Performance-Based Track-Quality Measures and Their Application to Maintenance-of-Way Planning

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A methodology for obtaining quantitative measures of track quality based on rail-vehicle performance is described. Loaded track geometry is treated as the driving impetus for vehicle dynamic response. It is interpreted in terms of system failure, which in turn is linked to basic maintenance requirements. In this treatment, the failure mode is restricted to high probability of derailment. This is felt to be an appropriate criterion since it drives the track-maintenance programs of most eastern freight railroads; however, this methodology could be used to evaluate track quality according to some other criterion, e.g., rider comfort. Given this framework, the benefits associated with various types of track maintenance can be analytically evaluated according to their respective costs. In addition, the relative contributions of various track degraders (e.g., heavy cars) can be measured and the appropriate charges made to shippers who use these vehicles.

To date, there exist no widely accepted numerical estimators of railroad track quality (1,2). This is quite surprising, since it has been believed for some time that a quantitative measure of track quality would permit railroad managements to allocate scarce maintenance-of-way resources and funds (3,4). More recently, new developments have added impetus to the effort to find measures that will be accepted by the railroad community. Mergers and acquisitions are making it difficult for management to be familiar with the condition and maintenance needs of the enlarged systems. The deregulation of the railroads makes it mandatory that realistic estimates of users' costs be developed. This is particularly true of that cost component associated with track maintenance and degradation.

Numerical estimators of track quality exist and can be classified according to the method by which they are derived (5). They include:

1. Observations of track's structural parameters, such as defective ties, drainage condition, rail size, and rail condition;
2. Measurements of track deflection, such as those obtained by the Decarotor (6), which are used to make inferences regarding track strength; and
3. Measurements of loaded-track geometry as obtained by an automated track survey vehicle (7).

Such raw data as those generated by the above methods may be quite numerous—as many as 10,000 data values per kilometer per parameter. Hence, they are often converted into more-tractable compact summaries that represent track condition over, say, a 1-km interval. Examples would be defective ties per kilometer, average track strength over a 1-km interval, or standard deviations of track gage over a 1-km interval. Expressed in this form, the summaries are called track-quality indices (TQIs).

A number of planning models have been developed and proposed for implementation by the railroads. The input consists of a specific set of TQIs, and the output is an allocation schedule for maintenance resources. Such models have been derived by using empirical formulations (8), regression analyses (9), and the calculus of variations. All the planning models assume that the TQIs are valid, that the indices are objective summaries of track condition, and that they are minimally affected by noise, statistical sampling uncertainty, and other extraneous influences.

DEFINITION OF TRACK QUALITY

The design of a parametric measure of track condition has as its point of departure a general definition of what is meant by track quality. Good track shall be considered to be that which can support desired train movement for at least some time after the quality has been determined. Implicit in this statement are two important concepts:

1. The classification of the track (good or poor) cannot be made independently of the type of service to be provided by that track. Thus, development of a measure of track quality requires analysis of the entire system, which includes vehicle configuration and speed as well as track characteristics. One might expect that the measure of track that applies to track that supports movement of mixed freight at 60 km/h would be quite different from that which applies to passenger operations at 120 km/h.

2. A careful distinction must be made between instantaneous measure of track quality, which relates to the present ability of the track to support the desired train movement, and the rate of change of track quality, which relates to the future ability of the track to support the desired train movement. Parameters that are good measures of track quality may provide little or no indication of the rate of change in track quality. However, many factors that are now used to estimate track quality provide excellent indicators of this rate of change.

The vehicle can be modeled as a predetermined mathematical operator whose input is displacement. If the velocity of the vehicle is known, this displacement can be expressed as a function of time and is shown as z(t) in Figure 1. The displacement of the wheel set (assuming that there is no wheel lift) is the loaded-track geometry. The vehicle model, i.e., the mathematical operator or filter, operates on z(t) to determine displacement x(t) and force g(t) at various points in the vehicle. The equation of motion to solve for x(t) and y(t) in terms of input z(t) is as follows:

\[ M(d^2y/dt^2) + b(dx/dt) + kx = 0 \]  

where y(t) = x(t) + z(t) and the other terms are defined in Figure 1. Use Equation 1 together with the following equation to solve for force g(t) into the wheel set:

\[ g(t) = m(d^2x/dt^2) - b(dx/dt) - kx \]  

