five times faster than the ratio for the NETSIM model (4).

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The development of a model such as that described in this paper represents the contributions of many people. In particular, we wish to extend our thanks to Guido Radelat and George Tiller of the Traffic Systems Division of the Federal Highway Administration; Barbara Andrews and Mark Yedlin of KLD Associates, Inc.; William McShane of the Polytechnic Institute of New York; and Fred Wagner of Wagner/McGee Associates. This work was performed under contract with the U.S. Department of Transportation.

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# Service Rates of Mixed Traffic on the Far Left Lane of an Approach

WILLIAM R. McSHANE AND EDWARD B. LIEBERMAN

**The effect of left-turning vehicles on approach capacity has been observed and studied, but a complete model has been lacking. As part of the development of the TRAFLO simulation model for the Federal Highway Administration, the capacity of a signalized intersection approach was modeled. The most complex component of this capacity model, that for left-turn lanes, is discussed. Three basic types of intervals for the vehicle discharge process are identified, and the probabilities of each are found. Expressions for expected vehicle discharge per cycle and for saturation flow rates per hour of green are reported. Reasonable results have been obtained.**

The problem of a left-turn lane that serves both turning and through vehicles has been addressed in various ways, as has the problem of lanes that serve turners only (1-3). The 1965 Highway Capacity Manual (4) provides some empiric assessment of left-turn capacity without special phasing; other treatments include equivalencies for left turners facing opposing traffic (5). But none of the existing treatments provides the level of information and detail necessary for inclusion in the TRAFLO model currently being developed for the Federal Highway Administration (FHWA).

The TRAFLO model includes a determination of service rates on approaches to a signalized intersection. (It also includes appropriate treatments of unsignalized intersections, but these are not treated in this presentation.) The discharge service rates of vehicles on the individual lanes are strongly linked to the mechanism by which the through vehicles distribute themselves among the lanes in a self-optimizing fashion. Computationally, the service rates of the mixed traffic lanes (i.e., those that contain turning and through vehicles) are functions of the percentage of turning vehicles in the lanes. This interdependence of service rates and the lateral distribution of vehicles on an approach creates a feedback that can be addressed by the following iterative computations:

1. Assuming that all vehicles in the outermost lanes turn, the service rates of these lanes can be computed from appropriate models to begin the procedure.
2. Based on these service rates, the principle of drivers optimizing their individual travel time is applied in order to compute the implied percentage of turns in each lane that serves mixed (i.e., through and turning) traffic.
3. If the lane percentages differ from the assumed values, the initial computation is repeated but the latest lane percentage is used.

The overall procedure implied in these steps is discussed and justified in a paper by Lieberman elsewhere in this Record. It has been shown that convergence is obtained and that few iterations are required.

This paper is restricted to the model of the lane that contains through and left-turning vehicles, as required in the statement "computed from appropriate models" in step 1 above. The complexity of the process makes a closed-form solution infeasible, but the formulation obtained is both complete and tractable. It is certainly well suited to the original purpose--inclusion in the TRAFLO model--and could potentially be a component in a general procedure for estimating intersection capacity.

LEFT-TURN-LANE MODEL

The far left lane at an intersection generally contains mixed traffic, of which a fraction $\lambda$ turns left and a fraction $(1 - \lambda)$ goes through. The activity during a particular green phase is handled by three submodels:

1. An A model, which handles the discharge of the subject lane while an opposing queue is discharging or a coherent oncoming platoon is discharging;
2. A B model, which handles the discharge of vehicles on the subject lane during unstructured opposing arrivals, usually after the A model; and
3. A C model, which handles the discharge from the subject lane by a number of left-turn laggers at the end of the green phase and a "jumper" at the beginning of the phase.

Each vehicle in the subject lane is assigned a probability $\lambda$ of being a left turner, except as explicitly noted. Gaps in the opposing flow are assumed to be exponential during the B model. This assumption is reasonable and has two important advantages: (a) If each gap in each opposing lane is exponential, it can be shown by order statistics that the resultant gaps are also exponential and (b) in the B model, truncated gaps and conditioned gaps play an important role. Note that the distribution of the conditioned gaps of an exponentially distributed sequence of gaps is also exponential.

