Generalized Procedure for Estimating Single- and Two-Regime Traffic-Flow Models

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Macroscopic traffic-flow models play an important role in the planning, design, and operation of transportation facilities. Evaluation of these models is often required to select the appropriate model that best represents prevailing operating conditions. For this purpose, a technique is needed that will enable the analyst to easily and quickly estimate model parameters. The technique should be easy to understand and use and inexpensive to apply and should generate results that reasonably represent actual traffic behavior. The development of such a technique is described. The proposed estimation procedure is based principally on the theoretical relations between model parameters and traffic-flow criteria. Such relations were developed for both single- and two-regime approaches. To facilitate use of the procedure, generalized nomographs were developed to model the complexity of the theoretical aspects involved. These nomographs are capable of directly providing the values of model parameters that satisfy specified evaluation criteria. This procedure significantly reduces the need for regression analysis in estimating model parameters and thus appears to be of particular use in a wide range of transportation applications.

Considerable research has been undertaken to model the interrelationships among traffic-flow variables, and researchers have developed several models that describe the behavior of traffic flow on highways. In general, traffic-flow models can be classified into two major classes: microscopic and macroscopic. Microscopic models consider the spacing and speed of individual vehicles as model elements. Macroscopic models, on the other hand, describe the operations of traffic flow in terms of the speed, flow, and density of the traffic stream.

The macroscopic models are generally adequate for most practical purposes and have been widely used in the planning, design, and operation of transportation facilities. Before any particular macroscopic model can be used, however, the analyst should estimate model parameters that best represent prevailing traffic characteristics.

There is a need for a technique that will enable the analyst to directly estimate model parameters. It is obviously desirable that such a technique exhibit several important features, including efficiency, flexibility, accuracy, and generality: It should be easy to understand and use and be inexpensive to apply, it should allow flexible treatment of the various variables and parameters involved and generate results that reasonably represent actual traffic behavior, and, most important, it should be general in nature and allow a wide range of transportation applications. With these features in mind, a generalized procedure for estimating single- and two-regime models has been developed.

This paper presents a background of microscopic and macroscopic modeling theories and briefly discusses the concept of the proposed procedure. A detailed description of the evaluation procedure for single-regime models is given, and the evaluation procedure for the two-regime models is described.

BACKGROUND

The general macroscopic theory of traffic flow is based principally on the microscopic (car-following) theory. These two classes of theories are described briefly below.

Microscopic Theory

The microscopic description of vehicular traffic flow was first formulated by Reuschel (1) and Pipes (2). They formulated the phenomena of pairs of vehicles following each other:

$$X_n - X_{n+1} = L + S(\dot{X}_{n+1}) \quad (1)$$

In this formulation, it is assumed that driver (n + 1) maintains a separation distance from driver n proportional to the speed of his or her vehicle ($\dot{X}_{n+1}$) plus a distance L. The factor L is the distance headway at standstill ($\dot{X}_n = \dot{X}_{n+L} = 0$). The constant S has the dimension of time, and the differentiation of Equation 1 gives

$$\ddot{X}_{n+1} = (1/S)(\dot{X}_n - \dot{X}_{n+1}) \quad (2)$$

where $\ddot{X}_{n+1}$ = the acceleration (or deceleration) rate.

This differential equation is generally referred to as the basic equation of the car-following theory. This basic stimulus-response relation was investigated further by Chandler, Herman, and Montroll (3), who formulated a linear mathematical model that took the following form:

$$\ddot{X}_{n+1}(t+T) = \lambda\,[\dot{X}_n(t) - \dot{X}_{n+1}(t)] \quad (3)$$

where T = the time lag of response to the stimulus and λ = the sensitivity factor.

This formulation was refined by Gazis and others (4,5), and a more general expression of the sensitivity factor was proposed:

$$\lambda = \alpha\left\{\dot{X}^m_{n+1}(t+T)/[X_n(t) - X_{n+1}(t)]^{\ell}\right\} \quad (4)$$

where α is the constant of proportionality.

The general expression for the microscopic theory thus becomes

$$\ddot{X}_{n+1}(t+T) = \alpha\left\{\dot{X}^m_{n+1}(t+T)/[X_n(t) - X_{n+1}(t)]^{\ell}\right\}[\dot{X}_n(t) - \dot{X}_{n+1}(t) \quad (5)$$

Macroscopic Theory

Gazis, Herman, and Rothery (5) have shown that, by integrating the generalized microscropic equation (Equation 5), the following expression is obtained:

$$f_m(u) = c' + cf_{\ell}(s) \quad (6)$$

where

u = steady-state speed of the traffic stream,
s = constant average spacing, and
c and c' = some appropriate constants consistent with physical restrictions.

The integration constant c' is related to free-flow speed u_f or jam spacing s_j, depending on the values of ℓ and m. The jam spacing s_j can be transformed to jam density k_j by $s_j = 1/k_j$.

By using this general solution of Gazis and others, May and Keller (6) developed a matrix of the steady-state flow equations for different ℓ and m values. This matrix was modified by Ceder (7) and has been further refined here to properly establish some regions of the matrix. The final version is shown in Figure 1.

The matrix shows the speed-density relations for different combinations of ℓ and m parameters in four "regions". In region 1 ($\ell < 1$ and $m > 1$), the boundary conditions are not satisfied. Models in region 2 ($\ell < 1$ and $m < 1$) have no intercept with the speed axis, $u_f \to \infty$. Models in region 3 ($\ell > 1$ and $m > 1$) have no intercept with the density axis, $k_j \to \infty$. Region 4 ($\ell > 1$ and $m < 1$) contains models that have intercepts with both axes.

It should be noted that this paper is concerned only with the three specially delineated parts of the matrix shown in Figure 1. These include region 4 and the two portions of regions 2 and 3 that correspond to $\ell = 1$ and $m = 1$, respectively. For consistency and ease of reference, models in region 4 will be referred to throughout as single-regime models, those in region 2 as congested-flow models, and those in region 3 as non-congested-flow models.

Single- and Two-Regime Approaches

There are generally two approaches to representing traffic-flow relations: single-regime and two-regime. In the single-regime approach, the entire range of operations is represented by a single model (normally from region 4) as shown in Figure 2(a), but one could represent the regimes of noncongested and congested flow by separate models, as shown in Figure 2(b). This two-regime representation, first proposed by Edie (8), provides a theory that accounts for the discontinuity often observed in traffic-flow data. As defined in this paper, in the two-regime approach the non-congested-flow regime can be represented by a model from region 3 or region 4, and the congested-flow regime can be represented by a model from region 2 or region 4.

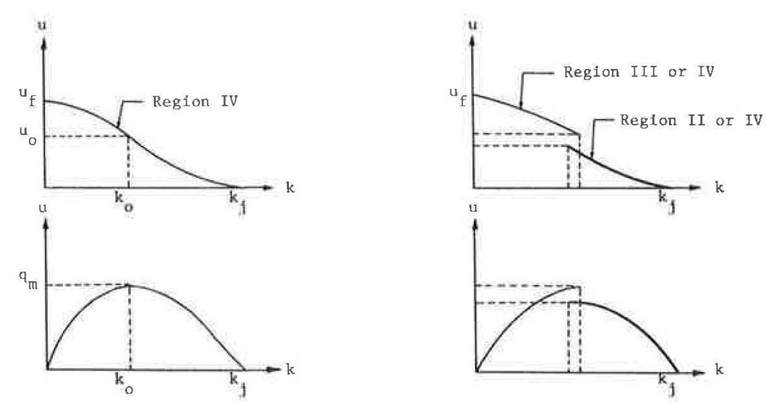
CONCEPT OF THE PROPOSED PROCEDURE

An illustration of the proposed estimation concept is shown in Figure 3. The procedure is based principally on the theoretical relations between traffic-flow criteria and model parameters. The traffic-flow criteria include free-flow speed (u_f), optimum speed (u_o), jam density (k_j),

Figure 1. Matrix of steady-state flow equations for different values of ℓ and m.

ℓ \ m	m < 1	m = 1	m > 1
$\ell < 1$	$u^{1-m} = ck^{\ell-1} - ck_j^{\ell-1}$ where $c = \alpha \frac{1-m}{1-\ell}$ and $\ell > m$	$\ln u = \alpha \frac{k^{\ell-1}}{1-\ell} + d_4$	$u^{1-m} = \alpha \frac{1-m}{1-\ell} k^{\ell-1} + d_3$
		(boundary conditions not satisfied)	
$\ell = 1$	$u^{1-m} = (1-m)\alpha \ln(k_j/k)$	$\ln u = \alpha \ln(d_1/k)$	$u^{1-m} = (1-m)\alpha \ln(1/k) + d_2$
$\ell > 1$	$u^{1-m} = u_f^{1-m}\ 1-(k/k_j)^{\ell-1}$	$\ln u = \ln u_f + \frac{\alpha}{1-\ell} k^{\ell-1}$	$u^{1-m} = u_f^{1-m} + ck^{\ell-1}$ where $c = \alpha \frac{1-m}{1-\ell}$ and $\ell > m$

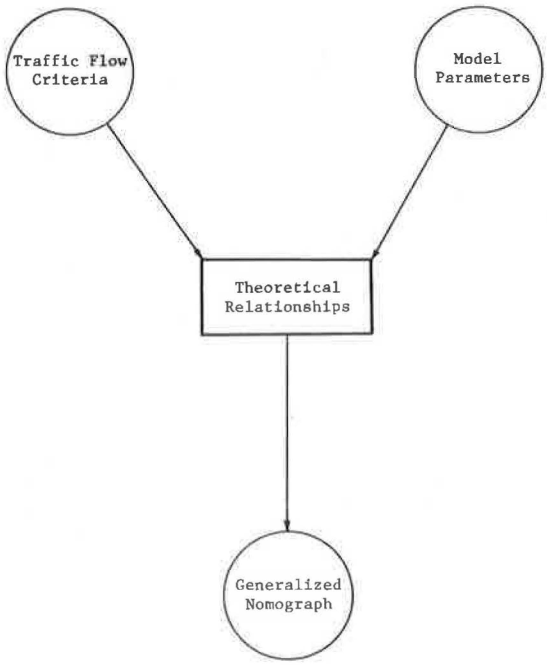
Figure 2. Single-regime and two-regime approaches.



