Estimating the Contribution of Various Factors to Variations in Bus Passenger Loads at a Point

ROBERT M. SHANTEAU

A procedure for estimating the relative contribution of various factors to the variation of passenger loads on buses at a point is described. These factors include unequal bus headways and the uncertainty in the number of passengers to arrive in a given time interval. Since overcrowding is undesirable, for a given passenger flow a bus company must use more buses if the variation of loads between buses is large than if it is small. It is assumed that bus arrivals are either so frequent or so unpredictable that passenger arrivals are independent of bus arrivals. It is also assumed that bus headway does not change much over a typical passenger-trip length. Data are presented to show that, in the typical case, unequal bus headways contribute far more to variations of passenger loads on buses at a point than all other factors combined. It is rare that headways are so well controlled that their contribution becomes comparable with that of other factors. Where headways are poorly controlled, the public would most likely benefit from investments in headway-control strategies. In principle, the cost of controlling headways can be balanced against the benefits to find an optimal level of control.

The number of buses an urban transit agency must provide on a busy line is usually determined by the peak passenger flow past the maximum load point. Most often, the objective of the bus company is to use the fewest number of buses while still providing an acceptable level of service, where level of service is defined in terms of overcrowding and/or passenger waiting time.

Overcrowding is undesirable because (a) it causes discomfort and inconvenience to the passengers and (b) it makes circulation within the bus difficult and thus causes the bus to spend more time loading and unloading. In this paper, passenger waiting time is not used as a measure of performance. Instead, passenger loads are used because the bus company itself is mostly concerned with overloading.

Most bus companies try to provide some excess capacity to compensate for variations in loads that occur from day to day. These variations are caused by the stochastic nature of passenger arrivals and by unequal headways (elapsed time between buses). It was recognized as early as 1916 (1, p. 156) that unequal headways can be a major cause of variations in passenger loads.

Since fewer buses are needed on a line for which variations in loads are small than on one for which they are large, it is usually to the economic advantage of the bus company to use a control strategy that attempts to equalize headways (2). However, because traffic conditions and passenger stops cause variations in the travel times of buses, it is difficult to prevent deterioration in the regularity of headways (3). It is useful for the bus company to know what improvements in variations of loads it can expect from a range of control strategies so it can properly allocate its resources between vehicles and control.

PROBLEM STATEMENT

The practitioner is faced with the problem of how to evaluate various strategies without actually implementing them. In this paper, it is assumed that the future is sufficiently like the past that we can predict what would happen under different strategies in the future by analyzing data collected in the past. Basically, we want to relate the load on a bus to its headway in such a way that we can predict its load for different headways but for the same general passenger arrival pattern. First, however, we must set up a structure for the problem in which we can carefully state our assumptions.

ANALYSIS

Bus Trajectories and Passenger Arrivals

Suppose, for a particular day in the past, we plotted the time of arrival, origin stop, and destination stop of every passenger on the line on a time-space diagram, as shown in Figure 1. Here, the x's represent passenger arrivals and the subscripts the destination stop. Multiple subscripts mean that several people arrived at a stop at the same time. On the same diagram, we also plot the trajectory of each bus.

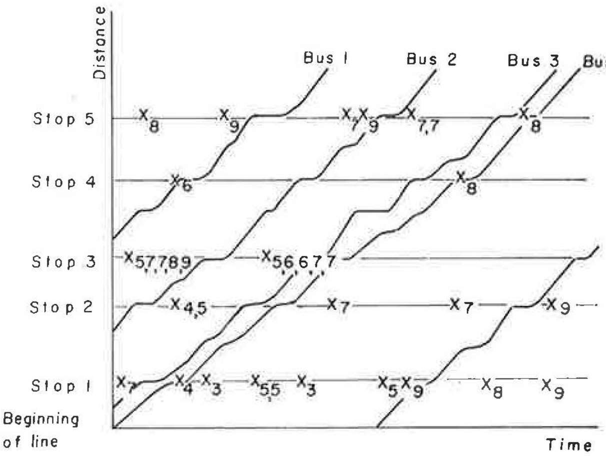
Usually, when a bus stops, it picks up all passengers who have arrived since the last bus left. For instance, in Figure 1, bus 3 picks up two passengers at stop 2, one each destined for stops 4 and 5. Assume that each bus always has room for all passengers waiting for it. Then the load on a bus at a particular point is the number of people who have arrived between that bus's trajectory and the previous one's and who desire to travel beyond that point. Passengers who got off prior to that point or get on after it are irrelevant. For instance, bus 3 is carrying seven passengers as it leaves stop 5. Note that, where bus drivers refuse passengers because their buses are already too full, the actual load may not match precisely the load that we have constructed. Note also that only departure times from stops are relevant and that the details of the trajectories between stops can be ignored.

