A Probabilistic Model of Gap Acceptance Behavior

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Although gap acceptance measures are often used in traffic engineering problems, there are no known, widely used techniques to account for the distribution of gap acceptance behavior at a specific site. The use of a simple-to-calibrate logit function to model the distribution of gap acceptance behavior is described. Because of the ease of modeling and the resulting good fit derived for this application of the model, it is believed that similar efforts could be conducted to model the actual distribution of gap acceptance behavior for use in specific design problems. However, further study with greater quantities of data and at other locations is recommended.

This paper describes an empirical study of the gap acceptance behavior of drivers merging from a stop on a one-way, single-lane street into the major flow on a one-way, single-lane arterial. The distribution of gap acceptance is a model with a logit function. By use of a logit function, the cumulative probability of accepting a gap of a specific length is modeled with a reasonably good fit.

Measures of gap acceptance enter into the calculation of the capacity of unsignalized intersections, warrants for stop signs, the capacity of weaving and merging areas, and other design problems. Although gap acceptance measures are important to design, there is no known, widely accepted technique that permits the design engineer to account for the distribution of gap acceptance behavior at a specific design study location. The Transportation Research Board's recent interim update of the Highway Capacity Manual (1) accounts for gap acceptance behavior with average, aggregate measures. These measures are used despite the fact that gap acceptance behavior is known to vary in different locations with respect to not only the mean length of gaps accepted but also the skew of the distribution of gap lengths accepted (2).

In cases where data on gap acceptance behavior have been collected for specific sites, they have traditionally been gathered by using cameras and pen recorders. Not only does this method imply great drudgery but also, as one recent gap acceptance study indicated, it cannot reliably measure gap lengths at smaller intervals than 0.5 s (3)--not to mention the possibility of error involved in transcribing data from films and pen recorders to a form usable for data processing.

Because of the cost, drudgery, and error involved in collecting data, and because there are no easy-to-use methods for synthesizing distributions of gap acceptance behavior even if data are collected, it is understandable that average, aggregate measures are often resorted to in design problems.

This paper presents a simple but robust means of synthesizing a distribution of gap acceptance behavior. The paper is divided into the following sections:

1. A description of the theoretical model that structures the inputs of the gap acceptance decision;
2. A description of the data collection site, the means by which data are collected, and the data themselves;
3. A description of the empirical model and its calibration; and
4. The conclusions derived from the empirical investigation.

CONCEPTUAL MODEL

To structure the model, a description of the behavior of drivers when they are confronted with a gap acceptance decision is theorized. The theory as defined is no more than fitting a rational decision process into a conceptual, mathematic framework.

Decision Inputs

In setting up a structure for a rational gap acceptance decision, two constraints are invoked:

1. No driver will accept a gap in the major stream that he or she believes will certainly lead to a collision.
2. No merging driver gains admittance to the major stream through intimidation of major-stream drivers.

These constraints are sometimes violated, but such types of behavior are considered irrational and are dropped from consideration. Given these two constraints, the inputs to the decision process can be postulated as follows.

Driver Risk

Risk is the value the driver places on the probability of collision during a merge with the major flow of traffic. All drivers are assumed to be adverse to accepting a gap in the major stream that implies a high degree of risk. The driver assigns a positive value to the risk of accepting all gaps, and the value of risk will become large as the gap length becomes small. A driver will decide to accept a gap only if the value of risk is less than the value assigned to the estimated delay of waiting for a larger gap.

