areas as well as to collect data regarding those areas is helpful.

4. There is a need to update and expand the Simpson-Curtin formula to account for inflationary effects on transit fare increases. There is also a need to include variables that account for travel cost changes in competing modes of travel.

5. There is a need to examine transit pass usage patterns. Delineating "convenience" users from "financial savings" users and obtaining information on their usage frequency would be helpful for marketing analyses and predicting revenue trends.

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Abridgment

Measured Fare Elasticity: The 1975 BART Fare Change

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By using the measured response of San Francisco Bay Area Rapid Transit (BART) patrons to a fare structure change in 1975, this paper shows that variance in empirical demand elasticities can be strongly and inversely related to the level of patronage aggregation considered and the relative change in fare. The 1975 fare structure change affords a unique opportunity to observe such variance with both increases and decreases in fare occurring for cases at different fare and patronage levels. Two levels of aggregation are considered. One is the systemwide total response aggregate; the other treats each origin-destination data element as a separate case. Different values are computed for elasticity and are found to be related to the level of aggregation. Elasticity functions are also derived from the cases for use in BART forecasting procedures. Analysis for the correct weighting factors to use in fitting the elasticity functions indicates that variance of the measured elasticities is related to the case patronage levels and the square of the difference in logarithms of the fares before and after the change. The fitted elasticity functions also demonstrate that divergences in values of elasticity can be a function of both model specification and the operating point selected for the calculation of elasticity from the function.

The objective of this study was to more accurately represent the varying response of San Francisco Bay Area Rapid Transit (BART) patrons from different market areas to fare changes through derived elasticity functions. Elasticity functions aid in the prediction of responses to fare structure changes at a more refined level of critical system screenlines. They also provide potential controls for studying level-of-service impacts during simultaneous fare and level-of-service changes on BART in mid-1980.

This paper presents results for both calculated constant aggregate elasticities and acceptably fitted elasticity functions that quantitatively demonstrate how much divergence can occur with such computations. An important reason for this divergence is the high variance in the response of trip making to a fare change that appears to be inversely related to the level of patronage. Variance may provide an additional explanation for controversial inconsistencies in elasticity estimates (1,2).

FACTORS IN DIVERGENT ESTIMATES

Depending on the level of aggregation used in computation, the data in this study yielded different values for average elasticity. This was not unexpected since Chan and Ou (1) had hypothesized that aggregate empirical elasticities based on coarse demand data would tend to underestimate the response while disaggregate calibrated elasticities, mostly based on zonal and household data, would overestimate. Thus the absolute value of the aggregate elasticity would be less than that of the disaggregate elasticity. The calculated aggregate elasticity did demonstrate this relation with respect to the average elasticity for the set of origin-to-destination cases that is a more disaggregated level of data. In this case, such differences appear to be an artifact of the method of computation and the aggregation of data.

Gomez-Ibanez and Fauth (2) offer three other explanations for such differences: variations in data accuracy, failure to capture characteristics differentiating markets, and different variables included in model specifications. Model specification does appear to be a significant factor for elasticity functions derived from mathematical models of demand. Ruiter (3) provides an excellent summary of many travel demand models along with derived elasticity functions. In most of the forms summarized, elasticity is not a constant. It is, instead, a function of the variables in the model, most often of the cost variable. Different sensitivities are thus implied for the value of elasticity. A departure from base values for the variables in the function results in a divergence in values computed for elasticity. If results in this paper can be extrapolated, differences on the order of 30 to several hundred percent can easily occur.

Measured elasticities attempt to describe the response of demand to a change in cost directly. An increase in trip cost can be expected to reduce trip making. Adjustments can be made for seasonality, trip purpose, accessibility to alternative stations, and perceived value of cost and its change with time. But adjustments cannot be made for all factors. Thus, erratic values for elasticity can be expected. The extent of the erratic behavior can be surprising. For example, elasticity for daily demand in the 793 selected cases ranged in value from

13.3 to -17.3 with a standard deviation of 2.129 about a mean of -0.781. Morning peak-period elasticity showed a greater range from 21.1 to -34.4 with a standard deviation of 4.121 about a mean of -0.710. Since the square of the deviation from the mean is strongly related to the inverse of the level of patronage, the greater variation in the peak may be related to the lower levels of patronage in the sample cases.