(a) Single Regime

(b) Two Regime

Figure 3. Concept of proposed estimation procedure.



optimum density (k_o), and maximum flow (q_m). These five criteria, shown in Figure 2(a), represent the critical points of the traffic-flow relations. In addition, model parameters include α, m, and ℓ, which are contained in the matrix of the general macroscopic models. To make the procedure easier and more flexible to use, the theoretical relations are translated into a generalized nomograph that can be used to directly determine (or output) model parameters that satisfy specified (or input) traffic-flow criteria.

This concept has been proposed and applied to the estimation of single-regime models by Easa (9) and is further extended in this paper to the estimation of two-regime models. The basic principles of the single-regime approach will be repeated here for purposes of integrity and because the single-regime approach is complementary to the two-regime approach, as will be discussed later in this paper.

SINGLE-REGIME APPROACH

As mentioned previously, the single-regime approach, as defined in this paper, is limited to models in region 4 of Figure 1. The generalized procedure for the estimation of models in this region is described here in four parts:

1. Establishment of the theoretical relations between model parameters and traffic-flow criteria,
2. Development of the nomograph,
3. Description of the procedure for establishing the feasible region of model parameters, and
4. A sensitivity analysis of various aspects involved in the procedure.

Theoretical Development

The steady-state flow equation for region 4, shown in the matrix in Figure 1, is as follows:

$$u^{1-m} = u_f^{1-m} [1 - (k/k_j)^{\ell-1}] \quad (7)$$

where u and u_f = steady-state and free-flow speeds, respectively; and k and k_j = density and jam density, respectively. This equation represents a single-regime model that has an x intercept (jam density) and a y intercept (free-flow speed) and corresponds to combinations of m and ℓ values so that m < 1 amd ℓ > 1.

Ceder and May (10) have shown that a relation between m and ℓ parameters and traffic-flow characteristics can be obtained. Such a relation includes k_j and u_f and optimum variables u_o and k_o of speed and density, respectively. From Equation 7, one can obtain the following relations, in which q is expressed as a function of k and u, respectively:

$$q = u_f k [1 - (k/k_j)^{\ell-1}]^{1/(1-m)} \quad (8)$$

$$q = u k_j [1 - (u/u_f)^{1-m}]^{1/(\ell-1)} \quad (9)$$

At maximum flow, dq/dk = 0. Therefore, by differentiating Equation 8 with respect to k and equating the derivative to zero, one obtains:

$$(k_o/k_j)^{\ell-1} = (1 - m)/(\ell - m) \quad (10)$$

In addition, at maximum flow, dq/du = 0. Therefore, by differentiating Equation 9 with respect to u and equating the derivative to zero, one obtains:

$$(u_o/u_f)^{1-m} = (\ell - 1)/(\ell - m) \quad (11)$$

Equations 10 and 11 are related as follows:

$$(u_o/u_f)^{1-m} = 1 - (k_o/k_j)^{\ell-1} \quad (12)$$

Rearranging to obtain m as a function of ℓ, (u_o/u_f), and (k_o/k_j) gives

$$m = 1 - \ln[1 - (k_o/k_j)^{\ell-1}]/\ln(u_o/u_f) \quad (13)$$

Substituting Equation 13 into Equation 10 gives

$$\ln(u_o/u_f) = [1/(\ell - 1)]\left\{[1/(k_o/k_j)^{\ell-1}]^{-1}\right\} \ln[1 - (k_o/k_j)^{\ell-1}] \quad (14)$$

The reader can see that Equations 13 and 14 relate model parameters ℓ and m to traffic-flow criteria k_j, u_f, k_o, and u_o. Furthermore, the maximum flow q_m is related to k_o and u_o, as follows:

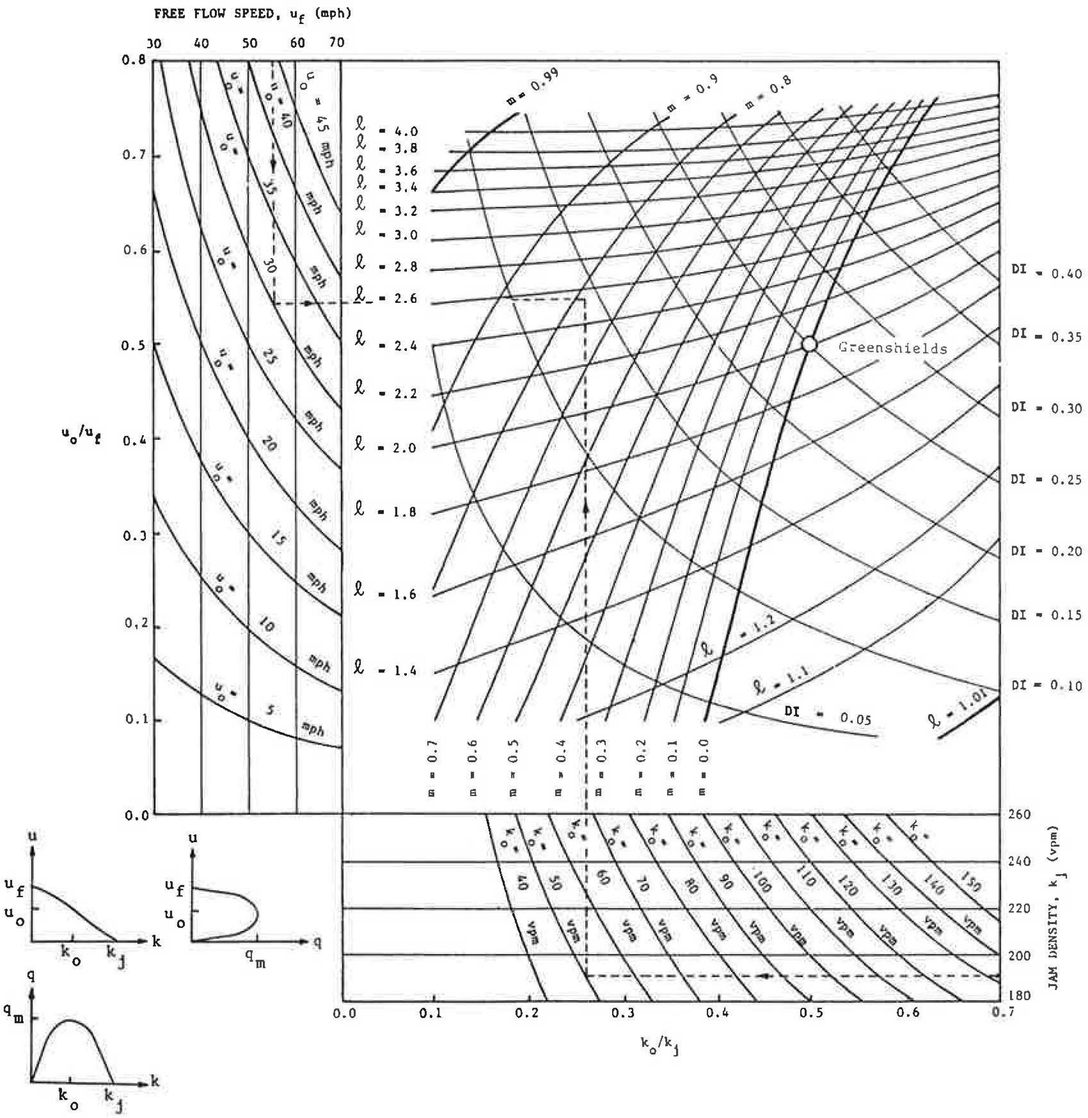
$$q_m = k_o u_o \quad (15)$$

Equations 13-15 now relate model parameters ℓ and m to the five traffic-flow criteria, k_j, k_o, u_f, u_o, and q_m. By establishing these criteria from traffic-flow data, one can use these equations to determine the corresponding parameter values. A generalized nomograph developed to simplify this process is described below.

