Optimal Peak-Load Pricing, Investment, and Service Levels on Urban Streets—A Numerical Example

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Socially optimal automobile tolls, bus fares, bus service levels, and street capacity can be determined by the use of an integrated peak-load pricing model. The objective of this paper is to develop such a model and to demonstrate the model's applicability and usefulness with regard to its implications for transportation policy. The model that is presented departs from previous ones in that it uses disaggregate travel demand models in order to derive empirically implementable pricing and investment rules for the provision of transit service. Our proposed model, as a whole, is concerned with maximizing the sum of the expected utilities derived by urban street travelers. Numerical results reveal that, given the cost and demand conditions posited, under all but the least-congested travel conditions considered, the travelers' welfare maximizing levels of automobile tolls far exceed those fees now collected by North American gasoline taxes and other automobile user charges. When the imposition of optimal automobile tolls appears impractical, the common practice of providing reserved bus lanes has much to recommend it, especially under the traffic and pricing conditions that prevail during peak hours in most North American cities. Given that automobile tolls are restricted to the gasoline tax, optimal provision of bus service implies mass transit subsidies, at least in peak hours. However, provision of reserved bus lanes would reduce substantially the travelers' welfare losses that result from subsidy reductions.

The objective of this paper is to model travel demand in a manner that derives the pricing and investment rules for a socially optimal provision of urban bus transit. The proposed model is primarily concerned with maximizing the sum of the expected utilities derived by travelers on urban streets. We use disaggregate travel demand models to derive the sum of these utilities. Disaggregate travel demand models are random utility models. They use individuals as the basic decision-making units in order to analyze travel behavior. The disaggregate demand models are used to derive the aggregate demand for different modes of transportation by the travelers on a given urban street.

Given the aggregate demand for different modes of transport, we use the functional forms for the expected utility and the equilibrium demand volumes and travel times of the modes of transport over a given urban street at peak and off-peak hours in order to express our objective function. Next, we use this objective function to derive the socially optimal bus fare, automobile toll, and bus service levels.

FRAMEWORK FOR A SIMPLE TRAFFIC ARTERY MODEL

In this numerical work, we limit our attention to the trips generated and ended along each side of a one-mile stretch of an urban two-way street. Suppose that N people per hour begin and N people per hour terminate trips along each mile of each side of this street. We assume that the origins and destinations of trips are uniformly distributed along each side. There are two modes of travel--automobile or bus. We define each trip to be M miles long. The demand for each mode is a function of the choice maker's socioeconomic characteristics and of the attributes of the alternatives, for example, trip costs and trip times. The travelers take the trip for work, shopping, or social or recreational purposes. We assume that workers take their trips in peak hours and nonworkers take their shopping or social and recreational trips in off-peak hours. Therefore, by assumption, the cross-elasticity between peak and off-peak demand travel is zero. This eases the analytical burden considerably.

Let us assume that the disaggregate demand models have the logit functional form that can be derived as a representation of utility maximization among a discrete set of alternatives under uncertainty. That is, the probability that individual s, selected randomly from the population, will choose mode i given by the multinomial logit model: $Pr_{is} = \exp(U_{is})/\sum_{j=1}^{I}\exp(U_{js})$, where Pr_{is} is the probability that individual s will choose alternative i from the set of alternatives available, and U_{js} is the utility of alternative j to individual s. Furthermore, we assume that the utility of alternative i to individual s (U_{is}) has the following functional form: $U_{is} = \alpha Z_{is} + \xi_{is}$, where α is a vector of parameters, Z_{is} is a vector of functions of the socioeconomic characteristics of individual s and of the attributes of alternative i, and ξ_{is} is a random variable that represents an unobserved disturbance or error term.

The multinomial disaggregate travel demand models used in this numerical work are those developed by the Metro Travel Commission (MTC)-Cambridge Systematics System for the San Francisco Bay Area (1). These models are based on data from the 1965 surveys by the Bay Area Transportation Study Commission (1,2).