DEFINITION OF ELASTICITY

The method used in this paper for computing elasticity (η) from a measured response to a change in fare is the log difference ratio evolved directly from the differential formulation for elasticity (4, p. 169): $\eta = (\log D_2 - \log D_1)/(\log F_2 - \log F_1)$ where D = demand, F = fare, and the subscripts represent before and after.

Elasticity functions are derived from commonly used demand model forms. Table 1 lists a selected set of demand models and the corresponding elasticity functions. Both case dependent and case independent variables are identified. K and S are intercept and coefficient scaling constants that may be functions of variables not represented by F. The trip cost F may represent fare or the ratio of fare to average annual income. Fitting most of the elasticity functions of Table 1 requires the assumption of a zero intercept. This is a serious and not always justified assumption (5, p. 13). Mixed-model functions are possible alternatives and are listed in Table 2. The corresponding implied-demand models shown were determined from the differential equations based on the elasticity function.

DESCRIPTION OF DATA

Each element of the origin-to-destination trip matrices for 33 BART stations based on daily and 2-h peak-period averages for representative weekdays in October and November 1975 is considered as a "case." Cases with no fare change, such as the round-trip cases representing system touring trips that enter and exit at the same station, and cases with zero patronage in any one of the time periods were rejected. These criteria left 793 acceptable cases. Table 3 provides selected statistics on total daily and morning peak data. Statistics for the afternoon peak were similar to the morning peak statistics.

The BART fare structure is distance-related with minimum fares for the central business district and among neighboring stations and with a decreasing marginal cost with distance. Surcharges are added to transbay fares, and adjustments are made for relative speed and required transfers (6). The new fare structure introduced on November 3, 1975, reduced some fares but increased most. The minimum fare changed from $0.30 to $0.25 and the maximum fare increased from $1.25 to $1.45.

BART patronage exhibits seasonality and a growth trend. Extensive analysis had been done on five years of monthly average patronage since the start of transbay service (7,8). Though statistically significant growth trends of more than 5.6 percent per year were identified for daily patronage, data used for elasticity functions were not detrended since, in addition to much lower regional population and employment growth rates, a possible explanation for the trend can be real dollar fare elasticity. Relative seasonal factors (8 p. 11) were used. Income and trip purpose data were available for each station as origin or destination from the May 1976 passenger profile survey.

Table 1. Alternative mathematical forms of demand models with corresponding elasticity functions with fare as independent variable.

Demand Model Form	Elasticity Function for D with respect to F	Function Ref. No.
Logarithmic or product		
$D = K F^{-a}$	$\eta = -a$	1.1
Exponential		
$D = K e^{-bF}$	$\eta = -bF$	1.2
Linear		
$D = K - bSF$	$\eta = -bS(F/D)$	1.3
Half-bell		
$D = K e^{-bF^2/2}$	$\eta = -bF^2$	1.4
Linear log		
$D = K - bS \log F$	$\eta = -bS(1/D)$	1.5
Log linear		
$\log D = K + bS(1/F)$	$\eta = -bS(1/F)$	1.6

Notes: Case dependent factors— D = demand; F = cost variable, fare or fare divided by income; K = scaling factor or intercept; and S = slope-scaling factor (for this analysis assume S = 1). Case independent parameters--a, b = constants.

Table 2. Mixed-model fitted elasticity functions.

