (10)$$

$$s_n(h) = m_s + j \quad \text{if h th sector entered by n th aircraft is sector j} \quad 1 \leqslant h \leqslant m_n \quad (11)$$

$$s_n(h) = m_s + m + k \quad \text{if n th aircraft is arrival at k th sink} \quad h > m_n \quad (12)$$

where m_n is the number of network sectors in the sequence for the nth aircraft. Then $\{s_n(\cdot)\}$ is a realization from a Markov chain of unknown order.

In the above formulation, the nodes and sources

Figure 4. Counts of departures, arrivals, and transitions in New York network.

	EWR	JFK	LGA	PHL	ACY	ILG	AVP	BGM	HAR	ABE	ELM	HPN	000	451	453	···	TOTAL
EWR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		53
JFK	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		52
LGA	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		58
PHL	0	0	0	0	0	0	0	0	0	0	0	0	0	22	1		36
ACY	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		9
ILG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2		5
AVP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		3
BGM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		3
HAR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		12
ABE	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		8
ELM	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		7
HPN	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		7
000	0	0	0	0	0	0	0	0	0	0	0	0	0	6	28		373
451	0	0	0	0	0	1	1	0	0	0	0	0	24	0	3		55
453	0	0	0	5	1	1	0	0	0	0	0	0	6	2	0		55
454	1	0	18	7	0	0	0	0	0	0	0	0	3	20	0		55
455	30	0	3	0	0	0	0	0	0	0	0	0	4	3	0		42
456	0	30	0	0	0	0	0	0	0	0	0	0	3	2	0		42
457	0	6	16	0	0	0	0	0	0	0	0	0	3	0	0		26
458	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0		50
459	0	0	0	0	0	0	0	0	0	0	0	0	13	0	0		32
460	0	28	0	0	0	0	0	0	0	0	0	0	15	0	0		47
461	4	0	0	0	0	0	0	0	0	0	0	0	1	0	0		53
462	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		40
463	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0		52
464	0	0	0	0	0	0	0	0	0	2	0	0	4	1	0		72
465	0	0	0	0	0	0	0	0	0	0	0	0	17	0	0		31
466	0	0	0	20	0	1	0	0	0	0	0	0	18	3	0		56
467	0	0	0	8	3	5	0	0	0	0	0	0	18	0	9		56
468	0	0	0	0	1	1	0	0	0	0	0	0	24	0	1		40
469	0	0	0	0	0	0	0	0	0	0	0	0	15	0	0		28
470	0	0	0	0	0	0	1	0	0	0	2	0	18	0	0		42
471	0	0	0	0	0	0	0	5	0	0	1	0	11	0	0		38
472	5	0	0	0	0	0	0	0	0	1	0	0	3	0	0		65
473	0	0	1	0	0	0	0	0	0	0	0	6	13	0	0		65
474	0	0	0	0	0	0	0	0	0	0	0	3	23	0	0		43
475	0	0	0	0	0	0	0	0	0	0	0	0	11	0	0		65
476	0	0	0	0	0	0	0	0	0	0	0	0	3	0	12		36
477	0	0	0	0	0	0	0	0	0	0	0	0	16	0	0		52
478	0	0	0	0	0	0	0	0	0	0	0	0	14	0	0		36
479	0	0	0	0	0	0	0	0	0	0	0	0	27	0	0		48
480	0	0	0	0	0	0	0	0	0	0	0	0	41	0	0		75
481	0	0	18	0	0	0	0	0	0	0	0	2	5	0	0		56
482	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0		18
483	0	0	0	0	0	0	0	0	0	0	0	0	7	0	0		29
TOTAL	40	64	56	40	5	9	2	5	0	3	3	11	373	59	56	···	2126

are transient states, whereas the sinks are absorbing. Further, the nodes form a communicating class that is accessible from the sources, but the sources (nonreturn states) are not accessible from any states in the chain. The characterization of the sequences will thus involve state transition matrices of very special form. Although all sequences begin in one of the source states, the probability of ever returning to those states is zero. In describing the sequences, we state first the initial distribution of $s_n(0)$ and then discuss the state-transition probabilities.

The initial distribution of $s_n(0)$ has parameter set

$$\underline{\theta} = \{\lambda_1/\lambda_s, \lambda_2/\lambda_s, \ldots, \lambda_{m_s}/\lambda_s\} \tag{13}$$

where

$$\text{prob}[s_n(0) = i] = \lambda_i/\lambda_s \tag{14}$$

$$\sum_{i=1}^{m_s} \lambda_i = \lambda_s \tag{15}$$

Thus the relative generation rates at the sources determine the probability distribution for $s_n(0)$ in a natural way.

To determine the movement of aircraft through the network, suppose that the sector sequences $\{s_n(\cdot)\}$ can be regarded as realizations of a Markov chain of order q. Then the distribution of $s_n(h)$ depends on the history of the sequence only through $s_n(h - 1)$, $s_n(h - 2)$, ..., and $s_n(h - q)$. To be more explicit, let

$$p_k = \text{prob}[s_n(h) = k] \tag{16}$$

$$p_{jk} = \text{prob}[s_n(h) = k|s_n(h - 1) = j] \tag{17}$$

$$p_{ijk} = \text{prob}[s_n(h) = k|s_n(h - 1) = j, s_n(h - 2) = i] \tag{18}$$

Then, if the sequences are zero-order Markov chains,

$$p_k = p_{jk} = p_{ijk} \tag{19}$$

For first-order Markov chains,

$$p_k \neq p_{jk} = p_{ijk} \tag{20}$$

For second-order chains,

$$p_k \neq p_{jk} \neq p_{ijk} \tag{21}$$

The extension to higher orders is direct.

In studying sequences of sectors, it is therefore necessary to determine both the order of the chain and all relevant transition probabilities. This is most easily handled by defining a series of transition matrices $P^{(1)}$, $P^{(2)}$, ..., where $P^{(q)}$, the q-step transition matrix, has element

$$p_{jk}^{(q)} = \text{prob}[s_n(h) = k|s_n(h - q) = j] \tag{22}$$

For a zero-order Markov chain,

$$P^{(q)} = [p_{jk}] \quad p_{jk} = p_k \tag{23}$$

For first-order Markov chains,

$$P^{(q)} = [P^{(1)}]^q \tag{24}$$

Of particular interest are both the limiting matrix,

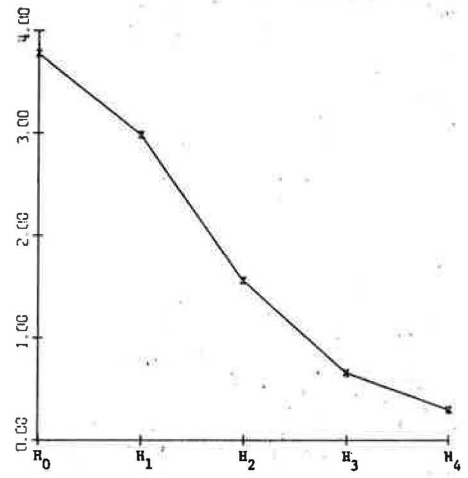
$$P^{\infty} = \lim_{q \to \infty} P^{(q)} \tag{25}$$

which can be used to determine the distribution of

Figure 5. Transition-count matrix for sector 451.

	000	453	455	464	466	479	ILG	AVP	TOT
000	1	0	1	0	2	0	0	1	5
453	0	0	0	2	0	0	0	0	2
454	15	2	0	0	0	0	0	0	17
455	3	0	0	0	0	0	0	0	3
456	1	0	0	0	0	0	0	0	1
464	0	1	0	0	0	0	0	0	1
466	1	0	0	0	0	0	1	0	2
PHL	0	0	0	6	5	10	0	0	21
TOT	21	3	1	8	7	10	1	1	52

Figure 6. Plot of conditional uncertainties in sector sequences.



exits from the system, given the entry sector, and the total-flow matrix

$$F^{(q)} = \sum_{r=1}^{q} \underline{\lambda} P^{(r)} \tag{26}$$

which measures the impact of given entries on sectors throughout the network for a given flow vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{m_s}, 0, 0, \ldots, 0)$ of dimension (1 x M).

Higher-step transition matrices determine the accessibility of sectors in the network. Since all flow originates at a source and ends at a sink, the only elements of the matrix that do not converge to zero as q becomes large are those that correspond to the source rows and sink columns. Let the limiting values of these elements be given by

$$e_{ik} = \lim_{q \to \infty} p_{i}^{(q)}; \ k\text{-}m_s\text{-}m \quad i = 1, 2, \ldots, m_s \quad k = 1, 2, \ldots, m_t \tag{27}$$

Then the sink-attraction rates $\mu_1, \mu_2, \ldots, \mu_{m_t}$ are related to the source-generation rates by

$$\mu_k = \sum_{l=1}^{m_s} \lambda_i e_{ik} \quad k = 1, 2, \ldots, m_t \tag{28}$$

This link between entry and exit rates is an important consideration in attempting to estimate system flows, given projected levels of both arrivals and departures at airports in the region. It is discussed more fully in the next section.

The problem of estimating transition probabilities in Markov chains has been studied by several authors (4-6). Suppose that there are available records of c independent sector sequences, each of which is assumed to be a random observation from a qth-order Markov chain with M states. Let n_{jk} be the number of times an aircraft enters state k from state j, n_{ijk} be the number of times an aircraft enters state k from state j after it has entered state j from state i, and so forth. If we assume stationary transition probabilities, n_{jk} forms a set of statistics sufficient for the state-transition probabilities. For a second-order Markov chain, n_{ijk} is sufficient. The results can be generalized to higher-order chains.

The maximum-likelihood estimates of the transition probabilities depend on the order of the Markov chain. For a second-order chain, $s_n(0)$ and $s_n(1)$ are assumed to be nonrandom, whereas $s_n(k)$, $k \geq 2$, are assumed to be random variables. Then the maximum-likelihood estimates of the transition probabilities in Equations 16-18 are given by

$$\hat{p}_{ijk} = n_{ijk} / \sum_{l=1}^{M} n_{ijl} \tag{29}$$

$$\hat{p}_{jk} = \sum_{i=1}^{M} n_{ijk} / \sum_{i=1}^{M} \sum_{l=1}^{M} n_{ijl} \tag{30}$$

$$\hat{p}_{k} = \sum_{i=1}^{M} \sum_{j=1}^{M} n_{ijk} / \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{l=1}^{M} n_{ijl} \tag{31}$$

Note that, since $s_n(0)$ and $s_n(1)$ are assumed to be nonrandom, the estimates of the transition probabilities involve summations over n_{ijk} rather than the direct use of n_{jk} and n_k (the results are not equivalent).

For the New York en route network, transition-count matrices that use n_{ijk} were obtained for each of the 32 sectors. Figure 5 is the matrix obtained for one of the sectors. The sector shown was evidently handling traffic that departed from Philadelphia (PHL).

To determine the order of Markov chain appropriate for a given set of data, a likelihood-ratio test was derived by Anderson and Goodman (4). The technique, however, can be applied effectively to ATC sector sequences only if the number of sectors is small and the number of observed sector sequences is quite large. For other situations, a graphical technique based on information theory given by Chatfield (5) (which can be related to the likelihood-ratio test) is all that the data will support. The technique involves plotting the conditional uncertainties about the next sector that an aircraft will enter if we are only given knowledge of its current sector, of the previous sector, of the two previous sectors, and so forth. The reduction in conditional uncertainties as more and more of the past is known helps to indicate the order of Markov chain necessary to characterize the sequences.

Figure 6 is a plot of the estimated conditional uncertainties in the sector sequences made by using all observed quadruplets in the sample. From the New York data, $N_4 = 777$ quadruplets were tabulated. The following formulas were used to calculate the conditional uncertainties:

$$H_0 = \log 44 \quad \text{(44 states in chain)} \tag{32}$$

$$H_1 = \log N_4 - N_4^{-1} \Sigma_{n_{i...}} \log n_{i...} \tag{33}$$

$$H_2 = N_4^{-1} \left(\sum_{i} n_{i...} \log n_{i...} - \sum_{i,j} n_{ij..} \log n_{ij..} \right) \tag{34}$$

Table 2. Specified source-generation rates and sink-attraction rates observed and computed by model, for airports.

Airport	Specified Source-Generation Rate[a]	Sink-Attraction Rate[a]	
		Computed	Observed
EWR	26.5	20.6	20.0
JFK	26.0	38.5	32.0
LGA	29.0	29.8	28.0
PHL	18.0	20.8	20.0
ACY	4.5	2.8	2.5
ILG	2.5	5.3	4.5
AVP	1.5	1.1	1.0
BGM	1.5	3.3	2.5
HAR	6.0	0.0	0.0
ABE	4.0	1.6	1.5
ELM	3.5	1.6	1.5
HPN	3.5	5.3	5.5
000	186.5	182.3	186.5
Total	313.0	313.0	305.5

[a] Aircraft per hour.

Table 3. Observed and computed sector-flow rates computed by model, for sectors.

Sector	Sector-Flow Rate[a]		Sector	Sector-Flow Rate[a]	
	Computed	Observed		Computed	Observed
451	29.5	27.5	469	13.9	14.0
453	28.1	27.5	470	22.7	21.0
454	29.3	27.5	471	24.3	19.0
455	20.1	21.0	472	34.3	32.5
456	21.9	21.0	473	32.0	32.5
457	12.8	13.0	474	21.3	21.5
458	23.5	25.0	475	33.5	32.5
459	15.2	16.0	476	19.4	18.0
460	25.1	23.5	477	24.8	26.0
461	25.5	26.5	478	18.1	18.0
462	20.8	20.0	479	26.0	24.0
463	25.6	26.0	480	38.0	37.5
464	36.9	36.0	481	26.7	28.0
465	13.4	15.5	482	7.8	9.0
466	31.0	28.0	483	17.1	14.5
467	28.4	28.0	Total	765.9	750.0
468	18.9	20.0			

[a] Aircraft per hour.

$$H_3 = N_4^{-1} \left(\sum_{i,j} n_{ij..} \log n_{ij..} - \sum_{i,j,k} n_{ijk.} \log n_{ijk.}\right) \tag{35}$$

$$H_4 = N_4^{-1} \left(\sum_{i,j,k} n_{ijk.} \log n_{ijk.} - \sum_{i,j,k,l} n_{ijkl} \log n_{ijkl}\right) \tag{36}$$

The sharp drop from H_0 to H_2 shows the importance of knowing the current sector when determining the next. The drop from H_2 to H_3 is almost as sharp, which indicates significant information in the previous sector. The drop from H_3 to H_4 may or may not be significant, but it does not appear to be so important as the earlier drops. No rigorous statistical tests were performed because of the large number of states in the chain and the consequently small number of counts for all observed pairs, triplets, and higher sequences during the 2-h sample period.

On the basis of the above analysis, it appears that second-order Markov chains are sufficient to describe the patterns observed in the sector sequences. The maximum-likelihood estimates of p_{ijk} can thus all be developed from the transition-count matrices by means of

$$\hat{p}_{ijk} = n_{ijk}/\sum_{l=1}^{c} n_{ijl} \tag{37}$$

where c is the number of columns in the matrix.

Traffic in the network is then completely described by arrival rates λ_i, conditional entry probabilities $p_{j|i}$, and transition probabilities p_{ijk}. The next section considers the use of such a formulation in predicting network flow patterns.

APPLICATION OF MODEL

To use the above method to predict sector flows, the arrival-rate parameters λ_i are specified and an arc-flow matrix $F^{(1)}$ of dimension (M x M) is formed from Equation 26 with $M = m_s + m + m_t$. The elements of the matrix are

$$f_{ij}^{(1)} = \begin{cases} p_{j-m_s|i}\,\lambda_i & \begin{cases} i = 1, 2, \ldots, m \\ j = m_s + 1, m_s + 2, \ldots, m_s + m \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{38}$$

After q transitions, the arc flows are given by

$$f_{jk}^{(q)} = \sum_{i=1}^{M} f_{ij}^{(q-1)}\, p_{ijk} \quad \begin{matrix} j = 1, 2, \ldots, M \\ k = 1, 2, \ldots, M \end{matrix} \tag{39}$$

After many transitions,

$$\lim_{q\to\infty} f_{jk}^{(q)} = \begin{cases} \mu_{k-m_s-m} & j = k = m_s + m + 1, m_s + m + 2, \ldots, m_s + m + m_t \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

In other words, all flow eventually reaches and remains in one of the sinks. Further, total sector flows are given by

$$f_j = \lim_{q\to\infty} \sum_{r=1}^{q} \sum_{i=1}^{M} f_{ij}^{(r)} \quad \text{for } m_s < j \leqslant m_s + m \tag{41}$$

Tables 2 and 3 show the observed sink-attraction and sector-flow rates computed by the above method with source-generation rates λ_i set equal to that estimated from the sample data. Good correspondence between the observed and computed rates resulted. After q = 10 iterations, 99.99 percent of the flow had reached a sink and there was little change in computed rates beyond that point. Any of various stopping criteria could be used to stop the iterative process.

To demonstrate the use of the model as a planning tool, the rate of traffic that departed from Newark (EWR) was increased by 50 percent, which yielded the computed flow rates shown in Tables 4 and 5. Increases in sector-flow rates of more than 10 percent occurred in sectors 454, 472, and 480. Although most of the additional traffic terminated in the en route sink, a certain proportion became

Table 4. Flow rates for EWR departures increased by 50 percent, for airports.

Airport	Specified Source-Generation Rate[a]	Computed Sink-Attraction Rate[a]	Percentage of Change[b]
EWR	39.75	20.6	0.2
JFK	26.0	38.6	0.3
LGA	29.0	29.9	0.5
PHL	18.0	21.4	2.7
ACY	4.5	2.9	4.2
ILG	2.5	5.3	0.1
AVP	1.5	1.1	0.0
BGM	1.5	3.6	8.2
HAR	6.0	0.0	0.0
ABE	4.0	1.6	4.6
ELM	3.5	1.6	0.0
HPN	3.5	5.6	4.7
000	186.6	194.1	6.4
Total	326.3	326.3	4.2

[a] Aircraft per hour. [b] Compared with computed rates in Table 2.

Table 5. Flow rates for EWR departures increased by 50 percent, for sectors.

Sector	Computed Sector-Flow Rate[a]	Percentage of Change[b]	Sector	Computed Sector-Flow Rate[a]	Percentage of Change[b]
451	31.7	7.6	468	18.9	0.0
453	28.3	0.9	469	14.2	2.3
454	33.9	15.6	470	23.6	4.1
455	20.2	0.2	471	24.7	1.7
456	23.0	5.1	472	41.3	20.4
457	12.8	0.0	473	33.5	4.7
458	23.6	0.2	474	21.8	2.3
459	15.2	0.0	475	36.8	9.7
460	25.2	0.4	476	20.5	5.8
461	28.0	9.8	477	25.4	2.6
462	21.3	2.4	478	19.7	8.5
463	26.4	3.1	479	26.1	0.5
464	38.0	2.8	480	42.2	11.2
465	14.0	5.1	481	26.9	0.5
466	32.0	3.3	482	7.8	0.0
467	28.6	0.7	483	17.6	2.5

[a] Aircraft per hour. [b] Compared with computed rates in Table 3.

Table 6. Results of combined forward and backward analyses, for airports.

Source and Sink	Target Rate[a]		Model-Specified Rate[a]	
	Source Generation	Sink Attraction	Source Generation	Sink Attraction
EWR	26.5	20.0	13.11	19.53
JFK	26.0	32.0	12.22	12.28
LGA	29.0	28.0	14.04	12.77
PHL	18.0	20.0	9.62	9.39
ACY	4.5	2.5	2.43	1.06
ILG	2.5	4.5	1.42	1.81
AVP	1.5	1.0	0.65	0.45
BGM	1.5	2.5	0.79	0.83
HAR	6.0	0.0	3.24	0.00
ABE	4.0	1.5	1.54	0.69
ELM	3.5	1.5	1.79	0.68
HPN	3.5	5.5	1.71	2.80
000	179.0	186.5	95.69	95.00
Total	305.5	305.5	158.25	147.29

[a] Aircraft per hour.

arrivals at other airports in the region. Although this is consistent with the observed behavior of the system, it points out the interdependencies between source-generation rates and sink-attraction rates.

Although departure rates from airports can be easily manipulated, given the above formulation, arrival rates cannot. Given a single source for all entries from outside the region, it is not possible to set the arrival rate at each of the airports. However, if the role of sources and sinks is reversed and the network is run backward, the sink-attraction rates (μ_k) can be set as desired and the source-generation rates determined from the analysis.

To perform a backward analysis, the following adjustments are necessary. Conditional exit probabilities must be estimated by

$$\bar{p}_{j|k} = c_{jk}/c_{.k} \tag{42}$$

where c_{jk} is the number of aircraft attracted to sink t_k directly from sector N_j. Transition probabilities must be estimated by

$$\hat{p}_{ijk} = n_{ljk}/\sum_{l=1}^{c} n_{ljk} \tag{43}$$

where c is the number of rows in the transition-count matrix for sector j. The initial flow vector must be estimated by

Table 7. Results of combined forward and backward analyses, for sectors.

Sector	Sector-Flow Rate[a]		Sector	Sector-Flow Rate[a]	
	Computed	Observed		Computed	Observed
451	29.1	27.5	468	19.3	20.0
443	27.3	27.5	469	14.4	14.0
454	27.6	27.5	470	21.6	21.0
455	20.6	21.0	471	21.1	19.0
456	19.7	21.0	472	34.1	32.5
457	12.4	13.0	473	30.9	32.5
458	24.1	25.0	474	20.5	21.5
459	16.5	16.0	475	32.7	32.5
460	22.9	23.5	476	17.6	18.0
461	25.5	26.5	477	24.6	26.0
462	19.4	20.0	478	16.4	18.0
463	24.8	26.0	479	25.3	24.0
464	36.0	36.0	480	37.0	37.5
465	14.4	15.5	481	26.0	28.0
466	29.4	28.0	482	6.8	9.0
467	28.0	28.0	483	14.7	14.5

[a] Aircraft per hour.

$$f_{jk}^{(1)} = \begin{cases} p_{j-m_s/k} & \begin{cases} j = m_s + 1, m_s + 2, \ldots, m_s + m \\ k = 1, 2, \ldots, m_t \end{cases} \\ 0 & \text{otherwise} \end{cases} \tag{44}$$

$$f_{ij}^{(q)} = \sum_{k=1}^{M} f_{jk}^{(q-1)} p_{ijk} \tag{45}$$

In practice, a decision maker who wishes to predict sector flows will most likely want to specify both arrival rates and departure rates. To do so, the forward and backward analyses may be combined. Suppose the desired (target) source-generation and sink-attraction rates are $\{\lambda_i^{(t)},\ i = 1, 2, \ldots, m_s\}$, and $\{\mu_k^{(t)},\ k = 1, 2, \ldots, m_t\}$. Then the total generations and attractions that result from the sum of the forward and backward analyses will equal their targeted values if the model analysis rates $\lambda_i^{(s)}$ and $\mu_k^{(s)}$ are set to satisfy

$$\mu_k^{(s)} + \sum_{i=1}^{m_s} \lambda_i^{(s)} e_{ik}^{(\text{forward})} = \mu_k^{(t)} \quad k = 1, 2, \ldots, m_t \tag{46}$$

$$\lambda_i^{(s)} + \sum_{k=1}^{m_s} \mu_k e_{ik}^{(\text{backward})} = \lambda_i^{(t)} \quad i = 1, 2, \ldots, m_s \tag{47}$$

in which e_{ik} is determined from Equation 27 for the forward analysis and in a similar manner for the backward analysis. Note that, since the above set of equations does not have a unique solution, only $m_s + m_t - 1$ rates can be specified separately; the other rate is determined by the fact that the sum of the source-generation rates must equal the sum of the sink-attraction rates.

Tables 6 and 7 summarize the results of applying the above procedure to the New York center at the observed source-generation and sink-attraction rates (the en route source rate has been adjusted down to make the sum of the source and sink rates equal). Again, close agreement with observed sector flows is evident. To predict sector flows under projected increased traffic rates in and out of the region, the decision maker need only select new target values and repeat the above procedure.

CONCLUSION

The characterization of ATC network flows by using second-order Markov chains provides a technique for predicting sector flows that, although it is relatively easy to apply, can readily indicate potential areas of excess traffic loading. Based on empirical data, the method preserves general patterns of network flow without specifying the actual geometry of each aircraft's flight. For ATC network planners, such a method for predicting traffic distribution could provide a useful tool for ensuring safe and efficient movement of air traffic.

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REFERENCES

1. N.W. Polhemus. Modeling Aircraft Flow in Air Traffic Control Systems. Proc., Transportation Research Forum, 1974, Vol. 15, pp. 526-539.
2. J.R. Ford and D.R. Fulkerson. Flows in Networks. Princeton Univ. Press, Princeton, NJ, 1962.
3. R.B. Potts and R.M. Oliver. Flows in Transportation Networks. Academic Press, New York City, 1972.
4. T.W. Anderson and L.A. Goodman. Statistical Inference About Markov Chains. Annals of Mathematical Statistics, Vol. 28, No. 1, 1957, pp. 89-110.
5. C. Chatfield. Statistical Inference Regarding Markov Chain Models. Applied Statistics, Vol. 22, No. 1, 1973, pp. 7-20.
6. G.T. Duncan and L.G. Lin. Inference for Markov Chains Having Stochastic Entry and Exit. Journal of the American Statistical Association, Vol. 67, No. 340, 1972, pp. 761-767.

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Analyzing Ticket-Choice Decisions of Air Travelers

SCOTT D. NASON

This paper examines the nature of the problem that faces air travelers confronted with choosing from among a variety of air fares, each associated with different service characteristics, and the problem of forecasting these decisions. A theoretical framework is developed that views the problem at the level of the individual traveler; the ticket-type choice is expressed in terms of the individual's socioeconomic characteristics, the characteristics of the trip in question, and the level of service associated with each available alternative. Logit models are suggested as the preferable functional form on the basis of theoretical and computational grounds, and the properties of logit models are briefly described. A pilot application of the method is presented for a two-alternative situation (full fare versus standby) by using a small sample of interview data collected from departing passengers at Boston's Logan Airport. A calibrated model is presented that demonstrates a statistically significant relationship between the ticket-type choice and the fare, fare differential, trip purpose, automobile ownership (as a proxy for income), and the passenger's perception of the delays that may be expected if flying standby. This application merely demonstrates a method and could easily be improved by using the airlines' onboard surveys for estimation.

Events during the last few years have substantially altered the air-travel-demand forecasting requirements of the individual airlines. Until recently, the number of different fares available was quite limited, and differences among the fare packages available from individual airlines were almost nonexistent. In this environment, the crucial requirements were for an aggregate estimate of the size of an individual city-pair market, which may or may not have been based on the level of service available in that market, and a carrier's share of the total, based on a measure of that carrier's frequency share (or a more-sophisticated model that took into account the timing of those flights).