Value of Delay Time

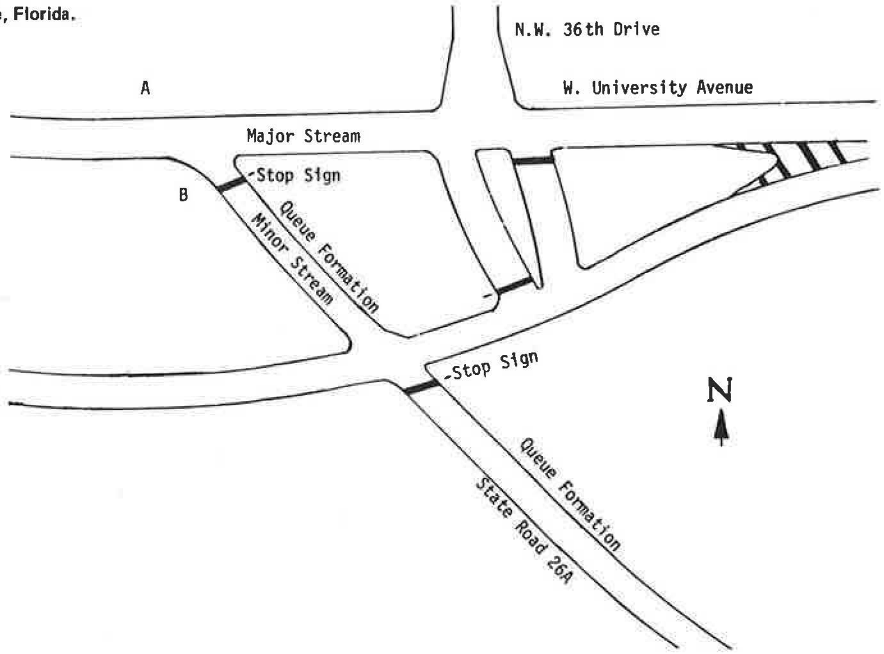
The value of delay due to gap refusal is the driver's estimated value of the time that will elapse until a suitable gap occurs. The driver's assessed time value of delay is always positive and is modified by (a) the length of time spent in the queue at the intersection and (b) the traffic volume in the major stream.

The length of time spent in the queue is a measure of exposure to irritation caused by the gap acceptance process. The degree of irritation the driver has been exposed to will modify the driver's weighting of the time value of delay. In other words, as the driver approaches the head of the queue, the estimated time value of delay will be marginally increasing with additional delay.

As traffic volumes in the mainstream increase, the merging driver understands that the delay caused by waiting for a larger gap will become greater. Therefore, the driver's estimated time value of delay due to gap refusal increases with increasing mainstream traffic volumes.

In another study (4), these modifications of driver behavior were loosely titled "pressure of traffic demand". Although the term "pressure" does conjure the correct image of what takes place as traffic demand increases, in reality pressure has nothing to do with gap acceptance behavior. What is actually being modified by increased traffic demand is the driver's estimate of the value of delay time.

Figure 1. Study intersection in Gainesville, Florida.



Decision Process

The two inputs into the gap acceptance decision process, the value of risk and the value of delay time, can be structured into a model of gap acceptance behavior. In the structure, risk is assumed to be independent of the length of the minor-stream queue and the major-stream traffic volume. If the value of risk assigned to accepting a gap is greater than the assigned value of delay due to not accepting a gap, then the gap is refused. If the value of risk assigned to accepting a gap is less than the assigned value of delay due to not accepting a gap, then the gap is accepted. This decision process is defined in Equations 1-3:

$$(VR_i) > (VT_i) \times f(Q_i, V_i)_{\text{gap refusal}} \quad (1)$$

$$(VR_i) < (VT_i) \times f(Q_i, V_i)_{\text{gap accepted}} \quad (2)$$

$$(VR_i) = (VT_i) \times f(Q_i, V_i)_{\text{undefined}} \quad (3)$$

where

VR_i = value assigned to the risk of accepting gap i,
VT_i = value assigned to the time penalty estimated for refusal of gap i,
$f(Q_i, V_i)$ = function that accounts for modifications in driver delay-time judgment,
Q_i = queue length in minor stream at time of gap i, and
V_i = mainstream traffic volume at time of gap i.

COLLECTION AND PREPARATION OF FIELD DATA

An unsignalized intersection in the western part of Gainesville, Florida, was chosen for the study of gap acceptance behavior (see Figure 1). This intersection was designed so that a single-lane, one-way, major-stream movement intersects a single-lane, one-way minor stream. The intersection was observed during the period of greatest congestion, the afternoon peak.

During the afternoon peak, the minor-stream traffic would back up and build a large queue. During the congested period, acceptable gaps in the mainstream became less frequent. Saturation of the intersection tested gap acceptance behavior under critical conditions.

