comes from travelers from outside Campinas proper, and this may not be substantiated under closer examination. Nevertheless, the results from both the demand model and the evaluation indicate that this link warrants further examination at a greater level of detail.

ACKNOWLEDGMENT

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Simple Equilibrium Analysis of the Dedication of a Freeway Lane to Exclusive Bus Use

YOSEF SHEFFI

In this paper, the dedication of an existing freeway lane to exclusive (with-flow) bus use is critically examined. A simple equilibrium analysis by means of a logit mode-choice model and typical volume-delay curves indicates that such projects might bring about the expected benefits only under extreme congestion. The benefits are measured in terms of the ratio of total person hours before to those after the implementation.

One of the many methods suggested in order to increase transit ridership is the dedication of a freeway lane for exclusive use by high-occupancy vehicles or buses. The rationale behind the so-called "diamond lane" is that by shifting the right number of users from private automobiles to buses, everyone would be better off. The automobile users, who are faced with higher congestion on a reduced-capacity freeway (and, it is hoped, who envy the free-flowing buses on the dedicated lane) would shift to transit. Naturally, it is hoped that there would not be a shift of so many users to transit that congestion would develop on the diamond lane. (It is reasonable to assume that the travel time on the diamond lane should be no longer than the travel time on the remaining lanes.)

The above-mentioned scenario seems to be a part of the underlying rationale for several diamond-lane projects throughout the country--for example, the Southeast Expressway in Boston and the Santa Monica Freeway in Los Angeles. In both of these projects no capacity was added to the system, but rather existing automobile lanes were reserved for high-occupancy vehicles. Neither of these projects achieved sufficient diversion to high-occupancy vehicles, possibly because they were terminated at an early stage for other reasons.

Obviously, many local factors, such as enforcement, marketing, and geometric design, have contributed to the early termination of such projects. However, this paper suggests that such projects might not be beneficial even if the flows are allowed to stabilize, due to the equilibrium characteristics of the problem. At the new equilibrium point, the total travel time (in person hours) might be higher than it was before.

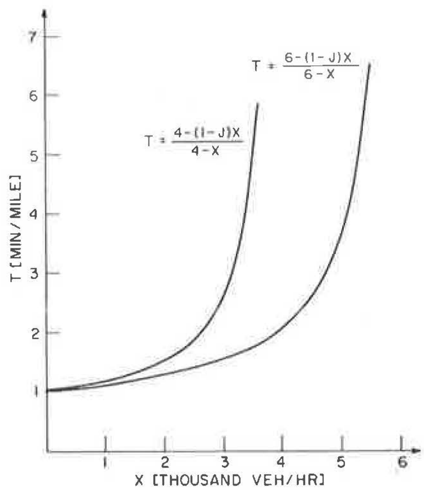
The analysis offered here is very simplistic and the actual results in a particular case would naturally depend on the actual demand and congestion functions involved. However, it seems that only under conditions of quite high congestion would benefits be realized.

A detailed analysis of priority lanes had been performed by May and others at the University of California in Berkeley (1-4) by using simulation methods. Such methods can obviously handle many more factors and considerations and (unlike the analysis presented here) are suited for a detailed design or a feasibility study.

Our analysis assumes two modes only (buses and cars) on one freeway segment. It can be extended to additional modes and more-realistic conditions at the expense of somewhat complicating the analysis. With the present scope of the analysis, the reader can follow the formulas and results with the aid of a pocket calculator.

The paper is organized as follows: The next section presents the equilibrium framework and the model from which the total travel time (before and after the implementation of the exclusive lane) can be computed. The performance measure and analysis of

Figure 1. Flow versus travel-time curves for three- and two-lane highway segment 1 km long (T_o = 1 min/km, J = 1/2).



some numerical examples are presented in the following section.

THE MODEL

Consider a three-lane freeway segment of length L miles that leads from Residence City to the central business district (CBD). Let the volume-delay curve associated with this freeway segment be as follows:

$$T_c = L \cdot T_o \{[6 - (1 - J)q]/(6 - q)\} \quad (1)$$

where

- T_c = automobile travel time per kilometer (h),
- q = flow of vehicles (in private-car units) (thousands of cars/h),
- J = parameter of the volume-delay function, and
- T_o = free-flow travel time (min/km).