Function Ref. No.	Fitted Elasticity Function	Implied Demand Model Form
2.1	$\eta = -a - bF$	$D = K F^{-a} e^{-bF}$
2.2	$\eta = -a - b(F/D)$	$D = K F^{-a} - [b/(a+1)]^F$
2.3	$\eta = -a - b F^2$	$D = K F^{-a} e^{-bF^2/2}$
2.4	$\eta = -a - (b/D)$	$D = K F^{-a} - (b/a)$
2.5	$\eta = -a - (b/F)$	$\log D = \log K - a \log F + b(1/F)$ or, $D = K F^{-a} e^{b/F}$

Notes: Case dependent factors—D = demand; F = cost variable, fare or fare divided by income; K = scaling factor or intercept; and S = slope-scaling factor (for this analysis assume S = 1). Case independent factors—a, b = constants.

Table 3. Selected statistics for October and November 1975 representative sample averages.

Patronage	Total Day		A.M. Peak Period	
	October	November	October	November
Total	124 942	118 090	33 848	31 447
Total patronage[a]	123 822	117 276	33 820	31 415
Average extracted fare ($)	0.632	0.753	0.657	0.778
Average trip distance (km)	21.1	20.9	22.3	22.0
Average time in BART (min)	27.6	27.3	28.3	27.9
Total 793-case sample patronage	97 685	90 335	27 302	24 686
Average extracted fare ($)	0.709	0.872	0.732	0.899

Note: Trip distance is on-board BART trip distance. Time in BART includes average waiting and transfer time plus nominal on-board travel time.

[a]Exclusive of round trips.

METHODOLOGY FOR DEVELOPING ELASTICITY FUNCTIONS

The elasticity functions were developed by fitting the curves of Table 2 with least squares regression. Patronage and trip cost variables were computed from data for each month and from averages for both months. Income, trip time, distance, work trip purpose, and BART system segment indicator variables were also considered. Best fits were obtained with patronage and both trip cost variables for October, the "before" month. Distance and trip time, which correlated with fare, provided good initial fits; other variables tested seemed to be irrelevant. The multiple correlation coefficients for the fits were very low despite very satisfactory F-values. An analysis of residuals revealed that the residuals were not normally distributed and were strongly related to the values of patronage, fare, and the abso- lute difference in the logarithm of fares. Weighted least squares was indicated.

The appropriate weighting factor to use is one that inversely relates the residual variance to some constant variance (5, p. 80; 9, p. 326). Of the various weighting schemes tried, three had some theoretical merit. The first used the square of the log difference in fares; this could be justified by considering measured elasticity to be a stochastic demand response divided by a constant that is the log difference in fares. The second used the level of patronage; this could be justified by considering it as a sample size for the measured response of individual trip makers. The third used a factor based on the proportion of trips changed; this would be applicable for the output of a Bernoulli process where 1 is a change in trip making and 0 is no change. All three reduced variance and produced better multiple regression coefficients, but the last yielded unsatisfactory distributions of residuals. Best results were obtained through a combination of the first two in the weighting factor (w_i), $w_i = D_{i1}\ (\log\ F_{i2} - F_{i1})^2$ where D_{i1} is the level of patronage of case i in the "before" month and F_{ik} is the fare for case i in month k.

FINDINGS AND DISCUSSION

Average Values for Elasticity

Selected averages for daily systemwide measured elasticities are given in Table 4. Except for the aggregated totals without round trips, averages are for 793 cases. The weighted averages are less in absolute value than the unweighted since the weight factors were heavier for cases with measured elasticities with smaller deviations from the mean value. The range of these estimates is noteworthy. In other results, morning peak elasticities varied less but indicated a more elastic aggregate response; afternoon peak elasticities varied more and were less elastic.

Table 4. Average measured elasticities of BART daily systemwide patronage related to fare change in November 1975.