The data collection was conducted during a Friday afternoon in the autumn over a period of 2-3 h. The equipment was set up on the north side of University Avenue, across the street from the merge area (point A in Figure 1). Three people were used to record gap acceptances. Two observers operated hand-held switches. One of these two observers recorded a signal whenever, in time, a mainstream vehicle entered the merge area, thus measuring the time length of all mainstream gaps. The second observer recorded, in time, the acceptance of a gap by a minor-stream vehicle. The data were stored by using the MEMODYNE system, which stores data on magnetic tape (cassette) and records the time lengths of gaps by means of an internal clock.

The third observer measured the length of the minor-stream queue at 1-min intervals. The queue-length data were later merged with the data stored by the MEMODYNE system.

The field data were transferred to disk storage at the University of Florida computer facilities. The data were then formatted for analysis with a standard statistical software package (5).

EMPIRICAL MODEL

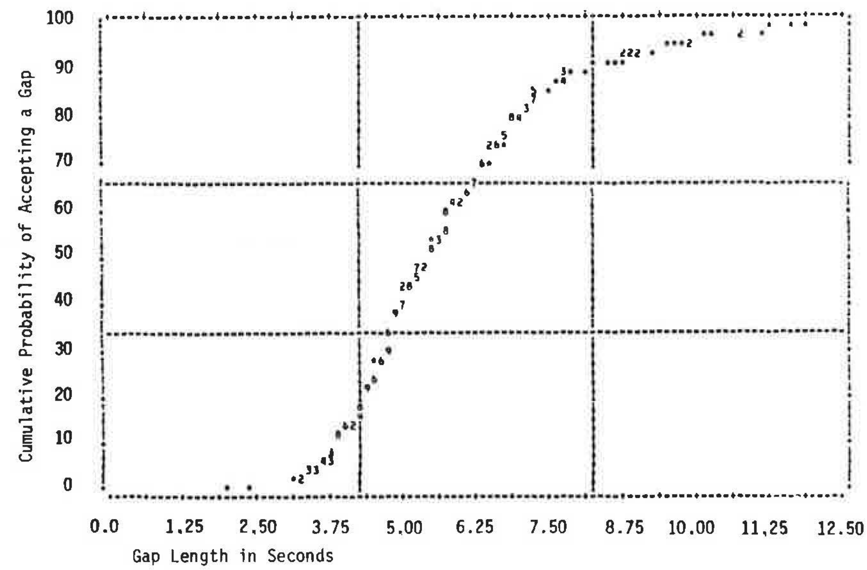
The conceptual decision inputs included an assigned value of risk and the estimated value of delay due to gap refusal. To say the least, measurements of such driver perceptions would be difficult to collect. Thus, the weights drivers placed on inputs are estimated through regression. The driver-perceived values are hypothesized to take the following form:

$$VR_i = H(t_i) \quad (4)$$

$$VT_i \times f(Q_i V_i) = G(t_i, Q_i, V_i) \quad (5)$$

where t_i is the time length of gap i and where it is observed that, (a) if $H(t_i) > G(t_i, Q_i, V_i)$, then the gap is refused and (b) if

Figure 2. Gap length versus cumulative probability of gap acceptance.



$H(t_i) < G(t_i,Q_i,V_i)$, then the gap is accepted. Since the model of gap acceptance behavior is only interested in gaps accepted, only events where formula b held are examined here. In addition, because the variable used to model the value of risk is a subset of the variables used to model time value of delay and because the events examined are a homogeneous set (only gaps accepted), the model can be condensed to the following:

$$H(t_i) - G(t_i, Q_i, V_i) = F(t_i, Q_i, V_i) \qquad (6)$$

Analysis Technique

Choice modeling is commonly done by using the cumulative probability of making a certain choice. In the case of gap acceptance, the choice is whether or not to accept a gap. Such choice phenomena are often modeled by use of either probit or logit analysis.