With the advent of deregulation, pricing freedom has emerged as a major factor that influences air-travel-demand decisions. Discount fares have stimulated new travel. Just as important to airline marketing departments is the impact on the yield per passenger or per passenger mile, which is affected by the passenger's choice of ticket type, as well as the impact of discount fares on the passenger's carrier-choice decisions. Passengers have always made minor distinctions between carriers on the basis of food, cabin attendants, or advertisements, but more and more there is a tangible economic incentive to choose one carrier over another. Examples include the unlimited-mileage tickets available on Eastern and Allegheny; the straight price reductions offered in some markets by National, Braniff, Texas International, World, and Transamerica (among others); and half-price coupon offers from United and American.

This paper examines the nature of these new decisions that face air travelers and proposes a technique that should prove useful in analyzing the passenger's ticket-choice decisions. The ticket-type choice is viewed within the context of the entire trip-planning process. Each individual's decision is based on that person's characteristics and the characteristics of each available alternative--travel time, price, reservation, length-of-stay restrictions, etc. This type of problem has exact parallels in other decision-making processes, and the modeling of personal preferences, which is well developed elsewhere, is adapted to the problem at hand.

TRIP-PLANNING PROCESS

There are several decisions involved in planning a trip by air; these include (a) a decision to travel somewhere, (b) a choice of destination or destinations and departure and return times, (c) a decision to fly in preference to other modes of travel, and (d) a selection of the least-expensive and most-convenient flight and ticket combination. For many trips, some of these decisions may be trivial or made simultaneously with other decisions. The first three (or even all four) are likely to be made simultaneously and without much hesitation in a number of cases, such as a necessary business trip from New York to Los Angeles. On the other hand, each decision may be distinct and nontrivial, as in the planning of a summer vacation.

The primary concern here is in analyzing the fourth decision, the flight and ticket-type choice. To that end, it is assumed merely that the first three decisions precede this one. That is, passengers determine the nature of their trip and then obtain the most-favorable flight and fare available. Problems that arise due to the less-common case in which flights or fares determine the destination choice or the duration of the stay must also be considered.

DETERMINANTS OF FLIGHT AND TICKET-TYPE CHOICE

Unlike standard air-travel-demand models, in which average population statistics and average fare and travel time characteristics are used to estimate aggregate demand, the ticketing problem should be viewed at an individual (or disaggregate) level. Each traveler's choice must be expressed in terms of the individual's characteristics and the characteristics of the choices available for the given trip.

Personal characteristics are used to approximate the different tastes of potential travelers. For example, it is generally conceded that individuals who have a high income place a higher value on travel-time savings and thus that income is a valuable indicator of how a traveler will trade off time or convenience versus money. A number of other personal characteristics may conceivably affect an individual's travel behavior; these may include age, sex, travel experience, or other factors.

A second class of personal characteristics relates to the particular trip that is being made. The purpose of the trip may serve as an important indicator of other underlying choice determinants, which include who is paying for the trip, the length of stay (which may affect eligibility for discount fares), and the flexibility of departure and arrival dates and times. The trip characteristics are likely to determine which fares, flights, or both are available to the traveler and may again affect the value of travel time or the certainty with which advance plans may be relied on.

Finally, the level-of-service (LOS) characteristics of the available alternatives are instrumental in the traveler's choice from among them. Alternatives will generally be differentiated by price and may be characterized by different amenities (aircraft type, food, drinks, movies, etc.); by different booking requirements (minimum and maximum stay; advance reservations, payment, or both; and cancellation fees); and by different trip-time characteristics (night flights, change of planes, lack of guaranteed space).

Naming these factors and acknowledging their importance is, of course, far easier than expressing the magnitude of their impact. The latter problem may best be handled, however, by obtaining information about the above-named factors from many travelers, which would include the flight and fare-type decisions they made, and by attempting to infer what other people would do as a function of their personal and travel characteristics.

LOGIT MODEL

The modeling of individual travel choices has progressed substantially in the last six to eight years. During this period a number of applications have helped to develop and refine the technical problems involved in modeling individual behavior. For a combination of theoretical, empirical, and computational reasons, the logit model has emerged as the most-common and practical model for problems of this type and appears to be well suited to the problem at hand. A more-complete discussion of logit and other disaggregate modeling techniques as well as the advantages and disadvantages of each has been given by McFadden (1) and by Richards and Ben-Akiva (2).

In order to apply a logit model to a choice problem, it is necessary to determine the set of alternatives that faces each individual and to define the attractiveness (utility) of each. The utility function for each alternative is expressed simply as a linear function of the traveler's socioeconomic characteristics, the trip characteristics, and the LOS variables for the given alternative in the relevant city-pair market. The sign and magnitude of the function coefficients are determined by means of an estimation process described below.

The utility functions (which express the relative attractiveness of each alternative) may be used to estimate the probability that any given alternative will be selected. This probability should, of course, increase with the utility of the alternative and should approach a maximum value of 1 for very high utilities or a minimum of 0 for very low ones. Also, the sum of the probabilities for the set of available alternatives must be 1 (i.e., only one alternative must be selected).

A number of possible functional forms meet the above criteria, but the logit model has generally been chosen for a variety of statistical reasons. In the logit model, the probability of selecting alternative i is expressed as follows:

$$P_i = \exp(U_i)/\Sigma\exp(U_j) \quad \text{for all j's} \tag{1}$$

This function yields an S-shaped curve that approaches 0 and 1 at extreme values of U_i. In addition, it should be noted that the curve is steepest (that is, changes in utility have the greatest effect) for values near i's proportional share of the market. Small changes in utility are relatively unimportant for very popular or very unpopular alternatives.

Computational features of the logit model make it relatively easy to set the probability of one or more alternatives to zero. Thus, for example, it would be necessary to make supersaver fares unavailable to travelers in markets in which such a fare is not offered. It would be prudent to make it similarly unavailable to business travelers who had little advance knowledge of the timing and destination of their trips. Such travelers do not assign a low utility to supersaver fares; they are simply unable to take advantage of them. Similarly, it may be argued that some alternatives are unavailable to many travelers because of a lack of knowledge of their existence.

DATA REQUIREMENTS

In order to estimate a logit model of passengers' ticket-type choice, it is necessary to have data on the decisions made by air travelers when faced with a known set of alternatives as well as data on passengers' socioeconomic characteristics. The requirements for information about the passenger and for a determination of which alternatives are actually available to a given traveler tend to argue against the use of waybill data for model estimation. All the necessary data could be obtained, however, by means of the airlines' on-board surveys.

These surveys should inquire about a few socioeconomic characteristics that may influence the passenger's decision, such as age, income, and travel experience. The major portion of the survey must elicit information about the trip being made; this would include its origin, destination, flight, fare class, purpose, duration, advance notice, flexibility, ticket purchaser, size of party, and other similar information.

The total number of data points obtained is not so important as the distribution of those observations. Specifically, it should be recognized that no inferences regarding an alternative may be made unless it was the choice of at least some of the respondents. That is, in order to identify the characteristics of trips made under the budget fares, for example, it is necessary to obtain observations from some who chose budget fares over other alternatives and some who chose other fares in preference to budget fares. It is the comparison of these two groups of people and their trips that yields the ability to predict what others may choose. Similarly, it should be recognized that carrier-choice models would require data from more than one carrier.

As a general rule, a random sample of a few thousand passengers would be more than adequate to support any such modeling effort that did not have specific goals regarding some lightly used alternative. Another option would be to use a choice-based sampling method, in which users of some alternative are specifically surveyed in order to ensure a minimum number of such observations. Known statistical methods can correct whatever biases may have resulted from nonrandom sampling.

Although coverage of every interesting alternative is necessary, it is not essential that every city-pair market be surveyed. In fact, the models should be totally independent of the markets for which they are calibrated. Market characteristics (such as fares, travel times, or distance) are factors in the ticket-type choice, but the effects of these variables should be uniform wherever basic behavior patterns are consistent. Thus an important feature of these models is that they may be estimated on one set of markets and applied to a different set of markets by substituting the appropriate personal and market data as input.

This geographic transferability is matched to a lesser extent by the temporal stability of the coefficients. So long as people do not alter their personal preferences with regard to the various LOS measures, the models will continue to hold their validity. This assumption will probably be valid for a few years. Shifts in use from one alternative to another will result (in the short run) from changes in the personal characteristics of travelers (such as changes in income or travel experience) or from other modeled variables such as the purposes of trips, the fares charged, or travel times.

ANALYZING NEW ALTERNATIVES

It follows from the discussion above that an alternative that does not exist and therefore that could not have been chosen in the sample requires special treatment. These options are, needless to say, perhaps the most interesting from a forecasting standpoint. Such alternatives fall into two classes and present differing magnitudes of concern.

First, many new alternatives are in fact only extensions of existing alternatives. A longer minimum stay or advance-purchase requirement, for example, may be analyzed within the present framework if the variables that reflect these requirements are cardinal rather than ordinal variables. Thus, an explanatory variable equal to the number of days before departure that the ticket must be purchased may be calibrated on observations of 7- and 14-day requirements and extrapolated (cautiously) to 30 days or some other length of time.

New fares or flights that do not have comparable existing examples require special treatment. In some cases it will be possible to infer reasonable values for the coefficients in the new utility function. More often, it is necessary to survey passengers regarding their hypothetical use of the new type of fare. There are, of course, dangers involved in inferring behavior patterns from individuals' stated actions in hypothetical situations, but such problems may be overcome if special care is taken.

SAMPLE APPLICATION

The modeling technique outlined above has been applied in one pilot study by using sample data collected for a hypothetical standby ticket. [A complete description of this study may be obtained from the author.] The limitations of such a data set should be recognized, and the following example should be viewed as a demonstration of a technique rather than a presentation of a usable model. In addition to this study, however, the modeling technique has been successfully applied to the mode-choice decisions of nonbusiness travelers in France by using a home-interview survey of 2000 households.

Passengers at Logan Airport were surveyed to determine the price differential required to induce them to forgo the benefits of the guaranteed seat. This value of a reservation varies with income and the passenger's perceived level of inconvenience if denied boarding. The two alternatives that face each passenger, therefore, are to fly under a full-fare ticket at a known fare and travel time or to fly standby at a lower fare and an estimated delay due to the possibility of not obtaining a seat. Each passenger survey must then be used to generate a set of ticket-type choices over a range of reservation prices. The passenger is assumed to choose the standby ticket for all reservation prices greater than the indicated value and a full-fare ticket for all smaller price differentials.

Utility functions for the two alternatives were defined by the following variables:

$$U = f(\text{FARE, EXPECTED DELAY, BUSINESS, CARS, FLORIDA, PROFESSION}) \quad (2)$$

where

EXPECTED DELAY = function of probability that a seat may not be obtained if flying standby (PNOSEAT) and length of delay if a seat is not obtained (DELAY);
BUSINESS = dummy variable: 1 for business trips, 0 otherwise;
CARS = number of cars in household;
FLORIDA = dummy variable: 1 if destination was Florida, Bermuda, or Caribbean (survey was conducted in February when these markets are well traveled); and
PROFESSION = dummy variables: PROFEM = 1 if employed, 0 otherwise; PROFR = 1 if retired, 0 otherwise (i.e., PROFEM = 0, PROFR = 0 for unemployed, students, etc.).

The dummy for business is an attempt to distinguish

Table 1. Some sample results.

Destination	Dummy Variables						Probability of Using Full Fare
	RESPR ($)	BUSINESS	PNOSEAT	DELAY	PROFEM	CARS	
St. Louis	40	No	0.75	5	1	1	0.35
	40	No	0.25	5	1	1	0.23
	20	No	0.75	5	1	1	0.68
	0	No	0.75	5	1	1	0.86
Los Angeles	40	No	0.75	5	1	1	0.94
	40	No	0.10	5	1	1	0.65
	40	No	0.10	5	1	2	0.74
	100	No	0.50	5	1	2	0.08
New York	5	No	0.50	1	1	2	0.58
	20	No	0.50	1	1	2	0.33

Note: Dummy variables are defined in Equations 2 and 5.

passengers who have not paid for their ticket. CARS, PROFEM, and PROFR are proxies for income, which were reluctantly used because true income figures were unobtainable.

There is no theoretically sound basis for inclusion of a dummy variable for Florida. In fact, one would expect that a sound data base would not require such a market segmentation for model estimation. The fact that it is necessary here reflects some difference between winter vacation spots and other destinations that is not captured by other variables. For the purposes of this application, it is helpful to view the set of all airline trips as being divided into business, winter vacation, and all other.

By using the formula in Equation 1, it is possible to express the probability of choosing full fare as follows:

$$P_{FULL\ FARE} = \exp(U_{FULL\ FARE})/[\exp(U_{FULL\ FARE}) + \exp(U_{STANDBY})] \qquad (3)$$

By dividing the numerator and the denominator by $\exp(U_{FULL\ FARE})$, it can be shown that

$$P_{FULL\ FARE} = 1/\{1 + \exp[-(U_{FULL\ FARE} - U_{STANDBY})]\} \qquad (4)$$

Thus the probability of choosing full fare depends only on the difference between the two utilities. This property of logit models is used to simplify the estimation process.

The equation to be estimated is given as follows:

$$\begin{aligned} U_{FULL\ FARE} - U_{STANDBY} = {} & A_0 + A_1(RESPR) + A_2(FULL\ FARE) \\ & + A_3(EXPECTED\ DELAY) \\ & + A_4(BUSINESS) + A_5(CARS) \\ & + A_6(FLORIDA) + A_7(PROFEM) \\ & + A_8(PROFR) \end{aligned} \qquad (5)$$

where RESPR is the difference between full fare and standby (or reservation price).

This model was estimated and the following model was obtained:

$U_{FULL\ FARE} - U_{STANDBY}$ = -2.247[-6.0] - 0.069 [-16.5]RESPR + 0.0211[8.79]FULL FARE + 1.178[4.35] EXPECTED DELAY + 0.9867[4.63]FLORIDA - 0.7511 [-2.81]BUSINESS + 0.5585[1.91]PROFEM + 0.0746[0.19] PROFR + 0.4332[4.10]CARS.

The numbers in brackets are t-statistics and indicate that all coefficients except the professional classes are significant at the 95 percent confidence level. In addition, all coefficients except the dummy for business have the anticipated signs. The statistics for the logit model's goodness-of-fit measures are (a) log of likelihood function = -356, (b) percent right = 88, and (c) -2 log-likelihood ratio = -1132 (with 9 df). These are comparable in some respects to the R^2- and standard-error measures from regression results.

By substituting this calculation into Equation 5, it is possible to express the probability of using full fare for any observation or prospective traveler. The estimated probabilities of choosing the full-fare option are shown in Table 1 for a range of values. The observed probabilities seem reasonable for values of the independent variables that lie near the center. This logit model breaks down, however, as RESPR goes to zero or as PNOSEAT goes to zero. Clearly, the probability of reserving should go to 1 and 0, respectively, for these two cases. That these two results cannot be achieved is due to the near-linear utility function, which does not capture the asymptotic effects as these limits are approached.

MODEL APPLICATIONS

In order to use such a model in a marketing framework, it is necessary that the model provide data for the population as a whole so that price elasticities and revenue estimates may be determined.

Direct elasticities can be calculated from the logit results by the following formula:

$$\epsilon_{ikt} = (\partial P_{it}/\partial X_{ikt})(X_{ikt}/P_{it}) = (1 - P_{it})(X_{ikt})(\beta_k) \qquad (6)$$

where

ϵ_{ikt} = elasticity for person i on mode t with respect to variable k,
P_{it} = probability that i will choose t,
X_{ikt} = value of kth variable for i on mode t, and
β_k = coefficient of kth variable.

For the first example in Table 1, this means that we can predict that this person will choose full fare 35 percent of the time and that a 1 percent increase in the price differential will cause a 1.8 percent decrease in that figure [ϵ = (1 - 0.35) (40) (-0.069) = -1.8]. In this range, reservations are price elastic; that is, passengers are highly sensitive to price changes. To an airline, which presumably would be concerned about encouraging full-fare passengers if revenues less the increased costs of carrying additional passengers will not decline, it is important not to operate in the highly price-elastic range. It can be shown that RESPR = $30 implies that the probability of traveling full fare = 0.52 and therefore that ϵ = (1 - 0.52) (30) (-0.069) = -1.00. The airline can thus expect to increase revenue (from the group of passengers represented by the first row of Table 1), since it decreases the price differential toward $30.

This method should prove particularly helpful in analyzing not only fare changes, but also such LOS characteristics as minimum-stay requirements, advance-purchase requirements, and preferences for certain types of aircraft. Once these variables have entered into the utility functions and coefficients have been estimated, it is possible to measure the impact of changes in these characteristics. Even more helpful will be the ability of the analyst to identify the reactions of various market segments, which will permit the type of price discrimination necessary to induce new business without diverting revenues from full-fare passengers.

In order to project aggregate demand for a given alternative, it is necessary to have information about the potential travelers and their potential trips. The total number of trips in a market and the characteristics of trip makers must be forecast externally from these ticket-type models by using the carrier's standard methods. This process may involve sophisticated models of trip generation or may be based on something as simple as a projected market-growth trend and an assumption that the socioeconomic distribution of passengers remains unchanged. In either case, aggregation methods require that the carrier's passengers be grouped into some number of relatively homogeneous cells. The model must then be applied to each cell separately; this will forecast the ticket-type choice of its members and accumulate the aggregate shares for each alternative.

CONCLUSIONS

The above discussion has set forth a proposed method for analyzing the many new factors that affect the airline-passenger flight and ticket-selection process. The model relies on a statistical technique that is well tested in other behavioral modeling disciplines and particularly in modeling transportation mode-choice decisions.

A pilot application of the model was performed on a set of survey observations by using a two-alternative choice set--full fare or standby--but is easily extended to any number of alternatives and is adaptable to many types of distinguishing characteristics, such as booking requirements, length-of-stay requirements, and time-of-day restrictions. A complete data set for estimation could easily be obtained by using the on-board surveys made by the carriers. With the richer data base, many of the simplifying assumptions in the pilot application could be relaxed, which would provide a sound model to aid carriers in their complex marketing decisions.

REFERENCES

1. D. McFadden. Conditional Logit Analysis of Qualitative Choice Behavior. In Frontiers in Econometrics (P. Zarembka, ed.), Academic Press, New York, 1973.
2. M. Richards and M. Ben-Akiva. A Disaggregate Travel Demand Model. Heath, Lexington, MA, 1975.

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Assessing the Safety and Risk of Air Traffic Control Systems: Risk Estimation from Rare Events

ALLEN C. BUSCH, BRIAN COLAMOSCA, J. STUART HUNTER, AND NEIL W. POLHEMUS

To assess the safety and risk of current and proposed air traffic control route-separation standards, it is necessary to estimate the frequency of occurrence of extremely rare events. Since direct estimates of collision risk from historical data require sample periods that are unacceptably long, alternative methods are necessary. This article describes a probabilistic model for collision risk and its use in the North Atlantic airspace; it includes a discussion of sequential testing designed to determine whether current navigational performance meets a specified target level of safety.

A problem frequently encountered by analysts who wish to determine the level of risk associated with a particular transportation system is that of estimating the frequency of rare events. Most catastrophic transportation accidents, such as the midair collision of two commercial airliners, occur so infrequently that estimates of the accident rate are difficult to obtain directly. Consequently, probabilistic models are often constructed to describe the various factors that must occur to cause an accident. Estimates of the rate of occurrence of these factors are then obtained separately and combined later in an overall-risk computation.

An example of such an indirect approach is that of collision-risk methodology, first proposed by Reich (1) to estimate the risk of midair collisions between aircraft strategically separated in the lateral, longitudinal, and vertical dimensions and subsequently applied to determine route spacing in the North Atlantic and Central East Pacific regions (2-4). Essentially, the model factors the occurrence of a collision into three events (lateral overlap, longitudinal overlap, and vertical overlap), all of which must occur simultaneously to create a collision. Since the frequency of each event is several orders of magnitude higher than the frequency of a collision, it can be estimated in a sufficiently short period of time. If we assume that the three events are independent, their probabilities can then be multiplied to estimate the probability of a collision.

This paper examines some of the important estimation problems raised in applying collision-risk methodology to oceanic environments. We concentrate first on the Poisson distribution, long used to model the occurrence of rare events, and discuss the problem of obtaining precise estimates of the rate parameter. The collision-risk model is then introduced and its use in risk estimation described. Emphasis is placed on the questions of validation and monitoring, both of which are necessary ingredients in establishment of a minimum navigational performance standard. The discussion includes both sequential and fixed-sample-size testing.

DIRECT ESTIMATION OF ACCIDENT RATES

The Poisson model for rare events states that the probability of x accidents in a period Δt follows the Poisson distribution

$$p(x) = (\lambda\Delta t)^x \exp(-\lambda\Delta t)x! \quad x = 0, 1, 2, \ldots \qquad (1)$$

where λ, the rate parameter, is expected number of accidents per unit of time. The expected value and variance of x are equal and given by

$$E(x) = \mathrm{var}(x) = \lambda\Delta t \qquad (2)$$

Figure 1 shows a plot of the Poisson distribution for $\lambda = 0.1 \times 10^{-7}$ and $\Delta t = 10^7$ track-system flying hours. The rate of 0.1 collision/10^7 flying hours has been selected by the International Civil Aviation Organization (ICAO) as the target level of safety (TLS) for use in setting oceanic navigational performance standards. Since the yearly number of track-system flying hours in the North Atlantic organized track system is currently less than 400 000, the chance of a midair collision in any given year at the TLS is extremely small.

Now suppose that one wishes to verify from historical data that the accident rate in the North Atlantic organized track system as currently structured does not exceed the TLS. Further, suppose we assume the rate to remain constant and begin monitoring the system. After observing T track-system flying hours with no midair collisions, the maximum-likelihood estimate for λ is $\hat{\lambda} = 0$ accident/10^7 flying hours. While $\hat{\lambda} = 0$ is the best point estimate for the accident rate, it is clearly unacceptable by itself, since some risk certainly exists. Of considerably more use is an interval estimate for λ that has upper and lower limits ($\hat{\lambda}_L$, $\hat{\lambda}_U$). Classical results for the Poisson distribution show that the limits of a 100 (1 - α) percent confidence interval for λ can be obtained from (5, p. 96)

$$\hat{\lambda}_L = (1/2T)\chi^2_{0,\alpha/2} \qquad (3)$$

$$\hat{\lambda}_U = (1/2T)\chi^2_{2,1-\alpha/2} \qquad (4)$$

where $\chi^2_{\nu,\alpha/2}$ is that value of a χ^2-distribution with ν degrees of freedom for which the probability of a larger value equals $\alpha/2$.

Figure 2 plots the upper 95 percent confidence limit ($\hat{\lambda}_U$) as a function of the length of monitoring period T. Note that, with no observed accidents, a period of approximately T = 400 million flying hours is required to bring $\hat{\lambda}_U$ below 0.1×10^{-7}. At current traffic levels, this corresponds to about 1000 years, which makes direct validation clearly impractical.

A slightly different perspective is obtained if one takes a Bayesian approach. If we begin with a noninformative prior distribution for λ, the posterior distribution for λ after T hours with no midair collisions is a χ^2-distribution with 1 df. Figure 3 plots the area of the posterior distribution that lies below 0.1×10^{-7} as a function of T, which shows how the degree of belief that the TLS is being achieved increases as the period of midair collisions increases. Again, the time required for that belief to reach an acceptable level is too long for practical purposes.

COLLISION-RISK METHODOLOGY

Because of the extremely long periods required to obtain precise estimates of the rate of occurrence of rare events, alternative formulations are necessary. In a series of articles, Reich (1) describes a probabilistic model for estimating collision risk in a system of parallel routes on which aircraft are strategically separated in the along-track, cross-track, and vertical dimensions. As subsequently developed and applied by the Federal Aviation Administration (FAA) and ICAO, the collision-risk model takes the following form:

$$N_{ay} = 10^7 [P_y(S_y)] P_z(0) \frac{\lambda_x}{S_x} \left\{ E_y(\mathrm{same}) \left[\left|\frac{\overline{\Delta V}}{2\lambda_x}\right| + \left|\frac{\overline{\dot{y}(S_y)}}{2\lambda_y}\right| + \left|\frac{\overline{\dot{z}(0)}}{2\lambda_z}\right| \right] + E_y(\mathrm{opp}) \left[\left|\frac{\overline{V}}{\lambda_x}\right| + \left|\frac{\overline{\dot{y}(S_y)}}{2\lambda_y}\right| + \left|\frac{\overline{\dot{z}(0)}}{2\lambda_z}\right| \right] \right\} \qquad (5)$$

where

N_{ay} = expected number of collisions in 10^7 track-system flying hours;
S_y = lateral separation between parallel tracks;
$P_y(S_y)$ = probability of lateral overlap, given lateral separation S_y;
$P_z(0)$ = probability of vertical overlap for co-altitude aircraft;
$\lambda_x, \lambda_y, \lambda_z$ = longitudinal, lateral, and vertical dimensions of a typical aircraft;
S_x = longitudinal dimension of proximity shell for measuring occupancy;
$E_y(\mathrm{same}), E_y(\mathrm{opp})$ = same- and opposite-direction occupancies;
$\overline{|\Delta V|}, \overline{|\dot{y}(S_y)|}, \overline{|\dot{z}(0)|}$ = average relative along-track, cross-track, and vertical closing velocities for same-direction traffic; and
$\overline{|V|}, \overline{|\dot{y}(S_y)|}, \overline{|\dot{z}(0)|}$ = average relative along-track, cross-track, and vertical closing velocities for opposite-direction traffic.

An excellent discussion of the model, which includes its mathematical development, is given by Busch, Colamosca, and Vander Veer (2).

The above model factors the occurrence of a collision into essentially three events--longitudinal overlap, lateral overlap, and vertical overlap--all of which must occur simultaneously to create a collision. Overlap in a given dimension is defined as a situation in which two aircraft deviate from their planned positions in such a manner that their centroids are within some critical distance (such as a wingspan) of each other in that dimension. One then proceeds to estimate the probability of the three types of overlap, which are assumed to be independent (an assumption that has been tested from empirical data and appears reasonable).