For the purposes of the work presented in this paper, we assume that discharges from both the subject lane and the opposing queue (if any) are deterministic at specified rates. An endless supply of vehicles is assumed in the subject lane, for it is the capacity value that is of interest in this work. Clearly, if the necessary demand is lacking, the actual output can be less than the levels that result from the computations.

It must be recognized that the model presented here was developed to be both logical and complete and to yield known sensitivities that are not represented in previous formulations and at the same time to remain tractable and computationally feasible. Some refinements are possible but were judged unnecessary for the precision with which the model would be used. For instance, duration for the A model is actually a random variable, but a simple deterministic computation is done to estimate its duration. Likewise, a weighted average headway is used in the B model to simplify the formulation and the computations.

## A Model

For simplicity, completely random (i.e., exponentially distributed) opposing arrivals are assumed in the A model. As Figure 1 shows, a queue forms on the opposing approach. The queue discharges during the "A period", effectively blocking any discharge of left turners on the subject approach during this time.

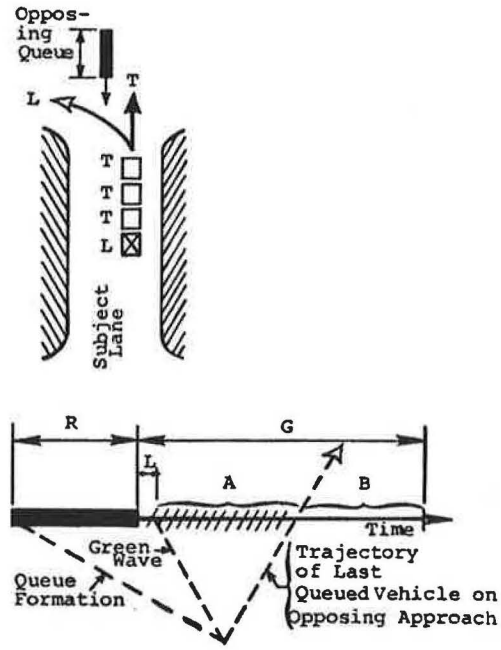
The A duration is given by

$$A = Q_{opp}(R + L)/(S - Q_{opp}) \quad (1)$$

where

$Q_{opp}$ = total opposing flow (vehicles/h),
R = red-phase duration (s),
L = total start-up lost time for discharging queue (s), and
S = total opposing saturation flow (vehicles/h).

Figure 1. The A model.




How many vehicles can be discharged from the subject lane during the A duration? Only leading through vehicles can discharge, since the first left turner must wait until the opposing queue is discharged. The maximum number of through vehicles that can discharge (M*) is

$$M^* \doteq (A/t_0) \quad (2)$$

where $t_0$ is the mean discharge headway for a through vehicle in the lane of interest (s/vehicle).

Consider the situation of X through vehicles preceding the first turner. The probability of there being x through vehicles preceding the first left turner is

$$P_r(X = x) = \begin{cases} (1 - \lambda)^{x\lambda} & x < M^* \\ (1 - \lambda)^{M^*} & x = M^* \end{cases} \quad (3)$$

and the expected value of x is

$$N_A = E[X] = \sum_{x=0}^{x=M^*} x \cdot P_r(X = x) \quad (4)$$

or

$$N_A = [(1 - \lambda)/\lambda] - [(1 - \lambda)^{M^*+1}/\lambda] \quad (5)$$

For small $\lambda$, the asymptotic form $N_A \simeq M^*[1 - (M^* + 1)\lambda/2]$ is used to avoid numeric problems with the division by $\lambda$ as it approaches zero.

It is necessary to compute the gap-distribution parameter $\alpha$ for the B model. This value is based on the total opposing flow minus that portion that is discharged during the A duration.