Probit analysis has been used in the past to synthesize distributions of gap acceptance behavior. Notable examples are Solberg's and Oppenlander's analysis of unsignalized intersections (2) and Drew's analysis of merging at freeway ramps (6). In these studies, a probit functional form was used to model the cumulative probability of accepting gaps of varying lengths. Probit analysis fits the dependent variable to a normal, cumulative probability distribution. To estimate a model of this functional form requires the use of maximum likelihood. Although maximum-likelihood procedures have been greatly improved, maximum likelihood is still cumbersome.

In a recent study of gap acceptance, Radwan and Sinha (3) modeled the cumulative probability of accepting gaps of varying lengths with logit analysis. They collected data by using time-lapse photography and a 20-pen recorder. By using data collected from a stop-controlled multilane intersection, Radwan and Sinha constructed a biased model by forcing a symmetrical logit function to fit what they admit is a skewed distribution (median is not equal to the mean). Although their model specification is biased, Radwan and Sinha have provided an example of the applicability of logit analysis in modeling gap acceptance behavior.

Because of the logit's simplicity of calibration, it is chosen for use in this study. The logit closely approximates the probit and may be linearly transformed to provide easy estimation of model parameters with linear regression. The simple, dichotomous choice logit functional form is

$$P = \{1/[1 + e^{F(x)}]\} \; -\infty \; F(x) < \infty \qquad (7)$$

and its linear transformation is

$$\ln[P/(1 - P)] = F(x) \qquad (8)$$

where

P = cumulative probability of accepting a gap,
x = variables related to the gap acceptance decision, and
F(x) = linear function.

Dependent Variable

The dependent variable, the cumulative probability of accepting a gap of a specific length, is calculated by using the following equation:

$$P_i = (d_i/N) \; 0 < P < 1 \qquad (9)$$

where

P_i = cumulative probability of accepting a gap of time length i,
d_i = number of gaps accepted of time length i or less, and
N = total number of events.

A plot of the values derived from the calculation of P_i (Equation 9) is shown in Figure 2. The uniform S-shape of the data points is the first clue that the study is on the right track in using a cumulative probability functional form to model gap acceptance.

The plot of the cumulative probability of accepting a gap, shown in Figure 2, is skewed. The mean length of the gaps accepted is greater than the value that coincides with the gap length that was accepted by 50 percent of the sample (median). The slope on the lower part of the curve is steep and then tends to flatten at the top. In other words, marginal change in the bottom of the curve is greatest, and the marginal change decreases as the curve is followed to the top.

In the data preparation, the problem of multiple drivers accepting one gap arose. Because the majority of gap acceptance observations are single-vehicle acceptances (261 events), the analysis is limited to acceptances of gaps by one vehicle. Presumably, with more data the model could be expanded to include multiple acceptance. However, the objective of this study is to show how a simplistic technique can be used to efficiently model the gap acceptance decision. Therefore, it is assumed that only modeling single-vehicle gap acceptance would prove the case for logit analysis.

Independent Variables

Drew (6) modeled the cumulative probability of accepting a gap with the following specification:

$$P_i = F_p[\alpha + \beta(\log t_i)] \quad (10)$$

where

α = intercept,
β = slope coefficient, and
$F_p(\ )$ = probit functional form.

Drew skewed his independent variable by using the logarithm of t_i instead of t_i. The model specified here is similar to Drew's, but the skew is accounted for in a different manner. The conceptual model specified in Equation 6 is specified by using the linear form given below:

$$P_i = F_L(B_0 + B_1 X_i + B_2 Q_i + B_3 V_i) \quad (11)$$

where

B = slope coefficient,
X_i = $(\bar{T}/t_i) - 1$,
$\bar{T}$ = mean time length of all gaps accepted, and
$F_L(\ )$ = logit functional form.

X_i is used as the independent variable in Equation 11 instead of t_i for two reasons:

1. $\bar{T}/t_i$ is used instead of t_i because, when $t_i \leq \bar{T}$, the changes in the values of $\bar{T}/t_i$ range from one to infinity and, when $t_i \geq \bar{T}$, the values of $\bar{T}/t_i$ range from zero to one. Thus, the marginal changes of $\bar{T}/t_i$ are greatest when $t_i \leq \bar{T}$, which is consistent with the skew of the distribution of gap lengths accepted (Figure 2).