For a specific model and vehicle, the validity of predicted responses can be ascertained by tests. Varying degrees of sophistication are possible; however, for the purposes of this illustrative exercise, the simple linear model of Figure 1 is used. The methodology described in this paper is being pursued under a Federal Railroad Administration (FRA) contract by using a more-appropriate model.
DEFINITION OF PERFORMANCE-BASED TRACK-GEOMETRY DESCRIPTOR

Immediate or instantaneous performance is determined on the basis of observed accelerations, relative displacements in the vehicle components, and force levels transmitted through vehicle and track members. These can be related to derailment potential, fatigue of track and vehicle components, riding damage, and ride comfort. This is not a new concept—it has served as the basis of numerous previous dynamic investigations, the goal of which was to relate geometry variations to vehicle behavior (12). This study differs from others in the following key aspects:

1. No a priori assumptions are made as to the nature of the track-geometry input.
2. Simple linear models are used to obtain the salient response modes of a given vehicle to track-geometry perturbations.
3. These response modes are converted to the frequency domain. There the response signature is inverted to ascertain what maximum magnitude of track-geometry perturbation may be permitted in order to keep the vehicle response within acceptable limits. Since the conversion from spatial to temporal frequency depends on vehicle speed, the vehicle response and inversion must be performed for all speeds up to and including posted track speed. Since the track must be maintained to permit trains to operate at all speeds up to track speed, the worst-permissible geometry perturbation (i.e., that which just causes the limiting vehicle response) must be found for each wavelength. These worst-permissible perturbations for each wavelength define an envelope that identifies the maximum amplitude of the geometry perturbations for each wavelength. Track must be maintained within this envelope in order to restrict vehicle behavior to the limiting response levels.
4. The final envelope so obtained for a given track speed is used to define the filter that processes track geometry to relate excessive variations in geometry to excessive responses in vehicles. For bidirectional traffic, this filter has either a symmetric or antisymmetric impulse response, depending on the response mode examined.
5. The filter defined by the enveloping procedure has a finite number of free parameters (at least three, at the most four) that can be adjusted to account for more-detailed and more-accurate models of vehicles. Alternatively, these filters can be adjusted on the basis of full-scale vehicle tests on special track sections or on shakers.

Alternatively, performance-based track-geometry descriptors (PBTGDs) can be extended to include high-speed passenger service in which passenger comfort is an added criterion. The undesired vehicle response would be that which results in excessive levels of acceleration experienced by a rider in the vehicle.

As cited in the introduction, the vehicle model that will be used to illustrate this approach to formulating a PBTGD of track quality will be extremely simple. The vehicle's configuration is that of point mass $M$ suspended by a linear spring-damper combination that has spring constant $k$ and damping coefficient $b$. It is driven by geometry perturbations $z(t)$ that act on a wheel that has mass $m$. This model is that schematized in Figure 1.

DEVELOPMENT OF PBTGDs BY USING SIMPLE VEHICLE MODEL

Long-Wavelength Control

Typical rail vehicles have bump stops and other nonlinear constraining elements. When the relative displacement between $M$ and $m$ reaches critical levels, this implies that certain undesirable responses are occurring in the vehicle-track system. For example, large upward displacements in the suspension may indicate that the vertical forces vanish so that wheel lift may result. In the horizontal direction, large displacements may indicate that bump stops are engaged and direct coupling occurs between $M$ and $m$, which results in large lateral forces at the contact zone between the wheel and the rail. Therefore, one measure of derailment tendency for this simple model is the relative displacement between $M$ and $m$. The equations of motion for the relative displacement between $M$ and $m$ are as follows:

$$M(d^2x/dt^2) + b(dx/dt) + kx = M(d^2z/dt^2)$$

where

$$z = z(t) = \text{inertial track-geometry input as a function of time } t,$$
\[ y = y(t) = \text{inertial response of mass } m \text{ as a function of } t, \text{ and} \]
\[ x = x(t) = \text{relative displacement between } M \text{ and } m \text{ as a function of } t = y(t) - z(t). \]