In a general context, the most important aspect of the above model is its description of a collision in terms of events whose probability can be estimated with acceptable precision in a reasonable time. For example, consider the probability of

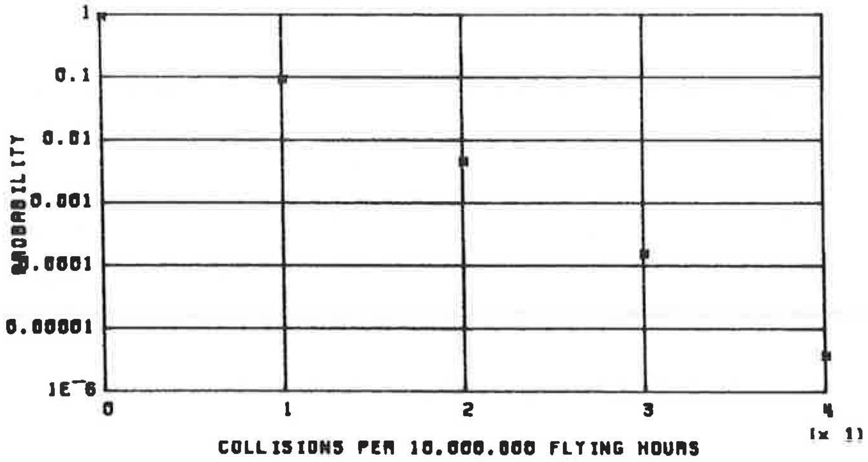


Figure 1. Poisson distribution rate 0.1 x 10^{-7}.

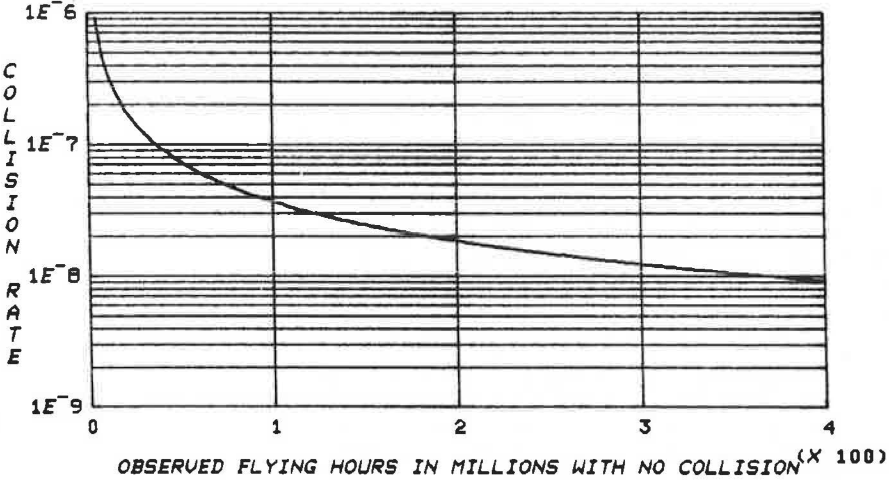


Figure 2. Upper 95 percent confidence limit for collison rate.

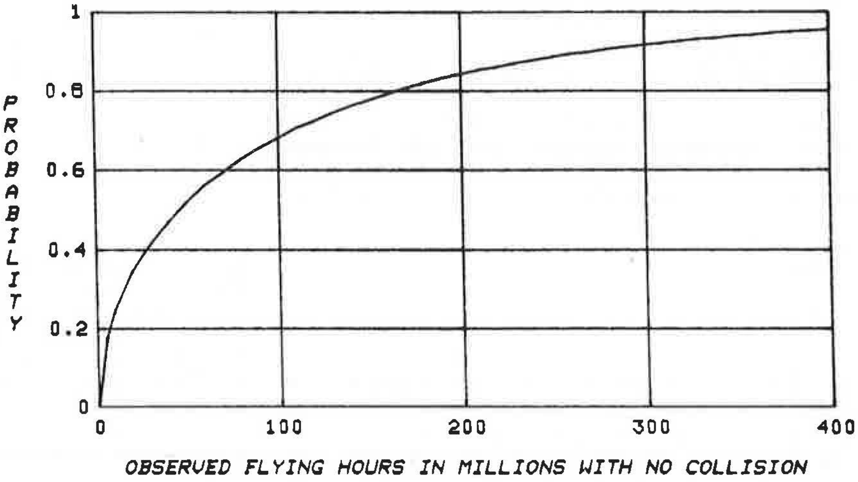


Figure 3. Degree of belief below the TLS.

lateral overlap $[P_y(S_y)]$, which is a function of the lateral distance S_y between two routes. If f(y) is the density function of lateral deviations from the track and if navigation of aircraft on adjacent tracks is independent, this probability is approximately

$$P_y(S_y) \doteq 2\lambda_y \int_{-\infty}^{\infty} f(y)f(S_y + y)dy \qquad (6)$$

which is a convolution of the distributions on the two routes.

Given estimated or concurred values (or both) for the other parameters in the collision-risk model (3), it was determined from Equation 1 that to meet the TLS in the North Atlantic requires a value of

$$C(S_y) = \int_{-\infty}^{\infty} f(y)f(S_y + y)dy \leq 6.45 \times 10^{-6} \qquad (7)$$

Kerstein (6) showed that if f(y) was symmetric with zero mean and unimodal with a small, slowly varying tail, then

$$C(S_y) \doteq 2f(S_y) \tag{8}$$

Use of this result for S_y = 60 nautical miles (a track separation currently under consideration for the North Atlantic) requires that the proportion of absolute lateral deviations between 50 and 70 nautical miles off track be below 1.3×10^{-4}; i.e.,

$$\text{prob}(S_y - 10 \leqslant |y| \leqslant S_y + 10) \leqslant 1.3 \times 10^{-4} \tag{9}$$

In the terminology of earlier reports, this requirement is called the zeta (ζ) criterion.

Since radar coverage is available only at the ends of the routes, only one measurement of the lateral deviation from path is available at the egress point of each flight. By making the conservative assumption that the observed distribution of lateral deviations at the end of a route is applicable across the entire route length, the observed lateral deviations could be used to determine whether the proportion in the band of 50-70 nautical miles meets the requirement given in expression 9. As discussed in the following section and given for some 120 000 flights per year in the organized North Atlantic route system, the time required to decide that such a standard is being met with reasonably high confidence is practical.

The requirement in expression 9 forms a part of the minimum navigational performance standard (MNPS) that is now being used in the North Atlantic and Central East Pacific organized track systems. (Additional specifications have been formulated to ensure that the assumptions employed to derive ζ are being met.) Full details of the MNPS requirement are contained in documentation from ICAO (4).

The next section discusses in detail sampling plans designed to determine compliance with the ζ-requirement.

COMPLIANCE EVALUATION

In testing compliance with a specified navigational performance standard, it is important to consider carefully the characteristics of any statistical test employed. In particular, suppose that it is decided to consider a separation between two adjacent routes of S_y = 60 nautical miles. At this separation, we formulate two hypotheses:

$$\begin{aligned} H_0&: \zeta = 1.3 \times 10^{-4} \\ H_1&: \zeta = 2.6 \times 10^{-4} \end{aligned} \tag{10}$$

The initiating hypothesis (H_0) corresponds to collision risk at the TLS, or 1.3×10^{-4}, while the alternative hypothesis (H_1) corresponds to a level deemed clearly unacceptable.

If S_y = 60 nautical miles, the problem faced by the decision maker is given in the following decision-analysis table:

	True State of Nature	
Decision	H_0 Is True	H_1 Is True
Accept H_0 and reduce separation	Correct decision	Type II error (unsafe)
Reject H_0 and do not reduce separation	Type I error (costly)	Correct decision

A type I error, whose probability is denoted by α, would mean that the decision to establish the separation of 60 nautical miles was rejected even though the risk at that separation met the TLS. Such an error can create serious economic penalties, particularly in use of fuel. A type II error, whose probability is denoted by β, creates an unsafe condition in that the risk, after a separation of S_y = 60 nautical miles has been decided on, exceeds the TLS.

In monitoring compliance with the standard, both α and β must be considered. The fixed-sample-size likelihood ratio test for the above hypotheses has the following decision rule: Reject H_0 if the proportion of deviations in the band of 50-70 nautical miles exceeds some value k. For fixed α and β, k and sample size N can be determined from

$$1 - \alpha = \sum_{x=0}^{k} (N\lambda_0)^x \exp(-N\lambda_0)/x! \tag{11}$$

$$\beta = \sum_{x=0}^{k} (N\lambda_1)^x \exp(-N\lambda_1)/x! \tag{12}$$

where $\lambda_0 = 1.3 \times 10^{-4}$ and $\lambda_1 = 2.6 \times 10^{-4}$. For the hypotheses in Equations 10, if α and β are both set at 5 percent, the solutions to Equations 11 and 12 are k = 22 and N = 120 900. Given current monitoring of 35 percent of the traffic in the North Atlantic, a sample size equal to 120 900 would require a sample period of approximately three years. Such a scheme is practical and could be implemented, although the time to decision is admittedly long.

If the actual proportion of deviations in the band of 50-70 nautical miles is either much less than λ_0 or much greater than λ_1, a decision could be made much sooner by using a sequential-probability ratio test. Such a procedure works as follows: Suppose N flights have been observed, of which x are between 50 and 70 nautical miles off track. Then (a) accept H_0 if $x \leqslant k_1$, (b) reject H_0 if $x \geqslant k_2$, and (c) continue sampling if $k_1 < x < k_2$, where k_1 and k_2 are functions of N. The testing starts with N = 1 and continues until a decision is made by means of either step a or step b.

For two simple hypotheses that involve Poisson parameters,

$$k_1 \doteq \{\log[\beta/(1-\alpha)]/\log(\lambda_1/\lambda_0)\} + [N(\lambda_1 - \lambda_0)/\log(\lambda_1/\lambda_0)] \tag{13}$$

$$k_2 \doteq \{\log[(1-\beta)/\alpha]/\log(\lambda_1/\lambda_0)\} + [N(\lambda_1 - \lambda_0)/\log(\lambda_1/\lambda_0)] \tag{14}$$

which are parallel lines with N plotted along the abscissa and x along the ordinate. For the hypotheses in Equations 10 and $\alpha = \beta = 0.05$, sampling continues as long as

$$-4.25 + (1.876 \times 10^{-4})N < x < 4.25 + (1.876 \times 10^{-4})N \tag{15}$$

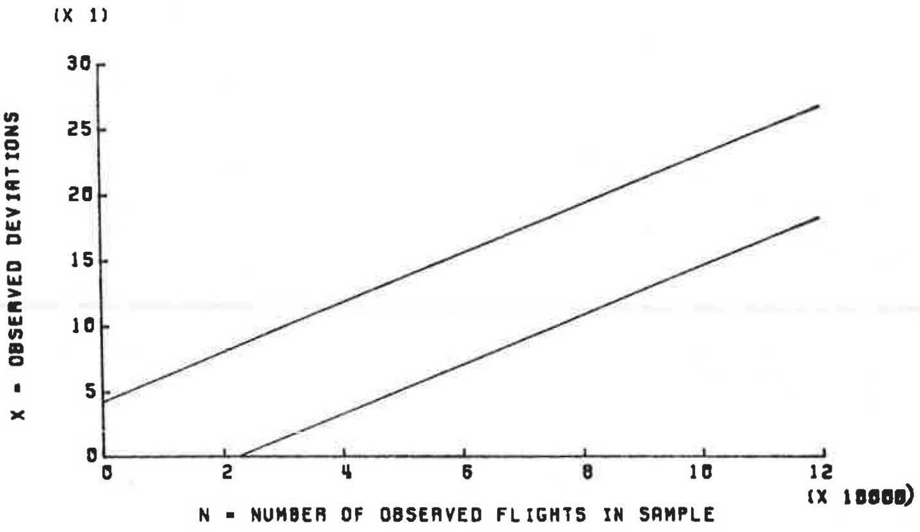
as illustrated in Figure 4. It will be observed that if the risk in the system is very large (that is, if $\zeta >> 1.3 \times 10^{-4}$), H_0 could be rejected very quickly. On the other hand, if the risk is actually much less than the TLS, a decision could be made to declare the 60-nautical-mile route separation acceptable in as few as 23 000 observed flights, given that zero aircraft were seen in the band of 50-70 nautical miles.

The expected sample size to reach a decision between the two hypotheses is given by

$$\begin{aligned} E(N|H_0) = \{ &\alpha \log[(1-\beta)/\alpha] + (1-\alpha)\log[\beta/(1-\alpha)]\} \\ &\div [\lambda_0 \log(\lambda_1/\lambda_0) - (\lambda_1 - \lambda_0)] \end{aligned} \tag{16}$$

$$\begin{aligned} E(N|H_1) = \{ &(1-\beta)\log[(1-\beta)/\alpha] + \beta \log[\beta/(1-\alpha)]\} \\ &\div [\lambda_1 \log(\lambda_1/\lambda_0) - (\lambda_1 - \lambda_0)] \end{aligned} \tag{17}$$

Figure 4. Sequential sampling plan for zeta.



which, in our example, yield $E(N|H_0) = 66\ 431$ and $E(N|H_1) = 52\ 770$, both of which are quite a bit smaller than the fixed-sample-size solution.

The above sequential-sampling plan was in fact adopted to assess whether route separation in the North Atlantic could safely be reduced to 60 nautical miles. As of the writing of this article, the data collection is in the continue-sampling region.

CONCLUSION

In estimating the risk of catastrophic transportation accidents (which occur very infrequently), direct estimation based on the frequency of occurrence of such events is not practical due to the extremely long sample periods needed to get reasonable estimates. In such cases, alternative methods, such as those described in this article, are necessary to reduce the required sampling period. In doing so, it is usually necessary to employ mathematical models based on assorted assumptions that, it is hoped, are reasonable. One of the basic tenets applied in developing and implementing collision-risk models to be used in practice was always to err when necessary on the conservative side, i.e., to make assumptions that, if not correct, would overestimate rather than underestimate the risk.

Careful design of statistical decision-making procedures is also important to control both types of wrong decisions. Too often, one or the other type of error is neglected, and the emphasis is placed solely on either safety or cost. The testing procedures described in this article and used in the North Atlantic systematically control the probability of both types of errors, which gives the decision maker assurance that the correct decision will be made with high probability.

REFERENCES

1. P.G. Reich. Analysis of Long-Range Air Traffic Systems: Separation Standards I, II, III. Journal of the Institute of Navigation, Vol. 19, 1966.
2. A.C. Busch, B. Colamosca, and J.R. Vander Veer. Collision Risk and Economic Benefit Analysis of Composite Separation for the Central East Pacific Track System. U.S. Department of Transportation, Rept. FAA-EM-77-5, 1977.
3. Methodology for the Derivation of Separation Minima Applied to the Spacing Between Parallel Tracks on ATS Route Structures. International Civil Aviation Organization, Montreal, Circular 120-AN/89/2, 1976.
4. Report of the Limited North Atlantic Regional Air Navigation Meeting, 1976. International Civil Aviation Organization, Montreal, Document 9182, 1976.
5. N.L. Johnson and S. Kotz. Distributions in Statistics: Discrete Distributions. Houghton Mifflin, New York, 1969.
6. A.R. Kerstein. Statistical Methods for the Estimation of Aircraft Collision Risk. Department of Civil Engineering, Princeton Univ., Ph.D. dissertation, 1976.

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Forecasts of Aviation Fuel Consumption in Virginia

ANTOINE G. HOBEIKA, A. BOONPUAN, AND F. TAMBERRINO

Aviation fuel shortages and their impact on airline services and fuel-tax revenues have encouraged transporters, suppliers, and state agencies to look more closely at future aviation fuel consumption. This paper, the work for which was sponsored by the Virginia Division of Motor Vehicles, determines forecasts for aviation fuel consumption in Virginia under various socioeconomic and airline-policy conditions. The forecasting method is an econometric model that consists of 17 basic components; the major components are population and economy. The state population and economic conditions are considered the major forces that affect travel behavior, airline service, and, in turn, aviation fuel consumption. The model clearly distinguishes between operations of air carriers and of general aviation. General aviation local operations and their itinerant piston-powered operations are considered to consume only aviation gasoline, whereas the remaining aviation operations consume jet fuel. A separate model was developed for each air-carrier airport, whereas an aggregated state model was built for general aviation operations. The scenarios tested include high gasoline prices, rising consumer price index for all goods, high air fares, improved fuel-efficient aircraft, and many other factors. The results show that aviation fuel consumption continues to increase but at different rates, which depend on the economic conditions in the state. Airline policies were also found to affect the amount of jet fuel consumption greatly.

Since 1950, there has been a steady rise in intercity travel by most major modes--private automobile and common carrier (rail, bus, and air). Among the common carriers, air travel increased from 14.2 percent in 1950 to nearly 85 percent in 1978 according to the Air Transport Association Facts and Figures of 1979. The growth in aviation is expected to continue, especially because of the recent deregulation of the airlines, the cut in air fares, and the provision of different air services.

At the state level, similar trends have been realized. In the last decade, aviation activities in Virginia have been growing at a considerable rate. In 1967, there were 1 773 814 domestic enplaned passengers, 1311 licensed aircraft, and 4991 licensed pilots. In 1977, there were 3 066 299 domestic enplaned passengers, 2465 licensed aircraft, and 10 724 licensed pilots. Concurrent with the growth in aviation activities is the growth in the demand for aviation jet fuel. The amount of aviation jet fuel consumed in Virginia had increased from 118 003 671 gal in 1968 to 210 547 896 gal in 1978.

However, the recent rise in fuel prices and the limited availability of aviation fuel have created a substantial impact on the Virginia air transportation system, so that many flights have had to be cancelled or rerouted. The Division of Motor Vehicles (DMV) of the commonwealth of Virginia is in charge of collecting the state tax revenues for fuel, which includes aviation fuel. They sponsored the work of determining the state's future demand for aviation fuel under various scenarios and the associated tax revenues in order to aid in the proper scheduling and allocation of future expenditures in this area.

Thus, we have developed a computerized econometric model for DMV to forecast aviation fuel consumption and its associated tax revenues. This paper also examines and discusses the impacts of alternative future scenarios such as high gasoline prices, rising inflation, and changing airline policies.

MODELING APPROACH

There are two major kinds of aviation fuel--aviation jet fuel and aviation gasoline. Aviation jet fuel is consumed mostly by air carriers and partly by general aviation (GA) aircraft with turbine-powered engines. Because of these two distinct kinds of fuel, aviation activities were separated into air-carrier and GA activities.

In determining air-carrier activities, each air-carrier airport is considered individually to encompass its own distinct characteristics and environment. An aggregated state model that jointly addresses all the air-carrier airports was eliminated from consideration, because it obscures the variability among the different airports and is not sensitive to the future conditions at each airport. There are now 11 air-carrier airports in Virginia: Charlottesville, Danville, Dulles International, Hot Springs, Lynchburg, Newport News, Norfolk, Richmond, Roanoke, Staunton (Shenandoah Valley), and Washington National. For purposes of tax revenue, Washington National Airport will not be considered in the model, since there is a state agreement that declares aviation fuel consumption at this airport to be nontaxable.

GA activities in Virginia take place at 84 airports. Similarly, for purposes of tax revenue, the model did not consider the GA activities at Washington National Airport. An aggregated state model that combined the GA activities at all airports was developed in this case because building a separate model for each individual airport was found unnecessary and the availability of data at each airport was limited. Since the quantity of aviation fuel consumption is directly related to the amount of aircraft operations, the model then focused on forecasting the number of aircraft operations in Virginia. Historically, GA operations at airports that had or did not have towers were not growing at the same rate. This fact led to the categorization of GA operations into towered and nontowered operations. Also, at each GA airport, two major types of operations were considered--itinerant and local.

MODEL STRUCTURE

The forecasting model is an econometric model that consists of 17 basic components, as shown in the flowchart in Figure 1.

The model was developed under the assumption that population and economic factors are the major forces that affect travel behavior, aviation fuel consumption, and (in turn) potential for tax revenue. This is clearly depicted in Figure 1, in which both population and economy are directly or indirectly related to each component of the model. Both are shown to affect air-carrier enplaned passengers and GA operations. Air-carrier enplaned passengers were converted to air-carrier departing flights by determining the number of enplaned passengers per departing flight, which is the multiplicative result of the seat-load factor and the available revenue seats. These departing flights were directly related to air-carrier jet fuel consumption. GA operations were classified into itinerant and local operations. All local operations were considered to be piston-powered operations, which consume aviation gasoline. All itinerant operations were considered to be both turbine- and piston-powered operations, which consume aviation jet fuel and aviation gasoline,

Figure 1. General relationships of aviation-fuel-consumption model.

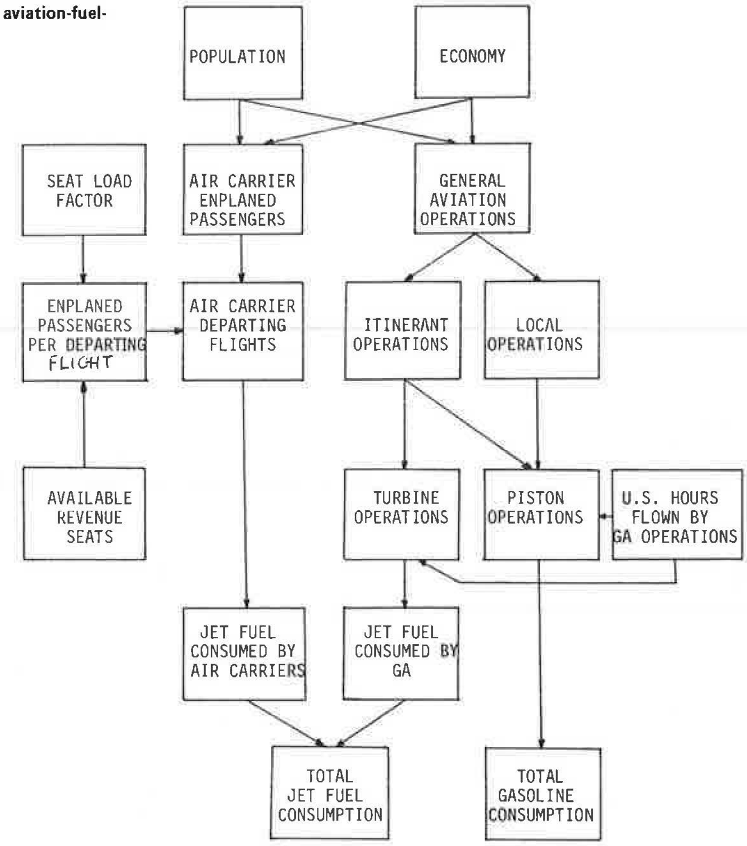

Figure 2. Airport service areas.

respectively. Total aviation jet fuel consumption is the sum of the consumption of both air-carrier jet fuel and GA jet fuel. Aviation gasoline consumed by GA piston-powered operations represented the total aviation gasoline consumption in Virginia. Aviation fuel-tax revenues were then easily determined by multiplying the aviation fuel consumption by the tax rate.

AIR-CARRIER DOMESTIC ENPLANED PASSENGERS

In determining the aviation jet fuel consumption at each air-carrier airport, the number of domestic enplaned passengers at each airport annually was forecast first. The number of enplaned passengers was considered to be influenced by the following variables: (a) service-area population, (b) service-area real per-capita personal income, (c) average air fare per revenue passenger mile, and (d) cost of 1 mile of automobile operation. The airport service areas are defined by the Division of State Planning and Community Affairs of the commonwealth of Virginia as the counties and cities located roughly within a radius of 60-70 miles from each airport. The service areas for each airport are shown in Figure 2. The real per-capita personal income is expressed on the base-year (1967) value. That is, real per-capita personal income for any particular year is equal to per-capita personal income of that year multiplied by the ratio of the consumer price index (CPI) for all goods in the base year to the CPI for all goods in that particular year. The cost of 1 mile of automobile operation was obtained from secondary sources (1). The

Table 1. Forecasting equations for air-carrier airports.

Airport	Forecasting Equation	R^2
Charlottesville	CHAPAS(I) = −143 438.3 + 1.411 92CHAPOP(I) + 6709.274[ACPM(I)/USFARE(I)]	0.943
Danville	DANPAS(I) = 10 049.59 − 747.0708USFARE(I) + 171.9913ACPM(I)	0.363
Dulles International	DULPAS(I) = −971 783.3 + 0.905 08DULPOP(I) + 191.2122DULPCI(I) + 37 462.33 [ACPM(I)/USFARE(I)]	0.987
Hot Springs	HOTPAS(I) = 5206.287 + 1.0791HOTPCI(I) −788.0317USFARE(I) + 180.9274ACPM(I)	0.217
Lynchburg	LYNPAS(I) = −106 442.6 + 0.7303LYNPOP(I) + 0.3440LYNPCI(I) + 30 602.15 [ACPM(I) ÷ USFARE(I)] − 12 162.02LYNDUM(I)[a]	0.873
Newport News	NEWPAS(I) = 84 478.09 + 0.185 75NEWPOP(I) + 54.045NEWPCI(I) − 19 939.13USFARE(I)	0.791
Norfolk	NORPAS(I) = −1 963 571 + 2.903 97NORPOP(I) + 143.2402NORPCI(I) − 12 720.67USFARE(I)	0.897
Richmond	RICPAS(I) = −52613.4 + 0.7574RICPOP(I) + 57.2030RICPCI(I) + 7581.388ACPM(I)	0.915
Roanoke	ROAPAS(I) = 640 679.1 + 0.7845ROAPOP(I) + 191.531ROAPCI(I) + 38 254.93 [ACPM(I)/USFARE(I)]	0.981
Staunton	STAPAS(I) = 11 777.796 + 0.005 06STAPOP(I) + 1.592STAPCI(I) + 3846.426[ACPM(I) ÷ USFARE(I)]	0.187

[a] LYNDUM(I) = dummy variable that represents the reduction of daily flights by Piedmont Airlines (prior to 1974 = 0; in 1974 and thereafter = 1).

average air fare per revenue passenger mile was obtained from the Civil Aeronautics Board (CAB) reports to Congress for various years.

The number of domestic enplaned passengers at each airport was considered to be a linear function of the above four socioeconomic variables. The typical relationship between the dependent variable (domestic enplaned passengers) and the independent variables at each airport by using the multiple-linear-regression technique is anticipated to be as follows:

$$\mathrm{XPAS(I)} = a + b * \mathrm{XPOP(I)} + c * \mathrm{XPCI(I)} + d * [\mathrm{ACPM(I)/USFARE(I)}] \qquad (1)$$

where

XPAS(I) = number of domestic enplaned passengers at airport X in year I,
XPOP(I) = population who live in airport X service area in year I,
XPCI(I) = real per-capita personal income of population in airport X service area in year I,
ACPM(I) = cost of 1 mile of automobile operation in year I,
USFARE(I) = average air fare per revenue passenger mile in year I, and
a, b, c, and d = regression parameters.

All the independent variables in the above equation should display positive correlation with the dependent variable. That is, if population or real per-capita personal income or both increase, the number of enplanements at that airport should increase. Similarly, the ratio ACPM/USFARE should have a positive coefficient, because if USFARE decreases with respect to ACPM for the same length of trip, it should induce the traveler to use air service more often.