2. One is subtracted from $\bar{T}/t_i$ so that, when $t_i = \bar{T}$, the value of the independent variable would be zero. This made the regression parameters easier to interpret.

Estimations

Neither mainstream traffic volume nor queue length is found to have statistically significant slope coefficients. Because they add nothing to the model, they are dropped. Not including them does not seem to affect the strength of the final estimate. The fact that the mainstream volume does not modify gap acceptance behavior does not seem as surprising as queue length not having a significant impact on gap acceptance behavior. Sometimes the queue was as long as 20 cars or more, a delay that would seem long enough to modify behavior. The bias in the model of deleting volume queue length is examined later.

The following linear model was estimated by using ordinary least squares:

$$Y_i = \alpha + \beta(X_i) + E_i \quad (12)$$

where $Y_i = \ln[P_i/(1 - P_i)]$ and E_i = stochastic error. The resulting parameter estimates and regression statistics are as follows:

$$Y_i = 0.422 - 0.965[(\bar{T}/t_i) - 1] \quad \text{t-statistic of } \beta_1 = 10.599$$
$$R^2 = 0.931$$
$$F = 3525.667 \quad (13)$$

Caution must be used in accepting the regression statistics as totally valid. In their calculation, it is assumed that the function estimated was linear. Still, the fit that is found by using $[(\bar{T}/t_i) - 1]$ is quite good, and the true statistics will be close to those formulated.

Bias Due to Deleted Variables

In this instance, the volume of mainstream traffic demand does not appear to have a significant impact on the length of gaps accepted. However, the lack of significance in this case does not mean that mainstream volume does not have an impact on gap acceptance in general. Study of other intersections

Figure 3. Gap length versus cumulative probability of gap acceptance for queue length of 1-5 automobiles.

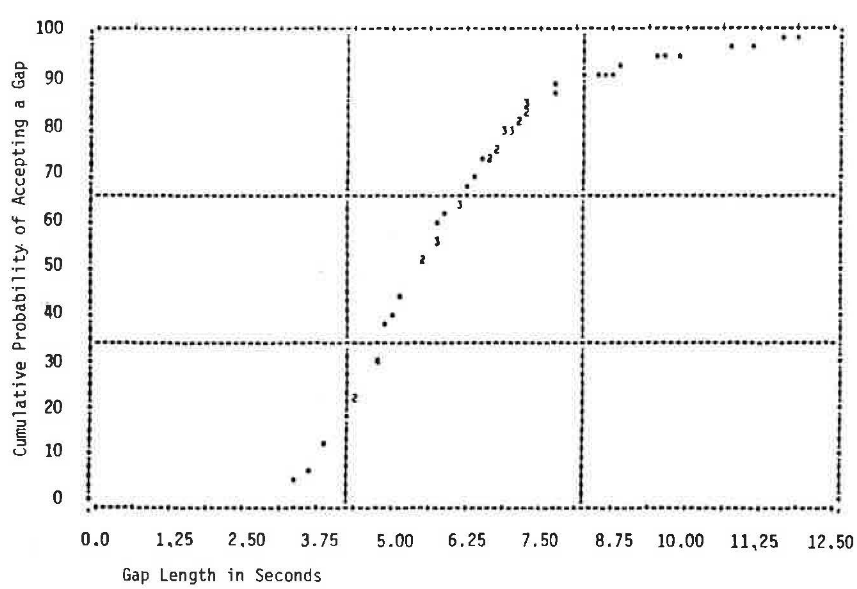


Figure 4. Gap length versus cumulative probability of gap acceptance for queue length of 6-10 automobiles.