For the demand models that are presented in this paper and for both the peak and off-peak periods, the expected utility of a utility maximizing individual chosen at random from the population is obtained, with an approximation, by applying a combination of the Clark (3) and the Lancaster (4) methods of aggregation (5). Clark presents a set of formulas for the first two moments of the maximum of two normally distributed random variables and the covariance of the maximum with a third normally distributed variable. Then Clark proposes that his solution for the moment be used as an approximation by assuming that the maximum is itself normally distributed. This approximation then permits solution of the expected utility. Lancaster approximates logistic distribution of standard deviation $\pi/\sqrt{3}$ of ξ by a normal distribution of standard deviation $(15/16)\ (\pi/\sqrt{3})$. Following the Lancaster method of approximation, we assume that (a) ξ_i has a normal distribution with mean zero and variance $(\pi^2/6)\ (15/16)^2$ and (b) ξ_i is independent from all ξ_j. We also assume that Z_i, the explanatory variables, are distributed multivariate normal with the row vector of means $E(Z_i)$ and covariance matrix Σ_Z. Then, we have $U_i \underset{\sim}{\sim} N[\alpha E(Z_i), \alpha\Sigma_Z\alpha^T + (15/16)^2 \cdot (\pi^2/6)]$. By following the Clark method of aggregation, the expected utility for the peak period (period 1) $[E(\tilde{U}^P)_1]$ can be computed as an integral part of the technique:

$$E(\tilde{U}^P)_1 = \iiint_{AST} \max(U_{A1}{}^P, U_{S1}{}^P, U_{T1}{}^P)\, g_{u_1}{}^P(U_{A1}, U_{S1}, U_{T1}) \cdot dU_{A1}dU_{S1}dU_{T1} \quad (1)$$

where

$g_{\tilde{U}^P}(\cdot)$ = the probability density function (pdf) of $U_1{}^P$,
A = drive alone,
S = shared ride, and
T = transit vehicle (bus) (5).

Similarly, for the off-peak period (period 2), the expected utility of a utility maximizing individual chosen at random from the population $[E(\tilde{U}^P)_{s2}]$ can be computed as follows:

$$E(\tilde{U}^P)_{s2} = \int\int_{AT} \max(U_{As2}{}^P, U_{Ts2}{}^P) g_{U_{s2}}{}^P (U_{As2}, U_{Ts2}) dU_{As2} dU_{Ts2} \tag{2}$$

where

$g_{U^P_{s2}}(\cdot)$ = the pdf of U^P_{s2},
A = automobile,
T = transit vehicle (bus), and
s = shopper or traveler who takes a social or recreational trip (4).

Given Equations 1 and 2, to complete the setting of a framework for the models that are presented in this paper, the next section focuses on the measured values of some of the independent variables that appear in the demand models and consequently in Equations 1 and 2.

MEASURED VALUES OF VARIABLES THAT APPEAR IN THE MODEL

Automobile Operating Costs

We assume that the measured automobile operating costs per mile in period t (Ca_t) represent a function of automobile travel time per mile in that period (ta_t). Specifically, the following functional form is selected for this study: $Ca_t = H_0 - H_1/ta_t$. The costs of each automobile trip are shared by A passengers. For an automobile traveler who originates a trip in period t, in a one-mile stretch of the artery total vehicle operating costs are

$$ACOST_t = (MCa_t + MFa + g)/A \tag{3}$$

where Fa is the automobile toll per mile and g is a fixed cost that is independent of the length of the trip.

Automobile Travel Time

Let us assume that buses and automobiles distribute themselves uniformly across the width of each side of each mile of the artery. Under this assumption, the measured travel time per automobile trip on each side of the artery, in period t, is a function of that side's volume to capacity ratio. Specifically, we select the following functional form for this study:

$$ta_t = ta_0 \{1 + a[(\delta X_t + M\,Na_t/A)/K]^b\} \tag{4}$$

where

ta_0 = travel time per mile at zero flow,
a and b = constants,
X_t = number of buses per hour for period t,
A = the average number of passengers per car,
M = trip length,
K = capacity of the road,
Na_t = number of automobile travelers who originate in a one-mile stretch of the road at period t, and
δ = the bus's automobile congestion equivalent.

MNa_t enters this expression because, in period t, any given point on the road is passed by automobile travelers who originated in each of the M miles that preceded that point. Therefore, the one-way in-vehicle travel time for an automobile traveler who takes trips in period t is equal to

$$ATIMET_t = M\left(ta_0\{1 + a[(\delta X_t + M\,Na_t/A)/K]^b\}\right) \tag{5}$$

Bus Travel Time

The measured travel time per bus trip has two components. First, when not engaged in stopping and starting maneuvers, a bus is assumed to travel at the same speed as an automobile (i.e., to require ta_t min/mile in period t). In addition, in each route mile, Nt_t travelers board and Nt_t leave X_t buses at Y or fewer stops. (Y is the number of uniformly spaced bus stops per mile.) Hence, $\mu_{ST} = 2Nt_t/X_tY$ is the average number of passengers that board or leave one bus at any one stop. Suppose that bus travelers make their decisions as to when to travel independently. Then, the probability that a total of n travelers will board and alight from any one bus at any one stop is given by the Poisson distribution with parameter μ_{St}. That is, $\Pr(n) = \exp(-\mu_{St})\mu_{St}{}^n/n!$. The probability that a given stop will be made, then, is $1 - \exp(-\mu_{St})$ (i.e., 1 - the probability that no one will have that stop as either origin or destination). The expected number of stops per mile is Y times this fraction. Therefore, the expected time required to travel one mile is equal to