Method of Computation	Type of Demand Adjustment: Unadjusted	Seasonality	Seasonality and Trend
Linear mean ratio			
Unweighted	-0.682	-0.773	-0.803
Log difference ratio			
Unweighted	-0.689	-0.781	-0.811
Weighted by -			
ADLF[a]	-0.559	0.621	-0.641
D[b]	-0.368	-0.443	-0.468
$D(ADLF)^2$	-0.348	-0.392	-0.406
Aggregated -			
793 cases	-0.377	-0.454	-0.479
Totals	-0.310	-0.399	-0.429

[a]ADLF = Absolute value of the difference in log fares. [b]D = Demand.

Selected Elasticity Functions

Selected fitted elasticity functions are shown in Table 5. High t-statistic values show that the independent variables are significant explanatory variables for measured elasticity even though coefficients of determination (r^2) are not large. Adding another variable did not improve the fit significantly in most cases. The functions listed are all preferable to a constant elasticity by regression criteria.

Some insight into the divergence of elasticity estimates given by differently specified models can be gained from the plots of elasticity functions against fare divided by income in Figures 1 and 2. Figure 1 shows functions that were directly fitted to the independent variable while Figure 2 shows those that were related to demand and had to be transformed by substitution of the appropriate demand function from Tables 1 and 2. The zero intercept functions are plotted to illustrate the implication of that constraining assumption. For example, curves 2.5 and 1.6 represent the same inverse cost independent variable of the log linear model but are different because curve 2.5 includes a significant intercept that shows the dominant influence of the product model. The null hypothesis is not supported for the plotted zero intercept cases and is rejected at the 0.01 significance level. On the other hand, the fitted straight-line function (2.1) derived from the exponential demand model effectively has a zero intercept. The null hypothesis that the intercept of this function is zero is maintained at 95 percent confidence.

Table 5. Selected weighted regression results for fitted measured elasticity function.

Table 2 Ref. No.	Independent Variable	Coefficient	t-Statistic	Intercept	Coefficient of Determination (r^2)
2.1	F/I	-11.294	-11.25	0.027 6	0.1380
2.5	I/F	0.015 67	12.27	-0.879 0	0.1598
2.3	$(F/I)^2$	-112.17	-9.78	-0.213 1	0.1079
2.2	(F/I)/D	-158.08	-6.49	-0.348 6	0.0506
2.1	F	-0.612 5	-8.58	-0.057 15	0.0851
2.5	1/F	0.216 7	10.39	-0.844 5	0.1202
2.3	F^2	-0.367 5	-6.75	-0.265 6	0 0545
2.2	F/D	-10.62	-5.97	-0.351 7	0.0431
2.4	1/D	-6.880 6	-5.56	-0.350 2	0.0376
-	T	-0.015 96	-9.12	-0.002 3	0.0952
-	L	-0.014 65	-7.84	-0.134 4	0.0721
-	F/I	-10.459	-6.30	0.040 1	0.1384
	T	0.001 783	-0.63		

Notes: F = October fare ($); I = income ($000s); D = October demand (trips); T = in-BART travel time (min), including wait and transfer time; L = on-board station-to-station travel distance (km).
Weighting factor is demand times the square of the difference in log fares.

The functions fitted directly and inversely to the cost variables yield the best and almost equivalent figures of merit. October fare divided by income is more efficient than fare itself and is preferred. The inverse cost function (2.5) produces behavior contrary to that implied by a log linear model and the regression coefficient would be rejected under a one-tailed test. The linear function (2.1) with its near zero intercept is the most acceptable with respect to its consistency with the postulated behavior of demand related to cost in an exponential model.

CONCLUSION

The measured elasticities and elasticity functions presented were all computed using acceptable methods from the same data base. Yet they give diverse values for elasticity. Variance in the response to fare change is one factor influencing this diversity. Also, the often used assumption of constant elasticity is not necessarily correct. Fare level or fare related to income are acceptable explanatory variables for the variation in measured elasticity.