Let $N_B$ equal the number of vehicles discharged from the opposing approach during the B duration on one cycle:

$$N_B = (Q_{opp}C/3600) \cdot \{1 - [(R + A)/C]\} = Q_{opp}B/3600 \quad (6)$$

where C is the cycle length in seconds. The rate of flow during the B duration is therefore

$$N_B/B = Q_{opp}/3600 \quad (7)$$

and, consequently, $\alpha = Q_{opp}/3600$ vehicles/s.

B Model

The B model, which is relatively complex, is presented in two parts: the "start" operation and the "normal" operation.

Start Operation

At the beginning of the B interval, there is a probability $P_R$ that the first vehicle is a left turner [this is the "left turner first" (LTF) situation]:

$$P_R = \sum_{x=1}^{x=M^*+1} (1-\lambda)^{x-1}\lambda = 1-(1-\lambda)^{M^*+1} \quad (8)$$

This left turner must wait through N gaps in the opposing flow that are too small, followed by a gap that is at least the minimum size. The probability that a gap is too small is

$$P_1 = (1-e^{-\alpha T}) \quad (9)$$

where $\alpha$ is the parameter for the opposing-flow gap distribution (1/s) and T is the gap required by the first left turner in a given gap (s). The expected value of the waiting time through the succession of such gaps is

$$E(W) = \sum_{i=0}^{\infty} i\mu_1 P_1^{\,i}(1-P_1) = \mu_1 P_1/(1-P_1) \quad (10)$$

where $\mu_1$ is the mean of the rejected gap, as shown in Figure 2a:

$$\mu_1 = \int_0^T t f_1(t)\,dt = (1/\alpha) - T(1-P_1/P_1) \quad (11)$$

After this, there is a gap that is big enough. The mean of this acceptable gap, denoted as $\mu_2$, is given by

$$\mu_2 = [T + (1/\alpha)] \quad (12)$$

which reflects the properties of the exponential distribution (see Figure 2b).

This same gap may also service other vehicles. If the following vehicle is another turner, it takes $H_o$ s to discharge. If it is a through vehicle, it takes $t_o$ s to discharge. To make the modeling tractable, it is assumed that both take the same time H, a weighted average:

$$H = \lambda H_0 + (1-\lambda)t_0 \quad (13)$$

The number of vehicles that can be accommodated in this gap is shown in Figure 3. It can be shown that the expected numbers of left-turn and through vehicles in this gap are as follows:

$$E(LT) = [1-(1-\lambda)U]/(1-U) \quad (14)$$

$$E(THRU) = (1-\lambda)U/(1-U) \quad (15)$$

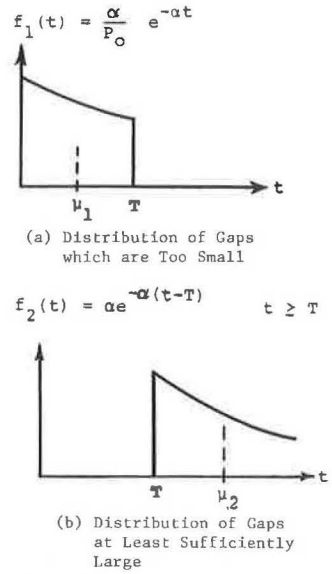
$$E(TOTAL) = 1/(1-U) \quad (16)$$

where $U = e^{-\alpha H}$. Note that these are the expected number of vehicles to be discharged in the time D:

$$D = E(W) + \mu_2 \quad (17)$$

defined by the LTF situation that begins the B interval.

Figure 2. Two types of gaps in the start of the B interval.




(a) Distribution of Gaps which are Too Small

(b) Distribution of Gaps at Least Sufficiently Large

Figure 3. Number of vehicles accommodated in a gap.

