Comparison of Two Integration Methods in Transportation Routing

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Two methods of integrating the only principle that produces an optimal transportation route are compared, where c is the criterion function whose path integral is to be minimized. (All quantifiable factors, including environmental factors, may be included.) The optimum curvature principle is a necessary condition that an optimal route (however obtained) must satisfy in any region where c is smooth. Classical routes such as linear, parabolic, and circular splines are approximations to optimal routes. An intrinsic-equation algorithm that may have the necessary smoothness is introduced and is compared with a previously presented arc-of-circle algorithm. In the example with known analytic solution, the arc-of-circle algorithm is an adequate approximation to the preferable intrinsic-equation algorithm, the latter of which reduces to the former in the case of constant curvature. The intrinsic-equation algorithm is an order of magnitude more accurate in the example and is preferable because it is easy to use and because the other algorithm does not satisfy the smoothness hypotheses. Discontinuities of the criterion function can be allowed.

This paper compares two integration schemes for establishing optimum transportation routes (referred to in this paper, for brevity, as highways) by the Optimum Curvature Principle (OCP). The OCP was described by Howard, Bramnick, and Shaw (1) and applied to a practical example by Howard and Shaw (2). For convenience, this paper describes the OCP. The first integration scheme uses a sequence of circular arcs joined together (as is sometimes done in highway routing), and the second uses a Taylor series expansion through cubic terms in which the path segments are joined together and there may be continuous curvature at the joints. The comparison reported in this paper is performed to investigate the practical significance of the possibly higher degree of smoothness of the second method. The hypothesis underlying the derivation of the OCP necessary condition by means of the calculus of variations implies the greater degree of smoothness. Error analyses are carried out in each case.

STATEMENT OF THE PROBLEM

The problem can be stated in two parts:

1. The mathematical problem is the numerical solution of a two-point boundary value problem in ordinary nonlinear differential equations, which represents the mathematical statement of the OCP described below. The practical problem is plan optimization of a highway between two given locations.
2. This paper is also concerned with studying a criterion field with a known solution for the optimum routes in order to establish the sensitivity and accuracy of the numerical integration schemes considered. Two integration algorithms are used: the new intrinsic-equation method introduced in this paper and Howard and Shaw's arc-of-circle method (2). Error analyses have been carried out (with the known analytic solution as standard of reference) to establish the correctness and the error bounds of the methods.

NOTATION

The following notation is used in this paper:

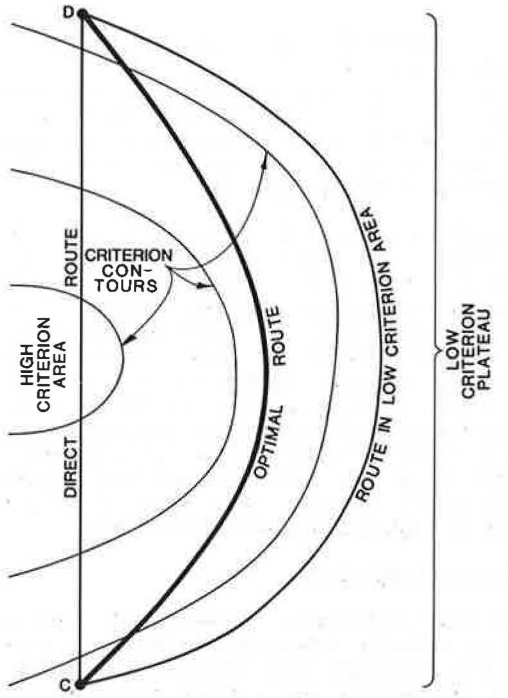
C^0 = segmented curve with discontinuous tangent;
C^1 = curve with smooth, continuous tangent;
C^2 = curve with smooth, continuous curvature;
$c = c\ (x, y)$ = criterion function at point x, y;
$c = \exp(0.05y - 0.2x)$ = equation of the exponentional cost function;
$c_x = \partial c/\partial x$ = partial derivative of c with respect to X;
$c_y = \partial c/\partial y$ = partial derivative of c with respect to Y;
$c_{xx} = \partial^2 c/\partial x^2$ = partial second derivative of c with respect to X;
$c_{xy} = \partial^2 c/\partial x\ \partial y$ = partial second derivative of $\partial c/\partial y$ with respect to X;
$c_{yy} = \partial^2 c/\partial y^2$ = partial second derivative of c with respect to Y;
$c' = dc/ds$;
$dc/ds_{\theta+\pi/2}$ = directional derivative of c perpendicular to route;
$E_d = \alpha h^k$ = discretization or truncation error in terms of step size h;
$E_r = \beta/h$ = round-off error in terms of step size h;
h = distance between adjacent points (step size);
$K = d\theta/ds$;
K_0, K_0' = K, K' values at point 0;
$K' = d^2\theta/ds^2$;
k = slope of log absolute error versus log step size on the coarse side of the mesh (large step sizes);
R = radius of curvature of optimum route;
s = distance along optimum route;
x, y = coordinates of the criterion function along X and Y axes;
x', y' = dx/ds, dy/ds;
δs = small increment in s corresponding to a small increase $\delta\theta$ in θ;
$\delta x, \delta y$ = small increments in x, y corresponding to increments δs and $\delta\theta$;
$\delta\theta$ = small increment in θ corresponding to a small increase δs in s;
θ = angle of the route with the positive X axis (rad);
θ_s, x_s, y_s = values of θ and coordinates at a point s along the optimum route;
θ_0, x_0, y_0 = initial values of θ_s, x_s, y_s;
θ_1, x_1, y_1 = values for the next point of θ_s, x_s, y_s; and
$\theta' = d\theta/ds$.