Nomograph Development and Use

The mathematical relations of Equations 13-15 were represented by the nomograph shown in Figure 4 (9), which incorporates contour lines for the five traffic-flow criteria and for model parameters ℓ and m. In the lower right-hand portion of the nomograph, values for jam density (k_j), ranging from 180 to 260 vehicles/mile, are provided. In addition, contour lines for optimum density (k_o) are provided for values ranging from 40 to 150 vehicles/mile. In the upper left-hand portion, values for free-flow speed (u_f), ranging from 30 to 70 miles/h, are given, and contour lines for optimum speed (u_o) are estblished for values ranging from 5 to 45 miles/h.

Figure 4. Generalized nomograph for single-regime models ($\ell > 1$, $m < 1$).



Contour lines for ℓ and m parameters were established by using these values of the traffic-flow criteria. Values of ℓ ranging from 1.1 to 4.0 are included. In addition, the ℓ contour corresponding to a value of 1.01 is provided and represents the limit after which the models would have no intercept with the y axis ($u_f \to \infty$) and would belong to region 2 of Figure 1. The m values range from 0.0 to 0.9, and a value of m = 0.99 is included to represent the limit after which models would have no intercept with the x axis ($k_j \to \infty$) and would belong to region 3. The thick line shown in the middle of the nomograph corresponds to a value of m = 0 and represents a lower limit of the m values. The negative values of m were considered undesirable; because such values have the effect of shifting the speed variable in the sensitivity term of Equation 4 from the numerator to the denominator, they were not included in the nomograph.

The final set of contours provided in Figure 4 is the set related to the maximum flow (q_m). Clearly, contours for q_m cannot be established in Figure 4, since this variable depends on k_o and u_o, which are provided as contours. To solve this problem, a variable DI was introduced. DI is defined as follows:

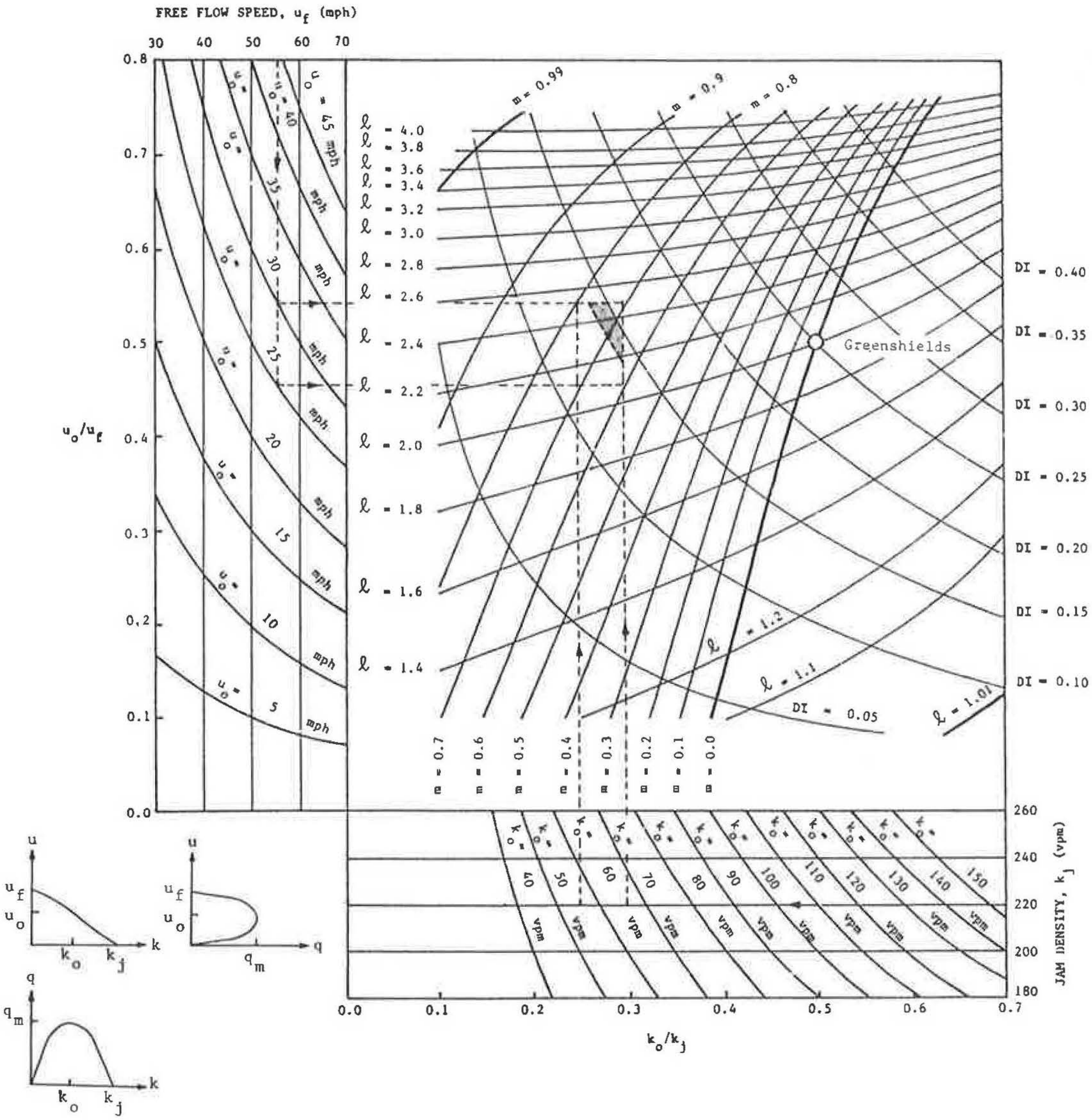
$$DI = (q_m / k_j u_f) = (k_o/k_j)(u_o/u_f) \tag{16}$$

By using this definition, contour lines for DI were constructed in Figure 4 for values ranging from 0.05 to 0.40. These contours are used to establish the maximum flow criteria, which will be described later. It should be noted that the Greenshields model (11) is a special case of single-regime models and corresponds to ℓ = 2.0 and m = 0.

To illustrate the use of the nomograph, let us consider an example. Suppose that the traffic-flow criteria are established as k_j = 190 vehicles/mile, u_f = 55 miles/h, k_o = 50 vehicles/mile, and u_o = 30 miles/h. The corresponding values of model parameters ℓ and m must be determined. To do this, the following steps are performed (Figure 4):

1. Enter at k_j = 190 vehicles/mile and draw a horizontal line that intersects with the contour corresponding to k_o = 50 vehicles/mile.
2. From that point draw a vertical line.
3. Enter at u_f = 55 miles/h and draw a vertical line that intersects with the contour corresponding to u_o = 30 miles/h.
4. From that point draw a horizontal line.
5. The intersection point of the vertical and

Figure 5. Establishing the feasible region for the single-regime approach ($\ell > 1$, $m < 1$).



horizontal lines established in steps 2 and 4, respectively, defines the required values of model parameters. It can be seen that this point corresponds to ℓ = 2.55 and m = 0.78.

6. Check the maximum flow value by reading the value of DI at the intersection point and multiplying it by k_j and u_f to determine q_m. From the diagram, DI = 0.14 and, therefore, q_m = 1463 vehicles/h (55 x 190 x 0.14). Obviously, in this simple example, q_m can be directly calculated from input values of k_o and u_o = 1500 vehicles/h.

With the values of ℓ = 2.55 and m = 0.78, the steady-state flow equations can now be defined. For example, the speed-density relation (Equation 7) can be described as follows:

$$u = 55\ [1 - (k/190)^{1.55}]^{4.55} \tag{17}$$

Establishing the Feasible Region

It should be noted that the procedure described above is intended for use when the traffic-flow criteria are specified as single values. In many situations, however, the analyst might be interested in information on the feasible range of model parameters that satisfy specified ranges (rather than single values) of the traffic-flow criteria. The estimation procedure for such cases can be described as follows, by using a hypothetical example.

Suppose that, based on a given set of traffic-flow data, one has established the following values of the traffic-flow criteria: k_j = 220 vehicles/mile, u_f = 55 miles/h, k_o = 55-65 vehicles/mile, u_o = 25-30 miles/h, and q_m = 1700-1800 vehicles/h. One must determine the feasible region of ℓ and m values that satisfy the above evaluation criteria. The estimation procedure, shown in Figure 5, consists of the following four basic steps:

1. Draw a horizontal line corresponding to the k_j value and two contour lines corresponding to the limits of the k_o range. The intersection of these lines defines two points.
2. Draw a vertical line corresponding to the u_f value and two contour lines corresponding to the limits of the u_o range. The intersection of these lines defines two points.
3. From the two points defined in step 1, draw two vertical lines and, similarly, draw two horizontal lines from the two points defined in step

2. The rectangular region defined by these lines includes the values of model parameters that satisfy the traffic-flow criteria k_j, k_o, u_f, and u_o.

4. Finally, establish the appropriate DI contours that correspond to the criteria range of k_j, u_f, and q_m. Since k_j and u_f can be expressed as ranges, Equation 16 is used with special considerations. Specifically, the lower and upper limits of DI are generally calculated as follows:

$$DI\,(\text{lower}) = q_m(\text{lower})/u_f(\text{upper})\,k_j(\text{upper}) \tag{18}$$

$$DI\,(\text{upper}) = q_m(\text{upper})/u_f(\text{lower})\,k_j(\text{lower}) \tag{19}$$

where lower and upper on the right-hand side refer to the limits of the established ranges of the traffic-flow criteria k_j, u_f, and q_m. (Note that, in this example, u_f and k_j are established as single values; the upper and lower limits are equal.)