[All quantities, such as car and bus travel times, flows, and occupancy factors, referred to in this paper are averages for the analysis period (say, peak) over a sufficiently large number of days.] Equation 1 has been suggested as a model of congestion by Davidson (5) and an estimation procedure has been reported by Taylor (6). This curve is shown in Figure 1. It is based on three lanes of freeway, each of which has an absolute capacity of 2000 vehicles/h. In Figure 1 we have assumed J = 1/2, L = 1 km, and T_o = 1 min/km.

We assume that the flow of vehicles consists of a flow of cars (F_c) and a flow of buses (F_b). If we denote the flow of car users by X_c and the flow of bus users by X_b, the vehicles and occupants flows are connected through the occupancy factors O_c and O_b for the cars and buses, respectively. In other words, $F_c = X_c/O_c$, and $F_b = X_b/O_b$. Let the total flow of users of the road segment under study be denoted by N, i.e., $N = X_b + X_c$. In Equation 1, we assume that $q = \alpha F_b + F_c$, where α is the equivalent of a bus in private-car units (typically 1.5 - 3.0).

In mixed-mode traffic, the bus travel time (T_b) equals the car travel time plus additional collection-distribution time (T_s). Thus $T_b = T_c + T_s$.

Let us assume that the mode split between the cars and buses is given by a logit mode-choice function. If we define the measured utility of the car and bus modes as V_c and V_b, respectively, the share of car users is given by

$$X_c/N = 1/[1 + \exp(V_b - V_c)] \quad (2)$$

where it is assumed that we are dealing with an aggregate mode-choice model or, alternatively, that the naive aggregation approach is used. [The logit function as a demand model is discussed by Domencich and McFadden (7), by Richards and Ben-Akiva (8), and by many other authors. The aggregation problem and in particular the naive aggregation approach have been discussed by Koppelman (9) and by Bouthelier and Daganzo (10).] Assume that a mode-choice model has been estimated for the problem under consideration and the resulting parameters are as follows:

$$V_c = -\Theta T_c + \Psi \quad (3a)$$

$$V_b = -\Theta T_b \quad (3b)$$

In this model, Θ is the coefficient of the (generically specified) travel-time variable, and Ψ includes all other parameters and variables in the model. It is reasonable to expect Ψ to be strictly positive since, at equal travel time, we expect the car share to be more than half. In fact, Ψ can be expressed in terms of the existing flows and the product of Θ and T_s. By using the logit formula with the definitions of Equations 3, it is not difficult to see that

$$\Psi = \log(X_c/X_b) - \Theta T_s \quad (4)$$

Now consider the dedication of one of the freeway lanes for exclusive bus use. Since congestion on the two remaining freeway lanes would increase, some users would divert to the bus, and the system would reach another equilibrium point.

The volume-delay curve that corresponds to a two-lane highway is given by

$$T_c' = L \cdot T_o \{[4 - (1 - J)F_c']/(4 - F_c')\} \quad (5)$$

The primed variable refers to the values of all the previously defined components after the introduction of the exclusive lane. The function given in Equation 5 is depicted in Figure 1 for J = 1/2, L = 1 km, and T_o = 1 min/km.

The third lane is reserved for buses, which operate at constant (not flow-dependent) speed. We assume that the bus travel time equals the free-flow car travel time plus some collection-distribution time; i.e., $T_b' = T_o + T_s$.

In order to keep the analytics trivial, we assume that the total number of person trips (N) remains fixed and so do the vehicle occupancy factors. The first assumption is reasonable for work trips, whereas the second assumes the typical behavior of a bus operator, i.e., keeping the load factor constant.