OPTIMUM CURVATURE PRINCIPLE

Description

The OCP is illustrated in Figure 1. Let us take a case of a route (e.g., a highway) to be built between points C and D. The criterion field is as shown and represents construction costs per mile of highway. There is a high-cost area to the left between the two end points and a low-cost plateau to the right of the figure. Such a situation could exist where a depression in the ground (with marshy conditions) is in the high-cost area; this is surrounded by rising ground, the condition of which

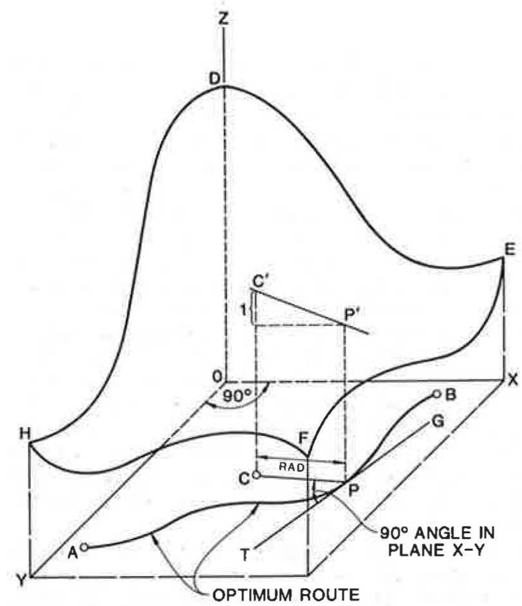
Figure 1. General illustration of optimum curvature principle.



gets gradually better, resulting in higher, drier ground. The low area would need an embankment to raise the roadway above flood criteria as well as suitable treatment for the marshy ground; this is surrounded by ground at gradually lower cost and eventually by good ground at constant low cost. There is a straight-line route passing through the high-cost area and a circuitous route in the low-cost area. Somewhere in between, there is an optimal route that is a compromise and that optimizes the path integral (total cost, in this case) of the route. The optimal route will be output by an OCP computer program that implements the OCP, provided that certain smoothness requirements of the criterion field are followed. Discontinuities of the criterion field can be allowed.

The OCP is described by Howard, Bramnick, and Shaw (1, Appendix 1). We may express the OCP as follows: At each point of an optimal route, the curvature is equal to the logarithmic directional derivative (or the percentage rate of change) of the local criterion function perpendicular to the route. That is to say that, at any point on the highway, the forward curvature is obtained by projecting up to the criterion surface at that point, measuring the slope of the criterion surface perpendicular to the local direction of the highway, and obtaining the curvature of the new highway path by means of the fundamental OCP equation. For any point on a route obtained by any method, the OCP can be applied to check whether the segment considered is part of an optimum route. If it is not, then a better route exists. This could be useful where a route is established for nonoptimal reasons and certain segments can be optimized. Figure 2 is adapted from Howard, Bramnick, and Shaw (1). The optimal route goes between points A and B; A and B are in the XY plane. HDEF is the ln[c(x,y)] function surface. Further details are given by Howard, Bramnick, and Shaw (1).