Unfortunately, the equation developed for some airports did not contain positive coefficients for all the parameters; thus the logical relationship was not represented correctly. In these cases, different combinations of the independent variables were employed, and those variables that provided the most-logical contributions were selected to represent the forecasting equations. The forecasting equations for all air-carrier airports considered are shown in Table 1. They were developed by using time-series data from 1967 to 1977.

Most of the equations displayed a high coefficient of determination (R^2), except for those for Danville, Hot Springs, and Staunton, where R^2 was quite low--0.363, 0.217, and 0.187, respectively. These airports had experienced fluctuations in the number of enplaned passengers over the last decade due to fluctuations in flight services and schedules and other economic conditions that the hypothesized equation was not able to capture. In spite of the low R^2-values, the plots of the residuals for these airports showed that the estimates were converging toward the actual data in the last five years. This positive indication plus the inability to produce better equations with the available data forced us to use the equations developed.

As stated earlier, aviation fuel consumption is dependent on the number of departing flights and the average amount of fuel consumed per departing flight. To translate the already-determined number of annual enplaned passengers at each airport into departing flights, the average number of available revenue seats and average-seat-load factor per departing flight had to be determined first.

Average Number of Available Revenue Seats per Departing Flight

The average number of revenue seats per departing flight at each airport in year I [XSEAT(I)] was determined by using the following relationship:

$$\mathrm{XSEAT(I)} = \Sigma_j \Sigma_k\ [A_{jk}(\mathrm{I})]\ [XB_{jk}(\mathrm{I})]/\Sigma_j \Sigma_k\ XB_{jk}(\mathrm{I}) \qquad (2)$$

where $A_{jk}(I)$ is the number of revenue seats per average departing flight by aircraft type (k) and by air-carrier group (j) in year I and $XB_{jk}(I)$ is the number of departing flights by aircraft type (k) and by air-carrier group (j) at airport X in year I. The A-values were obtained from CAB reports from 1971 to 1977. The B-values were obtained for the seven years from airport activity statistics published jointly by CAB and the Federal Aviation Administration (FAA).

Average-Seat-Load Factor per Departing Flight

An average-seat-load factor per departing flight at each airport for year I [XLF(I)] was determined by dividing the average number of enplaned passengers per departing flight at each airport by the average number of available revenue seats per departing flight [XSEAT(I)]. The number of enplaned passengers per departing flight was calculated by dividing the number of annual enplaned passengers at each airport by the total number of departing flights at that particular airport. Historical data for these two figures were available for each airport. Thus, average number of revenue seats and average-seat-load factors per departing flight at each airport were calculated for the years 1971-1977. These two figures were then projected into the future for the horizon year based on past trends.

Annual Number of Departing Flights at Each Airport

The annual number of departing flights at each airport [XOPN(I)] was then determined by dividing the annual number of enplaned passengers [XPAS(I)] by the average number of available revenue seats per departing flight [XSEAT(I)] and the average-seat-load factor per departing flight [XLF(I)] at that particular airport:

$$XOPN(I) = XPAS(I)/[XSEAT(I)]\ [XLF(I)] \quad (3)$$

Fuel Consumed per Departing Flight

In this model, the amount of fuel consumed per departing flight was considered to be the product of the average gallons of fuel consumed per block hour [$GALR_{jk}(I)$] and the total block hours [$BLOCK_{jk}(I)$] from ramp to ramp of average departing flights by aircraft type (k) and by air-carrier group (j). The latter value was determined as follows:

$$BLOCK_{jk}(I) = LENGTH_{jk}(I)/SPEED_{jk}(I) \quad (4)$$

where $LENGTH_{jk}(I)$ is the average stage length (miles) from ramp to ramp per departing flight by aircraft type (k) and by air-carrier group (j) in year I, and $SPEED_{jk}(I)$ is the average ramp-to-ramp speed (mph) per departing flight by aircraft type (k) and by air-carrier group (j) in year I. $LENGTH_{jk}(I)$, $SPEED_{jk}(I)$, and the average amount of fuel consumed per block hour [$GALR_{jk}(I)$] were obtained from CAB operating-cost and performance reports for the years 1971-1977. Thus, the average amount of fuel consumed per departing flight for these years by aircraft type (k) and by air-carrier group (j) in year I [$GALT_{jk}(I)$] was calculated by multiplying $BLOCK_{jk}(I)$ by $GALR_{jk}(I)$.

The average amount of fuel consumption per departing flight at each airport for year I [XFUEL(I)] was determined next by using the following relationship:

$$XFUEL(I) = \Sigma_j \Sigma_k\ [GALT_{jk}(I)]\,[XB_{jk}(I)] / \Sigma_j \Sigma_k XB_{jk}(I) \quad (5)$$

The above figure for XFUEL(I) was determined for each year from 1971 to 1977 and projected to the horizon year by using the trends of the seven years of data.

Finally, the total annual aviation jet fuel consumed at airport X was determined by multiplying the average amount of fuel consumed per departing flight at airport X (XFUEL) by its annual number of departing flights (XOPN). The total annual air-carrier jet fuel consumption in Virginia was then obtained by summing this value over the 10 air-carrier airports studied.

GA OPERATIONS

Several socioeconomic variables in Virginia are considered to influence GA operations. Among them are (a) the number of certified pilots, (b) the number of active GA aircraft, (c) the manufacturing investment, and (d) the CPI for gasoline. These socioeconomic variables were used in a series of multiple-linear-regression equations to determine the parameters of the estimating equations for GA operations. Future value of these independent variables had to be exogenously projected and then used as inputs into the model. In that respect, a regression analysis was also performed on the number of certified pilots and the number of active GA aircraft. The manufacturing investment and the CPI for gasoline were obtained from existing sources (1), which forecast their values to the year 1990.

There are four types of certified pilots: student, commercial, private, and airline. Airline pilots who operate air-carrier aircraft were not considered to influence the GA operations; the three other types were. In this model, each of the three types of pilot was forecast individually by using basically two independent variables--Virginia population and the corresponding national value of the variable under consideration. The national figures were obtained from the FAA statistical handbooks for the years 1967-1978. The projected value of these figures was also obtained from an FAA publication (2). The forecasting equation for student pilots in Virginia [VASPLT(I)] is as follows:

$$VASPLT(I) = -8123.678 + 0.001\,731\ VAPOP(I) + 0.017\,73 USSPLT(I) \quad (6)$$

where

VAPOP(I) = population in Virginia in year I,
USSPLT(I) = number of student pilots in the United States in year I, and
R^2 = 0.730.

The forecasting equation for commercial pilots in Virginia [VACPLT(I)] is as follows:

$$VACPLT(I) = -23\,085.52 + 0.005\,347 VAPOP(I) + 0.011\,51 USCPLT(I) \quad (7)$$

where USCPLT(I) is the number of commercial pilots in the United States in year I, and R^2 = 0.907.

Similarly, the forecasting equation for Virginia private pilots [VAPPLT(I)] is as follows:

$$VAPPLT(I) = -9091.199 + 0.002\,32 VAPOP(I) + 0.010\,48 USPPLT(I) \quad (8)$$

where USPPLT(I) is the number of private pilots in the United States in year I, and R^2 = 0.881.

The number of active GA aircraft in Virginia was considered to be dependent on the number of GA pilots in Virginia, on Virginia real per-capita income, and on the number of active GA aircraft in the United States. A series of multiple-linear regressions that used various combinations of the above variables was examined and analyzed. The equation that displayed the strongest relationship (statistically as well as theoretically) was chosen:

$$VAAAC(I) = -1121.012 + 0.1179 VAGAPL(I) + 0.2845 REAPCI(I) \quad (9)$$

where

VAAAC(I) = number of active GA aircraft in Virginia in year I,
VAGAPL(I) = number of GA pilots (student, private, and commercial pilots only) in Virginia in year I,
REAPCI(I) = Virginia real per-capita personal income in year I, and
R^2 = 0.943.

Once the forecasts of the socioeconomic variables that influenced GA operations in general were obtained, they were used in the GA operations forecasts at two categories of airports, towered and nontowered. This division of airports was found necessary because the growth rates of GA activities in these two types of airport were quite different.

Towered Airports

Two types of GA operations occur at towered airports--itinerant and local. Each of these operations was forecast separately. In itinerant operations, all independent variables were considered except the number of student pilots who perform mostly local operations (for training purposes). A series of multiple-linear regressions was performed between the dependent variable and the independent variables, which resulted in the selection of the following equation:

$$\mathrm{TOWITI(I)} = 158\,972.8 + 75.8859\mathrm{VAAAC(I)} \qquad (10)$$

where TOWITI(I) is the number of itinerant operations at towered airports in Virginia in year I, and $R^2 = 0.716$.

Similarly, in local operations, all independent variables were considered except for the number of commercial pilots. Only private and student pilots were considered to perform local operations. The best forecasting equation was found to be as follows:

$$\mathrm{TOWLOC(I)} = 32\,634.65 + 49.2940\mathrm{VAAAC(I)} + 116.3042\mathrm{MFGINV(I)} \qquad (11)$$

where

TOWLOC(I) = number of local operations at towered airports in Virginia in year I,
MFGINV(I) = amount of manufacturing investment in Virginia in year I ($000 000s), and
$R^2 = 0.700$.

Nontowered Airports

Again, the forecasts for two types of GA operations at nontowered airports in Virginia (itinerant and local) were developed individually.

Itinerant operations at nontowered airports were considered to be influenced by the same socioeconomic variables that influence such operations at towered airports. The selected forecasting equation is as follows:

$$\mathrm{NOTITI(I)} = 51\,347.61 + 177.2526\mathrm{VAAAC(I)} + 56.2954\mathrm{MFGINV(I)} \qquad (12)$$

where NOTITI(I) is the number of itinerant operations at nontowered airports in Virginia in year I, and $R^2 = 0.968$.

Similarly, local operations were regressed against the same independent variables as those used in towered airports, and the best forecasting equation developed is as follows:

$$\mathrm{NOTLOC(I)} = 117\,985.2 + 531.1408\mathrm{VAAAC(I)} \qquad (13)$$

where NOTLOC(I) is the number of local operations at nontowered airports in Virginia in year I, and $R^2 = 0.954$.

Both equations developed for nontowered airports are superior to those for towered airports because of the uniformity of the existing data. Besides, nontowered airports account for the most GA operations in the state--about 75 percent.

As stated earlier, the total GA itinerant operations, which take place at towered and nontowered airports, were considered to be both turbine-powered and piston-powered operations, which consume aviation jet fuel and aviation gasoline, respectively.

To determine the number of turbine-powered itinerant operations, the following relationship was adopted:

$$\mathrm{JETITI(I)}/\mathrm{ITINT(I)} = \mathrm{TURBIN(I)}/[\mathrm{TURBIN(I)} + \mathrm{PISTON(I)}] \qquad (14)$$

where

JETITI(I) = number of turbine-powered itinerant operations in Virginia in year I,
ITINT(I) = total GA itinerant operations in Virginia in year I,
TURBIN(I) = number of hours flown by GA turbine-powered operations in the United States in year I, and
PISTON(I) = number of hours flown by GA operations in the United States in year I.

The data and the forecasts for the two variables TURBIN(I) and PISTON(I) were obtained from the FAA statistical handbooks and the FAA aviation forecasts (2), respectively.

The piston-powered itinerant operations at all airports were simply determined by subtracting the turbine-powered itinerant operations from GA itinerant operations.

The GA local operations are composed of those operations at both towered and nontowered airports. Because local operations are mostly short-distance operations, they were assumed to be GA piston-powered operations, which consume only aviation gasoline.

Thus, the total GA piston-powered operations are composed of GA local operations and piston-powered itinerant operations.

GA FUEL CONSUMPTION

To translate GA operations into fuel consumption, a trend method was used in the case of piston-powered operations and a ratio method (which compared Virginia consumption with national consumption) was used in the case of turbine-powered operations.

The former method involved a simple linear regression between GA gasoline consumption as the dependent variable and GA piston-powered operations as the independent variable. The equation developed is as follows:

$$\mathrm{VAGAS(I)} = 3\,551\,434 + 1.068\,14\mathrm{GAPIOP(I)} \qquad (15)$$

where VAGAS(I) is the amount of GA gasoline consumption in Virginia in year I, and GAPIOP(I) is the number of total GA piston-powered operations in Virginia in year I. VAGAS was considered to represent the total quantity of aviation gasoline consumed in Virginia because there is a relatively small amount of aviation gasoline consumed by air carriers.

Since no data were available on the amount of GA jet fuel consumed in Virginia, the following ratio method that compares Virginia and the United States was used to determine the GA jet fuel consumed in Virginia:

$$\mathrm{GAJET(I)}/[\mathrm{VAGAS(I)} + \mathrm{GAJET(I)}] = \mathrm{USGAJT(I)} \div [\mathrm{USGAGS(I)} + \mathrm{USGAJT(I)}] \qquad (16)$$

where

GAJET(I) = number of gallons of GA jet fuel consumption in Virginia in year I,
USGAJT(I) = number of gallons of GA jet fuel consumption in the United States in year I, and
USGAGS(I) = number of gallons of GA gasoline consumption in the United States in year I.

USGAGS and USGAJT were both obtained from an FAA publication (2). GAJET was then added to air-carrier jet fuel consumption to yield the total aviation jet fuel consumption in Virginia. Aviation fuel consumption in Virginia (from DMV motor-fuel tax reports) is shown in Table 2.

SCENARIOS

One of the advantages of using an econometric model for forecasting is its flexibility for testing alternative future scenarios. A scenario is referred to here as a future condition in which the underlying assumptions of the model fail to hold true. The original conditions used in developing the model (base-case conditions) are presented in Table 3. The scenarios tested adequately cover the key elements and assumptions under consideration.

Sagging Economy

The originally projected rate of increase for manufacturing investment is 8.6 percent per year. If there is a slowdown of economic growth, aviation fuel consumption and consequently the tax revenues will be adversely affected. To test the hypothesis, a rate of increase of 8.0 percent per year was employed instead.

High Gasoline

In this scenario, CPI for gasoline will increase at 8.5 percent per year rather than 6.4 percent as was used in the base case. This scenario is of particular importance due to the uncertainty of the future gasoline supply and its price.

Table 2. Aviation fuel consumption in Virginia.

	Aviation Gasoline (gal 000 000s)		Aviation Jet Fuel (gal 000 000s)	
Year	Actual	Forecast	Actual	Forecast
1971	5.193	5.26	170.769	173.35
1972	5.286	5.44	174.813	177.02
1973	5.598	5.57	174.890	181.75
1974	6.247	5.59	184.297	180.78
1975	5.986	5.67	170.641	182.54
1976	6.010	5.81	178.654	185.26
1977	6.055	5.93	200.957	200.71
1978	6.290	6.12	210.648	204.77
1979		6.35		208.05
1980		6.58		211.99
1981		6.78		214.55
1982		6.96		217.32
1983		7.12		219.44
1984		7.30		221.82
1985		7.49		224.17
1986		7.67		226.72
1987		7.85		228.53
1988		8.03		230.64
1989		8.21		233.19
1990		8.40		235.52

Rising CPI for All Goods

Historically, the rate of inflation periodically rises at a faster rate than it does during average economic activity. It was decided to explore the impact of rising inflation on aviation fuel consumption. Thus, CPI for all goods (CPIALL) was assumed to increase at 8.5 percent per year rather than at 6.0 percent per year as in the base case.

High Gasoline and Rising CPIALL

It is very true that the high price of gasoline will simultaneously raise the cost of living. In this scenario, both CPIALL and CPI for gasoline (CPIGAS) were assumed to increase at 8.5 percent per year.

Rising CPIALL and High Air Fares

Rising inflation will induce airlines to raise the air fare in order to cope with the high cost of goods. In this scenario, CPIALL and air fares were assumed to increase at 8.5 and 6.5 percent per year, respectively.

RESULTS OF SCENARIOS

The results of the scenarios are shown in Table 4. Although they are self-explanatory, it would be beneficial to review some of the more-interesting results.

Aviation jet fuel consumption decreases under all but one scenario, high gasoline, which indicates the influence of adverse economic conditions. The high price of gasoline, according to the model, has had more influence on the operating cost of automobiles (ACPM), which in turn has induced intercity travelers to shift from automobiles to air carriers. The result is a greater number of departing flights and consequently a greater amount of fuel consumption.

Similarly, aviation gasoline consumption decreases under all but that same scenario. Since the model did not contain the price of gasoline as an explanatory variable to GA operations, it did not influence fuel consumption. However, it is expected that this variable will gain more importance in the future.

ADDITIONAL SCENARIOS

Several additional scenarios were performed to reflect all possible aviation conditions in Virginia and their impact on fuel consumption. They are discussed separately because some of the inputs to

Table 3. Base-case conditions at Virginia airports.

	Percentage Increase per Year at Airport										
Variable	All	CHA	DAN	DUL	HOT	LYN	NEW	NOR	RIC	ROA	STA
CPI											
Gasoline	6.4										
All goods	6.0										
Parking	5.75										
Air fare per revenue passenger mile	5.5										
Manufacturing investment	8.60										
Per-capita personal income (service area)		8.2	8.5	9.0	8.7	8.2	8.8	9.0	8.7	8.6	8.2
Available revenue seats per departing flight		1.8	0.46	1.15	0.46	1.61	1.37	1.97	2.15	2.71	0.46
Seat-load factor per departing flight		2.126	2.456	3.274	2.477	3.145	2.233	1.473	1.375	2.627	3.691
Fuel consumed per departing flight		2.435	2.343	1.945	2.486	3.576	1.331	2.352	1.258	3.168	3.096

Note: Airports are abbreviated as follows: CHA, Charlottesville; DAN, Danville; DUL, Dulles International; HOT, Hot Springs; LYN, Lynchburg; NEW, Newport News; NOR, Norfolk; RIC, Richmond; ROA, Roanoke; and STA, Staunton.

Table 4. Projected aviation fuel consumption for 1990 for each scenario.

Scenario	Jet Fuel Consumption (gal 000 000s)	Gasoline Consumption (gal 000 000s)
Base case	235.51	8.40
Sagging economy	235.47	8.38
High gasoline	236.77	8.40
Rising CPIALL	196.33	8.12
High gasoline and rising CPIALL	197.50	NA
Rising CPIALL and high air fares	194.71	NA

Table 5. Projected aviation fuel consumption for 1990 under base-case and additional scenarios.

Scenario	Jet Fuel Consumption (gal 000 000s)	Gasoline Consumption (gal 000 000s)
Base case	235.51	8.40
Introduction of commuter air service at specific airports	235.28	8.45
Expansion of Piedmont routes	236.08	NA
Improved efficiency of aircraft fuel consumption	225.76	NA
Competition between airports	238.12	NA
Increased seating capacity	224.15	NA

these scenarios are partly or totally performed outside the model. These scenarios are (a) introduction of commuter air service at specific airports, (b) competition between airports, (c) expansion of Piedmont routes, (d) improved efficiency of aircraft fuel consumption, and (e) increased seating capacity. The assumptions and the inputs under each scenario are presented first, and the results are discussed later for the sake of brevity.

Introduction of Commuter Air Service

The commuter air service is proposed to serve the airports of Danville, Hot Springs, Lynchburg, and Newport News.

At present, the number of air-carrier enplaned passengers at Danville and Hot Springs airports is declining. These two airports are exclusively served by Piedmont Airlines; it is assumed in this model that service will be terminated at either or both airports if the annual number of enplaned passengers drops below 1000. The termination of Piedmont service is expected to result in the introduction of commuter air service. It was also assumed that the number of users of commuter air service would reach 1500 at both airports in the first servicing year after termination of Piedmont service and would increase at 10 percent per year thereafter.

At Lynchburg Airport, Air Virginia is currently providing commuter air service, even when the number of enplaned passengers is still on the rise. At this airport, it was assumed that commuter air service would attract and accommodate up to 1800 passengers per year from 1980 to 1990.

Similarly, at Newport News Airport, a commuter service was assumed to replace Piedmont Airlines and to service nearby hub airports. This commuter service will attract an average of 2000 passengers per year from 1980 to 1990.

The aircraft used by commuter airlines are usually light and small. In this model it was assumed that PA-31 and Short 330 would be the representative aircraft; these have a two-piston engine and a two-turboprop engine, respectively. It was also anticipated that 50 percent of the users of the commuter air service will fly on Short 330s, and the remaining 50 percent will fly on PA-31s. The amount of fuel consumed per departing Short 330 or PA-31 flight was found to be 48.81 and 35.67 gal, respectively.

Competition Between Airports

Some air-carrier airports in Virginia have direct competitors in terms of better flight schedules and frequencies or facilities or both. One way of analyzing the competition within the model is to change the service area and the corresponding catchment-area population of the competitive airports.

In this scenario, it was anticipated that Charlottesville Airport would lose Fredericksburg to Dulles Airport, which has better facilities and flight schedules. It is assumed that Roanoke Airport would gain Bedford from Lynchburg Airport and Staunton Airport would lose its northern half of Page County to Dulles. Similarly, it was assumed that Norfolk Airport would attract Hampton and Richmond Airport would gain Williamsburg and James City and Gloucester Counties from the Newport News Airport service area.

Expansion of Piedmont Routes

Piedmont Airlines is in the stage of expanding and opening new routes. The expansion will affect air-carrier fuel consumption. It was assumed that only Norfolk, Richmond, Roanoke, and Dulles International airports would be affected by these route expansions. The opening and expanding of new routes (starting in 1980) will increase the stage length per departing flight, which thus affects the amount of fuel consumed per departing flight. After careful analysis and study of the existing flight schedules, flight frequencies, and stage lengths, it is expected that the amount of fuel consumed per departing flight at the above airports will increase from the base-case condition as follows:

Airport	Percentage Increase per Year
Dulles International	0.18
Norfolk	0.60
Richmond	0.60
Roanoke	0.60

Improved Efficiency of Aircraft Fuel Consumption

The limited availability and high price of aviation fuel are expected to entice the airlines to use more fuel-efficient airplanes. It was assumed that only Dulles, Norfolk, Richmond, and Roanoke airports, which are served by relatively large jet aircraft, would be affected by these fuel-efficiency improvements, starting in 1980. This scenario anticipates that the following gallons of fuel consumed per departing flight and the increase from the base case would be as follows in 1990:

Airport	Fuel Consumed (gal)	Percentage Increase per Year
Norfolk	2100	1.426
Richmond	1500	0.606
Roanoke	1400	2.459
Dulles International	6300	1.627

Increased Seating Capacity

Airlines are trying to improve the average number of available revenue seats per departing flight, which in turn will reduce the number of departing flights and the quantity of fuel consumption. It was assumed that the increase in available revenue seats starting in 1980 would occur in the same airports as in the previous scenario, with the addition of Lynchburg Airport. The following conditions at these airports were anticipated for 1990:

Airport	Seats Available	Percentage Increase per Year
Charlottesville	100	2.385
Dulles International	170	2.116
Lynchburg	90	2.257
Norfolk	140	1.967
Richmond	125	2.569
Roanoke	120	3.150

RESULTS OF ADDITIONAL SCENARIOS

The results of the additional scenarios are shown in Table 5. Again, the model's outputs respond logically to the conditions under consideration. The projected jet fuel consumption increases the most under the scenario for competition between airports. One reason for this result is that the more-competitive airports, which are the large airports in Virginia, would be attracting more passengers, which would result in more departing flights and consequently more fuel consumption. Also, these large airports have a higher rate of fuel consumed per departing flight, which adds to the total increase in jet fuel consumption.

Aviation gasoline is considered only under one scenario, introduction of commuter air service, which shows a small amount of increase in such fuel consumption. The introduction of commuter air service is expected to have little effect (as the results show) on aviation fuel consumption, because there would be relatively small numbers of departing flights and, in addition, the amount of fuel consumed per departing flight by small commuter aircraft is small.

The scenario for expansion of Piedmont routes produced a slight increase in the amount of jet fuel consumption. On the other hand, increased seating capacity and improved efficiency of fuel consumption would have a sizeable impact on the reduction of jet fuel consumption.

In conclusion, the amount of aviation fuel consumption in Virginia is primarily affected by the economic condition of the state and the nation. In addition, airline policies have a great effect on the amount of jet fuel consumption in the state.

REFERENCES

1. A.G. Hobeika, H.A. Goldstein, A. Boonpuan, and F. Tamberrino. An Integrated Revenue Forecasting Model for the Virginia Division of Motor Vehicles. Center for Urban and Regional Studies, Division of Environmental and Urban Systems, Virginia Polytechnic Institute and State Univ., Blacksburg, Aug. 1978.
2. FAA Aviation Forecasts, Fiscal Years 1979-1990. Office of Aviation Policy, Federal Aviation Administration, U.S. Department of Transportation, Sept. 1978.

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Estimating the Market Share of International Air Carriers

STEVEN R. GORDON

United States flag carriers and aviation authorities are currently participating in a large number of activities that promise to alter the structure of the international air-transport network. There is a pressing need to develop methods for estimating the share of traffic that U.S. carriers can expect to attract under the various alternatives being considered. To meet these needs, a new method called the international quality-of-service index (IQSI) has been developed. It is derived from the quality-of-service index (QSI) method developed by the staff of the Civil Aeronautics Board for domestic-route cases and augments the old QSI method by considering (in addition to frequency, aircraft type, and number of stops) the impact on market share of citizenship loyalty to flag carriers. Use of IQSI essentially eliminates the biases inherent in the old QSI method and reduces the average prediction error by more than 25 percent.

The U.S. international air-route system is in a state of flux. Sections of the system have been dramatically modified in recent Civil Aeronautics Board (CAB) regulatory proceedings. Recently concluded bilateral negotiations on international air rights between the United States and several major foreign powers have greatly affected both existing and potential route structures of U.S. and foreign flag carriers. The merger of Pan American and National airlines is likely to result in further changes to the system.

All evidence points to a continuation of the present state of flux. Several important bilateral negotiations are currently under way, and new international-route cases seem to appear before CAB as fast as the old ones can be resolved.

One of the most-important tasks in analyzing and selecting among alternative route structures is to estimate the resultant division of traffic between U.S. and foreign flag carriers. The division of traffic has a direct bearing on the profitability of U.S.-flag-carrier services on affected routes and indirectly on which services will be offered, the net benefit to the public, and the ultimate viability of the U.S. flag system as a whole.