Thus, the total travel time before the introduction of the exclusive lane is given by

$$T_t = (X_c + X_b)T_c + X_b \cdot T_s \quad (6)$$

or, substituting Equation 1 for T_c,

$$T_t = (X_c + X_b) \cdot L \cdot T_o \{[6 - (1 - J)q]/(6 - q)\} + X_b \cdot T_s \quad (7)$$

Substituting $q = F_b + F_c$ and the definitions of F_b and F_c in terms of X_b and X_c, respectively, the total travel time (in person minutes) becomes

$$T_t = (X_c + X_b) \cdot L \cdot T_o \left(\{6 - (1 - J)[\alpha(X_b/O_b) + (X_c/O_c)]\}/ \{6 - [\alpha(X_b/O_b) + (X_c/O_c)]\}\right) + X_b \cdot T_s \quad (8)$$

The total travel time with the exclusive lane is given by

$$T_t' = X_c' \cdot T_c' + X_b' \cdot T_b' \quad (8a)$$

Substituting T_c' and T_b' as in the derivation of Equation 6, the total travel time (in person minutes) becomes

$$T_t' = X_c' \cdot L \cdot T_o \{[4 - (1 - J)(X_c'/O_c)]/[4 - (X_c'/O_c)]\} + (N - X_c')(T_o L + T_s) \quad (9)$$

where $(N - X_c')$ replaces X_b'.

In the last equation, X_c', the equilibration flow of car users, is unknown. However, the equilibrium condition (Equation 2) holds after introduction of the exclusive lane as well and can be used to find X_c'; i.e.,

$$X_c'/N = 1/[1 + \exp(V_b' - V_c')] = 1/\{1 + \exp[\Theta(T_c' - T_b') - \Psi]\} \quad (10)$$

Substituting for T_c' and T_b', one obtains

$$X_c' = N \cdot \left\{1 + \exp\left[\Theta\left(LT_o\{[4 - (1 - J)(X_c'/O_c)]/[4 - (X_c'/O_c)]\} - (T_o L + T_s)\right) - \Psi\right]\right\}^{-1} \quad (11)$$

Equation 11 is a simple fixed-point problem in the equilibrium car flow X_c'. The equation can be easily solved numerically (by using, say, a programmable calculator) for X_c', given the values of L, N, Θ, J, O_c, T_o, T_s, and Ψ. Instead of using Ψ, one can alternatively use $\{\log[(N - X_b)/X_b] - \Theta T_s\}$ (see Equation 4), thus introducing the "before" bus-users' flow (or share) as a parameter in the model. In order to evaluate Equation 8, the parameters O_b and α must be specified as well.

We now examine the total travel time in the system before and after the introduction of the exclusive bus lane.

ANALYSIS

This section analyzes the mode split and the total travel time before and after the institution of the exclusive bus lane. We also change parametrically the values of all inputs to Equations 8, 9, and 11 in order to determine the ranges in which the exclusive bus lane is advantageous. The criterion used here is the ratio of the total travel time after the introduction of the bus lane to the total travel time before. Let R denote this ratio; i.e.,

$$R = T_t'/T_t \quad (12)$$

where T_t and T_t' are given by Equations 8 and 9, respectively. Note that the ratio specification eliminates L from Equation 12. It only enters through Equation 11, in which only the product $\Theta \cdot L$ affects the result.

Let us assume the following values of the model's parameters:

L = 20 km,
T_o = 1 min/km,
J = 0.5,
α = 3 private-car units,
O_c = 1.2 persons/car,
O_b = 40 persons/bus, and
T_s = 10 min.

These parameters can be thought of as site specific. We will now investigate the dependency of the ratio R on the total volume of users (N). In conjunction with the investigation of this function, we conduct a sensitivity analysis on the demand-model parameters (Θ and Ψ).

Figure 2 depicts R as a function of N for Θ = 0.05 and Ψ = 0.5, 1.0, 2.0, and 2.5. (Some of the values on which Figure 2 is based are given in Table 1.) Since R is defined as the ratio of total travel time after the implementation of the bus lane to the total travel time before, $R > 1$ indicates that the exclusive lane worsens the level of service. The lane exhibits net benefits only for $R < 1$.