Figure 2. Specific illustration of optimum curvature principle.



Derivation

Suppose that in three-dimensional space there are two points separated by layers of air that have different refractive indices. There are an infinite number of different paths that the light rays can take. If only the rays that travel between the two points are considered, there will be a number of paths that will locally minimize the time of travel and one global minimum that will be the shortest path possible. The calculus of variations has been applied elsewhere--for example, by Bliss (3)--to solve this problem.

The OCP is derived in the paper by Howard, Bramnick, and Shaw (1). Imagine a plastic medium formed in such a way that its refractive index at each point is proportional to the local cost function for the highway. Then, if a narrow beam of light is introduced at, say, the south end point of the highway and swept around a semicircle, the light rays will trace through the plastic, in accordance with Fermat's principle of least time, paths that correspond to our optimal routes. The equivalent thing is done on the computer by simulation. The mathematical expression of the OCP (plus the geometric relations required to complete the principle) is as follows:

$$d\theta/ds = (1/c)(dc/ds_{\theta+\pi/2}),\ dx/ds = \cos\theta,\ dy/ds = \sin\theta \quad (1)$$

where

θ = direction of the route,
$d\theta/ds$ = route curvature, and
c = local criteria function.

The subscript means that the direction of the derivative is taken in a direction perpendicular to the route direction. The integration methods are described later.

One characteristic of the OCP is that the optimal paths curve toward the area of the maximum criterion function. This can be understood if it is realized that the optimal route between two points separated by a high-criterion area will be better off if it skirts the high-criterion area and spends more of its path in the low-criterion area (1). The result is that the extrema tend to align themselves along the gradient to the criterion surface.

Example

1. From the exponential cost function in the example given by Howard, Bramnick, and Shaw (1), the optimal routes obtained by two different integration formulas were compared with the analytic solution. This enabled the error bounds to be studied. From the local criterion function surface generated on the computer, the OCP was used to determine the optimal routes between the two end points. To do this, a one-parameter family of optimal routes was generated by varying the starting angle from one of the end points, numerically integrating the OCP to determine the optimal route for each starting angle, and selecting the paths that terminate on the other end point. The optimal route is the best of this discrete set. A shooting method, outlined by Howard and Shaw (2), was used to narrow down the correct starting angle and successively diminish the increments in the starting angle, wherein the angle increments start at 0.1 rad and decrease by a factor of 10 for each successive graph.

2. The illustrative example given by Howard, Bramnick, and Shaw (1) was used to verify that the method did indeed give the optimal-path routes. This example uses an exponential criterion field of $c = \exp(0.05y - 0.2x)$. The optimal route for any starting angle--say, 0.8 rad--was calculated. [The details are not essential to an understanding of this paper, but the interested reader is referred to Equations 5-7 of Howard, Bramnick, and Shaw (1).] This was also used to get the error bounds for the new intrinsic-equation algorithm and the original arc-of-circle algorithm. Figure 3 is the computer printout for the optimal paths for which the intrinsic-equation algorithm was used. The computer printout for the arc-of-circle algorithm is so similar that it has been omitted. Each optimum starts at the same starting angle (say, 0.8 rad) but uses different step sizes. The error is the difference between the experimental and analytic value of the y-intercept when x is zero. Figure 4 shows the graph of log absolute error versus log step size for both algorithms.

INTEGRATION ALGORITHMS

The equations of the two integration algorithms compared in this paper are presented below. To save space, the method of derivation of each is described in sufficient detail that the results may be reproduced by anyone skilled in the art without listing all of the equations of the intermediate steps.