Although the need for a reliable means of estimating air-passenger route-specific traffic is clear and pressing, the best method now available is deficient in various respects. This method--commonly referred to as the quality-of-service index (QSI)--was developed by the staff of CAB for application in domestic-route proceedings (such as the investigation of Reno-Portland/Seattle nonstop service, May 1970) and, over a period of years, has gained acceptance in the decision processes of the board. It is also being used increasingly in the internal planning processes of the carriers for analyzing the profitability of alternative new routes and for assessing possible merger partners. The method assigns standard weights to flight frequencies (number of weekly flights) operated with individual aircraft types. Larger and faster aircraft are weighted more heavily than smaller and slower aircraft to accord with presumed passenger preferences. Higher weights are given to nonstop than to one-stop flights; higher weights are given to one-stop flights than to multistop flights. The weighted-flight-frequency data are then used for two purposes:

1. To estimate the share of passenger traffic that will be captured by individual flights and combinations of flights and
2. To estimate the change in passenger traffic that results from a change in the available market services.

The QSI method is deficient for application to the international sector for the following reasons:

1. The existing estimating relationships are based on the somewhat archaic domestic experience in the late 1960s before extensive wide-body operations and on the generally limited information on later models of then-operational aircraft types.
2. The existing estimating relationships are not well calibrated to long-haul market experience. Relatively short-haul domestic market data are dominant in the determination of standard weights, which results in an incorrect evaluation of the deterrent values of stops and connections and of the importance passengers attach to comfort and speed factors.
3. The method does not deal with operations of the supersonic Concorde or the Boeing 747SP.
4. The method gives no explicit recognition to the citizenship composition of air passenger demand or to the preference of citizens for services of national air carriers. This is a major shortcoming in view of observed preferences.

In the light of these identifiable problems and the requirements of carriers and U.S. authorities that participate in route cases and bilateral negotiations, a review of the existing passenger-estimating relationships for international services has been performed. As a result, a new method--called the international quality-of-service index (IQSI)--has been developed, and it has almost twice the predictive power of the QSI method. It differs from the QSI method primarily by using different weights for aircraft types and number of stops, by recognizing passenger preferences for their own flag carrier or carriers, and by accounting for the difficulties faced by fifth-freedom carriers in attracting a proportional share of the market.

DESCRIPTION OF IQSI METHOD

The IQSI method is a procedure that can be used to estimate international scheduled-carrier market shares. In order to apply the method to any market, the market's citizenship composition (U.S. citizens versus aliens) and the pattern of air service (frequency by aircraft type, by number of stops, and by carrier) it receives must be known. For retrospective estimation, citizenship composition can be obtained from U.S. International Air Travel Statistics, published monthly by the U.S. government, and patterns of air service can be obtained from the Official Airline Guide, published semimonthly by the Reuben Donnelley Corporation. For forecasting purposes, citizenship composition may be estimated from past trends or by using more-sophisticated techniques, and patterns of service must be part of the specification of the future scenario.

The IQSI method is simple to apply. It consists of the following four steps:

1. Compute IQSI values for each carrier based on the service it offers,
2. Reduce IQSI of the fifth-freedom carriers by 40 percent,
3. Allocate a portion of the U.S. passengers to U.S. carriers and a portion of the foreign passengers to the flag carriers of the foreign country according to the citizenship factors shown in Table 1 (in which NA indicates that either regional or nondirectional factors may be used), and
4. Divide the remaining (unallocated) passengers among the carriers in proportion to their IQSIs.

Step 1 of the computation of IQSI is the same as it is for the computation of CAB's domestic QSI except that different weights are used for aircraft type and number of stops, as shown below (CAB has not derived QSI weights for the B-747SP or for the supersonic Concorde; 1.50 is the QSI weight for the DC8-63):

Item	IQSI Weight	CAB QSI Weight
Aircraft		
Boeing 747	2.25	1.85
Boeing 747SP	2.15	NA
Three-engine wide body	1.50	1.50
Four-engine narrow body	1.00	1.00
Supersonic Concorde	1.59	NA
Number of stops		
0	1.00	1.00
1	0.42	0.55
2	0.31	0.40
3 or more	0.02	0.03

Each flight is assigned an IQSI value equal to the product of its weekly (or monthly) frequency, the IQSI weight associated with its aircraft type, and the IQSI weight associated with the number of stops it makes en route. IQSI values of all flights by a carrier are summed to derive the IQSI value for the carrier. Table 2 illustrates this procedure for Air France service from Paris to New York, November 1976.

Table 3 presents a hypothetical scenario for service from New York to Rio de Janeiro, slightly modified for illustrative purposes. In step 3 of the IQSI method, passengers may be allocated to carriers (on the basis of citizenship factors) in one of two ways. If the U.S.-flag and foreign-flag carrier shares of QSI are relatively independent of direction (to or from the United States), the nondirectional citizenship factors of Table 1 may be used, as illustrated in Table 4. The resultant market shares (U.S. share = 0.408, foreign = 0.515, fifth-freedom = 0.077) are averages over both directions of travel. If market shares in each direction of travel are desired or if the IQSI shares of the carriers differ significantly between travel to and from the United States, the market

Table 1. Citizenship factors for use in IQSI method.

Country or Region	Departing United States		Arriving United States		Nondirectional	
	U.S. Flag	Foreign Flag	U.S. Flag	Foreign Flag	U.S. Flag	Foreign Flag
Argentina	0.168	0.320	0.139	0.095	0.157	0.213
Australia	-0.318	0.200	-0.216	0.230	-0.281	0.218
Belgium	-0.055	0.743	NA	NA	-0.021	0.690
Bolivia	-0.590	0.576	-0.410	0.420	-0.470	0.497
Brazil	0.023	0.259	0.075	0.182	0.045	0.238
Caroline Islands	0.913	-0.551	0.930	-0.542	0.919	-0.546
Chile	0.067	0.365	0.069	0.178	0.061	0.272
Colombia	0.001	0.039	-0.041	0.142	-0.018	0.090
Costa Rica	-0.083	0.522	NA	NA	-0.032	0.310
Denmark	-0.093	0.575	-0.020	0.098	-0.054	0.349
Ecuador	0.043	0.140	0.036	0.002	0.094	0.071
El Salvador	-0.051	0.188	0.015	0.217	-0.015	0.205
France	-0.019	0.468	0.130	0.415	0.061	0.441
Germany	0.123	0.555	0.251	0.306	0.187	0.439
Greece	-0.104	0.684	-0.016	0.445	-0.032	0.570
Guatemala	0.232	0.055	0.169	0.008	0.196	0.026
India	0.024	0.594	0.002	0.676	-0.018	0.660
Iran	-0.134	0.746	NA	NA	-0.070	0.832
Ireland	0.003	0.531	0.044	0.294	0.021	0.430
Israel	-0.259	0.680	-0.178	0.887	-0.222	0.768
Italy	0.205	0.238	0.290	0.127	0.248	0.173
Japan	0.214	0.286	0.140	0.225	0.177	0.258
Mariana Islands	0.617	-0.500	0.538	-0.509	0.579	-0.503
Mexico	-0.048	-0.135	-0.043	-0.048	-0.046	-0.090
Morocco	0.087	0.153	NA	NA	0.062	0.149
Netherlands	-0.051	0.732	NA	NA	-0.028	0.681
New Zealand	-0.017	0.477	-0.064	0.592	-0.032	0.500
Nicaragua	-0.077	0.662	-0.004	0.488	-0.052	0.670
Pakistan	-0.514	0.797	-0.086	0.582	-0.332	0.767
Panama	0.502	0.002	0.342	0.054	0.418	0.033
Peru	0.261	-0.033	0.199	-0.253	0.230	-0.186
Philippines	0.069	0.227	-0.122	0.287	-0.031	0.258
Poland	0.123	0.602	-0.066	0.656	0.081	0.719
Portugal	-0.113	0.424	0.049	0.271	-0.001	0.324
Senegal	-0.496	0.431	0.570	-0.131	0.226	0.041
South Africa	-0.187	0.722	NA	NA	-0.100	0.757
South Korea	0.147	1.000	0.455	0.237	0.441	0.108
Spain	0.147	0.223	0.193	0.094	0.155	0.177
Switzerland	-0.039	0.650	0.065	0.175	0.019	0.436
Tahiti	-0.061	0.384	-0.053	0.185	-0.059	0.302
Taiwan	0.070	0.701	-0.126	0.728	-0.032	0.720
Thailand	-0.261	0.728	-0.019	0.461	-0.247	0.640
USSR	-0.361	0.797	-0.136	0.884	-0.241	0.813
United Kingdom	0.126	0.185	0.155	0.090	0.140	0.137
Venezuela	0.125	0.169	0.341	0.049	0.203	0.116
Central America	0.081	0.051	0.061	-0.008	0.069	0.023
South America	0.066	0.148	0.056	0.063	0.061	0.104
Europe	0.082	0.335	0.138	0.177	0.107	0.257
Africa	-0.049	0.247	0.189	-0.158	0.063	0.125
Middle East	-0.304	0.763	-0.163	0.873	-0.260	0.805
Far East	0.104	0.325	0.034	0.295	0.069	0.310
Oceania	-0.199	0.389	-0.199	0.381	-0.201	0.388

Table 2. Illustrative computation of IQSI.

Flight No.	Aircraft Type	No. of Stops	Monthly Frequency	IQSI Weight for Type	IQSI Weight for Stops	IQSI
3	B-747	1	14	2.25	0.42	13.23
15	B-707	0	29	1.00	1.00	29.00
77	B-747	0	30	2.25	1.00	67.50
Total						109.73

must be disaggregated by direction of travel, and the directional citizenship factors of Table 1 must be used. Table 5 illustrates this technique for same scenario as that used in Table 4. The market shares are shown below.

U.S.	Foreign	Fifth-Freedom
0.406	0.515	0.078
0.478	0.451	0.071
0.334	0.580	0.086

The citizenship factors in Table 1 are displayed alphabetically by country. For countries with no past competition between U.S. carriers and carriers of the country of interest, no citizenship factors could be derived. If passengers in the country of interest are expected to behave as their neighbors do, the regional citizenship factors found at the end of Table 1 may be used. If, instead, the analyst has a priori knowledge of citizenship behavior, predetermined values may be substituted. Finally, with no knowledge of the likely passenger behavior, citizenship factors of 0.0 should be used.

Positive citizenship factors imply that passengers from the given country fly on their own flag carrier more often than would be indicated by its IQSI share; this exhibits a form of loyalty to the flag carrier and explains the procedure of allocating some of the passengers to their national flag carrier before dividing the remainder according to IQSI. Negative citizenship factors imply that passengers are more likely to fly a foreign flag carrier than would be indicated by its share of IQSI. Reasons for such behavior vary; they range from negative citizen perception of their own flag carrier to ethnic interest in the foreign flag. (This might explain, for example, the negative citizenship factor for U.S. citizens who fly to Israel.) The application of a negative citizenship factor requires an imaginary allocation of a negative number of passengers to the country's flag carriers and consequently an increase in the number of unallocated passengers, who will then be divided among carriers according to IQSI shares. This procedure is illustrated in Table 6 for a hypothetical scenario of a flight from New York to Tel Aviv (the U.S. market share is 0.025 and the foreign-flag share is 0.975).

DERIVATION OF IQSI METHOD

To develop and validate the IQSI method, a data base

Table 3. Hypothetical service scenario, New York to Rio de Janeiro.

Carrier Flag	Direction	Aircraft Type	No. of Stops	Monthly Frequency	IQSI
U.S.	To U.S.	B-707	0	28	28.00
		B-747	0	28	63.00
				Subtotal	91.00
	From U.S.	B-707	0	28	28.00
		B-747	1	28	26.46
				Subtotal	54.46
	Total				145.46
Brazil	Each direction	B-707	0	32	32.00
		DC-10	0	26	39.00
				Subtotal	71.00
	Both directions				142.00
Fifth-freedom	Each direction	B-707	0	24	24.00
	Both directions			60 percent of each direction	14.40
				60 percent of both directions	28.80
Total	To U.S.				176.40
	From U.S.				139.86
	Both directions				316.26

Table 4. Illustration of passenger allocation, nondirectional method.

			Allocation Factor by Flag				Passengers Allocated by Flag		
Citizenship	Direction	Allocation Step	U.S.	Foreign	Fifth-Freedom	Unallocated Passengers	U.S.	Foreign	Fifth-Freedom
U.S.	Both	Preallocation[a]				178 200	0	0	0
		By citizenship[b]	0.045			170 181	8 019	0	0
		By IQSI[c]	0.460	0.449	0.091	0	78 283	76 411	15 487
Alien	Both	Preallocation[a]				233 800	0	0	0
		By citizenship[b]		0.238		178 156	0	55 644	0
		By IQSI[c]	0.460	0.449	0.091		81 952	79 992	16 212
Total	Both	Postallocation					168 254	212 047	31 699

[a] Hypothetical passenger volumes are about 10 percent above 1976; New York-Brazil traffic by citizenship.
[b] Source of citizenship factors is Table 1.
[c] Source of IQSI factors is Table 3.

Table 5. Illustration of passenger allocation, directional method.

			Allocation Factor by Flag				Passengers Allocated by Flag		
Citizenship	Direction	Allocation Step	U.S.	Foreign	Fifth-Freedom	Unallocated Passengers	U.S.	Foreign	Fifth-Freedom
U.S.	To U.S.	Preallocation[a]				89 100	0	0	0
		By citizenship[b]	0.075			82 417	6 683	0	0
		By IQSI[c]	0.516	0.402	0.082	0	42 527	33 132	6 958
Alien	To U.S.	Preallocation[a]				116 900	0	0	0
		By citizenship[b]		0.182		95 624	0	21 276	0
		By IQSI[c]	0.516	0.402	0.082	0	49 342	38 441	7 841
U.S.	From U.S.	Preallocation[a]				89 100	0	0	0
		By citizenship[b]	0.023			87 051	2 049	0	0
		By IQSI[c]	0.389	0.508	0.103	0	33 863	44 222	8 966
Alien	From U.S.	Preallocation[a]				116 900	0	0	0
		By citizenship[b]		0.259		84 623	0	32 277	0
		By IQSI[c]	0.389	0.508	0.103	0	32 918	42 989	8 716
Total	Both	Postallocation					167 382	212 337	32 281
Total	To U.S.	Postallocation					98 552	92 849	14 599
Total	From U.S.	Postallocation					68 830	119 488	17 682

[a] Hypothetical passenger volumes are about 10 percent above 1976; New York-Brazil traffic by citizenship.
[b] Source of citizenship factors is Table 1.
[c] Source of IQSI factors is Table 3.

Table 6. Illustration of passenger allocation with negative citizenship factors, nondirectional method.

			Allocation Factor by Flag				Passengers Allocated by Flag		
Citizenship	Direction	Allocation Step	U.S.	Foreign	Fifth-Freedom	Unallocated Passengers	U.S.	Foreign	Fifth-Freedom
U.S.	Both	Preallocation[a]				78 788	0	0	0
		By citizenship[b]	−0.222			92 613[d]	−16 825	0	0
		By IQSI[c]	0.195	0.805	0	0	18 060	74 553	0
Alien	Both	Preallocation[a]				30 784	0	0	0
		By citizenship[b]		0.768		7 142	0	23 642	0
		By IQSI[c]	0.195	0.805		0	1 393	5 749	0
Total	Both	Postallocation					2 628	103 944	0

[a] 1976 travel between New York and Israel.
[b] Source of factors is Table 1.
[c] IQSI factors are approximately those for August 1976.
[d] Note that the number of unallocated passengers has increased due to the imaginary allocation of -16 825 passengers to the U.S. flag carrier. The sum of allocated and unallocated passengers must equal the original number of passengers.

Table 7. Sample computation of citizenship factors.

Market	U.S. Citizens	Non-U.S. Citizens	Monthly Frequency	Aircraft Type	Stops	QSI
U.S. carrier flights						
Geneva-New York	451	311	12	4NB	0	12.000
Zurich-New York	1136	594	24	4NB	0	24.000
Total	1587	905	36			36.000
Swiss carrier flights						
Geneva-New York	817	727	13	4WB	1	12.285
Zurich-New York	2852	2664	30	4WB	0	67.500
Geneva-New York	361	443	11	4WB	0	24.750
Zurich-New York	373	447	12	4WB	1	11.340
Zurich-Boston	1111	790	28	3WB	0	42.000
Zurich-Chicago	804	701	24	3WB	1	15.120
Total	6318	5772	118			172.995
Total	7905	6677	154			208.995

Note: Aircraft types are abbreviated as follows: 4NB = four-engine narrow body; 4WB = four engine wide body; 3WB = three engine wide body.

was constructed by merging, for each flight number, passenger-volume and citizenship data collected by the U.S. Immigration and Naturalization Service (INS) and data on air-carrier schedules published in the Official Airline Guide. The merged data include the monthly frequency, the volume of U.S. citizens carried, the volume of foreign citizens carried, the type of equipment used, and the number of stops en route. The data were organized into four files--February, May, August, and November 1976. The May, August, and November files were used for model estimation and the February file was used for model validation. The files included data from all countries (except Canada and Caribbean points) that received competitive single-plane service from the United States. Canadian and Caribbean markets were excluded on the basis of their similarity to domestic rather than international travel.

Estimation of IQSI Weights

The first step was to obtain an initial estimate of citizenship factors. This estimate was derived by using the CAB QSI weights and the method described in the following section.

A computer program was then written to apply the IQSI method to the data to estimate, for each country, the market share of the U.S. carrier or carriers, the foreign-flag carrier or carriers, and the fifth-freedom carrier or carriers. This program was applied to the August data file several times and the IQSI weights were changed between runs to improve the fit between actual and predicted market shares. The IQSI weights that provide the best fit were given in the section that described the IQSI method. These weights were then used to reestimate the citizenship factors. Further sensitivity analysis on the IQSI weights, by using the new citizenship factors, resulted in no change to the IQSI weights.

The computer program to compare predicted with actual market shares was also responsible for the incorporation of the 40 percent reduction of fifth-freedom IQSI into the method. Without this reduction, fifth-freedom traffic was overestimated, on the average, by nearly 100 percent, no matter what IQSI weights were used. The reduction of fifth-freedom IQSI by 40 percent eliminated this bias.

Because of the small number of flights performed by the supersonic Concorde and the Boeing 747SP during the months analyzed, it was impossible to perform sensitivity analyses on IQSI weights for these aircraft types. Instead, markets served by these aircraft were isolated and weights were calculated so as to best predict the market share for the aircraft in the identified markets. This resulted in estimated IQSI weights of 1.59 for the Concorde and 2.15 for the B-747SP.

Estimation of Citizenship Factor

Let K be the fraction of U.S. passengers in a given market who will fly on the U.S. carrier or carriers regardless of its IQSI share. The remaining fraction (1 - K) is divided among flights according to IQSI. Let p be the number of U.S. passengers in the market; q, the U.S. IQSI share; and n, the number of U.S. passengers who use the U.S.-flag carrier. Then

$$n = Kp + q(1 - K)p \quad (1)$$

or

Table 8. Predictive ability of alternative methods.

Statistic	Method	Flag of Carrier U.S.	Foreign	Fifth-Freedom	All Carriers
Sum of residuals	1	2 583	−30 940	28 357	
	2	3 243	−28 600	25 357	
	3	14 360	−19 868	5 507	
	4	−2 033	3 942	−1 909	
Sum of squared residuals	1	5 422	6 450	3 734	15 606
	2	5 331	6 202	3 360	14 893
	3	5 267	5 713	1 158	11 638
	4	3 077	4 350	1 317	8 744
Average residual	1	0.005	−0.057	0.052	
	2	0.006	−0.053	0.047	
	3	0.026	−0.036	0.010	
	4	−0.004	0.007	−0.004	
Square root of average squared residual	1	0.100	0.109	0.083	0.098
	2	0.099	0.107	0.079	0.095
	3	0.098	0.098	0.046	0.084
	4	0.075	0.089	0.049	0.073
Average market share		0.418	0.516	0.064	0.333

$$K = (n - qp)/(p - qp) \qquad (2)$$

We can compute K for foreign carriers and passengers in a similar fashion by letting n be the number of foreign passengers who fly on the foreign-flag carrier; p, the number of foreign passengers in the market; and q, the foreign IQSI share.

Note that K can never exceed unity but that it can fall below zero if n is less than qp, that is, if the fraction of U.S. passengers who use a U.S. carrier is less than that carrier's share of the IQSI (or similarly for foreign passengers and carriers).

Table 7 illustrates how citizenship factors are computed. The example shown is for travel from Switzerland to the United States in November 1976. To calculate K for U.S. passengers, set n equal to 1587, q to 0.172 (36 divided by 209), and p to 7905. This results in a K of 0.035. Similarly, setting n to 5772, q to 0.828, and p to 6677 results in a K for Swiss passengers of 0.212.

Citizenship factors were computed as described above for the months of May, August, and November 1976 (results by month are available from the author). These factors were then averaged to derive the figures in Table 1. Note that citizenship factors for passengers returning to their own country are almost always greater than they are for passengers leaving their own country. Two reasons are hypothesized for this behavior: One is that passengers are eager to immerse themselves, when they start on a trip, in the culture of the nation they are to visit; conversely, by the time they are to return home, these passengers have satisfied their desire for a foreign culture and are eager to hear their own language and eat their national foods. A second possible reason is that the period between the making of reservations and the making of trips is shorter for the trip abroad than it is for the trip home (since most reservations for round trips are made at the point of origin). Consequently, passengers are more likely to encounter difficulty in making reservations for the flight of their choice on the trip abroad than on the trip home. It follows that the trip abroad is more likely to be distributed according to carrier capacity (and IQSI) than according to citizenship. The theory that capacity may in fact affect citizenship factors gains some support in the slightly lower citizenship factors observed in the peak month of August compared with those for the off-peak months of May and November. However, citizenship factors are sufficiently stable between months (generally varying by less than 0.1) to support the conclusion that psychological or behavioral patterns cause their directional variations.

VALIDATION

A computer program was written to predict the market shares of U.S., foreign, and fifth-freedom carriers for every market in the data base in both directions of travel. The program estimated market shares in the following four ways (hereafter called methods 1 through 4):

1. According to CAB method for domestic service,
2. Same as method 1 except that IQSI weights were used,
3. Same as method 2 except that fifth-freedom carrier IQSI shares were reduced by 40 percent, and
4. By applying citizenship factors and the 40 percent reduction of fifth-freedom IQSI.

These market shares were then compared with actual market shares, and the residual (the difference between the estimate and the actual) was computed. The residual and the squared residual in each market were weighted by the number of passengers in the market to account for the relative importance of correctly estimating market shares in major versus minor markets. Weighted totals and averages of the residual and squared residual were then computed for U.S. carriers, foreign carriers (excluding fifth-freedom carriers), fifth-freedom carriers, and all carriers.

The program was applied to the data for the months of May, August, and November (the months for which the method was calibrated) and for the month of February, which was used to validate the methodology. The February 1976 results are displayed in Table 8.

In each of the months, method 4, the IQSI method described above, produces a sum of squared residuals equal to approximately one-half that produced by method 1, the CAB domestic method. This is probably the best measure of fit, since the sum of squared residuals is an indication of the variance of the estimate around the actual value. In all months, method 3 (the same as method 4 except without the citizenship factors) also produces a substantial improvement over method 1 in terms of the sum of squared residuals. In general, its sum of squared residuals falls two-thirds of the way between methods 1 and 4 (closer to method 4).

Method 4 also produces substantially better average residuals than any of the other methods.

The average residual is a measure of the bias of the method. For example (as shown in Table 8), method 1 generally underestimates the market share of the foreign carrier by 0.057 and overestimates the market share of the third-country carrier by 0.052. Thus, a fifth-freedom carrier's market share of 0.01 would probably be estimated as 0.06, and a foreign carrier's market share of 0.55 would probably be estimated as 0.49. Methods 2 and 3 are not much better in this respect. By contrast, method 4 in no month biased any carrier's market share by more than 1.1 percentage points. Its bias is almost always far less than 1 percentage point.

Note that the sum of the biases in any method is always zero. If the actual market shares are x, y, and z and the estimated market shares are X, Y, and Z, the sum of the biases $[(X - x) + (Y - y) + (Z - z)]$ equals $(X + Y + Z) - (x + y + z) = 1 - 1 = 0$.

The square root of the average squared residual is an indication of the average magnitude of the error (difference between actual and predicted results). For example, if one method overestimates market share by 0.1 in one market and underestimates it by 0.1 in another, its average bias (average residual) is nil but its square root of the average squared residual is 0.1. If another method overestimates market share by 0.06 in one market and underestimates it by 0.02 in another, its average squared residual is 0.045. Whether the second method is better or worse than the first would, in this case, be open to question, since it produces more bias but a closer prediction. Among the methods examined, however, there is no question which is the better one, since method 4 performs better than any of the others in reducing both bias and absolute error.

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Forecasts of Passenger and Air-Cargo Activity at Logan International Airport

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This paper summarizes the results of a recently completed aviation-forecasting project conducted for Logan International Airport, Boston's major metropolitan air facility. Independent procedures are developed for forecasting certificated air-carrier (domestic and international), commuter, general aviation, and air-cargo traffic. Data are drawn from several sources, which include airport records and Federal Aviation Administration and Civil Aeronautics Board publications. To the greatest extent possible, multiple-regression techniques are employed to identify the factors responsible for historic changes in activity levels. Simple forecasting models are then used to predict aviation activity under alternative scenarios; these show that air-passenger and air-cargo volumes are likely to increase at the rate of approximately 5 percent per year. The exact growth rate will depend most heavily on changes in regional income and on costs and fares. Growth in aircraft operations will be lower, due to projected increases in airplane sizes and load factors but will still be significant. In addition to their primary use in planning at Logan, the results shed light on broad issues in aviation forecasting. One important implication is that the effects of rate and route deregulation on activity at major airports are likely to be minor in comparison with changes in economic conditions and fuel prices.

The long-term planning decisions now being made by airport authorities strongly depend on expectations of growth in aviation activity over the next two decades. There is currently considerable uncertainty whether the future will be characterized by the robust growth in airline activity observed from 1960 to 1969 and 1975 to 1978 or whether the experience of the last few years is a bubble that will burst and the commercial aviation industry will return to the modest secular growth rates of 1969-1975. Identifying the determinants of growth in the air-passenger and air-cargo industry is necessary for making reliable forecasts of airport activity. This paper summarizes the results of a recently completed aviation-activity-forecasting project conducted for Logan International Airport, Boston's major metropolitan air facility. Forecasting procedures have been developed independently for each of the major types of airport passenger activity and operations: certificated air carrier (domestic and international), commuter air, and general aviation (GA). A separate forecasting method has also been developed for air-cargo operations. For each type of service, statistical methods are employed wherever possible to isolate the factors that have caused variation in historical activity levels from available Logan-specific data. Scenarios of plausible future levels of these causal factors are then employed to derive activity forecasts over the next 20 years. [The consequences that result from alternative scenarios of future conditions have also been examined by Charles River Associates (CRA) (1).] These forecasts are explicitly demand oriented and do not incorporate the effects of potential capacity limitations.