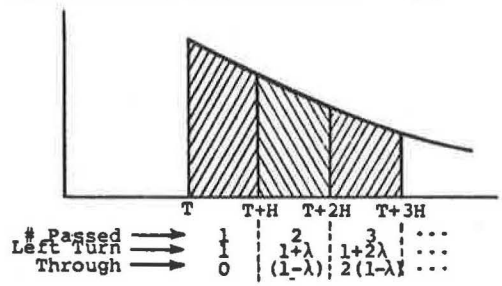
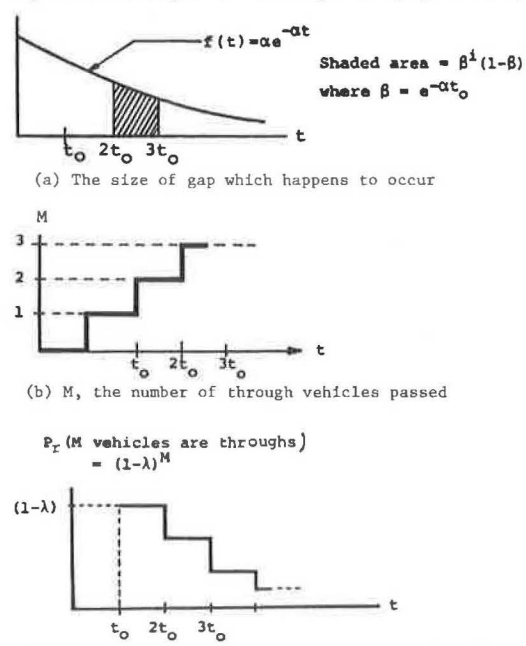



Figure 4. All through vehicles discharged during a gap in the opposing stream.




(a) The size of gap which happens to occur

(b) M, the number of through vehicles passed

(c) The probability of M consecutive through vehicles

A special case arises when the B duration is less than the acceptable gap (i.e., D > B).

### Normal Operation

After the time described above, other gaps occur. These can be referred to as normal gaps (i.e., not special, as the start situation was). Two distinct possibilities occur: (a) All vehicles processed are through vehicles, or (b) only the first M vehicles are through vehicles. These two possibilities will be considered here individually.

In the first case, note that the opposing gap is "used" by through vehicles only in the sense that they occupy the same time and preclude its use by others (i.e., left turners). Let $P_2$ be the probability of the gap "passing" only through vehicles and $N_2$ be the expected number of vehicles passed (given that all are through vehicles). It should be noted that $(1 - \lambda)^M\lambda$ is the probability string of exactly M through vehicles followed by cutoff and $\beta$, the probability that a gap will "see" only through vehicles, is equal to $e^{-\alpha t_o}$.

Figure 4 summarizes the sequence for an arbitrarily selected gap size: If the gap falls between $2t_o$ and $3t_o$, it can pass M = 2 through vehicles; the probability that there are two consecutive through vehicles is $(1 - \lambda)^2$. Summing over all possibilities, $P_2$ = the probability of M consecutive through vehicles times the probability that the gap is smaller. Thus,

Figure 5. Summary of activities in each phase of the model.