Arc-of-Circle Algorithm

The OCP gives the curvature of an optimal route at each point. A natural engineering approach is to form the route by joining small arcs of circles of curvature. The analytic equivalent of this geometric operation is the computation of the position and the direction of the route at the end of the small arc of length δs by means of the following equations:

$$\delta\theta = \delta s\,(c_y \cos\theta - c_x \sin\theta)/c \tag{2}$$

$$\delta x = \delta s \cos\theta\,(1 - \delta\theta^2/6) - 0.5\delta s\delta\theta \sin\theta \tag{3}$$

$$\delta y = \delta s \sin\theta\,(1 - \delta\theta^2/6) + 0.5\delta s\delta\theta \cos\theta \tag{4}$$

This algorithm was derived and applied by Howard and Shaw (2). The main steps in the derivation are as follows:

1. $\delta\theta$ is obtained from the OCP and definition of directional derivative (4).
2. δx and δy are obtained by resolving the chord of Figure 5 in the X and Y directions, respectively, by use of trigonometric identities and approximating $\sin\delta\theta$ and $\cos\delta\theta$ by their MacLaurin expansions through terms of order $\delta\theta^2$.

Joining small arcs of circles gives a route of class C^1 (smooth, continuous tangent) but discontinuous second derivative (curvature) at the joints. However, an optimal route (by derivation of the OCP) is of class C^2. The question arises as to whether (a) the intuitively clear arc-of-circle algorithm (vastly superior to the linear segmented C^0 approximation) is a sufficiently good approximation to the theoretical optimum for practical purposes or (b) it is possible to develop a smoother algorithm that is not too cumbersome and that yields significantly better results. This paper addresses this question through the following algorithm.

Intrinsic-Equation Algorithm

It is known from differential geometry (5) that a space curve is uniquely determined to within a congruence by its curvature and torsion (two scalar functions of arc length) and a plane curve by its curvature alone. The congruential ambiguity is resolved by specifying initial position and direction.

It is customary to expand the intrinsic equations in MacLaurin's series about a given point on the curve, referred to the intrinsic trihedral (tangent, normal, and binormal vectors). This gives useful local information, but to generate the entire curve it is necessary to resolve coordinates and directions back and forth between the moving trihedral system and the inertial system and to integrate the Frenet-Serret differential equations to obtain the rotation of the intrinsic trihedral as its origin moves along the curve.

In the two-dimensional problem of determining an optimal highway plan, it is more convenient to work directly and exclusively in the inertial frame of reference of primary concern. The algorithm consists of two steps:

1. Compute the curvature K and rate of change of curvature K' for the given point x_s, y_s, and direction θ_s from the following three equations:

$$K = \theta' = (c_y \cos\theta - c_x \sin\theta)/c \tag{5}$$

$$c' = c_x \cos\theta + c_y \sin\theta \tag{6}$$

$$K' = [c_{xy} \cos 2\theta + 0.5(c_{yy} - c_{xx}) \sin 2\theta - 2Kc']/c \tag{7}$$

where subscripts denote partial differentiation and the prime denotes total derivative with respect to arc length along the curve.

Equation 5 is the OCP, derived by Howard, Bramnick, and Shaw (1); Equation 6 is the chain rule of differentiation (4) plus the geometric relations $x' = \cos\theta$, $y' = \sin\theta$; Equation 7 results from differentiating and simplifying Equation 5.

2. Compute the coordinates (x_1, y_1) of the next point on the optimal route and the direction θ_1 of the route at that point, in terms of the distance $h = \delta s$ along the route, from the following three equations:

$$\theta_1 = \theta_0 + K_0 h + K_0' h^2/2 \tag{8}$$

Figure 3. Computer printout of optimal routes for intrinsic-equation algorithm.

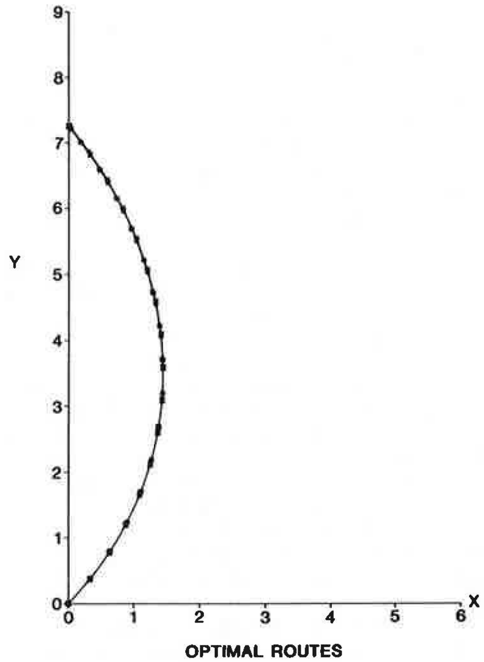


Figure 4. Error analysis.