By using Equations 18 and 19, it can be found that DI (lower) = 0.14 and DI (upper) = 0.15. The DI contours that correspond to these values are shown in Figure 5. The area between these contours that overlaps with the previously determined rectangular region defines the feasible region of model parameters. Clearly, values of ℓ and m within that region satisfy all of the traffic-flow criteria. It should be noted that the area below the DI (lower) contour includes all points with $q_m < 1700$ vehicles/h. Similarly, the area above the DI (upper) contour includes all points with $q_m > 1800$ vehicles/h.

The feasible region of model parameters determined above provides a range of models that satisfy specified ranges of the traffic-flow criteria and, consequently, confine the traffic-flow data from which these criteria are established. For this reason, the feasible region would be useful for a variety of transportation applications in which sensitivity to changes in the traffic-flow relations is of particular concern. It is also important to note that, for any model in the feasible region, the associated traffic-flow criteria are immediately defined. For example, for the model that corresponds to $\ell = 2.3$ and $m = 0.7$, it can be determined that $k_o = 61$ vehicles/mile and $u_o = 28$ miles/h. Noting that $u_f = 55$ miles/h and $k_j = 220$ vehicles/mile, the speed-density relation, for instance, can be described as follows:

$$u = 55\,[1 - (k/220)^{1.3}]^{3.33} \tag{20}$$

It is important to note that, in establishing the feasible region, both u_f and k_j were specified as single values. However, these two criteria can also be established as ranges, in which case additional computations are needed to establish the traffic-flow criteria associated with any selected model (9).

Sensitivity Analysis

In the estimation procedure previously described, a feasible region of model parameters was defined that includes all models that satisfy the established traffic-flow criteria. To investigate the likely variations among models of the feasible region, boundary models were investigated. Figure 6 shows three models in the previously defined feasible region that approximately bound other models in the region. Obviously, other models in the feasible region would lie somewhere within the band of models shown in Figure 6. It is interesting that, if the regression analysis technique (6) had been used to determine model parameters, the selected model with that technique would have lain within that band. This is essentially true, since such a model should fulfill the specified traffic-flow criteria. It is noted in Figure 6 that the expected maximum variations among models are relatively small and do not generally exceed the length of the speed criteria range (5 miles/h in this example).

Another important point related to the sensitivity of model parameters is worthy of note. The shape of model-parameter contours shown previously in Figure 4 clearly exhibits the sensitivity of ℓ and m values to variations in the traffic-flow criteria. To further illustrate the sensitivity of model parameters to these criteria, contours for k_o/k_j and u_o/u_f were established on an ℓ versus m diagram for the region of greater interest (ℓ = 1.8 to 2.8, m = 0.0 to 0.9), as shown in Figure 7. It is noted that the ℓ parameter is considerably more sensitive to u_o/u_f than to k_o/k_j, and this parameter tends to be almost insensitive to k_o/k_j at higher values of ℓ. On the other hand, the m parameter is slightly more sensitive to k_o/k_j than to u_o/u_f. These characteristics appear to be useful as guidelines for the relative effort to be expended in establishing the traffic-flow criteria. Figure 7 also shows the relative locations of high- and low-design facilities and the region of models that correspond to actual traffic-flow data analyzed in previous research work (12).

TWO-REGIME APPROACH

It should be remembered that models in region 4, (the region for which the generalized nomograph described above was developed) have intercepts with both speed and density axes and are designated as single-regime models. In the single-regime approach, the model determined by the nomograph is used to represent the entire range of operations. In addition to its use for the single-regime approach, the nomograph presented above can be used for the two-regime representation in which two models would be established--one for the non-congested-flow regime and the other for the congested-flow regime. In this case, the nomograph is used twice by using the traffic-flow criteria that correspond to each of the two regimes. Such a process is a straightforward application of the procedure described previously and will not be elaborated on further here.

In the two-regime approach, one can represent the non-congested-flow regime by a model from the region ($\ell > 1$, $m = 1$) and the congested-flow regime by a model from the region ($\ell = 1$, $m < 1$). The purpose of this section is to describe the generalized estimation procedure for the two-regime approach by using models from these two regions. This description is presented in three parts:

1. Development of the theoretical relations between model parameters and the traffic-flow criteria for the non-congested-flow and congested-flow regimes,
2. Presentation of nomographs for both regimes as well as a brief description of their intended use, and
3. Description of the procedure for establishing the feasible region of model parameters.

Theoretical Development

For the non-congested-flow region ($\ell > 1$, $m = 1$), the steady-state flow equation, shown in Figure 1, is as follows:

$$\ln u = \ln u_f + [\alpha/1 - \ell)]\, k^{\ell-1} \tag{21}$$

where

- k = density,
- α = constant of proportionality, and
- ℓ = model parameter.

Equation 21 represents a model that has no intercept with the density axis ($k_j \rightarrow \infty$). This relation can be used to establish the relations between traffic-flow criteria and model parameters. Such relations include free-flow speed u_f, optimum speed u_o, and optimum density k_o, as well as model parameters ℓ and α (note that α is referred to as model parameter).

From Equation 21, one can obtain the following relations, in which q is expressed as a function of k and as a function of u, respectively:

$$q = ke^{\ln u_f + [\alpha/(1-\ell)]\, k^{\ell-1}} \quad (22)$$

$$q^{\ell-1} = [(1-\ell)/\alpha]\, u^{\ell-1} \ln (u/u_f) \quad (23)$$

where e is the base of the natural logarithm.

At maximum flow, $dq/dk = 0$. Therefore, by differentiating Equation 22 with respect to k and equating the derivative to zero, one obtains

$$\alpha = 1/k_o^{\ell-1} \quad (24)$$

In addition, at maximum flow, $dq/d_u = 0$. Therefore, by differentiating Equation 23 with respect to u and equating the derivative to zero, one obtains

$$u_o/u_f = e^{-[1/(\ell-1)]} \quad (25)$$

It is interesting to note that Equation 25 can also be obtained from Equation 11 for the

Figure 6. Variations of models within the feasible region.

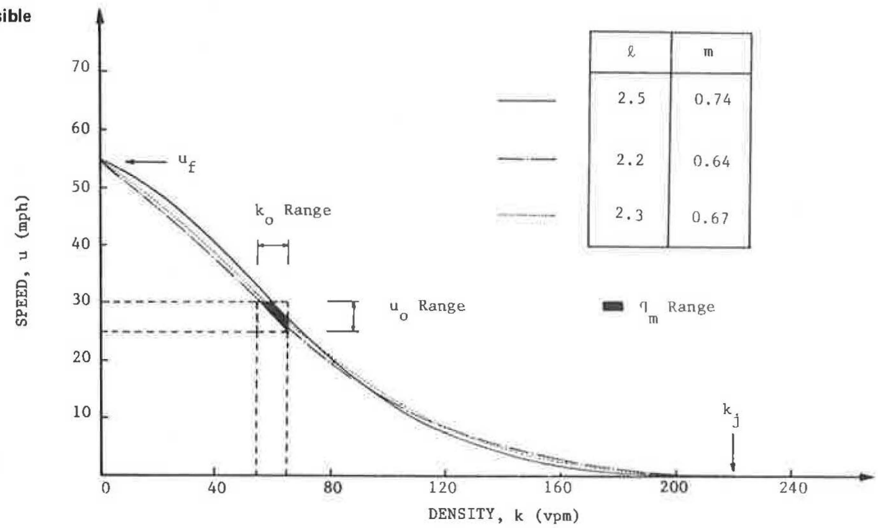
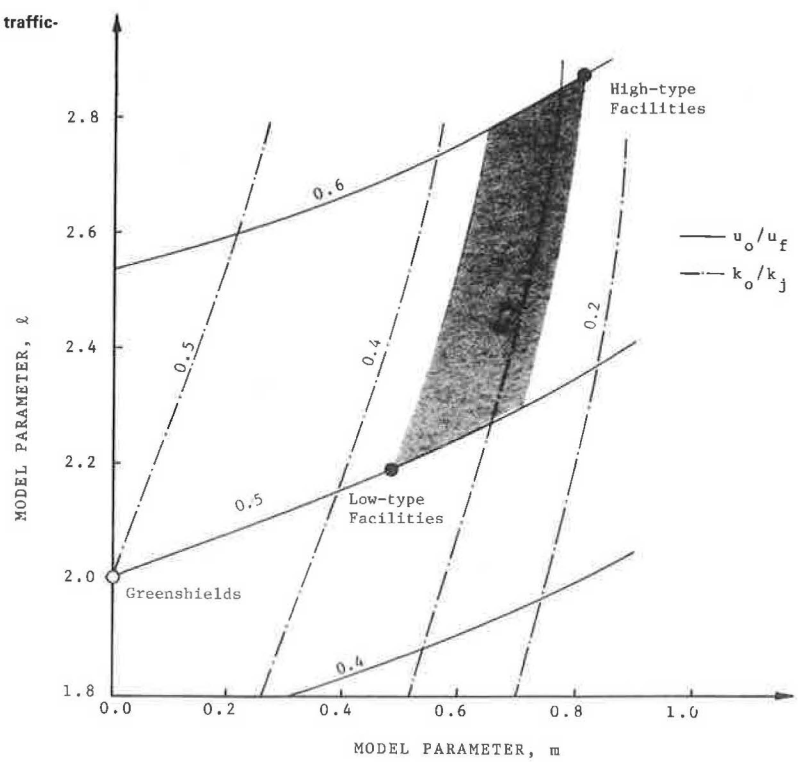