Aside from the practical application of these forecasts to the work of the Massachusetts Port Authority (Massport) planning department at Logan, the forecasting models estimated are of broader interest for several reasons:

1. The past two years have been marked by

substantial deregulation of rates and routes for both air-passenger and air-cargo traffic. There is general interest in the size of the effects of deregulation as they relate to other influences on the level of air-service demand.

2. Because Boston is a major city that has large sectors of each class of air service represented, forecasts specific to Logan may be indicative of likely trends elsewhere.

3. A large number of air-travel forecasting models were developed during the late 1960s that proved inaccurate, grossly overestimating future growth. It is of general interest to learn why these models performed poorly and how a model estimated on more-recent data compares with these models.

4. The air industry has moved to a relatively complicated and demand-sensitive pricing structure. It is important to know whether simple price measures such as average yield perform well in multiple-regression forecasting models or whether a more-complex series of price terms is needed to capture the range of prices that faces different customers.

FORECASTING PROCEDURES FOR PASSENGERS

Logan experienced double-digit growth in passenger traffic on certificated and commuter carriers from 1965 to 1970, a dramatic slowdown from 1970 to 1975, and a return to robust growth from 1975 to 1978. Because both international certificated and domestic commuter traffic were growing from a small base, their initial growth rates were especially large. The growth rates in annual numbers of flights generally mirrored the passenger growth rates, though the rates for 1970-1975 were much lower due to the introduction of wide-body jets.

The forecasting system developed is based primarily on a statistical examination of 1965-1978 data. Although including earlier data would have provided more data points, it would have been extremely difficult to isolate the effects of the introduction of jet aircraft on consumer attitudes toward and use of air travel. This examination is therefore confined to the jet era.

International and Domestic Certificated Carriers

Model Development

The models of certificated air-carrier passenger activity are estimated by using multiple-regression techniques. Many explanatory variables were considered for inclusion; the most basic of these is the population of the Boston area. All other things being equal, a larger population should produce more travel into and out of Boston. Another candidate variable is inflation-adjusted income, since numerous other studies have related air travel to income growth. A third important economic indicator is a cyclical variable, such as an index of capacity use, which may act as a measure of consumer sentiment. The inflation-adjusted level of air fares is another obvious candidate for inclusion; for long-range forecasts, one should also test the significance of the age distribution of the population if one believes that different age groups have inherently different propensities to travel by air. Finally, the occupational mix of the population in the Boston area may also be of interest, since it is well known that some groups travel extensively on business, whereas others make no business trips by air at all.

Mention should also be made of variables purposely not included in the demand models. We do not consider supply-side limitations such as runway or terminal capacity since this study is intended to produce unconstrained-demand forecasts for use by airport authorities. Frequency of air service is not included since discrete additions of flights to certificated-carrier markets are likely to produce only minor service-quality changes in an airport as large as Logan. The effect of changes in the relative performance of alternative modes is not included either. Relative travel times have remained roughly constant over the past 15 years and will be assumed to remain at or about current levels. Finally, the model ignores many possible short-term influences on air traffic, since it is intended to be long-term in orientation. For example, it is assumed that interruptions in fuel supply are temporary and that in the long run there will be adequate fuel supplies, although perhaps at a much higher price.

International Carriers

There are three basic types of international traveler at Logan. The first travels from Boston directly abroad; the second travels from some other point in the United States to Boston to connect with an international flight; the third travels from Boston to another gateway and then connects with an international flight. In a May 1979 study, Civil Aeronautics Board (CAB) origin-destination data, CAB portion-of-international-journey data, and Massport international-passenger data were used by CRA to break down Boston international travel into these three categories (see Table 1).

The level of direct Boston international travel grew rapidly from 1965 to 1971 and has grown more slowly since. Conversely, use of Boston as an international gateway has increased considerably since 1973 and by 1978 such use accounted for 22 percent of the total number of Boston international enplanements. This increase has undoubtedly been influenced by the growing congestion at Kennedy International Airport in New York and the marketing efforts of Massport. The congestion at Kennedy combined with changes in service levels from Boston has also led to a 60 percent decline in the use of other gateways by Boston travelers between 1973 and 1978.

The procedure used to forecast international passenger activity at Logan begins by predicting Boston-originating international travel. It is then assumed that the percentage of Boston travelers who

Table 1. Number of passengers by category of international traveler.

	Type of Traveler		
Year	A	B	C
1965	60 000	30 000	316 552
1966	80 000	47 500	375 952
1967	150 000	64 999	500 433
1968	210 000	82 499	547 717
1969	210 000	99 999	689 666
1970	210 000	117 499	870 433
1971	192 632	134 998	959 515
1972	206 842	152 498	1 160 344
1973	243 666	148 214	1 267 773
1974	186 565	253 115	1 187 778
1975	194 131	220 739	1 023 175
1976	169 982	261 458	1 206 481
1977	141 370	321 877	1 254 821
1978	98 153	398 477	1 385 701

Note: A = Boston international travelers who use international gateway other than Logan; B = travelers from outside Boston region who use Logan as international gateway; C = Boston travelers who fly abroad directly from Logan.

use other gateways (7 percent in 1978) continues to decline. To forecast international enplanements at Logan, such travelers must be subtracted from the Boston-originating total. Projections of non-Boston-originating passengers who use Boston as a gateway are then added to this subtotal. This procedure yields the total number of passengers who board international flights in Boston.

The left-hand variable in the basic forecasting equation is the true number of Boston-originating travelers. This represents international travel that originates in Boston regardless of gateway choice. Because this variable is specific to the Boston region, it should be primarily dependent on Boston region-specific factors. For forecasting purposes, we have chosen the regional definition of Boston used by the Bureau of Economic Analysis of the U.S. Department of Commerce; this region includes eastern Massachusetts, southern New Hampshire, and Rhode Island. Although this may be geographically slightly larger than the true Logan service area, past demographic and economic data and future projections are readily available for this region, and changes in this region's activity levels should correspond nearly perfectly with changes in the actual Logan market.

As outlined above, variables tested as possible explanatory factors for Boston international air travel included population, income, occupation mix, age mix, cyclical capacity use, and air fares (inflation-adjusted yield per international passenger mile). Well-known technical problems exist with use of this last variable since yield is not a fixed-weight price index. However, the problems are not thought to be as severe in this case, and no practical alternatives exist (2). Across a variety of alternative model specifications, variable combinations, and variable definitions, it was found that population, income, and fare levels prove to be the primary determinants of the volume of air travel. Independent of changes in population, income, and air fares, other potential determinants of international air travel are not statistically significant.

The reason for this finding is the extremely high correlation between regional income and regional international air travel, displayed in the correlation matrix in Figure 1, which uses Pearson correlation coefficients and the following variables: BOSNYPC, Boston-originating international air passengers per capita; RARIT, real average yield per international passenger mile (cents per mile in 1967 dollars); RINCPC, real Boston regional income per capita (thousands of 1967 dollars); CAPFRB, Federal Reserve Board index of capacity use; and AGEMXPC, percentage of Boston-region population 25-49 years of age. The correlation between international air travel (BOSNYPC) and real income (RINCPC) is extraordinarily high (0.97). The air-fare variable (RARIT) also has a high negative correlation with international air travel (-0.68). After changes in population, income, and air fares have been accounted for, there is little change in air travel left to be explained by other variables. It is therefore concluded that the following, rather simple forecasting model will prove reliable:

$$\text{BOSNYPC} = -33.31 + 431.3\ [146.0]\ \log(\text{RINCPC}) - 239.9\ [93.2]\ \log(\text{RARIT}),$$

where $R^2 = 0.96$ and the standard errors are in brackets. The choice of a specification with the log of variables only on the right-hand side is appropriate for a market characterized by extremely rapid growth in its early years followed by maturation. At the 1978 levels of the model's variables, the income elasticity of demand is 1.7 and the price elasticity is -0.94.

Domestic Carriers

In the domestic travel-forecasting procedure, a model is used to project true Boston domestic originations. The travelers who use Logan for a domestic portion of an international journey are then added to derive total domestic enplanements. As in the international travel model, socioeconomic variables specific to the Boston region defined by the Bureau of Economic Analysis are used. The same set of variables--population, income, air fares, occupation mix, age mix, and a cyclical variable--was tested for their explanatory power by using annual data from 1965 to 1978. In addition to average yield per passenger mile (RARDT) on domestic flights, a consumer price index (CPI) for air travel was tested. The Bureau of Labor Statistics CPI for air travel is based on a fixed-weight sample of air-carrier routes and, for that reason, is a theoretically superior price index. However, the sample of routes is not comprehensive and the measure did not perform so well as did the yield variable. Overall, across a large set of alternative model specifications, population, income, and air-fare levels again proved to be the dominant determinants of air travel. Other variables had coefficients that were extremely sensitive to model specification and that were, in most specifications, statistically insignificant. The correlation between domestic travel and income is 0.95 and between domestic travel and yield is nearly -0.97. These two variables by themselves explain nearly all the year-to-year variations in domestic travel, and a forecasting model based on these two variables can be expected to provide reliable forecasts:

$$\log(\text{PDCEP}) = 0.5597 + 1.4757\ [0.79]\ \log(\text{RINCPC}) - 0.700\ [0.54]\ \log(\text{RARDT}),$$

where

R^2 = 0.96 and the standard errors are in brackets,
PDCEP = Boston-originating domestic passengers per capita, and
RARDT = real average yield per domestic passenger mile (cents per mile in 1967 dollars).

Figure 1. Correlation matrix: regional income and regional air travel.

	BOSNYPC	RARIT	RINCPC	CAPFRB	AGEMXPC
BOSNYPC	1.0000	-0.6759	0.9697	-0.5354	-0.3909
RARIT	-0.6759	1.0000	-0.6358	0.3904	0.6881
RINCPC	0.9697	-0.6358	1.0000	-0.4867	-0.2474
CAPFRB	-0.5354	0.3904	-0.4867	1.0000	0.4376
AGEMXPC	-0.3909	0.6881	-0.2474	0.4376	1.0000

Table 2. Forecast of Logan trips.

Type of Trip	Number of Trips (000s) 1980	1985	1990	1995	2000
Domestic service					
Boston-based domestic	11 718	15 136	19 888	25 965	33 807
Boston-U.S.-international	+ 87	+ 70	+ 59	+ 71	+ 83
U.S.-Boston-international	+ 574	+ 1 155	+ 1 859	+ 2 995	+ 3 822
Total	12 379	16 361	21 806	29 031	37 712
International service					
Boston-based international	1 732	2 338	2 931	3 542	4 159
Boston-U.S.-international	- 87	- 70	- 59	- 71	- 83
U.S.-Boston-international	+ 574	+ 1 155	+ 1 859	+ 2 995	+ 3 822
Total	2 219	3 423	4 731	6 466	7 898

Because the model is estimated in log-log form, the coefficients are elasticities. The income elasticity of domestic travel is estimated to be 1.48 and the price elasticity, -0.70. Once again, this model forecasts pure domestic travel. The number of travelers who use Logan on a domestic leg of an international trip must be added to this subtotal to arrive at the number of domestic enplanements.

Baseline Scenario

The baseline scenario consists of forecasts of population, income, and air fares. The population and income projections are provided by CRA market service, which supplies regional disaggregation to the University of Maryland national macroeconomic model. The population of the Boston region is projected to grow at a rate of only about 0.6 percent per year, which is lower than that of the rest of the country. The baseline economic forecast shows that inflation-adjusted income per capita is growing at the following annual rates:

Years	Percentage of Growth
1978-1980	2.10
1980-1985	2.21
1985-1990	2.62
1990-2000	2.50

By comparison, U.S. real income per capita grew at the higher rates of 2.71 percent per year from 1960 to 1970 and at 4.20 percent per year between 1970 and 1973 (3, p. 383). However, the growth rate of income per capita from 1973 to 1978 was less than 1 percent per year. The baseline forecast assumes that long-term economic growth will be neither so robust as during the 1960-1973 period nor so weak as during the 1973-1978 period. Growth is expected to be slower than in the past because the easy-growth gains from urbanization, greater investment in education, and the large-scale entrance of women into the labor force have already occurred. Conversely, growth is expected to be more robust than in the past five years because a repeat of the worst postwar recession on record and a quadrupling of oil prices is not anticipated during every five-year period in the future.

However, the baseline forecast does allow for considerable growth in energy prices over the next two decades. Since energy accounts for less than 10 percent of total national final demand (4, p. 8) and because the consensus of energy modelers is that higher energy prices need not cripple long-term growth (5), the baseline economic-growth scenario is consistent with much higher real-energy prices and represents a realistic appraisal of future long-term economic growth.

Real domestic air fares declined at a rate of 2.7 percent per year over the entire 1965-1977 period and dropped nearly 7 percent between 1977 and 1978.

The baseline case assumes that air fares will continue to decline in relation to all other goods and services but that the rate of decline will be much less than that during the 1965-1977 period. The following specific changes in average yield per passenger mile relative to the overall inflation rate are assumed. (Since this paper was prepared prior to 1979 fuel-price increases, 1978-1980 fare declines are probably overstated, but long-term results are robust with respect to this short-term change.)

Years	Percentage of Change
1978-1980	-2.5
1980-1985	-1.8
1985-2000	-1.5

In this scenario, increases in average plane size, route rationalization, minor further increases in load factors, and the introduction of more-fuel-efficient jets are expected to more than offset real fuel-price increases. The assumption that the energy problem becomes one of high prices rather than limited supply is implicit.

Finally, the baseline case implicitly assumes that people will continue to want to travel (i.e., that no new invention or pastime will change broad personal attitudes toward travel) and that electronic communications will not replace the business need for person-to-person contact.

Baseline Passenger Forecasts

The baseline scenario leads to the baseline Boston-region air-travel forecast shown in Table 2 (calculated by CRA, May 1979). The numbers in the first row of the domestic and international categories represent Boston originations rather than enplanements. Enplanements are derived by accounting for the use of Boston as a gateway by passengers from other regions and the use of other international gateways by Boston travelers. The use of other gateways by Boston-originating international travelers declined from 30 percent in 1967 to 6.6 percent in 1978. Because of the increasing congestion at Kennedy and better air service at Logan, it is assumed that this percentage will fall to 5 percent by 1980, 3 percent by 1985, and 2 percent from 1990 to 2000.

Use of the Boston international gateway by residents of other regions has increased about 20 percent per year in recent years. The baseline case assumes that the annual growth rate is 20 percent from 1978 to 1980, 15 percent from 1980 to 1985, 10 percent from 1985 to 1995, and 5 percent from 1995 to 2000. (Inasmuch as the use of the Boston gateway depends on the level of Massport marketing activity, this portion of air-traffic growth may be considered a policy variable.) To calculate total Logan domestic passengers we add Boston-U.S.-international and U.S.-Boston-international travel to the Boston-based domestic travel. To calculate Logan international travel, we start with Boston-based international travelers, subtract those who use other gateways, and add travelers from elsewhere who use Boston as a gateway.

Baseline Operations Forecasts

Given forecasts of annual passenger volumes, it is possible to derive forecasts of aircraft operations by establishing likely average airplane sizes and load factors.

Based on estimates made by the Boeing Company regarding growth in the average seating capacity of certificated domestic aircraft divided by the 1977 average of 124.2 seats per airplane, the following values are derived:

Year	Average Increase, Seats per Plane
1977	1.00
1980	1.17
1985	1.30
1990	1.32
1995	1.37
2000	1.43

This analysis shows relatively rapid growth in average airplane sizes during the years 1977-1985 as wide-body airplanes become more dominant. Beyond 1985, the projections show a more-stable average airplane size. Of course, these projections are only an educated guess, because post-1985 airplanes have not yet been ordered. More-rapid growth in airplane sizes after 1985 must be considered a strong possibility, particularly in the presence of prolonged fare competition.

It is assumed that average load factors at Logan will approach an upper bound of 64 percent as follows: 1977, 50.5 percent (actual); 1980, 61 percent; 1985, 63 percent; and 1990-2000, 64 percent. Beyond 1980, the assumption is implicit that large-discount off-peak fares will continue to be offered and that airlines will tend to avoid low load-factor segments as the contribution per passenger declines in the presence of fare competition.

Domestic

The baseline operations forecasts are calculated as the projected number of passengers divided by the product of base-year (1977) passengers per flight, the ratio of future-year to base-year airplane size, and the ratio of future-year to base-year load factor. The projected domestic passenger volumes and calculation of growth in the number of domestic certificated flights (as calculated by CRA in May 1979) appear in Table 3.

The 1980 figure may be an underestimate that reflects the assumption of a more-rapid upgrading of the air fleet than may actually occur at Logan. However, the long-term trend is clear. The number of domestic certificated flights will stay relatively stable until 1985 and then increase significantly due to the slow increase in average airplane size thereafter.

International

While the above method could theoretically be used to project the number of international flights, a significant number of international passengers use foreign-flag carriers for which load-factor data are not available. Therefore, less-precise estimates of future international flights are made under the following two assumptions:

1. On the average, airplane sizes on international flights will grow two-thirds as much as airplane sizes on domestic flights. This assumption is made because roughly half the international flights already use wide-body aircraft.
2. On the average, international load factors will grow by 15 percent more than domestic load factors because there are fewer peak-hour and time-of-day problems involved in scheduling international flights.

These assumptions allow us to construct in Table 4 projections of numbers of international flights that correspond to the estimates of domestic flights in Table 3, given baseline scenario passenger projections. These calculations (made by CRA in May 1979) show that the number of international flights will more than double between 1980 and 1995 and will grow faster than domestic flights.

Caveats

While the calculations of number of flights for domestic and international travel provide our best forecast, there are two reasons why they may prove to be biased upward. First, current estimates of future airplane types and sizes based on current orders of the airlines lead to the assumption of a

Table 3. Estimates of certificated domestic flights at Logan, 1977-2000.

Variable	1977[a]	1980	1985	1990	1995	2000
Domestic passengers (000s)	9906	12 379	16 361	21 806	29 031	37 712
Base-year passengers per flight	59.0	59.0	59.0	59.0	59.0	59.0
Growth in no. of seats per airplane	1.00	1.13	1.31	1.31	1.36	1.42
Growth in load factor	1.00	1.21	1.25	1.27	1.27	1.27
No. of flights	167 898	153 451	169 347	222 151	284 884	354 434

[a] Data for 1977 are actual data.

Table 4. Estimates of international flights at Logan, 1977-2000.

Variable	1977[a]	1980	1985	1990	1995	2000
International passengers (000s)	1634	2219	3423	4731	6466	7898
Base-year passengers per flight	85.8	85.8	85.8	85.8	85.8	85.8
Growth in seats per airplane	1.00	1.08	1.20	1.20	1.24	1.28
Growth in load factor	1.00	1.24	1.29	1.31	1.31	1.31
No. of flights	19 040	19 312	25 772	35 076	46 393	54 897

[a] Data for 1977 are actual data.

rather gradual trend in average airplane size after 1985. However, there is a chance that fare competition will lead to greater production and use of larger aircraft, which would lower projections of the number of flights.

The second cause for possible upward bias is that we project an equilibrium load factor of 64 percent. This seems to be in the middle of a range of projections that extend from below 60 percent to 70 percent. More-sophisticated peak-load pricing or airline scheduling practices or both may be able to bring the average load factor higher. A combination of larger airplanes and higher load factors could significantly reduce the number of projected flights in future years. Even under these more-liberal assumptions, however, aircraft operations at Logan would be projected to grow substantially.

Commuter Air Service

In recent years, the commuter air market has experienced extremely rapid growth in New England in general and at Logan specifically, as well as across the rest of the United States. This growth has been aided by abandonment of some routes by certificated carriers, broader acceptance of favorable joint interline fares with the larger carriers, more-favorable financial treatment by lending institutions, and greater awareness of the commuter lines by the flying public. In future years, certificated carriers are unlikely to abandon many more routes, not many more unserved pockets of latent demand will surface for commuters, and growth will no longer be from a small base, so growth rates should decline. During the next 5-10 years, expansion is also likely to be severely constrained by a national shortage of suitable aircraft. Although we expect the commuter industry to continue to grow more rapidly than the certificated carriers, growth will become more difficult and passenger growth rates such as the 40 percent observed from 1977 to 1978 will not be sustained.

In a market environment of rapid growth but expected maturity and external constraints, formal econometric models often fail to provide a satisfactory forecasting tool. [An example of an econometric forecast of 1988 Boston commuter activity made in 1977 that had already been exceeded by 1979 is reported in a 1977 Federal Aviation Administration (FAA) report (6).] Instead, the method chosen for this study examines each segment between Logan and other airports served by each commuter airline and uses the experienced judgment of John W. Drake, a consultant to the aviation and air commuter industry, to forecast commuter air activity. The method and results are described in much greater detail in a CRA publication (1).

It should be noted that the air commuter designation excludes Air New England, which recently became a certificated carrier. Because Air New England was omitted from the above analysis of certificated carriers, the latter part of this section projects growth for Air New England alone.

Baseline Passenger Forecasts

Existing Routes

Four key market factors are taken into account in the forecasts of air commuter activity on existing segments: (a) traffic type (feeder versus local), (b) equipment type, (c) potential service improvements, and (d) carrier competition.

Based on a detailed consideration of these factors, the following aggregated forecasts of number of air commuter passengers per year at Logan were developed: 1977, 267 478; 1990, 650 182; and 2000, 1 405 671. These levels represent a 7.1 percent compounded annual growth rate from 1977 to 1990 and an 8.0 percent annual rate from 1990 to 2000.

Because of the extremely high annual growth rate observed during 1978, it might appear that these forecasts are somewhat low. However, an analysis of the commuter traffic growth during 1978 shows that much of the growth was due to extremely aggressive and nonsustainable expansion by only two firms. Given this consideration, the forecasts of commuter traffic will arbitrarily incorporate a 15 percent gain for 1979, a 10 percent gain for 1980, and then interpolate a constant-percentage growth rate between the forecasts given for 1980 and 1990 and for 1990 and 2000.

Although the judgmental technique for forecasting commuter growth rates provides conservative passenger growth rates relative to recent history, forecast long-term growth is nonetheless robust. Commuter passenger traffic at Logan in 1990 is forecast to be nearly 2.5 times the 1977 level, and by the year 2000, traffic will be more than five times the 1977 level. The reader should remain aware, however, that these projections are inherently softer than the certificated forecasts.

Air New England

Though legally a certificated carrier, Air New England has a route structure similar to that of a large regional commuter line and was therefore omitted from the projections for certificated carriers. However, since Air New England currently serves nearly as many passengers as all commuter lines combined, its inclusion is critical.

Unfortunately, it is always risky to make long-term market projections for a single air carrier or individual firm in any industry. Differences in managerial efficiency between Air New England and its competitors could cause significant redistributions of commuter-type activity among firms. Therefore our forecasts, which are based on the current traffic of Air New England and relationships between its growth and the growth of true commuter traffic, should produce accurate forecasts for the commuter-type market as a whole, though the firm-specific disaggregation should be treated with caution.

The resulting CRA forecast of total Logan commuter-passenger traffic is presented in Table 5. (In both Tables 5 and 6, the reader should treat the market total projections as more reliable than the commuter-Air New England market disaggregation.)

Commuter Air-Carrier Operations

Passenger forecasts have been translated into aircraft movements on a segment-by-segment basis through application of a number of standard industry practices regarding service frequency, equipment choice, and so forth (1).

CRA projections of total operations by passenger commuter airlines are given in Table 6. Due to use of larger airplanes and to improving load factors, it is anticipated that commuter operations at Logan will grow much less rapidly than will passenger volumes. Still, by the year 2000, they will be more than double the 1978 levels.

General Aviation

GA activity is the most difficult segment of the Logan air passenger market to forecast. Forecasting difficulties arise first from concerns about the

Table 5. Forecast of commuter-passenger traffic at Logan.

	Number of Passengers (000s)			
Year	Existing Commuters	New Routes	Air New England	Total
1978	327	0	369	696
1980	414	17	441	872
1985	519	25	568	1112
1990	650	36	736	1422
1995	956	55	1032	2043
2000	1406	74	1427	2907

Table 6. Forecast of commuter-passenger flights at Logan.

	Number of Flights (000s)			
Year	Existing Commuters	New Routes	Air New England	Total
1978	50	0	27	77
1980	55	3	28	86
1985	60	4	31	95
1990	65	5	35	105
1995	84	6	43	133
2000	108	7	52	167

accuracy of historic GA time-series data specific to Logan, since it is believed that inconsistent procedures for tabulating empty arrivals, flybys, etc., may have been employed. Unfortunately, more-reliable recent data are contaminated by significant levels of local helicopter traffic. These factors greatly reduce the validity of statistical models of itinerant GA activity based on Logan data. The use of statistical models is further hindered by the relatively unique character of Logan GA traffic, which restricts the transferability of results derived from other data. Therefore, it was concluded that scarce resources ought not to be devoted to statistical model development.

Instead, the GA forecast is made on the basis of trend extrapolation. GA operations increased by 5.4 percent annually between 1973 and 1978, a period that included the most severe postwar recession. We forecast a baseline growth of 5.6 percent per year for GA activity from 1978 to 1985, 5 percent per year from 1985 to 1990, and 4 percent per year from 1990 to 2000. The rate of growth of GA activity is expected to be dampened by further improvements in commuter air service to smaller cities.

Rocks and Zabronsky have given an example of a set of GA forecasts that allows for capacity constraints (7).

This forecast is defined as one that assumes that GA activity will be unconstrained by Logan's capacity or policies. In fact, it is highly likely that some form of constraint on the growth of GA traffic will have to be imposed within the next two decades. Such constraints might take the form of higher landing fees, time-of-day restrictions, or a slot-reservation system.

The following levels of GA operations at Logan are forecast:

Year	No. of Flights
1978	53 542
1980	60 160
1985	80 507
1990	102 570
1995	125 011
2000	152 095

FORECASTING PROCEDURES FOR CARGO

The air-cargo forecasts are based on actual cargo tonnages and growth trends at Logan during the 1960s and 1970s. Future air-cargo demand at Logan is projected by using a two-part model: (a) a set of single-equation air-cargo demand functions estimated separately for several subcategories of cargo and (b) a set of split ratios that are used to allocate the projected totals to types of aircraft and to enplanements and deplanements.

Model Development

The demand for air-freight movements depends on the level of general economic activity, regional specialization in activity conducive to air freight, air-freight rates, the quality of air-freight service, and the price and quality of freight services provided by competing modes. Quality-of-service considerations include scheduled frequency, speed of the mode, capacity offered, reliability of delivery service, and probability of loss and damage.