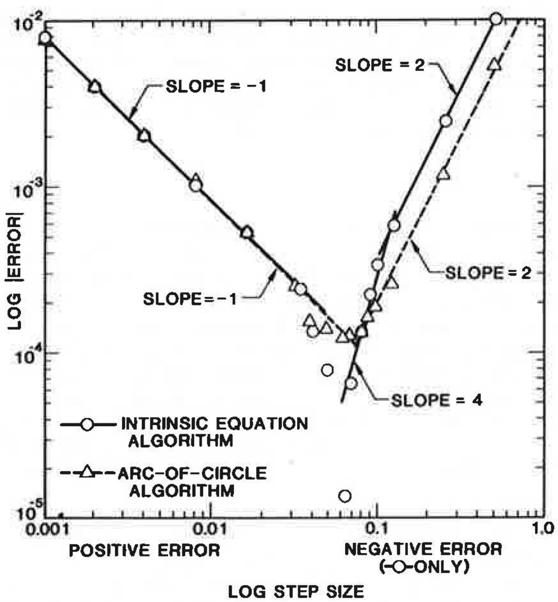
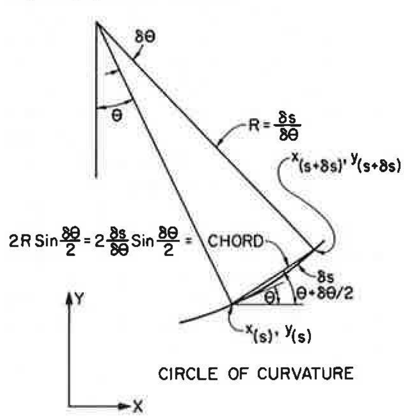


Figure 5. Circle of curvature.



$$x_1 = x_0 + h \cos\theta_0 - 0.5h^2K_0 \sin\theta_0 - h^3(K_0' \sin\theta_0 + K_0^2 \cos\theta_0)/6 \quad (9)$$

$$y_1 = y_0 + h \sin\theta_0 + 0.5h^2K_0 \cos\theta_0 + h^3(K_0' \cos\theta_0 - K_0^2 \sin\theta_0)/6 \quad (10)$$

Equations 8-10 are the Taylor series expansions of the respective functions θ_h, x_h, y_h about the values θ_0, x_0, y_0; previously derived relations are used to obtain expressions for the coefficients in the Taylor expansions. They are the practical equivalent of the intrinsic equations of the curve, referred to inertial axes.

Although it appears that x and y are computed to one higher order of accuracy in δs than is θ, they all are computed to the same order of accuracy in $\delta\theta$ and in K--namely, through K'. In fact, if K' = 0 (implying constant K), then the intrinsic-equation algorithm reduces to the arc-of-circle algorithm. Thus, the difference between the two is that the intrinsic-equation algorithm takes into account rate of change of curvature whereas the arc-of-circle algorithm does not.

ERROR ANALYSIS

It is known that the computational error in numerical integration on the digital computer is the sum of two components: (a) the discretization error and (b) the round-off error. These errors behave as follows:

1. The discretization error dominates on the coarse side or large step sizes. It is known that

$$E_d = \alpha h^k \quad (11)$$

where

h = numerical integration step size,
k = order of the method, and
α = factor that depends on the problem being solved.

2. The round-off error dominates on the fine side or small step sizes. It is known that

$$E_r = \beta/h \quad (12)$$

where β is a factor that depends on computer word size.

For the present example, with known analytic solution, the error was calculated by subtracting the computed value from the experimental value. The absolute value of the error is plotted against step size on log-log paper in Figure 4.

The arc-of-circle algorithm produces the neater curve, showing slope 2 where discretization error is dominant; a slope of -1 occurs where round-off error is dominant; the optimal step size (h) is about 1/16. The total error does not change sign, and there is a true minimum error.