Figure 7. Sensitivity of model parameters to traffic-flow criteria.



single-regime approach. When m approaches 1 in Equation 11, u_o/u_f becomes

$$\lim_{m\to 1} (u_o/u_f) = \lim_{m\to 1} [(\ell - 1)/\ell - m)]^{1/(1-m)} = \lim_{m\to 1} [(1 + \{1/[(\ell - m)/(m - 1)]\})^{(\ell-m/m-1)}]^{-[1/(\ell-m)]} = e^{-[1/(\ell-1)]} \tag{26}$$

which is the same as Equation 25. This feature indicates the continuity of the u_o/u_f ratio between the single-regime models and the non-congested-flow models. Now, rearranging Equation 25 to obtain ℓ as a function of u_o/u_f, one obtains

$$\ell = 1 - [1/\ln(u_o/u_f)] \tag{27}$$

Furthermore, the maximum flow $q_m (= u_o k_o)$ can be expressed as follows:

$$q_m = k_o u_f e^{-[1/(\ell-1)]} \tag{28}$$

It can be seen that Equations 24, 27, and 28 relate model parameters ℓ and α to the traffic-flow criteria u_f, u_o, k_o, and q_m (note that k_j does not exist).

For the congested-flow region (ℓ = 1, m < 1), the steady-state flow equation, shown previously in Figure 1, is as follows:

$$u^{1-m} = \alpha(1 - m) \ln(k_j/k) \tag{29}$$

where k_j = jam density and m = model parameter.

Equation 29 represents a model that has no intercept with the speed axis ($u_f \to \infty$). By following a procedure similar to that described for the non-congested-flow regime, the relations between the traffic-flow criteria k_j, k_o, and u_o and model parameters m and α can be established. From Equation 29, one can express q as a function of k and as a function of u, as follows:

$$q = k [\alpha(1 - m) \ln(k_j/k)]^{1/(1-m)} \tag{30}$$

$$q - uk_j \exp\{-[u^{1-m}/\alpha(1 - m)]\} \tag{31}$$

In addition, α and k_o/k_j can be expressed as follows:

$$\alpha = u_o^{1-m} \tag{32}$$

$$k_o/k_j = e^{-[1/(1-m)]} \tag{33}$$

It is worthy of note that, by taking the limit of k_o/k_j in Equation 10 (presented for the single-regime approach) when ℓ approaches 1, one would obtain Equation 33. This can be proved in a way similar to that described earlier for the non-congested-flow models. Similarly, this feature indicates the continuity of the k_o/k_j ratio between the single-regime models and the congested-flow models. Now, rearranging Equation 33 to obtain m as a function of k_o/k_j, one obtains:

$$m = 1 + [1/\ln(k_o/k_j)] \tag{34}$$

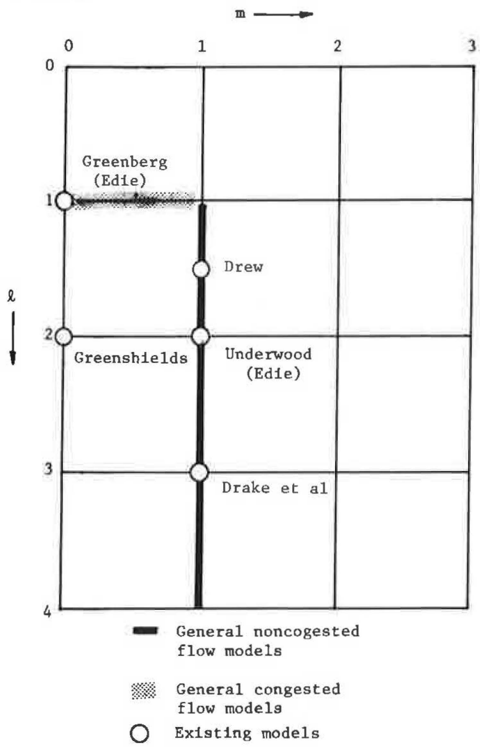
The maximum flow $q_m (= u_o k_o)$ is expressed as follows:

$$q_m = u_o k_j e^{-[1/(1-m)]} \tag{35}$$

It can be seen that Equations 32, 34, and 35 relate model parameters m and α to the traffic-flow criteria k_j, u_o, k_o, and q_m (note that u_f does not exist).

It is important to note that existing macroscopic

Figure 8. Locations of generalized and existing two-regime models on ℓ-versus-m matrix.



models are special cases of the formulations generalized above. Figure 8 shows the locations of the generalized and existing two-regime models. As seen, the three models developed by Drew (13), Underwood (14), and Drake and others (15) correspond to ℓ = 1.5, ℓ = 2, and ℓ = 3, respectively, on the general non-congested-flow models. In addition, the model developed by Greenberg (16) is a special case of the general congested-flow models when m = 0. The key elements of both the general and existing two-regime models are summarized in Figure 9. Clearly, the generalized formulations presented in Figure 9 provide a wider range of models and effect a more flexible treatment in the estimation process for two-regime models.

Development and Use of Nomographs

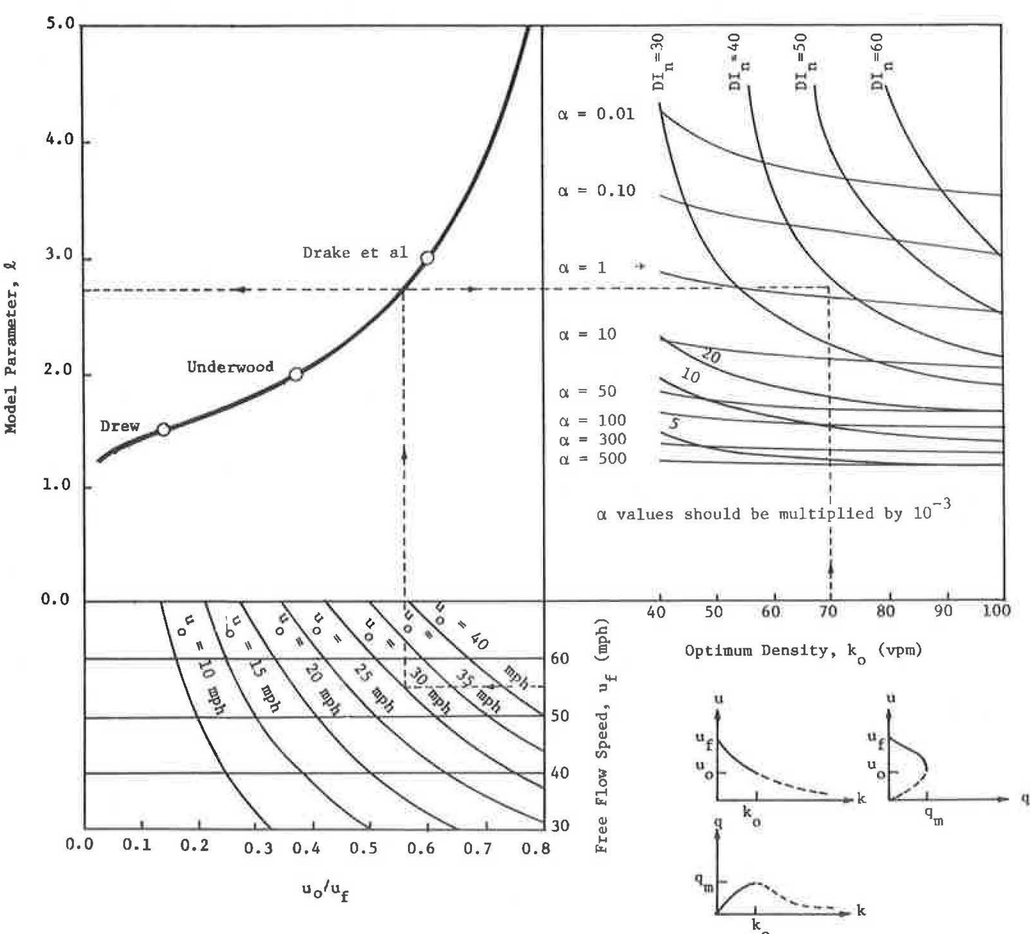
Once the theoretical relations between model parameters and traffic-flow criteria were established, as described above, these relations were translated into a practical tool, and generalized nomographs for non-congested-flow and congested-flow regimes were developed to graphically represent the theoretical aspects involved.

For the non-congested-flow regime (ℓ > 1, m = 1), Figure 10 shows a generalized nomograph that relates model parameters ℓ and α to the traffic-flow criteria u_f, u_o, k_o, and q_m. The nomograph encompasses the basic relation between ℓ and u_o/u_f (thick curve) and three sets of contours for u_o, α, and q_m. The basic relation (Equation 27) was established for values of u_o/u_f ranging from 0.02 to 0.78, which correspond to ℓ values of approximately 1.2-5. The locations of existing non-congested-flow models are shown on the curve. Note also that the curve has an inflection point at $u_o/u_f = 1/e^2$, which corresponds to ℓ = 1.5 [Drew model (13)]. The contour lines for optimum speed u_o were established for values ranging from 10 to 40

Figure 9. Characteristics of generalized and existing two-regime models.