\[ |X(f)/Z(f)|^2 = \frac{f^4}{f^2 + 2(2p^2 - 1)f^2 + (f')^2} \]  \hspace{1cm} (4)

where
\[ f = \text{temporal frequency (Hz)}, \]
\[ v = \text{forward speed}, \]
\[ \phi = \text{spatial frequency (cycles/km)} = \frac{f}{v}, \]
\[ Z(f) = \text{Fourier transform of track-geometry input (time to spatial frequency)}, \]
\[ X(f) = \text{Fourier transform of relative displacement}, \]
\[ f' = \text{resonant frequency; } f'^2 = \frac{k}{(2\pi)^2} M, \text{ and} \]
\[ p = \text{damping factor; } p^2 = \frac{b^2}{4kM}. \]

For a resonance of 0.5 Hz, a damping factor of 0.2, and a speed of 100 km/h, the response characteristic shown by the line in Figure 2 is produced. It is characterized by a concentrated peak at a wavelength of 55 m, which tapers off at unit gain (1 cm input = 1 cm relative displacement output) for wavelengths shorter than resonance and which rolls off dramatically at wavelengths longer than resonance.

The transmissibility can be inverted to prescribe the peak value of geometry input that produces a fixed acceptable level of relative displacement. The track geometry that causes 2.54 cm of relative displacement in the same vehicle that travels 100 km/h is shown by the curve in Figure 3. Reducing the vehicle's forward speed produces additional curves, indicated by appropriately labeled broken lines in Figure 4, and these serve to illustrate the enveloping procedure for 100-km/h track. The en-
Figure 4. Enveloping for track speeds up to 100 km/h.

Figure 5. Cross-level requirement for 16-m boxcar that has high center of gravity for several track speeds.

Envelope that results from varying the speed over all speeds through 100 km/h is shown by a solid line.

The enveloping procedure is repeated to get the PBTGD for other track speeds. The result is the family of curves shown in Figure 5. (Note that the ordinate of Figure 5 has been shifted in anticipation of some numerical results from a study by the Transportation Systems Center (TSC) to be discussed later.) Aside from the issue of scale, these curves exhibit the following salient features:

1. They are characterized by the need to maintain the track-geometry perturbations at a constant level of amplitude that extends from the wavelength of resonance at maximum permissible speed to arbitrarily short wavelengths $\lambda$;

2. At longer wavelengths, the need to control geometry amplitude tails off dramatically and, ultimately, the relaxation of amplitude grows at $1^2$ in the long-wavelength limit; and

3. The initial rapidity with which control of wavelengths longer than top-speed resonance may be relaxed is controlled by damping factor $b$.

The wavelength response characteristics of measurement tools and machines used to support track maintenance are such that the first feature listed above will have the greatest impact on maintenance costs (11,12).

Short-Wavelength Control

The descriptor derived in the preceding section has a mathematical form that is defined for both long and short wavelengths. As stated in item 1 above, the descriptor developed by limiting relative displacement $x(t)$ in the vehicle permits a geometry perturbation that has a 1.6-cm amplitude for arbi-
profundely short wavelengths. Thus, according to this formulation, a geometry deviation that has this amplitude and a wavelength of 0.5 m is permissible. A short-wavelength deviation of this magnitude is clearly not acceptable to a real vehicle; it is therefore concluded that a response mode other than relative displacement controls the short-wavelength input. Hence, the high-frequency dynamic force augment at the contact zone between the wheel and the rail is examined. Dynamic augment is defined as the added force above static loading needed to make all the vehicle masses follow the geometric irregularities in the track. Because of the isolation between mass components provided by spring in the suspension system, the effective mass loading at the contact zone between the wheel and the rail can vary significantly as a function of frequency, speed, and suspension parameters. This component of vehicle performance contributes to high derailment tendency through accelerated wear, fatigue, and flaw growth in both the rail and the vehicle components.