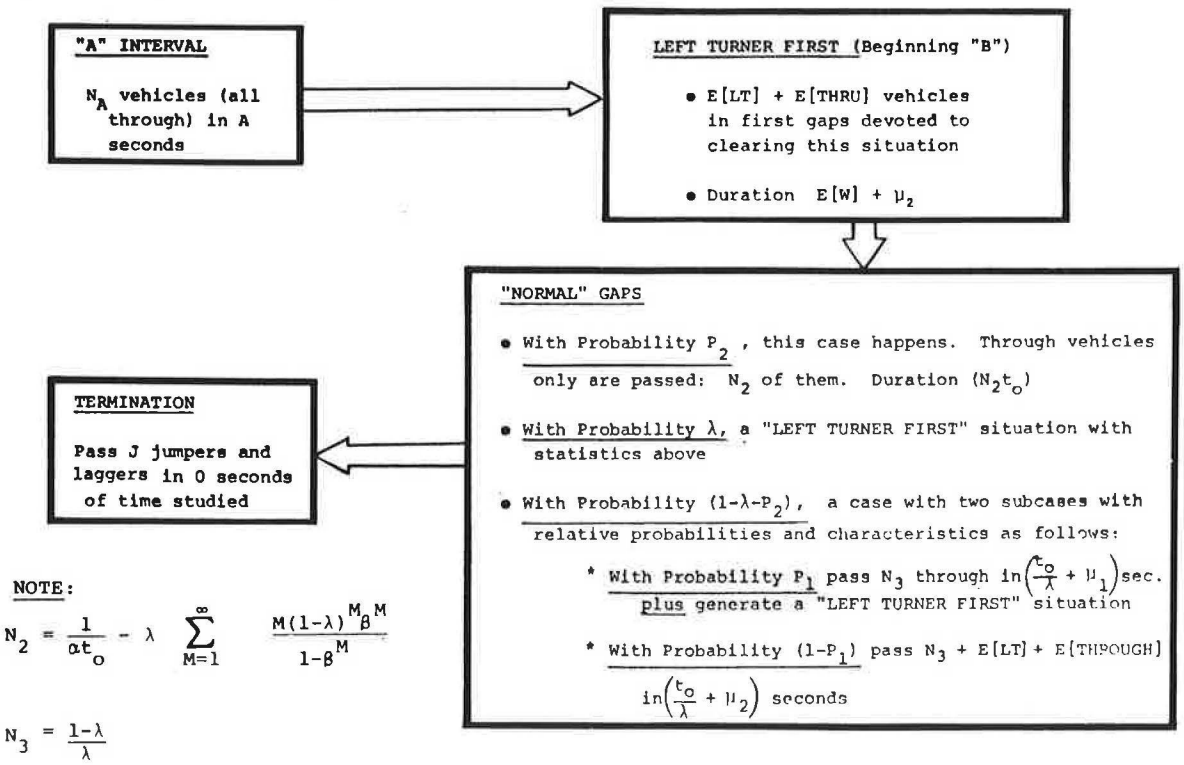
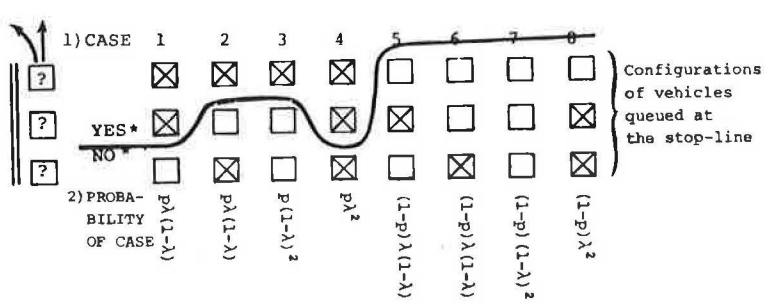



Figure 6. Vehicles at the end of the phase: the C interval.




☒ Denotes a left-turn vehicle

☐ Denotes a "through" vehicle

YES* indicates those who are judged to "push" through intersection

NO* indicates those who will not "push" through intersection

$$P_2 = \sum_{M=1}^{\infty} (1-\lambda)^{M\lambda}(1-\beta^M) = (1-\lambda) - \{\lambda\beta(1-\lambda)/[1-\beta(1-\lambda)]\} \quad (18)$$

One can similarly develop an expression for $N_2$.

For the second possibility--that only the first M vehicles are through vehicles--there are three subcases: (a) M = 0, (b) M > 0 and the first left turner cannot be passed, or (c) M > 0 and the first left turner can be passed. In the interests of brevity, a detailed development is not presented here.

Figure 5 shows a summary of these and other subcases. The relative frequency of the different subcases, and how one begets the other, are summarized later in this paper. Clearly, if M = 0, the first vehicle is a turner and the situation is identical to all LTF situations, as in the case of the B model.

C Model

Let there be J left-turn "jumpers" or "laggers" per cycle. One must find an expression for J.

At the end of the cycle, several configurations of left turners may "push" themselves through, as shown in Figure 6. It is plausible to assume that no more than 2 left-turn laggers/cycle will push across the stop line. From Figure 6, it can be estimated that there are $(p + p\lambda)$ expected laggers at the end of the cycle, where p is the probability that the vehicle or vehicles indicated in Figure 6 are left turners.