ELEMENT	NONCONGESTED FLOW REGIME ($\ell > 1$, $m=1$)				CONGESTED FLOW REGIME ($\ell=1$, $m < 1$)	
	GENERAL	DREW	UNDERWOOD	DRAKE ET AL	GENERAL	GREENBERG
Equation of State	$u = u_f e^{\frac{\alpha}{1-\ell}k^{\ell-1}}$	$u = u_f e^{-2(\frac{k}{k_o})^{0.5}}$	$\mu = \mu_f e^{-(\frac{k}{k_o})}$	$\mu = \mu_f e^{-\frac{1}{2}(\frac{k}{k_o})^2}$	$\mu^{1-m} = \alpha (1-m) \ln(\frac{k_j}{k})$	$\mu = \mu_o \ln(\frac{k_j}{k})$
Constant of proportionality	$\alpha = \frac{1}{k_o^{\ell-1}}$	$\frac{1}{\sqrt{k_o}}$	$\frac{1}{k_o}$	$\frac{1}{k_o^2}$	μ_o^{1-m}	μ_o
Parameter ℓ	$\ell = 1 - \frac{1}{\ln(\frac{\mu_o}{\mu_f})}$	1.5	2	3	1	1
Parameter m	$m = 1$	1	1	1	$1 + \frac{1}{\ln(\frac{k_o}{k_j})}$	0
Optimum speed	$\mu_o = \mu_f e^{-\frac{1}{\ell-1}}$	$\frac{u_f}{e^2}$	$\frac{\mu_f}{e}$	$\frac{\mu_f}{\sqrt{e}}$	$\alpha^{\frac{1}{1-m}}$	α
Optimum density	$k_o = \alpha^{-\frac{1}{\ell-1}}$	$\frac{1}{\alpha^2}$	$\frac{1}{\alpha}$	$\frac{1}{\sqrt{\alpha}}$	$k_j e^{-\frac{1}{1-m}}$	$\frac{k_j}{e}$
Capacity	$q_m = k_o \mu_f e^{-\frac{1}{\ell-1}}$	$\frac{k_o u_f}{e^2}$	$\frac{k_o \mu_f}{e}$	$\frac{k_o \mu_f}{\sqrt{e}}$	$\mu_o k_j e^{-\frac{1}{1-m}}$	$\frac{\mu_o k_j}{e}$

Figure 10. Generalized nomograph for non-congested-flow regime ($\ell > 1$, $m = 1$).



miles/h. As shown, the free-flow speed u_f associated with these contours ranges from 30 to 70 miles/h.

Contours for the constant of proportionality α were established based on Equation 24, which relates α to both ℓ and k_o (k_o ranges from 40 to 100 vehicles/mile). As noted, values of α contours range from 10^{-5} to 0.5. The final set of contours are those related to q_m. To establish contours representing q_m, a variable DI_n was introduced (based on Equation 28). This variable is defined as follows:

$$DI_n = q_m/u_f = k_o e^{-[1/(\ell-1)]} \tag{36}$$

From Equation 36, it should be noted that DI_n is a function of ℓ and k_o. By using this equation, contours for DI_n were established for values ranging from 5 to 60. As will be described later, these contours are used to ensure that the criteria range for q_m is satisfied.

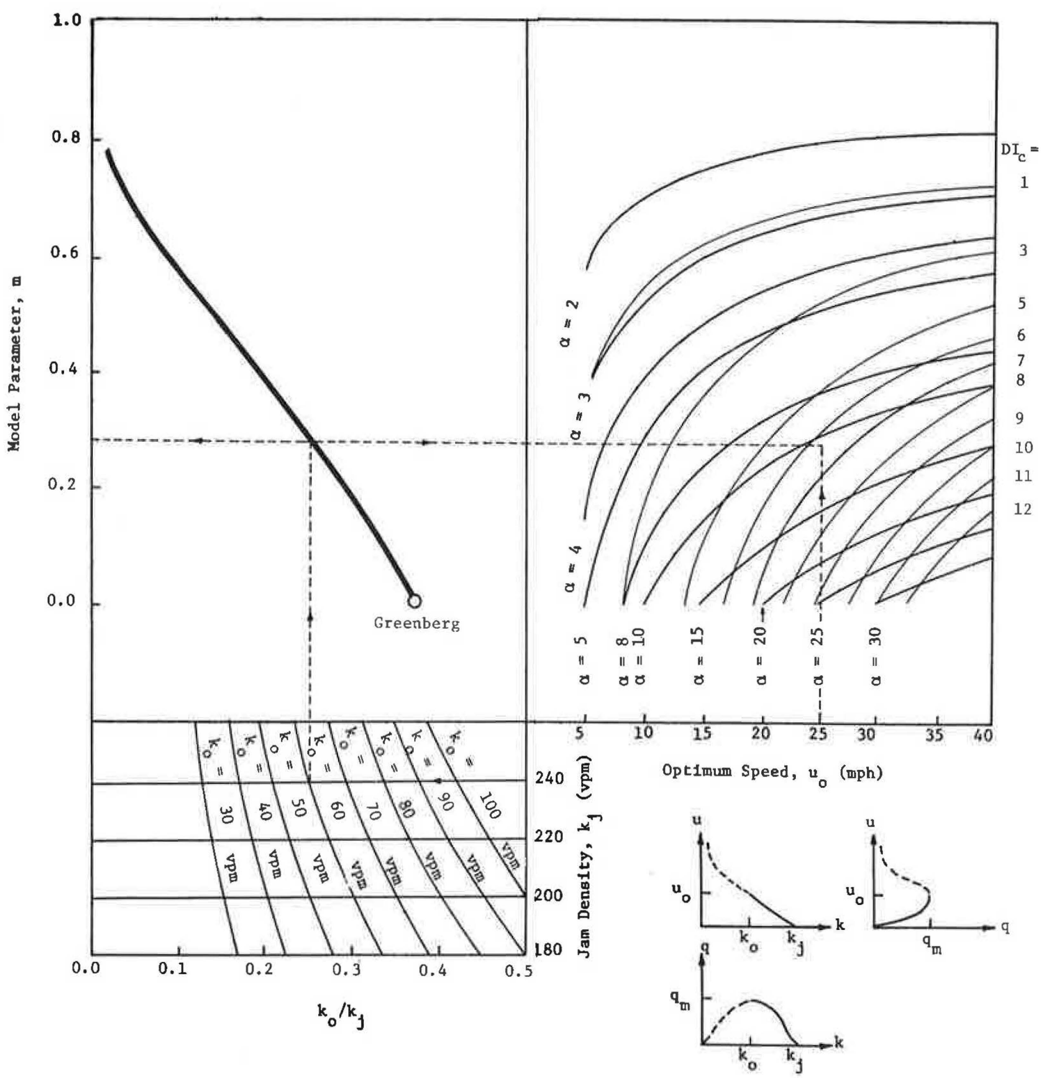
To illustrate the use of the nomograph, let us consider an example. Given that u_f = 55 miles/h, u_o = 30 miles/h, and k_o = 70 vehicles/mile, one must determine the corresponding model parameter ℓ and constant of proportionality α by performing the following steps (Figure 9):

1. Enter at u_f = 55 miles/h and draw a horizontal line that intersects with the contour corresponding to u_o = 30 miles/h.
2. From that point, draw a vertical line that intersects with the basic (thick) curve.
3. At the intersection point, draw a horizontal line (to the left) and read the value of ℓ = 2.6. Extend this horizontal line to the right.
4. Enter at k_o = 70 vehicles/mile and draw a vertical line that intersects with the horizontal line in step 3 above. At the intersection point, read the value of $\alpha = 0.9 \times 10^{-3}$.
5. Check the maximum flow value by reading the value of DI_n at the intersection point and multiplying it by u_f to determine q_m. From the diagram, DI_n = 39; therefore, q_m = 2145 vehicles/h (39 x 55). Obviously, in this simple example q_m can be directly calculated from input values of k_o and u_o as $k_o u_o$ = 2100 vehicles/h.

Having determined the values of ℓ = 2.6 and $\alpha = 0.9 \times 10^{-3}$, one can now define the steady-state flow equations. For example, the speed-density relation (Equation 21) can be described as follows:

$$u = 55 \exp(-0.56 \times 10^{-3} k^{1.6}) \tag{37}$$

Figure 11. Generalized nomograph for congested-flow regime (ℓ = 1, m < 1).



A similar nomograph was developed for the congested-flow regime ($\ell = 1$, $m < 1$) and is shown in Figure 11. The nomograph represents the relations between model parameters m and α and traffic-flow criteria k_j, u_o, k_o, and q_m and encompasses the basic relation of m and k_o/k_j and three sets of contours for k_o, α, and q_m. The basic relation (thick curve) is based on Equation 34. The curve has an inflection point at $k_o/k_j = 1/e^2$, which corresponds to $m = 0.5$. In addition, the Greenberg model is located at a point corresponding to $m = 0$. It should be noted that the curve is not extended beyond the Greenberg model for values of $m < 0$. This was used because such negative values have the effect of shifting the speed variable in the sensitivity term of Equation 4 from the numerator to the denominator, which is considered undesirable.