As seen from Figure 2, the ratio is rising at moderate levels of congestion, peaking, and decreasing as the total population increases. Beyond a certain level of congestion, the exclusive lane becomes favorable. As congestion increases (N increases), one can note two competing effects. Even though the share of car users drops with increasing N (and relative to the car share before), as is evident from Table 1, the number of users increases with N. Those car users are realizing conditions that are worse than before. It is reasonable to believe that the last effect is stronger than the former one, thus explaining the increase in R. The parameter that controls this effect in the demand

Figure 2. Ratio of total travel time before and after instituting preferential lane versus total flow for different values of Ψ.

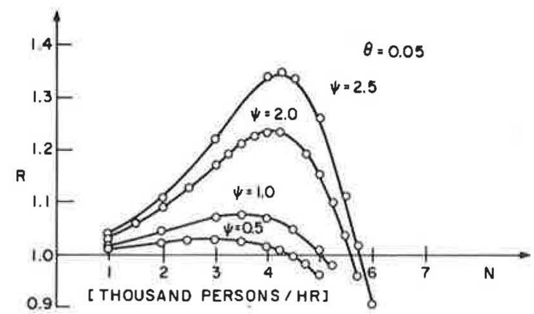


Table 1. Predicted statistics before and after the project as Ψ and N vary.

Ψ	N	X_c'	X_c	T_c'	T_c	R
0.5	1	0.714	0.731	21.746	21.172	1.010
	2	1.379	1.462	24.032	22.655	1.021
	3	1.972	2.193	26.973	24.592	1.027
	4	2.464	2.924	30.548	27.229	1.014
	5	2.841	3.655	34.506	31.030	0.966
1.0	1	0.802	0.818	22.006	21.310	1.020
	2	1.558	1.635	24.805	23.015	1.045
	3	2.231	2.453	28.687	25.326	1.069
	4	2.773	3.270	33.685	28.633	1.069
	5	3.158	4.088	39.225	33.762	1.007
2.0	1	0.916	0.924	22.357	21.485	1.034
	2	1.801	1.848	26.003	23.488	1.089
	3	2.611	2.772	31.927	26.337	1.169
	4	3.244	3.697	40.857	30.713	1.233
	5	3.621	4.621	50.700	38.289	1.152
	6	3.817	5.545	58.825	54.605	0.873
2.5	1	0.947	0.953	22.457	21.533	1.039
	2	1.872	1.905	26.392	23.620	1.105
	3	2.736	2.858	33.253	26.630	1.216
	4	3.416	3.810	44.679	31.348	1.337
	5	3.785	4.763	57.279	39.804	1.258
	6	3.955	5.715	66.808	59.351	0.907

Note: $L = 20$, $T_o = 1$, $J = 0.5$, $O_c = 1.2$, $T_s = 10$, $\alpha = 3$, $O_b = 40$, and $\Theta = 0.05$; variables are defined in text.

function is Ψ, which can be interpreted as the pure car bias. This now also explains why, in Figure 2, R increases with increasing Ψ.

Nevertheless, beyond a certain point (given Θ and Ψ), the number of car users stabilizes and the fact that more and more users choose the bus causes the ratio to start decreasing. Note, however, that no congestion on the exclusive lane is included in the model, and thus the R-values for the congested part of the figure are somewhat biased in favor of the exclusive-lane proposition.

Figure 3. Ratio of total travel time before and after instituting preferential lane versus total flow for different values of Θ.

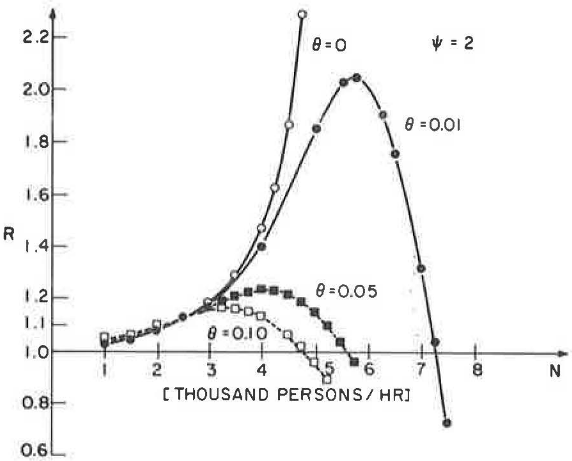
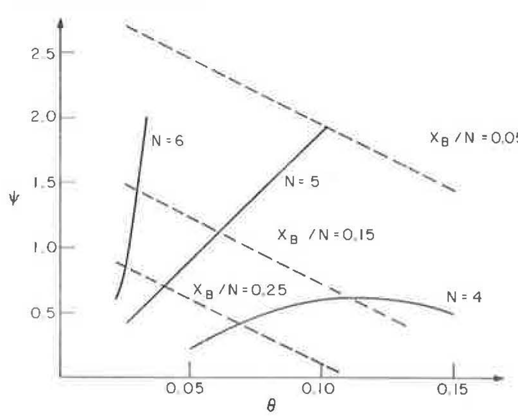