The intrinsic-equation algorithm is a case where the error changes sign and has a zero value. Since log 0 = $-\infty$, there is a sharp dip in the curve. The slope is -1 where the round-off error dominates and 2 where the discretization error dominates. The optimal step size is between 0.05 and 0.075. Theoretically, there is an h when the error is zero, but it is not practical to identify the exact value. If one examines the area of the sharp dip, the intrinsic-equation algorithm is an order of magnitude better than the arc-of-circle algorithm for optimal step sizes of each.

DISCUSSION OF RESULTS

1. The results show that the intrinsic equation procedure and the various error analyses carried out, together with the application to a problem with a known answer, verify the validity and accuracy of the method.

2. From the application of the OCP (with the two integration algorithms considered) to the illustrative example given by Howard, Bramnick, and Shaw (1), the methods were found to give correct results, with reasonable error bounds for the value of y when x = 0 as checked by the analytic solution in the same paper. For the intrinsic-equation integration, the error bound is about ±0.000 013 in a y of about 7.2. For the segment of a circle integration, the error bound is about +0.000 12 for the same y at about the same step size.

The error analyses, log absolute error versus log step size, are shown in Figure 4; they use, respectively, the arc-of-circle and the intrinsic-equation algorithms. The superiority of the latter is shown by the fact that it changed sign at a step size of about 0.06. A slope of -1 was clearly shown, for a step size of less than 0.03, on the fine mesh side of the graph as well as unusual stability for the random round-off error, which may be irregular. On the coarse side of the mesh, a slope of 2 was clear for step sizes greater than 0.13. For step sizes between 0.03 and 0.13, because of proximity of the change in sign, and mixing of the effect of round-off and discretization error, there was a steepening of the slopes. This steepening of the slopes and the change in sign show the superiority of the intrinsic-equation algorithm over the arc-of-circle algorithm in this example.

FUTURE RESEARCH

Since the OCP is a new optimizing tool, there are many areas in which further work can usefully be done. These areas include generation of the criterion function, integration of the OCP, and generalization of the OCP in various directions, including discontinuities in the criterion function, generalized end point conditions, and the like. The work of Howard, Bramnick, and Shaw (1) has been drawn on for some of the suggestions that follow.

Local Criterion Function

Revenue is lost to the community when taxable land is lost to right-of-way. If this factor is to be usefully included, some planning projections are needed of the future possible use the land might have had if no expressway had been built. The impact of the expressway itself on future land use could be included in the planning study.

The local criterion field could be generalized to include the effect of discontinuities such as sudden variations in right-of-way costs and environmental factors or sudden change from four to six lanes. Howard and Shaw (2) treat this problem together with the Weierstrass-Erdmann Corner Condition (which we hope to present in later papers).

When the OCP has been generalized for practical application to three-dimensional problems, factors that depend on direction and location relative to factors immediately adjacent to the point considered could be included. Some of these factors are cut and fill and other factors dependent on vertical alignment.

User costs and environmental and ecological factors can be included in (and can dominate) the criterion function. The method encompasses whatever factors are considered.

Theoretical Problems

The application could be extended to three-dimensional problems such as hilly country where cut and fill becomes important. The OCP remains valid, as shown by Howard, Bramnick, and Shaw (1), but, for simultaneous plan and profile optimization, a torsion principle [developed by Shaw (6)] is needed. It is planned to present this in a later paper.

The class of admissable arcs could be extended from continuously differentiable to piecewise smooth, and the boundary conditions could be extended by using such lemmas as the Weierstrass-Erdmann Corner Condition; the transversality condition could be included to allow for variable end points, such as cities being two-dimensional regions rather than points, or to find a route to a major river or political boundary.

The intrinsic-equation method, together with its error bounds, could be studied to improve the application of the OCP, particularly if some of the research suggested above were carried out. The theory of splines appears to have potential application in this regard.

Sufficient conditions for a minimum could be investigated. The OCP is one of several necessary conditions that make possible constructive determination of local extrema. Theorems discussed by Bliss (3), Akheizer (7), and others provide sufficient conditions for a weak extremum (extremum over the class of differentiable arcs) and sufficient conditions for a strong extremum (extremum over the larger class of piecewise smooth arcs), but the practical usefulness of these theoretical conditions is not clear without further study.