Contours for k_o and α for the congested-flow regime were established in a way similar to that for the non-congested-flow regime. In addition, to establish contours representing q_m, a variable DI_c was introduced (based on Equation 35). This variable is defined as follows:

$$DI_c = q_m/k_j = u_0 e^{-[1/(1-m)]} \tag{38}$$

DI_c, which is similar to DI_n for the non-congested-flow regime, is used to ensure that the criteria range for q_m is satisfied. The use of the nomograph is similar to that for the non-congested-flow regime. Figure 11 further illustrates the use of the nomograph. In this example, the input values of the traffic-flow criteria are k_j = 240 vehicles/mile, k_o = 60 vehicles/mile, and u_o = 25 miles/h. By using these values, the reader can ascertain the value of $m = 0.28$ and $\alpha = 10.5$. Substituting these values into the speed-density equation (Equation 29), for example, yields

$$u = 16.60\,[\ln(240/k)]^{1.39} \tag{39}$$

Establishing the Feasible Region

To this point, the procedures outlined above are intended for use in cases where the traffic-flow criteria are established as single values. Knowledge of the feasible region of model parameters is important for two-regime, as for single-regime, models. Such a feasible region is obtained when the traffic-flow criteria are established as ranges. The procedure for such cases can be described as follows.

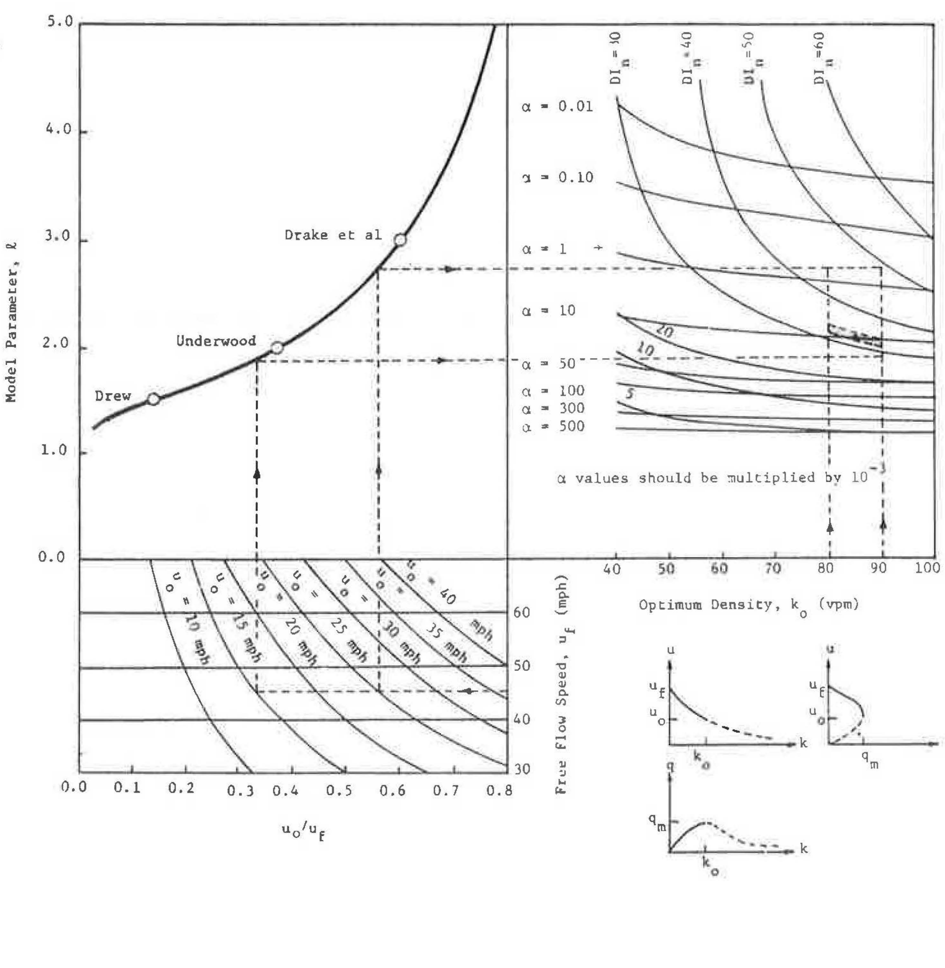
The use of the various steps involved in establishing the feasible regions for the non-congested-flow and congested-flow regimes can be illustrated by means of an example. Suppose that, based on a given set of traffic-flow data, one has established the ranges of the traffic-flow criteria for the non-congested-flow and congested-flow regimes as follows (k_j and u_f are established as single values):

Criterion	Non-Congested-Flow Regime	Congested-Flow Regime
k_j (vehicles/mile)	-	250
u_f (miles/h)	46	-
k_o (vehicles/mile)	80-90	70-80
u_o (miles/h)	15-25	15-20
q_m (vehicles/h)	1450-1550	1300-1400

One must now determine model parameters that satisfy these criteria.

For the non-congested-flow regime, the feasible region of model parameters ℓ and α that satisfy

Figure 12. Establishing the feasible region for the non-congested-flow regime ($\ell > 1$, $m = 1$).



given traffic-flow criteria can be established as shown in Figure 12. The procedure consists of the following basic steps:

1. Draw a horizontal line corresponding to u_f = 46 vehicles/mile. This line intersects with the two contour lines that correspond to the range of u_o (15 and 25 miles/h).
2. From the intersection points, draw two vertical lines that intersect with the basic (thick solid) curve at two points.
3. From these points, draw two horizontal lines to the right.
4. At the limits of the k_o range (80 and 90 vehicles/mile), draw two vertical lines that intersect with the two horizontal lines drawn in step 3 above. The rectangular region defined by these lines contains model parameters that satisfy traffic-flow criteria u_f, u_o, and k_o.
5. Finally, establish the appropriate DI_n contours that correspond to the criteria range of u_f and q_m. In general, the lower and upper limits of DI_n can be calculated (based on Equation 36) as follows:

$$DI_n(\text{lower}) = q_m(\text{lower})/u_f(\text{upper}) \quad (40)$$

$$DI_n(\text{upper}) = q_m(\text{upper})/u_f(\text{lower}) \quad (41)$$

where "lower" and "upper" on the right-hand side refer to the limits of established ranges of the traffic-flow criteria q_m and u_f.

By using Equations 40 and 41 and noting that u_f (lower) = u_f (upper) = 46 miles/h, it can be found that DI_n (lower) = 32 and DI_n (upper) = 34. The DI_n contours corresponding to these values are drawn in Figure 11. The area between these contours that overlaps with the previously determined rectangular region defines the feasible region of model parameters. Clearly, values of ℓ and α within that region satisfy all four traffic-flow criteria: u_f, u_o, k_o, and q_m. It should be noted that the area below the DI_n (lower) contour contains models with q_m < 1450 vehicles/h, whereas the area above the DI_n (upper) contour contains models with q_m > 1550 vehicles/h. In addition, for any model in the feasible region, one can immediately define the associated traffic-flow criteria. As an example, for the model corresponding to ℓ = 2.05 and $\alpha = 10^{-2}$, the associated criteria of k_o and u_o are determined as 85 vehicles/mile and 18 miles/h, respectively. For these values, the speed-density relation (Equation 21), for example, can be described as follows:

$$u = 46 \exp(-9 \times 10^{-3} k^{1.05}) \quad (42)$$

For the congested-flow regime, the feasible region is established in a way similar to that for the non-congested-flow regime. The feasible region is established as shown in Figure 13 by using the traffic-flow criteria given in the in-text table above. It should be noted that contours for DI_c