Note that the probability p is not the typical probability of a left turner, $\lambda$. It depends on the fact that the configurations in Figure 6 are created by the B interval activity. The value of p is discussed in the summary of model and time allocation.

The "NO" in Figure 6 also gives insight into the probability of a left turner leading the A interval: $p\lambda^2$. Based on observations made in Washington, D.C., the probability of that vehicle jumping is taken as 1/3. Thus, a reasonable estimate of the number of jumpers and laggers per cycle is

$$J = p[1 + \lambda + (\lambda^2/3)] \quad (19)$$

SUMMARY OF MODEL AND TIME ALLOCATION

Figure 5 summarizes the activity in each of the key intervals of the phase. However, this does not provide an estimate of how many events happen in the phase, from which the productivity can be computed.

Assume that there are $\eta$ events in the B interval, not including the initial LTF situation. Figure 7 shows the sequence in which these events occur.

There are $[1 + \lambda\eta + (1 - \lambda - P_2)P_1\eta]$ LTF situations in a green phase, which leaves the following time (in seconds) for other situations:

$$B - [P_R + \lambda\eta + (1-\lambda-P_2)P_1\eta][E(W) + \mu_2] \quad (20)$$

There are $(1 - \lambda)\eta$ other situations from the illustration above, each of which averages f seconds. From Figure 5, f can be computed as follows:

$$f = \{P_2(N_2 t_0) + (1-\lambda-P_2)P_1[(t_0/\lambda) + \mu_1] + (1-\lambda-P_2)(1-P_1)[(t_0/\lambda) + \mu_2]\} / [P_2 + (1-\lambda-P_2)P_1 + (1-\lambda-P_2)(1-P_1)] \quad (21)$$

Further simplification is possible, but not necessary, given the anticipated numeric solution. Thus, the situations use $(1 - \lambda)f\eta$ seconds.

The following equality,

$$(1-\lambda)f\eta = B - [P_R + \lambda N + (1-\lambda-P_2)P_1\eta][E(W) + \mu_2] \quad (22)$$

leads to

$$\eta = \{B - [E(W) + \mu_2]P_R\} / \{(1-\lambda)[\lambda + (1-\lambda-P_2)P_1][E(W) + \mu_2]\} \quad (23)$$

Knowing $\eta$, one can obtain expressions for the expected number of through vehicles (THRU) and left-turn vehicles (LEFT) in a phase:

$$THRU = N_A + P_2N_2\eta + E(THRU)[P_R + (1-P_2)\eta] + N_3\eta(1-\lambda-P_2) \quad (24)$$

$$LEFT = J + E(LT)[P_R + (1-P_2)\eta] \quad (25)$$

Given these, simple multiplication by (3600/green time) will yield the components of the saturation flow for the lane, in vehicles per hour of green.

Figure 5 and the separation of cases illustrated

Figure 7. Description of $\eta$ events.