Although theoretical considerations argue for estimating and using a fully specified air-cargo model as a Logan planning tool, practical considerations have led us to develop a more-streamlined model. Therefore, the basic forecasting model projects air-cargo demand on the basis of future economic growth and air-cargo rates only. [See the CRA report (1) for a detailed description of the estimated equations.] It has been found that demand for all types of cargo is significantly affected by regional income levels and by cargo rates. The income and price elasticities of the several cargo categories estimated are presented below as calculated by CRA in March 1979:

	Elasticities	
Cargo Category	Price	Income
Domestic freight and express		
Total	-1.68	0.68
Certificated carrier	-1.74	0.56
International freight and express (average)	-2.89	1.62

Since the econometric model provides a basis for the forecast of total tonnage only, separate estimates were made of the split between enplanements and deplanements and the proportion of tonnage in all-cargo freighters. These estimates were based on historical data at Logan and comparisons with forecasts for other major airports.

Baseline Scenario

As was the case in the passenger forecasts, it is necessary to define a baseline set of future conditions. The rate of income growth projected for the New England region (the assumed Logan service area for cargo) in the baseline scenario is consistent with that projected for the Boston area in the passenger forecasts given earlier. Real rates are expected to decline by an annual average of 2.0 percent from 1978 to 1985, and thereafter at 0.8 percent per year due to the combined effects of projected increased load factors, increased containerization, improved terminal technology, economies of scale, and the introduction of new specialized systems. (As in the case of passenger fares, cargo rates in 1980 are probably understated due to recent fuel-price increases. However, long-term results are robust with respect to this short-term change.) This equals an average annual decline of 1.1 percent from 1978 to 2000. In

Table 7. Forecast of Logan air-cargo tonnage, 1978-2000.

Type of Service	Cargo (tons) 1978 Actual	1980	1985	1990	1995	2000
Domestic freight and express						
Certificated carrier						
Arrival	60 660		91 308	107 534		145 560
Departure	75 894		109 568	129 040		174 670
Total	136 554	153 066	200 876	236 574	276 810	320 230
Commuter service						
Arrival	2 541		5 858	8 843		17 037
Departure	2 517		5 858	8 843		17 037
Total	5 058	6 730	11 716	17 686	25 412	34 074
International freight and express						
Arrival	18 154		29 107	34 691		47 560
Departure	29 603		43 660	52 036		71 340
Total	47 757	55 078	72 767	86 727	102 864	118 900
Domestic mail						
Arrival	19 282		22 645	23 188		23 644
Departure	21 923		22 645	23 188		23 645
Total	41 205	42 883	45 290	46 376	46 938	47 289
International mail						
Arrival	1 740		3 008	4 047		7 145
Departure	2 041		3 532	4 751		8 388
Total	3 781	4 660	6 540	8 798	11 919	15 533
Total, all types of service	234 355	262 417	337 189	396 161	463 943	536 026
Total, all-cargo freighters[a]	86 077	93 945	113 970	126 375	139 183	151 695

Note: Data for 1978 come from Green Sheets supplied by Massport Aviation Department. Data for other years are calculated from simulations made by CRA, March 1979.

[a] Included in the total above.

general, it is assumed that no dramatic changes will occur in the structure of the industry, the nature of supply constraints that face Logan operations, or the political environment in which the industry operates.

Baseline Forecasts

The cargo projections presented in Table 7 forecast that cargo and mail tonnage will more than double by the year 2000, from 234 000 tons in 1978 to 536 000 tons in 2000. The figures suggest that the explosive growth rates of the 1960s are not likely to be seen at Logan during the next two decades, though tonnages will not stagnate, as was the case in the recession of the mid-1970s.

The growth in traffic projected for domestic cargo at Logan is nearly identical to FAA and American Public Transit Association national projections. Because domestic cargo constitutes 75 percent of all freight and express at Logan, the forecast implies that Logan's share in the national air-cargo market should remain roughly constant over the forecast period. The growth in international cargo at Logan is projected to be lower than the national average due to the relative maturity of the Logan market.

SUMMARY

Passenger counts and air-cargo tonnage at Logan are expected to grow at about 5 percent annually during the next 20 years. This is roughly the average of the 1960-1978 period taken as a whole. We expect that the number of aircraft movements will grow more rapidly than they did during the 1960-1978 period. This more-rapid growth will occur because there will not be an increase in average airplane size comparable with that which occurred when wide-body jets were introduced and because a growing percentage of air operations will be performed by relatively small commuter aircraft. We conclude that air traffic will grow neither so rapidly as during 1960-1969 nor so slowly as during 1969-1975.

The greatest uncertainty in the forecast is related to the rate of secular economic growth. A 1 percent difference in this rate translates into about a 30 percent difference in the year 2000 air activity forecasts. A second major uncertainty concerns the rate of increase in aviation fuel prices. Since fuel costs are now greater than 20 percent of total operating costs for airlines, differences in future energy price trends can have significant effects. Energy price forecasts for the year 2000 that differ by a factor of 2 result in air activity forecasts that differ by nearly 20 percent.

Some uncertainty is also introduced by reductions in rate and route regulation. However, although this issue is far from settled, it appears that the uncertainty for airport planners relating to future activity levels is much less than the uncertainty caused by different economic-growth and fuel-price scenarios. Off-peak discount pricing is already extensive, load factors are already up, and certificated airplane orders that can be filled for roughly the next five years are already placed. Although it is not difficult to believe that competitive strategies can increase load factors and reduce costs per passenger even further, likely scenarios do not show that the impact on certificated passenger traffic is likely to be major in comparison with other effects.

There is one sector, however, in which airline regulatory reform is already severely affecting the airport planning process. This is air-commuter passenger traffic. Regulatory reform has allowed certificated carriers that have large airplanes and infrequent service to be replaced in smaller cities by commuter carriers that have more frequent service and smaller airplanes. In addition, regulatory changes have included legislated interline fares that make feeder commuter-air connecting service to full-fare certificated flights extremely inexpensive. Partly as a result of these changes, commuter passenger traffic at Logan increased by 40 percent between 1977 and 1978.

Such volatility in the presence of not previously experienced changes is, of course, difficult to forecast, and many projections made in the mid-1970s dramatically underestimated growth of commuter traffic. However, aside from this type of unforeseeable effect, many earlier (i.e., late 1960s) forecasts systematically overpredicted future activity. Although this paper does not detail an explanation of these inaccuracies, the analysis undertaken in this study isolated two principal causes. First, economic growth was projected to be much higher than the actual experience of the 1970s. Second, early forecasting models had income elasticities that are now believed to be too high and air-fare elasticities that are believed to be too low.

Another finding during the course of the study has been the limited amount of consistent and reliable data available for site-specific forecasts. Differences in the categorization of activity between data sources and between available and desired data items often constrain modeling efforts. For example, the adequacy of average yield as an explanatory variable in demand-forecasting equations during years when the rate schedule is complex and demand oriented was subject to some doubt. Although we found that average yield provides an adequate measure of fare levels and find no evidence of large backcasting residuals when it is used, the unavailability or the known inaccuracies and biases of data that measure both activity levels and causal factors often make informed judgment an appropriate forecasting tool.

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REFERENCES

1. Charles River Associates, Inc. Logan International Airport Passenger and Air Cargo Forecasts. Massachusetts Port Authority, Dec. 1979.
2. P.K. Verleger, Jr. Models of the Demand for Air Transportation. Bell Journal of Economics and Management Science, Vol. 3, 1972, pp. 412-457.
3. 1975 Statistical Abstract of the United States. Bureau of the Census, U.S. Department of Commerce, 1975.
4. R.W. Gilmer. Services and Energy in U.S. Economic Growth. U.S. Department of Energy; Institute for Energy Analysis, Oak Ridge, TN, 1977.
5. Charles River Associates, Inc. Review and Evaluation of Selected Large-Scale Energy Models. Electric Power Research Institute, Cambridge, MA, 1977.
6. G. Deosaran, H. Sweezy, and R. Van Duzee. Forecasts of Commuter Airlines Activity, Appendix C. Federal Aviation Administration, U.S. Department of Transportation, Rept. FAA-AVR-77-28, 1977.
7. J. Rocks and H. Zabronsky. Study and Forecast of General Aviation Operations at 60 Medium-Hub Airports. Federal Aviation Administration, U.S. Department of Transportation, Rept. DOT-FAA-77WAI-726, 1978.

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Forecasting Method for General Aviation Aircraft and Their Activity

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This paper describes the formulation and application of a general aviation (GA) forecasting model within the context of the North Central Texas regional airport system planning process. The objective of the model was to provide a means of forecasting registered county-level GA aircraft ownership and the activity of those aircraft (hours flown) that allows public policymakers and planners to assess the impact of policy and economic growth alternatives on GA demand. The bottom-to-top econometric and time-series model developed through this effort achieved these objectives with statistical results that varied across the 19 counties and four aircraft types. Finally, a feature of this model uncommon to other GA forecasting models is that the demand for aircraft is specified to be (among other things) a function of the demand for air travel (hours flown).

The North Central Texas Airport System Plan, adopted by the Regional Transportation Policy Advisory Committee (RTPAC) on November 16, 1974, presented the findings of a comprehensive two-year analysis of existing and future activity in the 19-county area defined by the North Central Texas and Texoma state planning regions. The plan identified existing airport facilities, forecast aviation demand through the year 1990, and recommended the staged development of a system of public airports (to include improvements to both proposed and existing airports) to meet that demand.

With the adoption of the plan, efforts of the RTPAC staff focused on assisting local governments in plan implementation. In addition, efforts were made to update the plan in response to changing conditions within the aviation community and within local communities. An outgrowth of these efforts was a realization that the technical planning process underlying the system plan did not allow for a rapid, comprehensive response to technical or policy issues that were raised by elected officials, airport managers, and the general public.

For example, a major issue raised by groups opposed to new airports was whether there was a need to build new general aviation (GA) airports if fuel prices continued their rapid climb: The assumption was that rapid fuel price increases were equated to an equally rapid decline in GA activity. Another issue was what effect either delaying the construction of or not building a recommended airport would have on existing facilities. In each case, neither the plan nor the technical planning process provided a mechanism for developing an analytically based response to the issue raised.

The identified weaknesses in the existing plan and planning process led to the development of a definition for a new technical GA airport system planning process for the North Central Texas region. This paper presents one component of that process: a forecasting method for GA aircraft and their activity. A necessary feature of any aviation forecast is the identification and measurement of the factors that affect aviation growth. The indicators of regional GA growth in this forecasting method are the number of aircraft (by type of aircraft) registered with the Federal Aviation Administration (FAA) and the hours flown by those aircraft. The model is based on the hypothesis that regional growth is, in general, a function of fuel costs, income, and general credit conditions. As with most forecasting models, data limitations prevent the method presented here from being a complete representation of all factors that affect GA growth.

The ultimate goal of a regional aviation forecasting effort is the generation of airport-specific activity forecasts. Therefore, a bottom-to-top approach to forecasting was assumed in the model's construction; i.e., the smallest geographic area was used for which sufficient data were available to support the forecasting effort. This was each of the 19 individual counties. When data limitations made direct estimates of county-level activity impossible, county-level estimates were derived as marginal products of regional estimates.

The forecasting model presented here does not provide future estimates of local and itinerant operations, which are variables of interest to federal, state, and local planners. There is a threefold reason for not forecasting aircraft operations directly. First, accurate historical operational data at the level of specificity required are simply not available. Second, the costs associated with initiating such a data collection effort are beyond the financial resources of a regional airport system planning effort. A third and more fundamental reason for not forecasting operations directly is that, in the perception of the aircraft owner, the total hours the aircraft has been flown represents the amount of available aircraft service that has been consumed over the lifetime of the aircraft.

It can therefore be argued that the theoretical measure of GA activity is total hours flown. Airport operations represent a complementary benefit in the consumption of GA hours flown and need not (and perhaps should not) be forecast directly. Since we recognized that (theory aside) we still needed forecasts of airport operations for planning purposes, data by which forecasts of total hours flown could be converted to aircraft operations were derived from a survey of registered aircraft owners in the region performed by the North Central Texas Council of Governments (NCTCOG). Further, NCTCOG is developing a GA demand and assignment model that will allow county-level aviation forecasts to be assigned to specific airports. This will enable planners to assess future needs at individual airports.

FORECASTING METHODS AVAILABLE

There are three established long-term forecasting methods employed in GA forecasting at the regional level: (a) the regional-share method, (b) the time-series (trend) method, and (c) the econometric method. It is common practice to use all three methods in a forecast to evaluate the best method or combination of methods. This formal exercise has resulted in some general agreement about the effectiveness of each method or combination of methods by aviation planners. More specifically, the consensus favors a combination of methods (b) and (c) while regarding method (a) as more or less a descriptive statistic.

The regional-share forecast is performed by observing alternative national forecasts from which one can compute the future alternative regional forecasts by calculating the region's percentage share of the national total and extending the series into the future. The implication of a constant market structure is the flaw that aviation planners reject (1). Regional share is really the outcome of regional aviation growth. Consequently, the results of regional GA forecasts are often illustrated by computing the regional share only for purposes of comparison.

A popular forecasting method among aviation planners today is the time-series (trend) method. In this method, the variable of interest is expressed as a function of time or as a function of its own variation over time. These models take on a wide variety of functional forms and incorporate different estimation methods. They can generally be classified as deterministic or stochastic (2). The Box-Jenkins models developed by FAA to forecast quarterly itinerant and local operations are examples of stochastic models (3). In general, deterministic and stochastic time-series models do not predict well in the long run because of their intrinsic nature; i.e., the forecast variable is not related to any causal variable over time. For this reason, their use is generally confined to short-term forecasts.

The econometric method is most often used to make long-term forecasts. Econometric models attempt to simulate economic behavior in the real world based on well-established relationships with other variables over time. An econometric model may consist of one or more equations, and each equation may consist of one or more variables. These models are typically constructed to explain the movement in one or more key variables (endogenous variables) by the movement in other outside variables (exogenous variables). By altering the values of the exogenous variables in the model, the forecaster can simulate real-world events based on alternative assumptions about the future. The examination of alternative long-term trends is a useful planning exercise for establishing confidence limits in the uncertain future.

The forecasting model constructed for GA activity combines both the econometric and the time-series methods in its structure. In this hybrid approach, the error terms in the econometric model are used to construct a time-series model, which in turn is used to adjust the original econometric model for any temporal systematic bias in the original data (4). The resulting transformed model is then used for forecasting.

DATA SOURCES

The FAA aircraft registration master file provided data on the number of aircraft and reported hours flown by aircraft type and county. In addition,

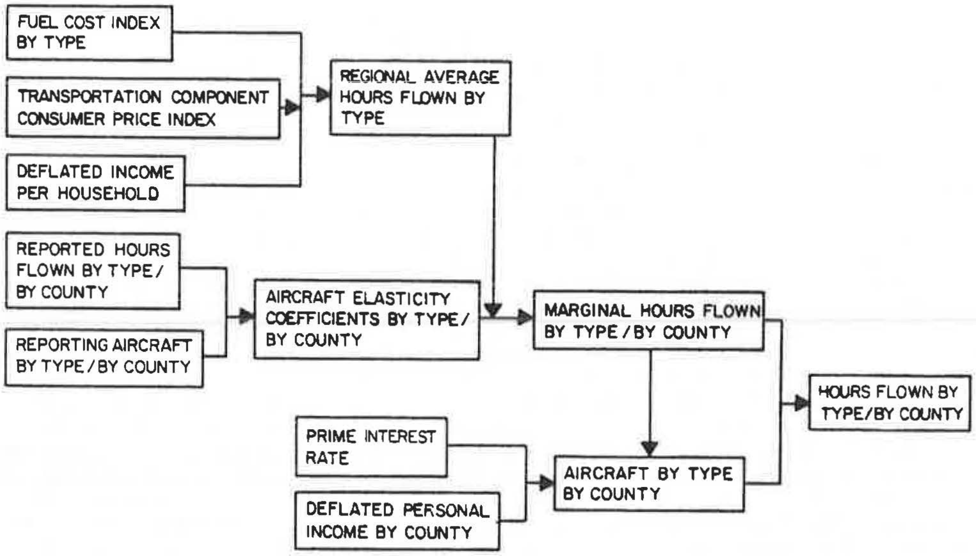


Figure 1. Economic model structure for estimating GA activity.

information from the file on the make, model, and year of manufacture of the regional aircraft fleet was used to identify fuel-consumption rates of a sample of GA aircraft to construct a fuel cost index. Data on these variables were available for December 31, 1970, to December 31, 1977.

Other data sources included all the items and the transportation component of the consumer price index (CPI) from the Bureau of Labor Statistics for 1970-1977. The Aircraft Blue Book was also used to obtain fuel-consumption rates (gal/h). County and regional income data were taken from Sales and Marketing Management magazine's annual survey of buyer income. Finally, the prime rate of interest charged by banks was obtained from the 1978 economic report of the President.

THE MODEL

An illustration of the structural relationships in the GA forecasting model is presented in Figure 1. This structure assumes that activity is determined by general regional economic conditions and those economic factors particular to aviation. First a production relationship is established between reported hours flown and the number of aircraft units that report hours flown in each county. Aircraft elasticity coefficients are obtained from these estimates, which are used to calibrate regional average hours flown at the county level. Regional average hours flown for each aircraft type is determined by a fuel cost index, the transportation CPI, and deflated household income. The calibrated average hours flown estimates (marginal productivities) are used with deflated county income and national prime-interest-rate data to forecast the number of aircraft by aircraft type in each county. Finally, the calibrated estimates of average hours flown are applied to the aircraft forecasts to obtain total hours flown.

Production Function

The production function is specified in nonlinear form as follows:

$$RH = \alpha_0 RA^{\alpha_1} \qquad (1)$$

where

RH = reported hours flown,
RA = aircraft that report hours flown, and
α_0 and α_1 = constants.

The parameter α_1 is interpreted as the long-term aircraft input elasticity coefficient. In general, input elasticity of any factor of production measures the percentage response of output from a percentage change in the factor of production. Thus, aircraft elasticity provides a measure of the sensitivity of hours flown from a change in the regional aircraft fleet size. To perform regression analyses, Equation 1 is linearized by logarithmic transformation into

$$\ln RH = \ln\alpha_0 + \alpha_1 \ln RA \qquad (2)$$

If we assume that Equation 2 represents the true relationship between total hours flown and total aircraft, Equation 1 can be rewritten as follows:

$$H = \alpha_0 A^{\alpha_1} \qquad (3)$$

where H represents total hours flown and A represents total aircraft.

According to general-production theory, labor and other resources as well as capital are included in the production function. These constitute the variable production inputs in the short term when the capital stock is assumed constant. In this case, labor (pilots) and other resources (fuel) are not known. The omission of these variables from the equation may result in biased estimates of aircraft elasticity; i.e., the normal probability distribution of α_1 may not have as its mean value the true population value of α_1. However, the effect of the specification bias must be weighed against the following:

1. The magnitude of any possible specification bias that results from omission of variables is unknown.
2. Any unknown bias that results from omission of variables implies the presence of multicolinearity in the regression coefficients and results in biased least-squares parameter estimates if the variable is included (5).
3. The objective in forecasting is to obtain estimates of minimum-variance parameters and not necessarily unbiased estimates of parameters.

In practice, it is the extent of the bias that is most important in model specification. If a high correlation exists between aircraft and pilots, the coefficient α_1 will be biased whether or not pilots are included in the equation. With reference to statement 3, a biased regression coefficient that results from an omitted variable has been proved to be more efficient (has smaller variance) than the regression-coefficient estimate when the omitted variable is included in the equation (2). For forecasting purposes, minimum-variance estimators are typically chosen over all other estimators in small samples because they are more precise than an unbiased estimator with high variance.

Another factor that potentially affects the elasticity coefficients is the type of use to which the region's aircraft fleet is applied.

In the course of model development, aircraft use was carefully examined and ultimately rejected by NCTCOG staff in the specification of the production function. County cross-tabulations of hours flown by primary use yielded sample sizes in many counties that were too small for accurate modeling and prediction. In addition, most aircraft types were found to be dominated by one or two primary uses; e.g., air-taxi and executive uses are predominant for jet aircraft and personal and instructional uses are predominant for single-engine piston-powered aircraft (6, p. 10). This suggests that, for a given aircraft type, changes in use are minimal over time. Differences in equipment, pilot-licensing requirements, and aircraft performance may present limitations to an aircraft's use. Therefore, it was assumed that changes in primary use for each aircraft type would not be significant during the forecast period.

Demand Equation for Average Hours Flown

The equation for regional average hours flown has been specified to include both price and income effects on GA activity. Specifically, its functional form appears as

$$H/A = \beta_0 - \beta_1 FI/TCPI + \beta_2 INC/HH \quad (4)$$

where

H = reported hours flown in the region,
A = aircraft that report,
FI = regional fuel cost index,
TCPI = regional transportation component of the CPI,
INC = regional income deflated by CPI for all goods, and
HH = number of households in the region.

The ratio FI/TCPI represents the real cost of aircraft operation and thus its expected sign is negative. Since GA operating costs are not included in TCPI, the ratio is a good measure of the relative impact of changes in aviation variable costs to other transportation costs. When FI increases relative to TCPI, average hours flown decreases, and when TCPI increases relative to FI, average hours flown increases. Thus, a substitutional relationship is hypothesized between general aviation and other modes of transportation. The expected sign of INC/HH is positive. The hypothesis here is that as income per unit (household) increases, average hours flown increases.

Average hours flown is used as the measure of GA activity demand only because total hours flown is not known. Treating reported hours flown as a sample drawn from the regional population of aircraft owners, the appropriate measure of demand is the average or mean value of the sample. The assumption involved here is that the sample average is equal to the population average for the region. At the county level, approximately 50-60 percent of aircraft owners reported hours flown, but the sample sizes at the county level for the less-populated counties are very small. Therefore, average hours flown is estimated at the regional level in order to use a larger sample size.

To measure the variable cost impact on the demand for hours flown, a regional fuel cost index was constructed for the period 1970-1977. The construction of this index served two purposes: (a) to convert the cost per gallon of fuel into a measure of the cost per hour of flying over time and (b) to account for changes in fuel consumption rates per flight hour due to variations in the number of aircraft types over time. Specifically, the Ratchford (7) index was used for this purpose, and it appears as follows:

$$FI_t = \left[\sum_{i=1}^{n} (H^i_{t-1} C^i_t G_t) \Big/ \sum_{i=1}^{n} (H^i_{t-1} C^i_{t-1} G_{t-1}) \right] FI_{t-1} \quad (5)$$

where

FI = fuel cost index,
C^i = fuel consumption rate (gal/h) for the ith aircraft make and model,
G = regional price per gallon of aviation gasoline or kerojet fuel,
H^i = hours flown by the ith aircraft make and model, and
t = time.

Demand Equation for Aircraft Investment

Unlike other regression models used to forecast the GA fleet, the aircraft demand equation included in this model specifies GA aircraft to be (among other things) a function of GA activity (8). In this context, the demand for aircraft can be considered as a derived demand for air travel. In other words, the desired stock of aircraft does not reflect the demand for aircraft per se but a demand for the flow of services that aircraft can provide over time, i.e., hours flown.

The theory employed in the aircraft investment equation is a variant of the general flexible accelerator model developed by Jorgenson and Siebert (9). In this model, a variable relationship is derived from the production function, which relates the increase in hours flown to the level of the regional aircraft stock. First, it is assumed that the aircraft stock will expand until the marginal product of aircraft equals the real user cost of aircraft. In the competitive case, the real marginal user cost of adding one more aircraft to the regional aviation fleet is equal to the market price of aircraft. The marginal product of aircraft derived from the production function is as follows:

$$dH/dA = \alpha_1 \alpha_0 A^{\alpha_1}/A = \alpha_1 (H/A) \quad (6)$$

Then, in user equilibrium, real marginal user cost is equal to marginal product:

$$\alpha_1 (H/A) = (C/P) \quad (7)$$

where C is the price of aircraft and P is the CPI. When we solve for A in Equation 7, the equilibrium capital stock is as follows:

$$A = \alpha_1 PH/C \quad (8)$$

As number of hours flown increases, the aircraft stock increases by the multiple of α_1, and as the cost of aircraft (C) increases, the aircraft stock decreases. In this form, the number of aircraft is related to its own activity as well as to its market value.

In general form, the complete specification of the aircraft demand equation (which includes other variables) is as follows:

$$A = A(H, C, P, R, Y, A_{t-1}) \tag{9}$$

The specific form of demand in Equation 8 requires aircraft to be a function of total hours flown (H). However, average hours flown was actually used in the estimation procedure since total hours flown is unknown. The additional variables are the prime interest rate (R), county-level deflated income (Y), and a stock-adjustment variable (A_{t-1}). The interest rate is hypothesized to have a negative impact on the growth in the aircraft stock because purchases of most durable assets are sensitive to changes in the price of credit. The number of aircraft should also be positively related to economic activity. An increase in income should result in an increase in aircraft demand. The stock-adjustment principle in A_{t-1} is included to determine the time rate of change in the aircraft stock as it adjusts to new levels of demand.

The stock-adjustment principle is intended to measure the response of the aviation industry to a change in aircraft demand. It is assumed that net additions to the aircraft stock reflect the desired demand for a minimum aircraft fleet size. When demand is stable, the stock-adjustment coefficient obtained when regressing aircraft in the present period against aircraft in the previous period is positive and lies between 0 and 1 (9). If this value is greater than 1, the demand for aircraft becomes explosive and increases at an exponential rate. When the stock-adjustment coefficient is less than 0, the demand for aircraft becomes oscillatory; i.e., it periodically fluctuates rather than increases at a steady rate. Initially, the lagged value of aircraft was included in the demand equation but was dropped when both explosive and oscillatory results were obtained.

It is often assumed in this type of investment growth model that the prices of capital goods increase at the same rate as does the general price level. When this occurs, the only fluctuation in the capital stock results from changes in the use of the stock. In terms of Equation 8, when P and C increase at the same rate, the real price of aircraft is constant and only variations in hours flown account for the variation in aircraft. This hypothesis was tested by leaving C and P in the equation. Generally, this ratio varied only slightly and was found to be insignificant. Therefore, aircraft price (book value) and CPI were excluded from the final model.

Estimates of the propensity to own aircraft can be read directly from the model by observing the coefficient of income. These propensities vary by county and by aircraft type. Since the expected sign of the propensity to own aircraft is positive, an increase in income will result in an increase in aircraft ownership and a decrease in income will result in a decrease in aircraft ownership. This represents the direct relationship between general aviation and the regional economy.

The final form of the investment demand equation used in the county-level forecasts is

$$A = \beta_0 + \beta_1 \alpha_1 (H/A) + \beta_2 Y + \beta_3 R \tag{10}$$

where α_1 is the aircraft elasticity coefficient. By substituting future values of H/A, Y, and R into Equation 10, the forecast values for aircraft will be obtained. Forecast values for average hours flown (H/A) are taken from the average hours flown in Equation 4. The specification of future values for Y and R constitutes the judgmental assumptions made.