CONCLUSIONS

Howard, Bramnick, and Shaw (1) established the principle and theoretical feasibility of the OCP. The OCP is a practical aid in the location and optimization of highways in certain cases. An improved intrinsic-equation integration process is originated in this paper, and its usefulness is demonstrated. An error analysis has been carried out to study the error in cases where the true end value is known.

Bounds can be established on the accuracy of the optimal routes in a practical way. It is shown that the error bounds are well within engineering tolerances.

The reasons for comparing the two integration algorithms are theoretical and practical. The Hilbert differentiability condition (satisfied in optimal routing problems) proves that optimal routes must be at least as smooth as the criterion function, which must be as smooth as class C^2 for the derivation of the Euler-Lagrange equation and the OCP. But the joins of circular arcs, parabolas, and straight lines often used in highway design are only of class C^1 and thus cannot be optimal routes, whereas the intrinsic equations of a curve can be carried to as many terms as necessary to ensure any required degree of smoothness. On the other hand, in this one analytic example, the circular arcs approximate the true optimal route to a sufficient degree of engineering accuracy. This is comforting since, once the optimal route (or routes) has been established and accepted, the highway engineer can approximate it with curves and straight lines in his or her usual way. The same program can then be used to find the cumulative effect of this departure (or any other departure) from the optimal route. But it must be emphasized that the optimal route that does satisfy all of the hypotheses must be obtained first as a standard of reference.

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Effect of Increased Truck Size and Weight on Rural Highway Geometric Design (and Redesign) Principles and Practices

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A summary is presented of a study of the effects that an increase in legal truck limits would have on highway geometric design elements and of the cost implications should various segments of the Texas highway system require redesign and modification to facilitate their safe and efficient operation. The paper includes (a) a review of past and current research concerning the effects of a possible change in legal vehicle dimensions and weights on the geometric design elements of rural roads, (b) an identification of those geometric elements most affected by a change in truck dimension and weight, (c) an assessment of the effects a change in legal truck size and weight will have on these geometric design elements for a variety of operating conditions, and (d) an estimate of the cost required to redesign and modify the highway section.

A set of issues surrounding the legal limits on sizes and weights of motor vehicles has become a primary policy concern of government and the trucking industry. Such concern is reflected by current federal initiatives (stemming from the Surface Transportation Act of 1978), related study activities, and actions of several state transportation agencies.

Fuel shortages and rapidly increasing fuel prices have provided an impetus for resolving many of the problems associated with vehicle sizes and weights. The underlying notion is frequently reflected in the following simple relation: Large vehicles can carry more freight per unit of fuel. However, although fuel conservation is important, it is only one of many measures that may be used in an analysis of size and weight issues.

Today's highway network is the result of an evolutionary process that represents, among other things, a mix of geometric design principles and practices. Any significant change in vehicle operating characteristics should require an assessment of geometric design practices and the impact on the existing highway system in terms of operational aspects and safety. Also needed would be an estimate of the cost required to redesign and modify the current network or segments of the network to accommodate the larger vehicles.

In Texas, a study is under way to evaluate some of the effects of operating larger and heavier vehicles on the highway system. Initial results, determined by using a study technique modified from the National Cooperative Highway Research Program (NCHRP) (1), showed estimated pavement costs, bridge costs, truck operating cost savings, and fuel savings that would result from increases in limits on axle weight and gross vehicle weight (GVW) coupled with corresponding changes in vehicle unit length and width. No change in the height of vehicles or trailers is considered in this study. The work reported in this paper focuses on the costs of the geometric design and redesign requirements associated with increases in vehicle size (length and width) as well as weight.

SCOPE OF THE RESEARCH

As an initial assumption, four different vehicle combinations (2) and two highway classification schemes (cases 1 and 2) are considered. The four vehicle scenarios are shown in Figures 1 and 2.

In case 1, the three functional rural highway systems are considered in the analysis: (a) the Interstate highway system, (b) the U.S. and state highway system, and (c) the farm-to-market (FM) road system. Case 1 represents a traditional approach that fits the Texas highway network of about 60 000 miles.

Case 2 differentiates on the basis of road use. In case 2, the following rural functional classes, or combination of classes, are also considered in