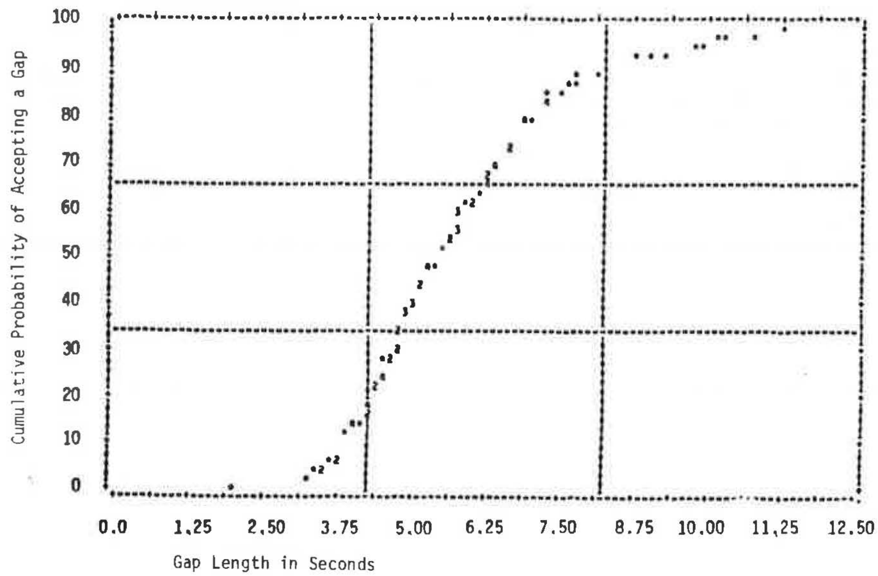


Figure 5. Gap length versus cumulative probability of gap acceptance for queue length of 11-15 automobiles.

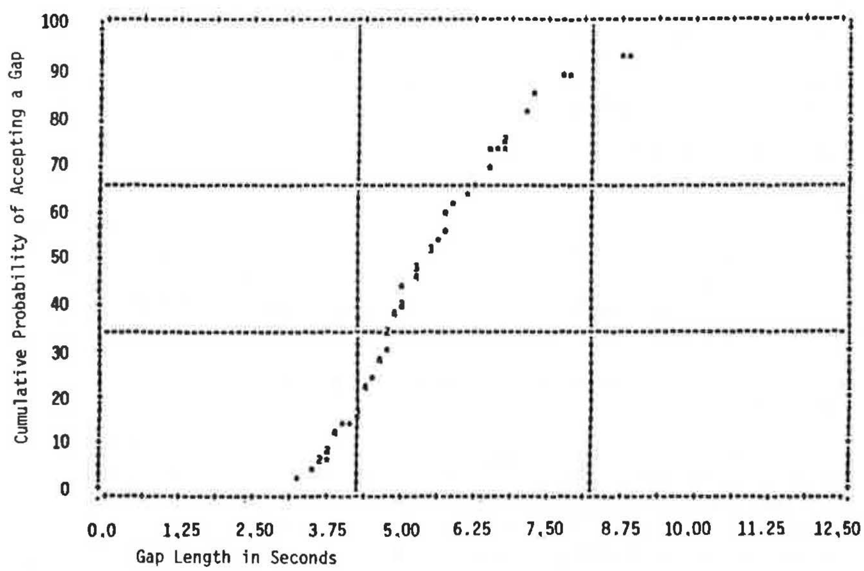


Figure 6. Gap length versus cumulative probability of gap acceptance for queue length of 16-20 automobiles.