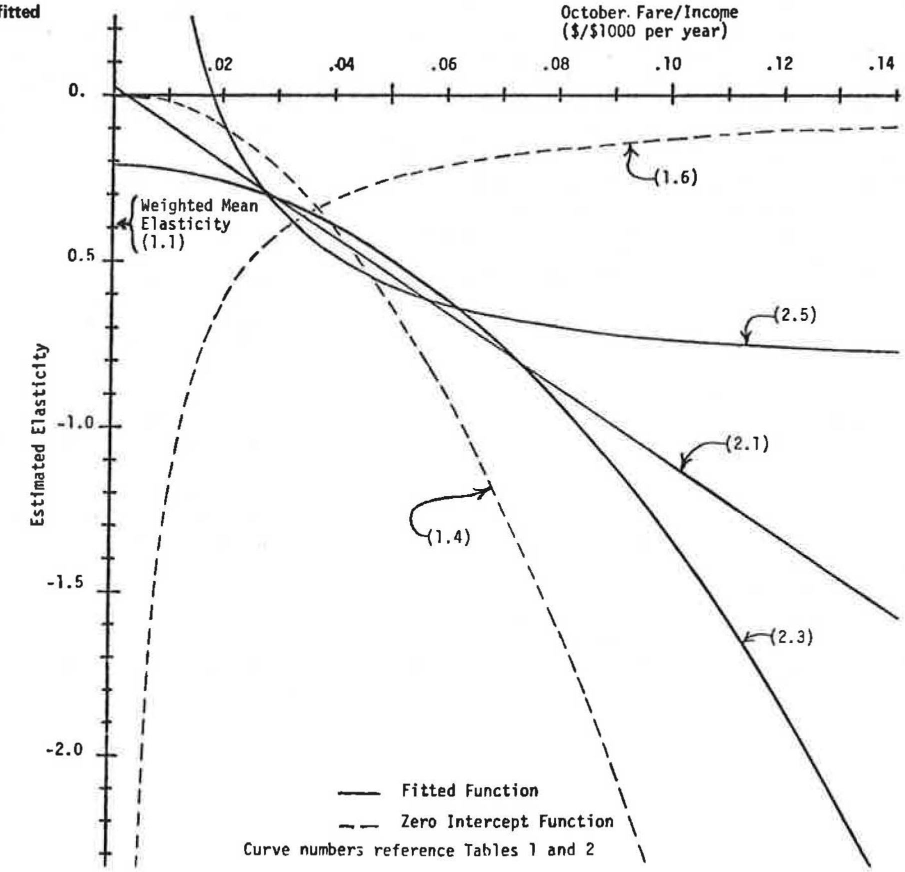

Figure 1. Elasticity functions directly fitted to October fare divided by income.