Table 2. Predicted statistics before and after project as Θ and N vary.

Θ	N	X_c'	X_c	T_c'	T_c	R
0.01	1	0.889	0.891	22.272	21.430	1.027
	2	1.770	1.782	25.843	23.337	1.078
	3	2.636	2.673	32.176	26.008	1.178
	4	3.454	3.564	45.647	30.017	1.399
	5	4.099	4.455	78.485	36.702	1.846
	6	4.397	5.345	129.098	50.096	2.005
	7	4.502	6.2263	171.095	90.452	1.319
0.05	1	0.916	0.924	22.357	21.485	1.034
	2	1.801	1.848	26.003	23.488	1.089
	3	2.611	2.772	31.927	26.337	1.169
	4	3.244	3.697	40.857	30.713	1.233
	5	3.621	4.621	50.700	38.289	1.1522
	6	3.817	5.545	58.825	54.605	0.873
0.10	1	0.940	0.953	22.436	21.537	1.040
	2	1.831	1.905	26.168	23.620	1.100
	3	2.586	2.858	31.680	26.630	1.160
	4	3.080	3.810	37.912	31.348	1.134
	5	3.344	4.763	42.971	39.804	0.960

Note: L = 20, T_o = 1, J = 0.5, O_c = 1.2, T_s = 10, α = 3, O_1 = 40, and ψ = 2; variables are defined in text.

Figure 4. Regions of demand-function parameters in which exclusive-lane project is advantageous.



When the values of Ψ are very low, this second effect is more pronounced. A low value of Ψ means that users react principally to travel-time differences. Our example would correspond in this case to fixing the travel time on an existing highway lane at ($T_s + LT_o$) and eliminating congestion effects on this lane. This, of course, is an unrealistic scenario. By using Equation 4, one can get a feeling for which values of Ψ are associated with different preimplementation mode-split levels. For Θ = 0.05, a bus share of between 25 and 5 percent is associated with values of Ψ between 0.6 and 2.4, respectively. For such values, the exclusive lane is appropriate only for N between 4.5 and 5.7. Such a use level of the facility corresponds to congestion that approximately doubles to triples the free-flow travel time.

We now turn to investigate the model's sensitivity to the values of Θ. Figure 3 depicts R versus N for Ψ = 2 and Θ = 0.01, 0.05, and 0.10. (Table 2 gives some of the values on which Figure 3 is based.)

The general shape of the curves is similar to that in Figure 2. A low value of Θ means that travel time is not a major determinant in the mode-choice decision. The associated values of the ratio R would be high, since individuals would keep choosing the automobile mode even though the car travel time is growing as congestion grows. At the extreme (Θ = 0), the curve would not have a downward-sloping part at all.

At higher values of Θ, users respond more and more to the travel-time differences and the share of bus riders grows; this leads to a reduction in R. (This effect was discussed in the context of Figure 2.) From Figure 3 one can see that for Ψ = 2, the exclusive lane becomes appropriate for N = 4800 users/h (which corresponds to Θ = 0.10) and N = 7300 users/h (which corresponds to Θ = 0.01). These values correspond to travel times on the remaining two car lanes that are between two and nine times the free-flow travel times.