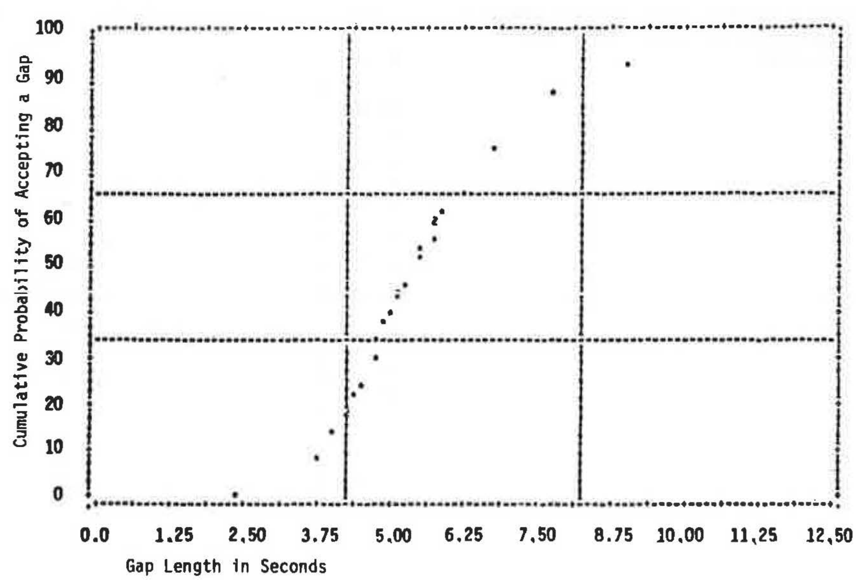
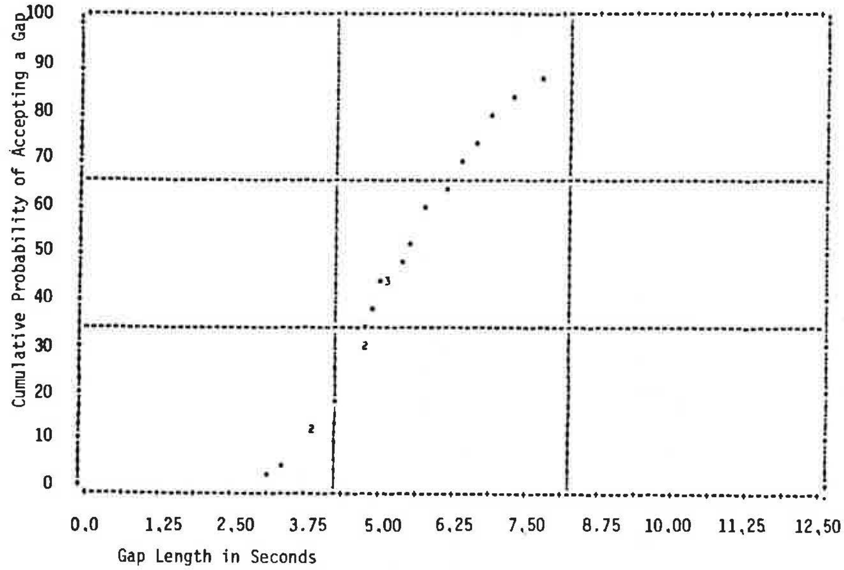


Figure 7. Gap length versus cumulative probability of gap acceptance for queue length of ≥20 automobiles.



is needed before concrete evidence will be available.

More surprising is that the length of the queue does not seem to affect the length of the gap accepted. The median gap lengths accepted as queue lengths increased (minimum gap lengths accepted by 50 percent of the sample) are given below:

Queue Length (no. of cars)	Median Gap Length (s)
1-5	5.50
6-10	5.55
11-15	5.58
16-20	5.60
≥21	5.58

There appear to be no significant differences with increasing queues. In Figures 3-7, the cumulative probabilities of accepting gaps of various lengths stratified with respect to queue length are plotted. All plots appear to have relatively the same distributions. If the observations are consistent with the theory of the marginally increasing value of additional delay time, then the plots with longer queues should appear steeper and closer to the left-hand side. This does not appear to be the case. Thus, it is assumed that there is minimal bias due to the deletion of queue length from the model. However, the real proof of bias would only come with a more data-intensive effort.

CONCLUSIONS

This paper describes an empirical study of the gap acceptance behavior of drivers merging from a stop on a one-way, single-lane street into the major flow on a one-way, single-lane arterial. When a logit function is used, the cumulative probability of accepting a gap of a specific length is modeled with a reasonably good fit.

This study shows the effectiveness of using a simplistic logit form to model gap acceptance behavior. However, more work should be done to account for variables deleted in this study. In addition, the acceptance of gaps by multiple drivers and under other circumstances, such as in freeway weaving and merging areas, should be investigated. However, the simplistic method described here of accounting for the distribution of gap acceptance behavior would permit the design engineer to have better knowledge of driver behavior at the design location with a minimum of effort.

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