FORECAST ASSUMPTIONS AND RESULTS

Alternative Energy Scenarios to Address Fuel Uncertainties

The long-run projections of fuel prices in the model were provided by the U.S. Department of Energy (DOE) 1977 annual report to the Congress (10). In general, the prices of aviation gasoline and kerojet fuel are expected to increase dramatically through 1982 as a result of phased decontrol of domestic crude-oil prices, the continued decline in domestic petroleum reserves, and higher prices for imported oil. Included in this report are alternative energy scenarios for prices of all fuels used in all sectors of the economy. The two extreme cases--high energy demand and low supply and low energy demand and high supply--were used in the GA forecast to derive regional alternative projections of aviation demand. The two scenarios refer to the overall energy supply and demand in the economy and should not be confused with the supply and demand for aviation fuel exclusively.

As mentioned earlier, the fuel cost indices measure changes in fuel prices and fuel consumption rates over time. An examination of changes in fuel efficiency from 1970 and 1977 indicated a slow but consistent trend toward better fuel efficiency in general aviation. Relatively greater changes were found in the turboprop and turbojet category due primarily to weight reductions in newer models (11). If the present trend continues, reductions in consumption (gal/h) may only amount to 5-10 percent for all of general aviation. Therefore, it is expected that improvements in fuel efficiency will not offset future price increases in fuel. [This has also been assumed by others (3, pp. 34-42).]

Throughout the forecast period, it must be assumed that fuel supplies will be available. Currently, there are no actual data on total fuel consumption by the aviation industry. However, there are three factors whose consideration lends some judgmental credibility to this assumption. First, on theoretical grounds, it is reasonable to expect some increase in fuel supplies as a result of crude-oil decontrol. Even in the high-demand and low-supply scenario development by DOE, an increase in fuel supplies is expected. Second, the current government fuel-allocation program favors the production of distillates, which includes kerojet fuel, over motor gasoline (12, p. 3). The allocation of distillates has been set equal to 1978 production levels as opposed to an allocation reduction for gasoline. Finally, aviation gasoline price controls were lifted in February 1979.

Economic Growth

Economic growth in the region is expected to slow through the remainder of 1979 and during all of 1980. Regional family income adjusted for inflation is projected to decline by 2.8 percent over this period and afterwards to increase at its historical rate of 1.5 percent. The growth of total inflation-adjusted personal income for most counties is expected to slow to an average of from zero to 2 percent in 1979 and to return to individual historical growth rates by 1981. The primary cause of the expected decline in economic growth is the anticipation of double-digit inflation through 1982. Inflation is expected to average 10.5 percent during the period 1979-1982, the primary cause being rising energy prices and previously built-up inflationary expectations.

Prime Interest Rate

It is assumed that the prime interest rate charged by banks will reach its peak in 1979 and the average for the year will be 11.25 percent. Any decline in the prime rate will be slow through 1982, primarily as a result of high inflation. The average for the 1980s should be about 8.5 percent as compared with the average 7.9 percent during the 1970s. Results from the model indicate that interest rates had a slight dampening effect on regional aircraft investment.

Forecast Results

The model structure described provides county-level estimates of aircraft types. For the 19-county North Central Texas area, this provided 152 separate sets of forecast results. For the purposes of reporting the model results, the 19-county forecasts have been summed to reflect a regional forecast.

Figures 2-9 provide a graphic presentation of the forecasts for the North Central Texas and Texoma state planning regions. Although the curves exhibit an overall upward trend over the forecast period, each reflects the anticipated negative influences of rapid increases in fuel prices due to deregulation in the early 1980s coupled with continued high inflation. For example, the total regional aircraft stock is expected to grow at an annual rate of 3.5-4.2 percent over the seven-year period from 1978 to 1984, compared with a 6.1 percent annual growth rate from 1971 to 1977.

Most important, the distinguishing feature of

Figure 2. Estimated number of single-engine piston aircraft.

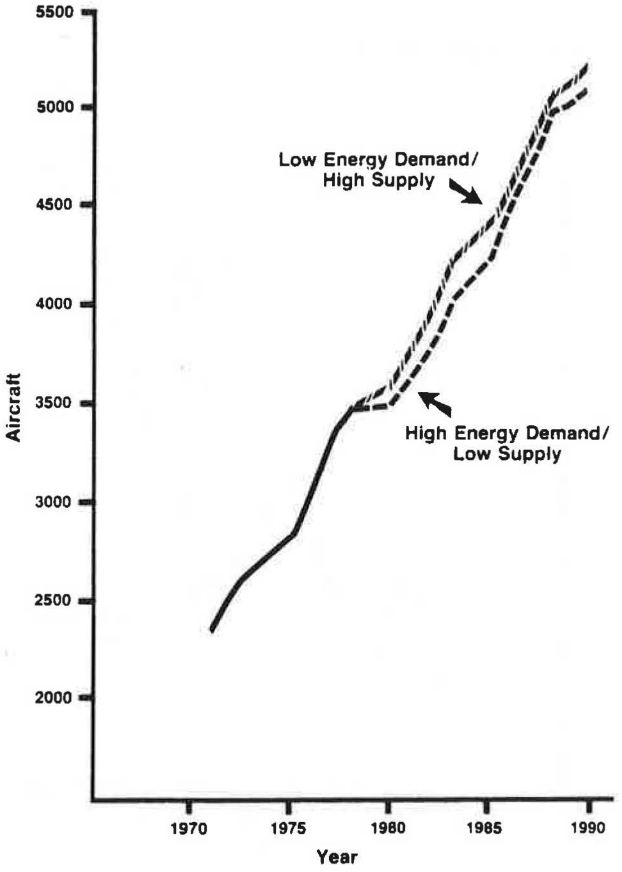


Figure 3. Estimated total hours flown by single-engine piston aircraft.

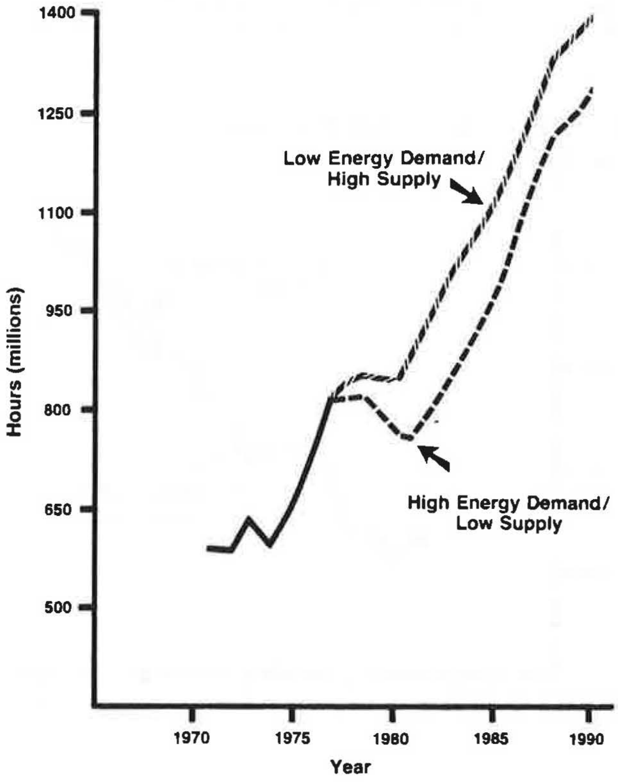
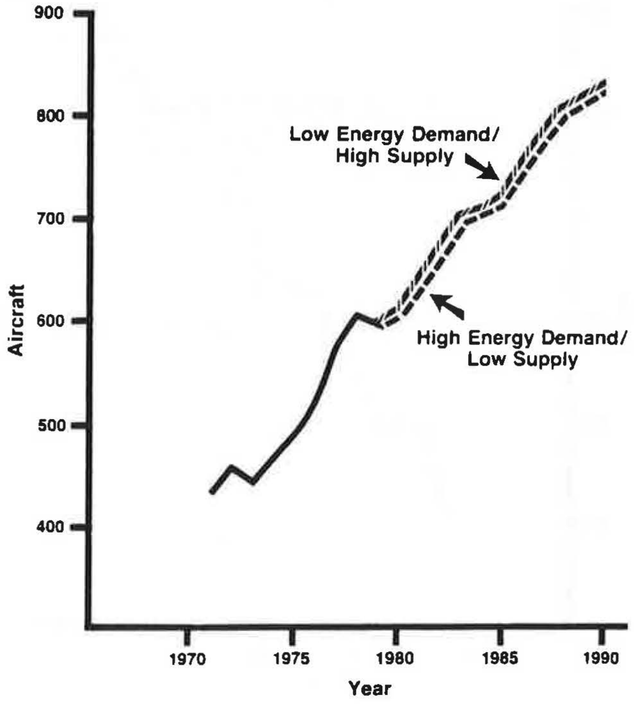


Figure 4. Estimated number of multiengine piston aircraft.



this model--the influence of hours flown on aircraft stock--is clearly reflected in each pair of figures. With the exception of turboprop aircraft, the curves for hours flown and for number of aircraft are identical in shape for each type of aircraft.

Empirical Results

The empirical results of the application of the GA forecasting model to Dallas County are provided in Equations 11-26, which are categorized by aircraft type. The t-statistic for each estimated coefficient is in brackets after each coefficient. Other statistics included are the adjusted coefficient of determination (R^2), the mean-square error (MSE), and the F-test for the original least-squares estimates. In addition, the error models for each equation appear in backward-shift operator notation, in which $B = e_{t-1}$ and $B^2 = e_{t-2}$, and the autoregressive parameters for the error process U_t are included.

Single-Engine Aircraft

$$\ln H = 2.930[1.565] + 1.351[4.694] \ln A \quad (11)$$

where R^2 is 0.72, MSE is 0.003, F is 22.03, and the error model is as follows:

$$(1 - 0.07B) \ln U_t = \ln e_t \quad (12)$$

$$A = -943.12[-2.975] + 1.180[1.008] (H/A) + 0.000\,18Y[8.229] - 1.29R[-0.252] \quad (13)$$

where R^2 is 0.97, MSE is 584.775, F is 42.12, and the error model is as follows:

$$(1 - 0.41B - 0.20B^2)U_t = e_t \quad (14)$$

Multiengine Aircraft

$$\ln H = -0.419[-0.106] + 2.160[2.854] \ln A \quad (15)$$

where R^2 is 0.43, MSE is 0.009, F is 8.14, and the error model is as follows:

$$(1 + 0.27B) \ln U_t = \ln e_t \quad (16)$$

$$A = -53.95[-0.975] + 0.228[2.162] (H/A) + 0.000\,002\,4Y[2.564] + 0.578R[0.253] \quad (17)$$

Figure 5. Estimated total hours flown by multiengine piston aircraft.

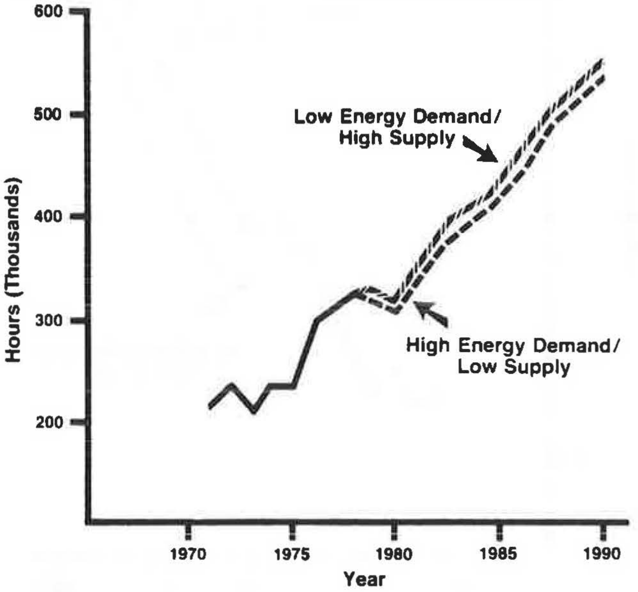


Figure 6. Estimated number of turboprop aircraft.

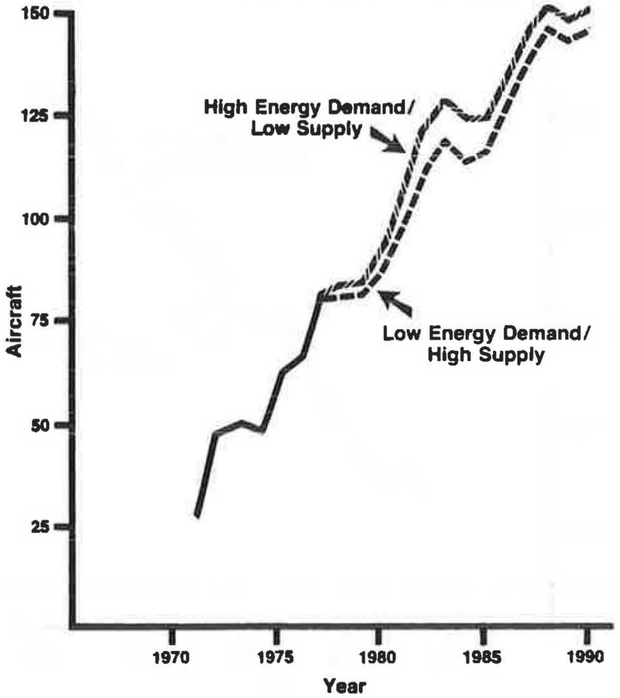
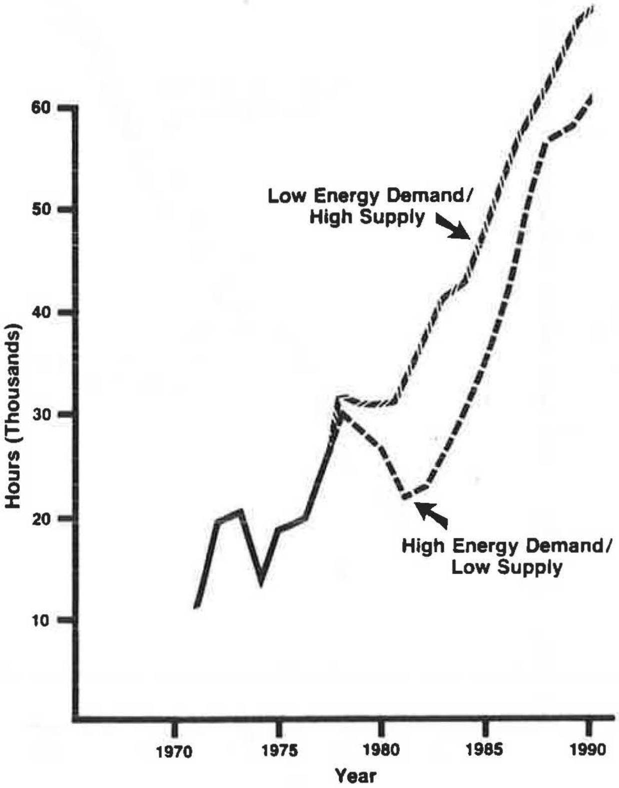


Figure 7. Estimated total hours flown by turboprop aircraft.



where R^2 is 0.93, MSE is 38.451, F is 20.34, and the error model is as follows:

$$(1 - 0.23B - 0.03B^2)U_t = e_t \quad (18)$$

Turboprop Aircraft

$$\ln H = 7.148[6.919] + 0.698[2.249] \ln A \quad (19)$$

where R^2 is 0.26, MSE is 0.042, F is 5.06, and the error model is as follows:

$$(1 + 0.11B) \ln U_t = \ln e_t \quad (20)$$

$$A = -105.06[-6.440] - 0.061[-2.908] (H/A) + 0.000\,016Y[11.477] + 0.845R[3.627] \quad (21)$$

where R^2 is 0.99, MSE is 2.574, F is 246.54, and the error model is as follows:

$$(1 - 1.13B + 0.57B^2)U_t = e_t \quad (22)$$

Turbojet Aircraft

$$\ln H = 4.445[1.965] + 1.570[2.370] \ln A \quad (23)$$

where R^2 is 0.29, MSE is 0.067, F is 5.62, and the error model is as follows:

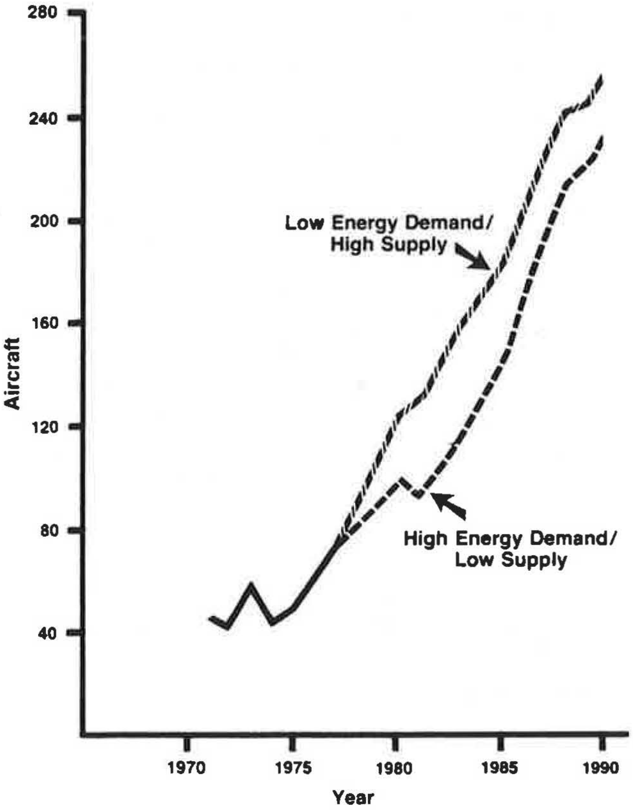
$$(1 - 0.25B) \ln U_t = \ln e_t \quad (24)$$

$$A = -239.92[-5.222] + 0.129[4.137](H/A) + 0.000\,025Y[6.155] - 3.701R[-3.054] \quad (25)$$

where R^2 is 0.97, MSE is 3.840, F is 45.79, and the error model is as follows:

$$(1 - 1.19B + 0.75B^2)U_t = e_t \quad (26)$$

Figure 8. Estimated number of turbojet aircraft.



Equations 27-34 give the results for regional average hours flown.

Single-Engine Aircraft

$$H/A = 211.761[4.118] - 0.861[-1.954]\ GF/TCPI + 0.0029[0.756]\ INC/HH \quad (27)$$

where R^2 is 0.20, MSE is 36.853, F is 2.17, and the error model is as follows:

$$(1 - 0.10B + 0.27B^2)U_t = e_t \quad (28)$$

Multiengine Aircraft

$$H/A = -59.852[-0.403] - 0.390[-0.283]\ GF/TCPI + 0.0203[1.552]\ INC/HH \quad (29)$$

where R^2 is 0.26, MSE is 241.74, F is 2.53, and the error model is as follows:

$$(1 - 0.07B - 0.13B^2)U_t = e_t \quad (30)$$

Turboprop Aircraft

$$H/A = 1244.01[7.484] - 8.002[-4.771]\ KF/TCPI \quad (31)$$

where R^2 is 0.69, MSE is 859.70, F is 22.76, and the error model is as follows:

$$(1 + 0.75B - 0.50B^2)U_t = e_t \quad (32)$$

Turbojet Aircraft

$$H/A = 1565.50[5.049] - 10.395[-3.344]\ KF/TCPI \quad (33)$$

where R^2 is 0.48, MSE is 4687.95, F is 11.18, and the error model is as follows:

$$(1 - 0.01B - 0.34B^2)U_t = e_t \quad (34)$$

Figure 9. Estimated total hours flown by turbojet aircraft.

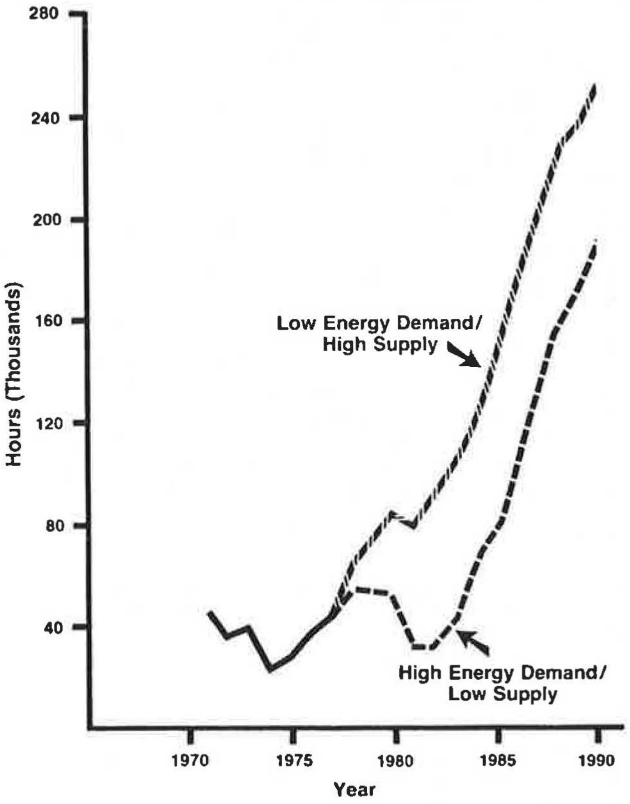


Table 1. Sensitivity analysis of dependent variables for selected years.

Dependent Variable	Percentage Change from Baseline Forecast Relative to a 1% Increase in Fuel Cost Index					Percentage Change from Baseline Forecast Relative to a 1% Increase in Deflated County Income				
	1979	1981	1983	1985	1990	1979	1981	1983	1985	1990
Aircraft										
Single-engine	−0.09	−0.08	−0.07	−0.07	−0.08	1.26	1.25	1.21	1.16	1.11
Multiengine	0.00	0.00	0.00	0.00	0.00	0.67	0.62	0.57	0.55	0.48
Turboprop	0.00	0.00	0.00	0.00	0.00	2.47	2.02	2.54	2.61	2.76
Turbojet	−1.94	−1.56	−1.27	−1.11	−0.78	3.88	3.13	2.53	2.22	1.96
Hours flown										
Single-engine	−0.14	−0.30	−0.32	−0.47	−0.74	1.20	1.20	1.18	1.14	1.09
Multiengine	−1.07	−0.13	−0.12	−0.11	−0.28	0.71	0.67	0.62	0.60	0.53
Turboprop	−10.66	−9.47	−9.16	−9.98	−10.68	1.90	1.58	2.77	2.82	2.83
Turbojet	−2.94	−6.81	−6.16	−5.12	−4.51	3.69	2.97	2.43	2.08	1.89

These equations indicate that the real cost of aviation fuel has a significant impact on average hours flown. However, the t-statistics for each equation are only asymptotically valid for the transformed equations. The low adjusted R^2-values indicate that the equations have little explanatory power, which may be due to (a) the small number of observations, (b) omission of other relevant data (such as fuel availability), or (c) measurement error in average hours flown due to incomplete reporting of hours flown. In any event, the objective of econometric forecasting is not to maximize the adjusted R^2 but to minimize the error variance (13). In addition, multicolinearity was found to be present in the turboprop and turbojet equations between the real cost of fuel and average deflated income per household. The latter variable was dropped from these equations since the purpose of the forecast was to examine the alternative energy scenarios projected by DOE.

The strength of the GA forecasting model lies in the individual county estimates of the demand equations for hours-flown production functions and for aircraft investment. Comparisons of the mean-square errors of the county versus the regional demand equations for aircraft investment found the error of the county estimates to be smaller than that of the regional estimates for each type of aircraft.

In order to measure the responsiveness of the model to the forecast assumptions, a sensitivity analysis was undertaken to assess (a) the relative impacts of the exogenous variables and (b) the effect of a possible deviation in the forecast assumptions. The percentages reported in Table 1 are interpreted as elasticity coefficients. They are derived by allowing the exogenous variable of interest (the fuel cost index or personal deflated income) to increase by 1 percent above the baseline forecast while all other exogenous variables in the model are held constant. This reveals the responsiveness of the endogenous variables to the specified exogenous variables. A table for the prime interest rate is not included because a 1 percent change in this variable was discovered to have no effect on any dependent variable.

CONCLUSION

The forecasting model defined in this paper allowed the development of a quantitative means of assessing the impact of major economic forces that influence GA growth. Perhaps more important, experience gained through development of the model led to greater recognition by all parties of the role of general aviation within the North Central Texas region. From an analytical viewpoint, it is clear that there is considerable room for improvement and refinement in the model's structure and statistical strength. However, while we recognize that the research conducted to date on the development of county-based GA forecast models has been limited and, further, that the funding made available through FAA for the Continuous Airport System Planning Process program has been limited, the model is nevertheless offered as a useful first step in a continuing effort to strengthen the analytical basis for conducting regional airport system planning.

REFERENCES

1. Proc., Third Annual FAA Forecast Conference (Washington, DC, June 1978). Federal Aviation Administration, U.S. Department of Transportation, 1978, pp. 29-30.
2. R.S. Pindyek and P.L. Rubinfeld. Econometric Models and Economic Forecasts. McGraw-Hill, New York, 1976.
3. FAA Aviation Forecasts: Fiscal Years 1979-1990. Federal Aviation Administration, U.S. Department of Transportation, Sept. 1978.
4. A.R. Gallant and J.J. Goebel. Nonlinear Regression with Autoregressive Errors. Journal of the American Statistical Association, Vol. 71, 1976, pp. 961-967.
5. P. Roa and R.L. Miller. Applied Econometrics. Wadsworth Publishing Co., Belmont, CA, 1971, pp. 341-406.
6. S.G. Vahovich. General Aviation Aircraft: Owner and Utilization Characteristics. Federal Aviation Administration, U.S. Department of Transportation, Nov. 1976, Chapter 6, Section 1.
7. B.T. Ratchford. A Model for Estimating the Demand for General Aviation. Transportation Research, Vol. 8, Aug. 1974, pp. 193-203.
8. T.F. Henry, J.C. Tom, and R.A. Jeter. Federal Aviation Forecasts: Fiscal Years 1978-1989. Federal Aviation Administration, U.S. Department of Transportation, Sept. 1977, pp. 60-63.
9. D. Jorgenson and J. Siebert. Investment Behavior in U.S. Manufacturing, 1947-1960. Econometrica, Vol. 35, 1967, pp. 169-220.
10. Annual Report to Congress: Projections of Energy Supply and Demand and Their Impacts. U.S. Department of Energy, DOE/EIA-003612, Vol. 2, 1977, pp. 107-115.
11. Aircraft Blue Book. Aircraft Dealers Service Association, Aurora, CO, 1978.
12. Weekly Announcements. Energy Information Administration, U.S. Department of Energy, Aug. 1979.
13. J.S. Armstrong. Long-Range Forecasting: From Crystal Ball to Computer. Wiley, New York, 1978.

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