The vehicle model used for illustrative purposes is still that shown in Figure 1, but now spring and damping constants appropriate for vehicle response to profile and alignment geometry perturbations are used. For typical mixed freight that responds to profile, the unsprung mass becomes that part of a wheel set and side frame seen at one wheel, and the spring mass becomes that share of bolster, car body, and lading load applied to one wheel. The pertinent parameters and equation of motion are given below:

\[
M(\ddot{g}/t^2) + b(\dot{g}/dt) + k\dot{g} = \text{Mmd}^2(\ddot{z}/dt^2) + (M + m)(\ddot{z}/dt^2) + (M + m)k(\ddot{z}/dt^2)
\]

where \( g = g(t) \) = force transmitted across wheel-rail contact zone and other terms were defined earlier.

\[
\begin{align*}
\vert G(q) \vert^2 &= \left(2a_4 + 2a_2 q^2 + a_0 q^4 + (2\gamma - 1)q^4 + (\gamma^2)\right) \\
&\quad + \left[a_6 q^4 + 2(2\gamma - 1)^2 q^4 q^4 + (\gamma^3)\right]
\end{align*}
\]

(6)

where

- \( G(q) \) = Fourier transform of wheel-rail force,
- \( q^2 = (1 + r)p^2 \) = effective damping factor for numerator transition,
- \( (\gamma) = (1 + r)(\gamma) = \) effective resonant frequency for numerator transition, and
- \( r = M/m \) = ratio of sprung to unsprung mass.

The associated performance-based envelope for this response mode is again developed by inverting the transmissibility. This envelope prescribes the extent to which geometry variations must be controlled in order to keep the dynamic augment plus static wheel load less than 143 kN—a value chosen for illustrative purposes only. The results of using this performance criterion for a variety of operating speeds are shown in Figure 6. Note that the curves do not interpenetrate each other as speed is changed (as they did in Figure 4), so the envelope for a given track speed is the vehicle requirement when it is running at that speed.

Control of Combination of Short and Long Wavelengths

The total performance requirements of the vehicle are now obtained by combining the results of Figures 5 and 6. The principle used is that the resultant envelope for a given track speed is the envelope of the envelopes obtained for all the different failure modes for that speed.

On this basis, the family of curves shown in Figure 7 is obtained. For the speed regime displayed in Figure 7, the following observations are made:

1. For long wavelengths (>100 m) the amplitudes can increase dramatically and ultimately approach a \( \lambda^2 \)-characteristic;
2. For intermediate wavelengths, the level of control is constant; and
3. For short wavelengths (<7 m) progressively greater levels of control are needed; this increasing control also approaches a \( \lambda^2 \)-characteristic.

The Figure 7 control characteristics can be squared so that they can be interpreted as power levels of allowed geometry deviations. In this form, they can be compared with empirical and analytical representations of the power spectral density (PSD) of rail profile as is done for 64-km/h track and shown in Figure 8 [based on a study conducted for TSC (13)]. When this is done, it is observed that the control properties of the PBTGD produce geometry variations in profile the power content of which resembles that of existing track. This is a key result since (a) no a priori assumptions were made with respect to the track input; yet (b) safe vehicle performance does not require a track the PSD of which is materially different from that found for existing track. Stated another way, existing track structure has geometry variations the form of which is suited to vehicle performance.

TSC has investigated freight-car roll response to cross-level input (14). Based on preliminary findings, the lowest (worst-case) roll resonance is of the order of 0.5 Hz, and wavelengths shorter than resonance must be held to 1.6 cm. This is the rationale for the scaling on the ordinate of Figure 5.

Figure 6. Allowed profile deviations based on static plus dynamic wheel load < 143 kN enveloped for 100-km/h operation.
It should be noted that this descriptor differs from those currently used in two key aspects:

1. The wavelength response of the new descriptor varies with track speed, whereas the wavelength response of current descriptors (warp of a given length) is constant with respect to track speed; and
2. The comparison threshold of current descriptors varies with respect to track speed, whereas the comparison threshold of this descriptor remains fixed.