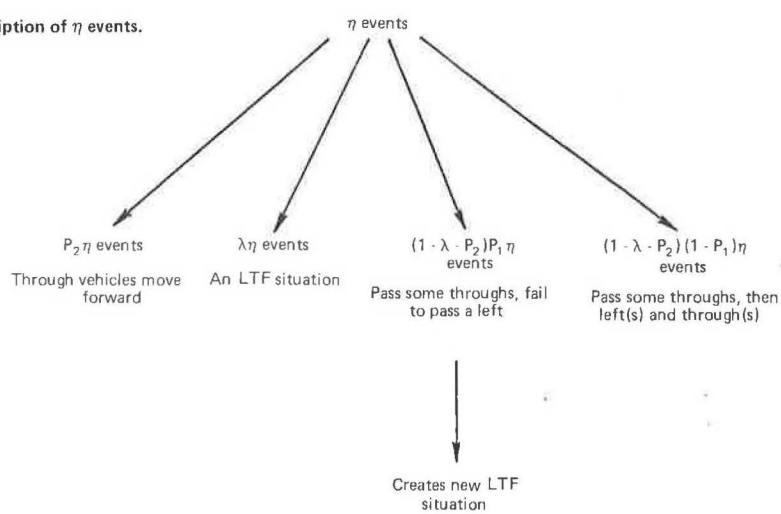
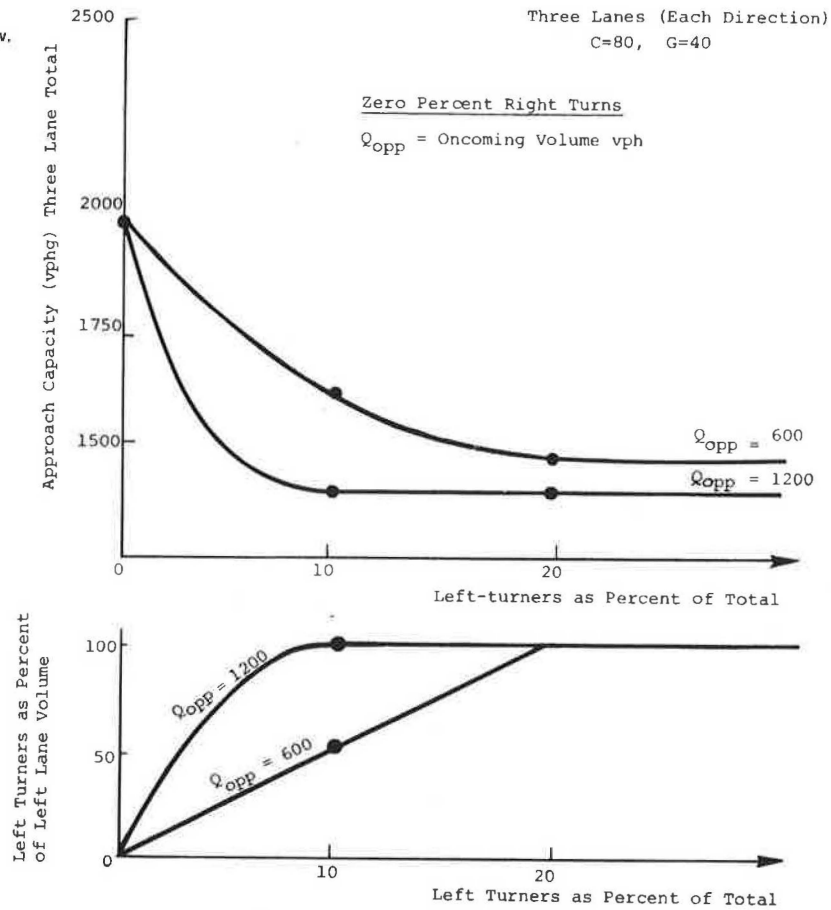


Figure 8. Approach capacity and lane distribution as a function of percentage of left turners and opposing flow.




for the η events can also be used as the basis for estimating the p of the previous section:

$$p = P_R + \lambda\eta + (1 - \lambda - P_2)P_1\eta/P_R + \eta \quad (26)$$

USE AND EFFECTIVENESS OF THE MODEL

The detailed model formulated here is well suited to a computational model such as TRAFLO. It exposes the basic elements of left-lane use, identifies the probability of each case, and is extremely useful in considering sensitivities and interactions. For saturation conditions in the far left lane, our computations have resulted in flow rates that are within 5 percent of values observed in a data base collected by other researchers for FHWA.

Figure 8 shows the effect of the subject model. A six-lane arterial is considered, with a cycle length of 80 s and a green phase of 40 s facing the approach. The fairly rapid fall-off of the discharge capacity with increasing turns replicates patterns observed in actual data better than alternative models. The sensitivity to the opposite-direction flow is interesting. Of particular interest is the fact that through vehicles quickly avoid the far left lane as the opposing flow or the number of turners increases.

For simplicity, Figure 8 does not show the interaction effect of right turners on the far left lane and vice versa. The complete TRAFLO approach model does have a right-turn component model.

ACKNOWLEDGMENT

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The right-turn model cited in the text was created by Charles Berger of Sperry Systems Management, who also participated in the TRAFLO model work. Guido Radelat is the FHWA project manager.

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