Assumptions

Suppose that passengers' arrival times, origins, and destinations in no way depend on buses' departure times or on the strategy being used. That is, in Figure 1, if the buses had arrived at different times, the location of the x's and the subscripts would not change. This would happen, for instance, if buses arrived either so frequently or so unpredictably that passengers did not check the time before leaving for their stop. Then if we were to hypothesize a different set of trajectories for the buses, we could still easily construct the load each bus would have carried.

One difficulty with the procedure presented so far is that we need to know the passenger arrival times and the bus departure times at every stop. Suppose that bus trajectories are parallel or, in particular, that the time between a bus's departure at one stop and its departure at the next is the same for every bus (but not necessarily the same at every stop). If we now observe the departure times of all buses at one stop, we know the departure times of all buses at all stops. Essentially, we have assumed that a bus's headway does not change significantly over a distance comparable to the length of an average passenger trip. This allows the headway to change slowly. On a well-controlled bus line the headways do in fact change slowly.

Figure 1. Time-space diagram of a bus line.



This assumption also makes the description of passenger arrivals easier. In particular, it implies that the bus travel time from any point x to the point in question, x_0, is fixed. Let us call this travel time $\tau(x,x_0)$. Suppose we observe two passengers, one who arrives at stop x at time t and another who arrives at stop x_0 at time $t + \tau(x,x_0)$. Because bus trajectories are parallel and the arrival time of both passengers is the same relative to a given trajectory, both passengers catch the same bus. For our purposes, the two passengers are equivalent, and we can replace the original arrival process by an equivalent one at x_0 for which the arrival time of a passenger at x is shifted by $\tau(x,x_0)$. (Note that passengers who alight before the bus reaches x_0 are ignored.)

If we know the passenger arrival times for the equivalent process, then for any set of bus departure times from x_0 we could construct the loads on the buses at x_0. This procedure, however, still depends on the exact bus departure times (as opposed to simply the headways), since, for instance, the average rate of passenger arrivals might change over time and the loads on two buses with the same headway might be different. Let us assume that passenger arrivals are stationary, so that their average arrival rate is consistent. (The extension to nonstationary arrivals requires only that a headway be replaced with the time necessary to pick up a given number of passengers.) Then the expected load on a bus is simply proportional to its headway. That is, if $L|H = h$ is the random variable associated with the load on buses that have a headway h, then the expected value (or average) of the loads on these buses should be

$$E(L|H = h) = mh \tag{1}$$

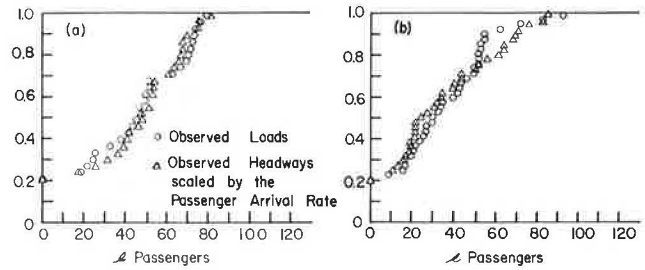
where m is the average passenger arrival rate. Because of the uncertainty in the number of passengers to arrive during a headway h (which is here called the uncertainty in passenger arrivals), the load on a bus with headway h will vary about mh.

Similarly, the average load over all buses, regardless of headway, is

$$E(L) = mE(H) \tag{2}$$

Because of unequal headways, uncertainty in passenger arrivals, and perhaps other factors, the load on an individual bus will vary about this mean. Suppose for the moment that only unequal headways contribute to the variability of loads on buses--i.e., passengers arrive in a steady stream. Then a bus with headway h would carry exactly mh passengers. If we were to plot the distribution of loads under this supposition and compare it with the distribution of actual observed loads, then any difference observed would be caused by factors other than unequal headways.

Figure 2. Comparison of actual load distribution with distribution of observed headways scaled by the passenger arrival rate: (a) 7:55-8:30 a.m. and (b) 8:45-9:30 a.m.



Example

Figure 2 shows a comparison such as that proposed above for data gathered for two different time periods on a bus line in Oakland and Berkeley, California. [The data were collected in November 1977 at the maximum load point of the northbound 51 line of Alameda-Contra Costa County (AC) Transit. This load point is downstream from a transfer point of the Bay Area Rapid Transit system.] The details of the data collection are given elsewhere (4). Basically, loads and headways of buses at the maximum load point were observed during two time periods when the passenger arrival rate was nearly constant. In this case, the maximum load point was downstream from a rapid transit station and any difference between the two distributions represents the effect not only of the uncertainty in the arrival of bus passengers but also of the uncertainty in the arrival of train passengers upstream of the transfer point and the uncertainty as to whether a bus picks up a batch of transfers from the rapid transit station at all.