Figure 4 shows regions of values of the demand-model parameters Θ and Ψ in which the exclusive-lane project would be warranted. (The values of the rest of the variables are identical to those fixed in Tables 1 and 2.) In general, for a given number of total person trips, the project would be favorable when Θ is high and Ψ is low. Thus, for a given N, the project is favorable when the values of Θ and Ψ are located to the right and below the corresponding N-value curve.

The dashed lines in Figure 4 indicate combinations of Θ and Ψ in which the preproject bus mode share (X_b/N) is 5, 15, and 25 percent. Based on these shares and the total volume, one can get an idea of the probability of success of the exclusive lane, given the values of all the rest of the mode parameters as defined in the beginning of this section.

DISCUSSION OF RESULTS

In this paper, we have tried to show that, under general assumptions, dedicating a freeway lane for bus use yields net benefit only under conditions of relatively heavy congestion.

So far, only the sensitivity of our model to the demand-function parameters was discussed. The other parameters of the problem were fixed at the values presented at the beginning of the section on analysis. The effect of these parameters can be determined from the model's equations. This is discussed next.

Increasing the segment length (L) or the free-flow travel time would have an effect that is quite similar to the effect of increasing θ, i.e., a lower R-value and favoring the project at lower volumes. This can be seen from Equation 11. The effect of increasing the collection-distribution times (T_s) is similar to the effect of increasing Ψ, which is contrary to the effect of increasing θ. The effects of the car-occupancy parameter (O_c) and the congestion-curve parameter (J) are similar; both cause the congestion curves to be effectively lower. Lowering the congestion curves has a similar effect to using lower volumes to enter these curves and thus the exclusive lane would be less favorable if either O_c or J is increased, all other parameters being equal. The private-car-unit parameter (α) and the bus-occupancy parameter (O_b) would not substantially affect the results. In general, as α/O_b increases, the flow (in private-car units) in the base case, for a given N, is larger. Thus the ratio R would tend to be lower and the project more favorable.

The model presented in this paper is very simple and does not pretend to capture the subtleties of the real situation. However, it is suggested only as a framework for a more-complete analysis on the subject, which should precede the implementation of a similar bus project. Such a simple analysis can capture, in many cases, the important elements of equilibrium attained through the interaction of demand and performance (supply) relationships and be used for a first-cut or sketch-planning tool in other contexts. In the context of bus priority lanes, such analysis should indicate that a more-comprehensive in-depth study should be carried out since the benefits of such projects as bus priority lanes are not obvious.

The model presented in this paper can be trivially extended to include a carpooling model and a lane for high-occupancy vehicles rather than a lane for buses. One should also include a calibrated demand model and congestion function as well as a more-accurate aggregation method. This, however, extends the analysis and one would require more than a programmable calculator to carry out the model estimation, aggregation, and equilibration.

In closing, we note that extending the analysis method to include carpooling on the high-occupancy-vehicle lane would mean that our no-congestion assumption on the exclusive lane would become questionable, especially at the high congestion levels at which the project seems attractive. Note also that at higher congestion levels there is more accident potential, a fact that was not included in our model but whose effect would be to make the exclusive lane an even less-desirable project.

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Car-Ownership Forecasting Techniques in Great Britain

A. D. PEARMAN AND K. J. BUTTON

The prospect of continuing changes in the relative prices of different energy sources and of energy as a whole with respect to the general price level has heightened interest in the forecasting of car ownership and use. In Great Britain, two main schools of thought exist concerning aggregate forecasting techniques. The longer-established of these uses straightforward projections from a logistic curve of car ownership per capita calibrated mainly on the basis of national-level time-series data. This technique, however, has lately been subject to increasing criticism. As a result, a second approach, closer to recent American work and based largely on cross-sectional calibration, has now emerged and is increasingly finding favor in government circles. The developments that have taken place in Great Britain in national-level forecasting techniques are described and assessed. Then recent advances in local-level forecasting are described and particular reference is made to a detailed study of 10 000 households in the West Yorkshire conurbation. Special emphasis is placed on the role of family structure and employment status in influencing car ownership and also on the importance of accessibility to facilities by public transport. In the final section, those areas in which further work is particularly needed and the importance of intrahousehold interaction and the relations among accessibility, public transport provision, multicar ownership, and energy prices are discussed.