EXTENSION TO REAL VEHICLES AND MORE-ELABORATE MODELS

It is hypothesized here that these salient characteristics outlined above are ascribable to all vehicle models regardless of how complex or nonlinear they may be. In other words, long-wavelength performance is such that the amplitudes of geometry deviations must be maintained constant. The longest wavelength for which this must be done is determined by track speed. The nature of maintenance machines and practices is such that roll-off or relaxation of control over longer wavelengths is not critical. This hypothesis states that the curves in Figure 4 are not restricted to a simple linear model and that they can be applied to more-elaborate and representative nonlinear vehicle models. This hypothesis is being tested as part of a research project sponsored by FRA and Consolidated Rail Corporation (Conrail) in which this methodology will be applied to more-accurate vehicle models to determine effective threshold and resonances for four specific freight vehicles. The validation will be conducted as follows.

First, a single sine-wave track-geometry input will be tested. The amplitude will be adjusted so that it complies with descriptor requirements.
Emphasis will be on frequencies near key vehicle body resonances and for various phase relationships that are input at each end of the vehicle.

Following this, the vehicle model will be subjected to pairs of sine waves of different frequencies the combined magnitude of which does not exceed the descriptor requirement. The two frequencies, \( f_1 \) and \( f_2 \), are chosen on the basis that they are related as a rational harmonic, i.e.,

\[
|n_1 f_1 - n_2 f_2| < p f'
\]

where \( n_1 \) and \( n_2 \) are integers, \( p \) is the damping factor, and \( f' \) is the resonant frequency.

**PERFORMANCE-BASED TRACK-QUALITY INDICES**

**TQIs as Summaries of PBTGDs**

The PBTGD provides a mapping between the track geometry at a point and vehicle performance. The PBTGD generates as many data values as are recorded by the geometry car. For FRA's T-6 vehicle, this means that there will be more than 5000 values for the PBTGD per kilometer of track. This is clearly far too much data to be of use in long-range track maintenance planning. Although a section-gang foreman, who is responsible for the daily upkeep of, say, 81 km of track, would be interested in each value of the PBTGD, an engineer or planner who has a wider responsibility requires some type of statistical summary that is representative of track quality over large track segments. Such summary statistics, as mentioned earlier, are defined as TQIs. Deriving the TQIs from PBTGDs allows the former to be interpreted as indicative of vehicle performance.

**Types of Summaries**

Both root-mean-square (RMS) and exception-type statistics have been used as TQIs; however, by 1980, only Southern Railway had formulated a TQI that could be called performance-based (7). The most significant drawback of RMS-type TQIs (mean, standard deviation, etc.) is that information regarding discrete geometry deviations is totally suppressed, but when these deviations are not directly linked to poor vehicle performance, their retention is not critical. Concurrently, the use of a rank-order TQI requires the definition of one or more thresholds of acceptability for the magnitude of the geometry deviations. With the exception of track gauge (and Southern's thresholds for track warp), these levels have been set arbitrarily (7); i.e., they have not had vehicle performance as their basis. Hence, in the case of TQIs that are derivatives of PBTGDs, it appears favorable to use RMS statistics.

However, the situation changes when a PBTGD, which directly relates the amplitude and wavelength of a geometry perturbation to probability of vehicle failure, is used as the basis for a TQI. In this case it is desirable for the TQI to retain some information regarding particularly poor values, i.e., high probability of derailment, of the PBTGD. Likewise, there now exists a rationale for assigning thresholds to be used in formulating TQIs from rank-order statistics.

**Use of Histogram to Develop TQI**

Figure 9 shows a hypothetical histogram of values that might be developed for some PBTGD as a result of a track-geometry survey. Each recorded value of the PBTGD is on the histogram. Since a particular track-geometry input evokes a definite response in the vehicle model, failure in the model (Figure 1) was deterministic. The transition from model behavior to real vehicle behavior introduces uncertainties that arise in two ways:

1. For a given vehicle type, the dynamical characteristics, e.g., spring constants, will vary from the nominal according to some probability distribution; and
2. For a given vehicle response, the probability of vehicle derailment is not binomial [for example, for this study, the level of acceptable wheel-rail force was defined as 143 kN; obviously, this does not imply that track will always sustain a loading of 142.9 kN and will certainly fall at a force of 143.2 kN].