That the distributions nearly coincide for each of the two time periods shows that the contribution of factors other than unequal bus headways to the variability of loads on buses is unimportant. In particular, uncertainty in passenger arrivals contributes little. Only if the headway distribution were considerably narrower than it was for these buses would uncertainty in passenger arrivals become significant. That is, unequal headways cause most of the variation in loads.

Variance Calculations

The phenomenon cited above can be quantified. If buses arrive either so frequently [i.e., at headways less than about 10 min (5)] or so unpredictably that passengers arrive independently of buses, then the variance of the load on a bus [Var(L)] is related to the variance of the headway of a bus [Var(H)] and a function characteristic of the passenger arrival process [r(h)], as follows (4):

$$\mathrm{Var}(L) = m^2\,\mathrm{Var}(H) + mE[Hr(H)] \tag{3}$$

Note that the variance and expected value are taken with respect to the headway distribution. In the second term, the expected value is taken of the function hr(h), where r(h) is the variance-to-mean

Figure 3. Typical shape of r(h) for AC Transit bus line in Oakland and Berkeley, California.

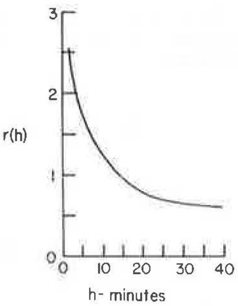


Table 1. Contributions of various factors to variance of load on a bus.

Factor	Value of Contribution (passengers^2)	
	7:55-8:30 a.m.	8:45-9:30 a.m.
Unequal bus headways	665	643
Uncertainty in the arrival of nontransferring passengers	63	50
Unequal rapid-transit-train headways	7	1
Uncertainty in the arrival of transferring passengers	14	2
Uncertainty that a bus picks up a batch	47	41
Total	796	737

ratio of the number of passengers to arrive in a time interval of length h, taken as a function of h. If passengers arrived in a Poisson process, for instance, then r(h) would be a constant, 1. It has been shown elsewhere (4) that, for values of h near a bus headway, r(h) is closer to 2 or 3. Further, r(h) is not constant. Its shape is typically as shown in Figure 3.

If there were no variability in headways, Var(H) would be zero; if there were no variability in the arrival of passengers, r(H) would be zero. Thus, the first term [m^2 Var(H)] is the contribution to load variance of unequal headways, and the second term is the contribution to load variance of uncertainty in passenger arrivals.

The equation analogous to Equation 3 for the data analyzed in Figure 2 is somewhat more complicated and is given elsewhere (4). Table 1 gives the numerical results for the various contributions. Again we see that unequal headways, represented by Var(H), cause most of the variability in the load on a bus. In this case, Var(L) $\simeq$ 750 passengers^2, and the standard deviation of the load is about ±27 passengers. That is, even though sufficient buses might be dispatched so that the average load is equal to the number of seats on a bus, an individual bus quite easily could carry as many as 27 standees or have 27 empty seats.

Coefficient of Variation of Bus Headways

The equation for Var(L) can be rewritten as follows:

$$\begin{aligned}\mathrm{Var}(L) &= C^2(H)\, m^2\, E^2(H) + mE[Hr(H)] \\ &= C^2(H)\, E^2(L) + mE[Hr(H)] \qquad (4)\end{aligned}$$

where $C^2(H) = \mathrm{Var}(H)/E^2(H)$ and is the square of the coefficient of variation of the headway. Note that the calculation of E[Hr(H)] requires knowledge of r(h) as well as the complete headway distribution. For the purpose of approximation, let us assume that r(h) is a constant (even though we know it is not). Let this constant be r. Then mE[Hr(H)] $\simeq$ rmE(L), and

$$\mathrm{Var}(L) \simeq [C^2(H)\, E(L) + r]\, E(L) \qquad (5)$$

Now the analysis of whether headways contribute more or less than passengers to the variation in loads comes down to whether $C^2(H)$ E(L) is large or small relative to r.

Rule of Thumb for Buses and Rapid Transit

The rule is this: If $C^2(H)$ E(L) is comparable to r, then headways and passengers contribute about equally to variations in the load on a bus. Note that this statement is also true if we take square roots: If C(H) $[E(L)]^{1/2}$ is comparable to $r^{1/2}$, then headways and passengers contribute about equally. For the two time periods analyzed above, C(H) was about 0.63 and 0.81, and E(L) was 41.4 and 32.0 passengers, respectively. Thus C(H) $[E(L)]^{1/2}$ was 4.05 and 4.58. As seen in Figure 3, r(h) is about 2-3 for headways of 2-6 min (the average headway during both periods was about 4 min), so we will let r be 2.5. Then $(2.5)^{1/2} \simeq 1.2$ is far less than C(H) $[E(L)]^{1/2}$ for either time period. Thus, we conclude that for these time periods unequal headways contribute far more to the variation in the load on a bus than does uncertainty in passenger arrivals.