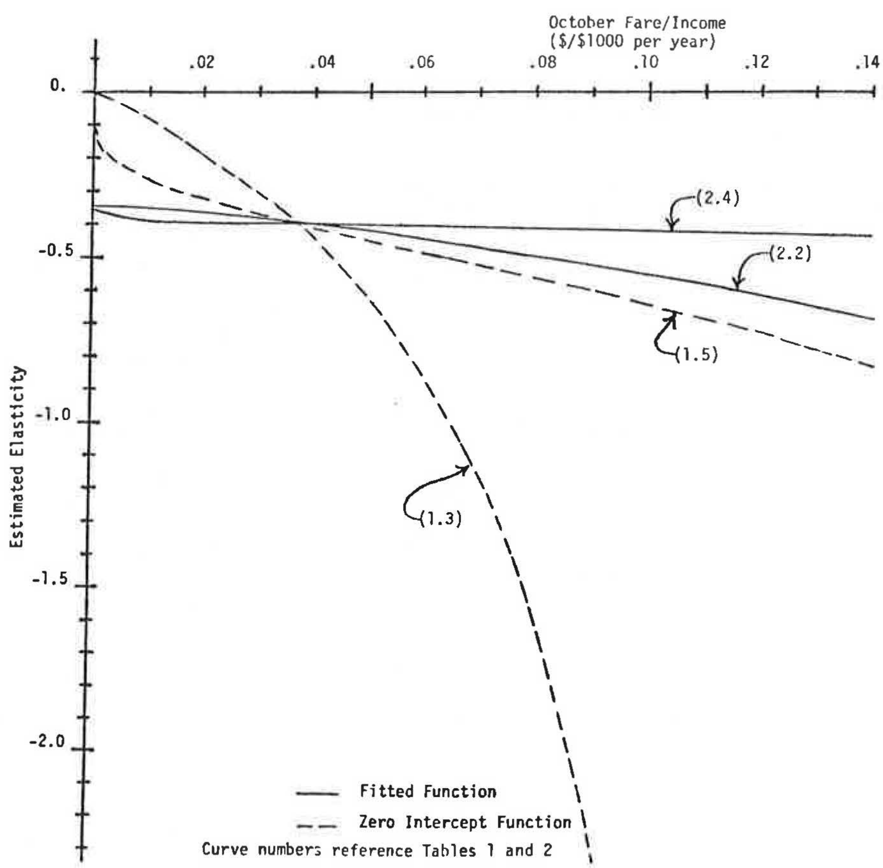

Figure 2. Elasticity functions derived from functions fitted to inverse demand or fare divided by income and by demand.

Elasticities derived from some demand models, including the disaggregate models, do consider such cost variables. Transferability of results would be affected by the specification of these and other possibly important variables not yet identified.

The variation in elasticity data is large relative to the mean value and inversely related to the demand level and the square of the log difference in fares. The indications are that the demand weighted mean value of the cases approaches the aggregate elasticity, which may provide a better estimate of the expected response if a single value for change in demand is sought.

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Further Evidence on Aggregate and Disaggregate Transit Fare Elasticities

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This paper presents new evidence on transit fare elasticities from experimental demonstrations and demand models. Mean values and standard deviations of fare elasticities are analyzed for both aggregate and disaggregate ridership categories. Aggregate fare elasticities for fare-free, fare prepayment versus cash payment, and promotional fare reductions are presented. Fare elasticities are also disaggregated by mode, trip length, route type, period of the day, and income and age groups. A review of the methods used in elasticities estimation is also presented.

Over the past few decades, transit operators have relied on the Simpson and Curtin formula (1) for predicting the impact of fare changes on transit ridership. The Simpson and Curtin formula, which predicts the percentage decrease in ridership as a function of the percentage increase in fares, has reverted to the rule of thumb that transit ridership will decrease (increase) 0.3 percent for every 1 percent increase (decrease) in transit fares.

Although the Simpson and Curtin rule of thumb is generally correct in highlighting the fact that transit ridership is inelastic, its indiscriminate use can lead to serious miscalculations of the ridership impacts of fare changes. This problem was brought out by two American Transit Association (ATA) studies of losses in passenger traffic due to transit fare increases between 1950 and 1967 (2,3). Both studies, while finding an average shrinkage ratio of -0.33, showed wide variances in the range of elasticities estimated, ranging from -0.004 to -0.97. Dygert, Holec, and Hill (4) have shown that in slightly more than half the cases the shrinkage ratio estimated by ATA was below Simpson and Curtin's rule of thumb.

The existence of such a wide variation in transit fare elasticities has prompted many transportation analysts to present evidence of disaggregate ridership response to fare changes (5-7). This paper presents new information on the size of aggregate and disaggregate transit fare elasticities obtained from demonstrations and demand models. In addition, this paper cautions the reader in interpreting the demand elasticity estimates from data containing no fare change.

APPROACHES TO ESTIMATING TRANSIT FARE ELASTICITIES

Nature of Approaches to Demand Estimation

Two broad approaches to estimating fare elasticities may be distinguished. These approaches include (a) monitoring fare changes or demonstration studies, or those that rely on data generated either by a practical demonstration of an actual change or by monitoring an actual change in current fares; and (b) nonexperimental approaches, or those that rely on a data base either devoid of an actual change in current fares or where actual changes are part of historical trends.