In Figure 9 a hypothetical density function for probability of vehicle derailment has been mapped onto values of the PBTGD. In practice, this function must be estimated by using empirical techniques. This figure also indicates that there are two hypothetical thresholds: \( T_1 \), which is the acceptance standard for new or rebuilt track; and \( T_2 \), which represents the threshold for immediate spot maintenance. The latter threshold is set by management personnel, who determine these thresholds based on what is considered to be the acceptable risk of derailment.

Figure 9 clarifies the problems associated with selecting a TQI. A section-gang foreman is interested in every incident in which the PBTGD is equal to or exceeds \( T_2 \). A TQI that suppresses these individual locations is of no particular use. However, a chief engineer who is planning next season's production maintenance program would be more interested in the general shape of the histogram (sample moments) and in the rate of migration of points to the extremes. The engineer would not, however, be totally uninterested in the number of points that lie outside \( T_2 \). This dual interest seems to lead to a hybrid TQI, one that combines the characteristics of RMS and exception-level statistics. For example, the critical threshold \( T_2 \) could be expressed as

\[
T_2 = \bar{x} + n s, \text{ where } \bar{x} \text{ is the sample mean, } s \text{ is the sample standard deviation.}
\]
Figure 10. Viscous-blob model of track degradation by using PBTGDs.

Figure 11. Hypothetical change in track quality with time.

standard deviation, and n is a pure number. The value of n would indicate when the maintenance program for a track segment changed from basic to production (n will decrease as the track deteriorates).

Figure 10 depicts three successive PBTGD histograms, each separated by a time interval, for a segment of track that is deteriorating with accumulating time or tonnage. The histogram tends to flatten, as would a viscous blob resting on a horizontal surface. The PBTGD histogram or its derivative TQIs say little (except for the hypothesis that poor track degrades faster than good track) about the rate at which this flattening process occurs. In order to determine the rate of track deterioration for a given level of track use, structural characteristics must be incorporated. Consider, for example, two contiguous track segments A and B that hypothetically receive identical use (annual tonnage, wheel-load distribution, train speed, etc.) and that both have 25 percent of the ties defective. Segment A is tied and surfaced so that no defective ties remain in the track. Simultaneously track segment B is surfaced but none of the defective ties are replaced. The behavior of a geometric TQI is qualitatively indicated in Figure 11.

Although it is a common and expeditious assumption to assume that track structure is elastic, it is the deviation from elastic behavior that results in deterioration of the track surface. Nonelastic behavior occurs primarily in the tie fastenings (spikes) and in the ballast and subgrade. Efforts are under way to develop constitutive relations for track surface (6,15) that could provide the basis for an analytical model of track-surface deterioration. Another approach, which is being pursued as a part of the joint FRA-Conrail maintenance-of-way planning research program, involves the empirical observation of the deterioration of selected revenue trackage (9).

Many parameters have heretofore been used to estimate track condition even though they lack a direct relation to vehicle performance, e.g., defective-tie counts. Some of these will prove to be excellent indicators of the rate of change of the quality of the track surface. We are thus led to the following scenario:

1. A purely geometric TQI, which provides a macroscopic measure of track quality and which is a derivative of PBTGDs,
2. A function that describes track deterioration as the change in this TQI, $\Delta TQI = f(S, U, P, M)$, where $S$ is significant structural parameters (e.g., rail weight, track stiffness, and tie condition); $U$ is use factors, which include annual tonnage and its wheel-load spectrum; $P$ is present track quality; and $M$ is any miscellaneous parameter, e.g., unusual rainfall.

CONCLUSION

At the risk of oversimplifying the developmental problems, a framework for formulating measures of track quality has been described. The method advocated requires a clear distinction between measures of present track quality and indicators of the rate of change of track quality. It is the thesis of this paper that the former be made up only of data that represent loaded-track geometry since this is what determines the behavior of a given vehicle traversing the track.

The actual development of PBTGDs is quite a bit more complicated than presented here. It requires detailed analysis of combinatorial geometry inputs, i.e., multiple frequencies of a given geometry and simultaneous excitation of two geometry inputs or combinations of geometry deviations (16).

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