In order for headways not to dominate, C(H) $[E(L)]^{1/2}$ would have to be about 1.2. For a full-sized bus, E(L) $\simeq$ 50, so C(H) would have to be about $[2.5/E(L)]^{1/2} \simeq 0.22$. Such a small variability in headways is rare, although not unknown. For instance, in a study of the Newcastle-upon-Tyne 33 route in December 1973 (6), C(H) at the Lonsdale Terrace stop was 0.57. Three years later, it was found to be 0.20.

The same analysis can be used for systems that have larger or smaller average passenger loads per vehicle. For instance, a rapid transit system might schedule its trains so that they carry about 1000 passengers. Then C(H) would have to be about 0.05 for headways and passengers to contribute equally to Var(L).

CONCLUSIONS

In effect, C(H) is a measure of the variability of the headway (specifically, the standard deviation) on a scale of the mean. This analysis shows that, for a bus line, if C(H) is above about 0.30, then unequal headways contribute almost exclusively to the variability of loads on buses. In this case, we would say that headways are poorly controlled. On the other hand, when C(H) is below about 0.3, unequal bus headways and uncertainty of passenger arrivals contribute about equally. In this case, headways are well controlled. In fact, it does little good to reduce C(H) below about 0.2 because under that value the uncertainty in passenger arrivals, over which the bus company has little control, starts to dominate.

When headways are poorly controlled, it might pay the bus company to invest in control strategies that reduce the variance in headways. If it can thus reduce the variance in loads, it can either use fewer buses and tolerate the same amount of overcrowding or it can reduce overcrowding and use the same number of buses. In the first case the bus company saves money, and in the second case the public receives better service. In either case, the public benefits (2).

This paper does not discuss the cause of unequal headways or the cost of controlling headways. It does illustrate that, once unequal headways occur, they are the dominant cause of variations in loads. A strategy to control headways, of course, must be based on a knowledge of why unequal headways occur. Once effective strategies are developed, then in principle their cost can be balanced against the benefits, as derived from this paper, to find the optimum level of control.

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Proposed Approach to Determine Optimal Number, Size, and Location of Bus Garage Additions

THOMAS H. MAZE, SNEHAMAY KHASNABIS, KAILASH KAPUR, AND MANI S. POOLA

A proposed technique for determining the location, size, and number of new bus-garage additions is described. First, different cost components (nonrevenue transportation cost, operating cost, and construction cost) related to new garages (location, size, and number) are identified, and it is shown how most of the current techniques fail to consider the full ramifications of all of these cost elements. Second, an optimization model is presented that includes the full range of cost components that deserve consideration in decisions related to the number, location, and size of new garages. A case study is also presented in which the implications of the full range of cost components are tested on an actual fixed-facility problem. The case study uses the proposed technique in its most fundamental state. The analysis shows that some of the less visible but recurring nonrevenue cost components may significantly affect the total annual garage cost. On the other hand, the more prominent, one-time construction cost may be of marginal importance in the annual cost of the garages distributed over the life of the facility.

Determining the location, size, and number of new bus garages is a problem commonly faced by expanding transit agencies. However, little independent research has been devoted to developing a standard and accurate technique to determine the least-cost number, size, and location of garage facility expansions. The importance of the use of a standard and accurate technique for such purposes is twofold:

1. The addition of a new garage (or garages) represents a long-term commitment to a costly portion of the transit system. The following costs are quite important with respect to other system costs and can vary considerably in magnitude according to the prospective garage number, location, and size alternatives: (a) the costs of nonrevenue travel to and from work assignments, (b) the cost of operating the garage, and (c) the costs of new construction.
2. Locating and sizing a new bus garage is often one of the more controversial aspects of transit planning. Bus garages often occupy prime industrial sites but, because bus operators are public agencies, they do not enhance the local tax base. Furthermore, the movement of buses into and out of a garage often has a disrupting effect on traffic flow on adjacent arterials. For these reasons and others, proposals for new bus garages often meet with strong local opposition. Thus, it seems only prudent that the decision maker should have accurate information relative to the total cost ramifications to justify his or her choice of the location and size of a proposed garage or the number, location, and size of proposed garages.

This paper reviews methods that transit authorities have used to locate and size garage additions. The analysis techniques are described so that the reader can contrast existing techniques with the proposed technique. Next, a proposed technique is presented, along with a case study, to portray the possible cost saving resulting from its use. Finally, directions for future development of the proposed technique are outlined.

PROBLEM STATEMENT

The basic goal of all transit agencies is to provide transit service in the most equitable and cost-effective manner. The development of criteria defining the number, size, and location of fixed facilities constitutes a key element in the realization of this goal. A mislocated or improperly sized facility can, over a few years, account for millions of dollars in wasted funds. Conversely, the dollars saved by optimally locating and sizing these facilities can be more effectively used in other areas of system operations.