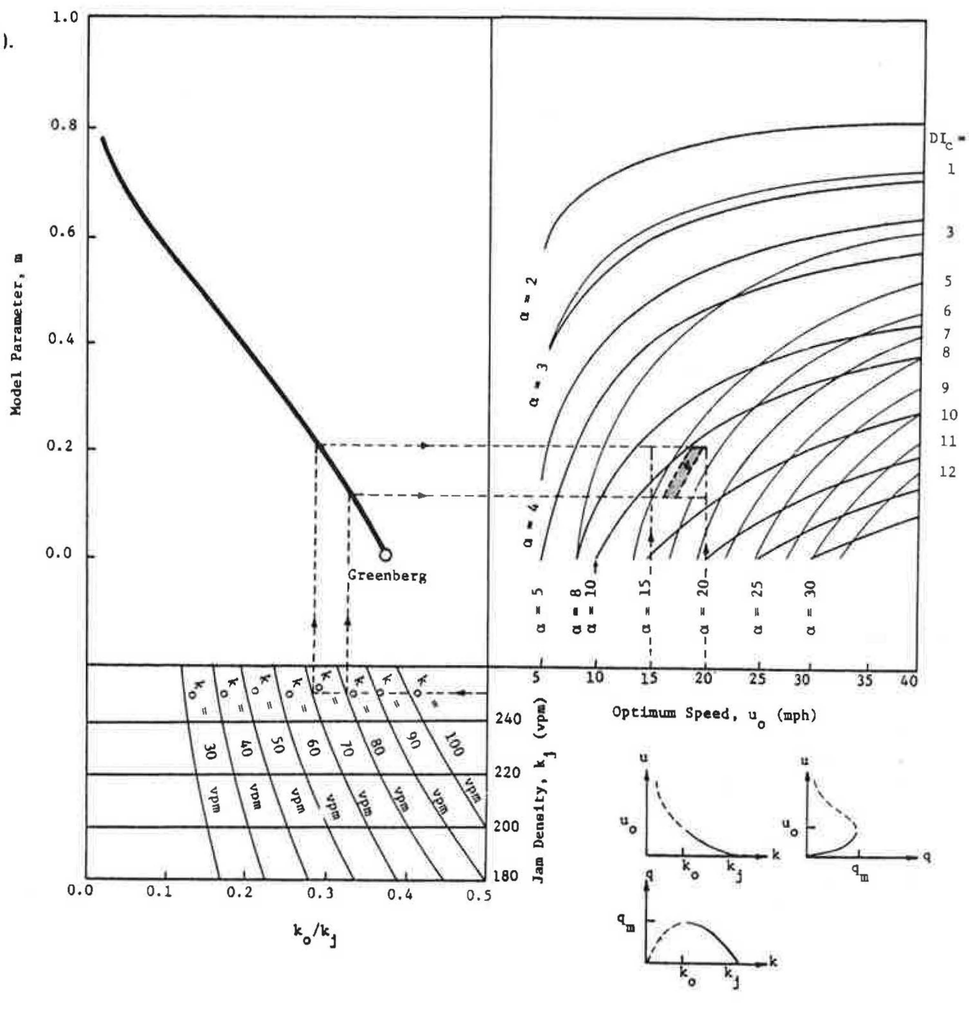


Figure 13. Establishing the feasible region for the congested-flow regime (ℓ = 1, m < 1).

are generally established (based on Equation 39) as follows:

$$DI_c(\text{lower}) = q_m(\text{lower})/k_j(\text{upper}) \quad (43)$$

$$DI_c(\text{upper}) = q_m(\text{upper})/k_j(\text{lower}) \quad (44)$$

By using these equations and noting that k_j (lower) = k_j (upper) = 250 vehicles/mile, one can find that DI_c (lower) = 5.2 and DI_c (upper) = 5.6. It should be noted that, for any model in the feasible region, the associated traffic-flow criteria can be immediately defined. For example, for the model corresponding to m = 0.19 and α = 10.5, the associated traffic-flow criteria are u_o = 18 miles/h, k_o = 75 vehicles/mile, and k_j = 250 vehicles/mile. For these values, the speed-density relation (Equation 29), for example, can be described as follows:

$$u = 14.03\,[\ln(250/k)]^{1.24} \quad (45)$$

It should be remembered that the feasible regions of model parameters established for the non-congested-flow and congested-flow regimes are intended for use in cases in which a range of models representing the traffic-flow data is of interest. In addition, u_f and k_j were specified as single values in an attempt to simplify the procedure. However, these criteria can be specified as ranges, in which case the estimation procedure would involve some additional computations to determine the associated traffic-flow criteria.

CONCLUSIONS

This paper presents a generalized procedure for estimating single- and two-regime traffic-flow models. The procedure is based principally on the theoretical relations between model parameters and traffic-flow criteria. Emphasis has been given to translating the theoretical aspects into practical analysis tools. Generalized nomographs developed for both modeling approaches are capable of directly providing the user with the values, or the feasible region, of model parameters that satisfy specified traffic-flow criteria.

Based on this research work, a few important observations can be made:

1. The input to the nomograph procedure is rather simple. It includes the traffic-flow criteria, which can be established from traffic-flow data for a particular facility. The output of the nomograph includes model parameters that satisfy these criteria. Clearly, if the traffic-flow criteria are carefully selected, the resulting model is likely to provide a reasonable representation of the data. Similarly, a good selection of the criteria ranges would result in a feasible region of model parameters that is more representative of the data characteristics.
2. The nomograph procedure should be viewed as complementary to rather than as a substitute for the existing regression analysis procedure for model estimation. The nomograph procedure is intended for use in situations in which a reasonable estimation of model parameters would suffice. When a relatively high degree of accuracy is required, the regression analysis procedure should be used.
3. The nomograph procedure appears to represent a powerful and flexible estimation tool that enables the analyst to adjust the evaluation criteria and to directly determine their effect on the evaluation results.
4. It should be emphasized that the nomograph procedure is based solely on theoretical aspects and so is general in nature; it is not a site-specific procedure. As a consequence, it appears to be of particular value for a wide range of transportation applications.

Future research work should be devoted to the following areas:

1. The nomograph procedure appears to provide a basis for the development of a facility design index for various highway types. Such an index would characterize highway facilities by specific combinations of model parameters. To this end, the variables DI, DI_n, and DI_c used in this paper could be further investigated by using real traffic-flow data.
2. Guidelines for the selection of single- or two-regime approaches under varying operating conditions should be developed. Future work to investigate alternative methods of modeling the two-regime approach is required.
3. A similar nomograph procedure for the remaining portions of the matrix of macroscopic models should be developed.

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Projected Vehicle Characteristics Through 1995

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The U.S. Department of Transportation has established fuel-consumption standards for passenger vehicles and light trucks that will result in increasingly fuel-efficient vehicles in the future. Projections of characteristics of the mix of vehicles on the road that can be expected to change as a result of industry compliance with the standards are presented through 1995, based on a variety of government and industry publications. The average mass (weight), power, and engine size of passenger vehicles—including light trucks with a maximum gross vehicle weight of 3860 kg (8500 lb)—will obviously decrease during this period. Fuel economy will continue to improve steadily, and average acceleration performance will not decline appreciably after the 1983-1985 period. The characteristics of recreational vehicles will change, mostly in the next few years. All of these changes in on-the-highway averages will be brought about through "replacement" of heavy and high-performance vehicles by others of more modest weights and powers rather than through the introduction of very small or low-performance vehicles, which will lead to a more homogeneous vehicle population.

The fuel embargo of 1973 and 1974 and the spot fuel shortages of the summer of 1979 have aroused wide public reaction and contributed to a change in consumer buying habits. Vehicle purchasers are, on the average, seeking more fuel-conserving cars. In response to this demand and to U.S. Department of Transportation (DOT) mandates, the automobile industry is gradually changing its fleet mix to produce vehicles that generally have better fuel-consumption characteristics. This is being accomplished primarily through size and weight reductions as well as a shift to smaller engines (with accompanying performance impacts). To improve overall efficiency, other changes in vehicle technology are also being introduced.

It is of interest to project the long-range impact of these changes on vehicle operations, traffic safety, and overall fuel consumption. To do this, it is first necessary to predict the distributions of the characteristics of vehicles that will be on the road in future years. This prediction process and the results obtained are the subject of this paper. The process assumes an orderly progression of changes based on present rule making and associated projections. It does not consider possible catastrophic events, such as curtailment of automobile production, cessation of fuel imports, or imposition of fuel rationing. The projected characteristics can then be used in analyses or models to estimate impacts of interest.

This paper deals with two basic vehicle categories: passenger and recreational vehicles. The first category includes American and imported automobiles as well as all light trucks (e.g., pickups and vans) with a gross vehicle weight (GVW) of as much as 3860 kg (8500 lb). Recreational vehicles include motor homes, pickup campers, and passenger-vehicle/trailer combinations.

POPULATIONS OF PASSENGER VEHICLES

The aim of this study was to estimate the average characteristics of vehicles that will be on the road in future years as well as the distributions around the averages. The estimation process required, first, breaking down each year's sales into identifiable vehicle categories, each described in terms of such factors as weight, engine size and power, and production. Then all of the sales over the 15-year period prior to the year of interest were accumulated. This accumulation process accounted for the scrappage rates of the vehicles as well as the decreasing annual mileage with age. Finally, averages and other quantities were determined on a travel-weighted basis (that is, vehicles driven more kilometers in the year of interest counted more heavily in the averaging process). Thus, the averages and distributions should be representative of what one would find by measuring all vehicles passing a given location. The assembly process, which involved summing over 3000-4000 identifiable vehicle categories, was made feasible by using specially written computer programs.

For convenience, passenger vehicles were generally divided into three groups: American cars, foreign cars, and light trucks. Then detailed vehicle characteristics were assembled only for selected model years (because of the rather painstaking process required). The characteristics for intervening years were estimated by the computer program, by use of interpolation. The following subsections provide more detail about the assembly process.

Data on Vehicle Characteristics

The most important determinant of acceleration performance is the ratio of a vehicle's net engine power to its mass [commonly, but imprecisely (from a technical viewpoint), called its weight]. Other characteristics, such as transmission and axle ratios, frontal areas, and aerodynamic drag coefficient, also have an effect. Unfortunately, these latter characteristics are not generally available other than on a special-case basis. Therefore, performance capability was estimated solely on the basis of power-to-mass ratio. More specifically, for each vehicle model identified, the maximum net power of the engine and an appropriate vehicle mass (weight) were recorded. For automobiles, this was taken as the curb weight (empty vehicle weight plus fuel and coolant) plus a