$$tt_t = ta_t + 2N_t\epsilon/X_t + \S\, Y[1 - \exp(-\mu_{st})] \tag{6}$$

where ϵ is the time required to board or unload a passenger once a bus has stopped and § is the amount by which the time required to traverse a route segment is increased by each stop and start maneuver. Therefore, the one-way in-vehicle travel time for a bus rider who takes trips in period t is equal to

$$TIME\,T_t = M\{ta_t + 2Nt_t\epsilon/X_t + \S\, Y[1 - \exp(-\mu_{st})]\} \tag{7}$$

Access Time for Bus Riders

Most bus travelers neither live nor have their destinations on the traffic arteries traversed by the bus they use. Rather, a typical bus rider must walk to the route from an origin and from the route to a destination. Once a traveler reaches the artery, he or she must walk to the nearest bus stop. If origins and destinations are uniformly distributed between stops and there are Y uniformly spaced bus stops per mile, the one-way walking time for the traveler who uses the bus will be ht + 60/2SY min, where ht is the walking time (min) for a typical bus rider to the route from an origin and from the route to a destination and S is the average bus passenger's walking speed (5).

If the average length of a bus passenger's wait at a stop is a fraction (β) of the headway between buses ($1/X_t$), the average measured waiting time in period t is $60\beta/X_t$ min. Therefore, the average total of measured one-way access time by bus in period t is equal to (ht + 60/2SY) + ($60\beta/X_t$) min.

MODEL 1: THE BASIC MODEL

Let us suppose that government's objective is to maximize the total net benefits received by travelers, over a period of a day, from traveling on the artery. That is, given the assumptions and definitions of the last two sections, the objective function can be written as follows:

$$\text{Max } W = U - Ct - Cr \tag{8}$$

where

U = total benefits received by travelers over a period of a day from traveling on a road (Equations 1 and 2),
Ct = total daily operating costs of the bus company, and
Cr = total daily rental costs of that road.

Given the equilibrium flow pattern and the travel demand volume, this model finds those values of X_t, Ft_t, Fa_t, and K that satisfy all the Kuhn-Tucker conditions for maximizing W; that is, we compute those values of X_t, Ft_t, Fa_t, and K that satisfy $\partial W/\partial X_t = \partial W/\partial Ft_t = \partial W/\partial Fa_t = \partial W/\partial K = 0$ for t = 1, 2. The model's statement about street capacity ($\partial W/\partial K = 0$) implies that arterial street capacity is expanded to the point where the value of the marginal product of the last unit of capacity produced just equals the marginal costs of providing that unit of arterial street capacity. The model's statements about bus service at each period of time ($\partial W/\partial X_t = 0$) imply that bus service at each period of time is provided to the point where the marginal benefit of the last unit of a service produced, at that period of time, just equals the marginal costs of providing that unit of service.

We choose the socially optimal automobile tolls at period t (Fa_t) equal to the congestion charge at that period [i.e., the losses an additional automobile traveler's trip imposes on (a) the existing automobile travelers (by increasing the time and vehicle operating cost of their trips), (b) bus travelers, and (c) the bus company].

Similar considerations apply to the socially optimal bus fare. We choose the price of bus in period t (Ft_t) equal to the congestion charge at that period [i.e., the losses an additional bus traveler's trip imposes on (a) the passengers already aboard the bus on which he or she travels, (b) automobile travelers, and (c) the bus company].

MODEL 2: OPTIMALITY CONDITIONS UNDER PRICING CONSTRAINTS

Mohring argues that the provision of bus service involves economies of scale (6). This argument is based on the following example: Suppose that a bus company responds to a doubling of demand along route N by doubling the number of buses that serve the route (X). Given the modal split (Nt/N), if road capacity is allowed to expand such that the arterial volume to capacity ratio remains unchanged, the average number of passengers per mile served by a bus, and hence the average travel time per trip and bus company costs per passenger served, will all remain unchanged. However, such an expansion of road capacity would cut headway between buses in half, and thereby cut the costs of waiting time per passenger in half. Nevertheless, in the framework of our simple model, the argument is not such a straightforward one. In fact, for the demand volumes that prevail under equilibrium conditions, the average number of passengers per mile served by a bus, and hence the average travel time per trip and bus company costs per passenger served, may all change. The magnitude and direction of these changes, which are sensitive to the parameter values in the model, determine whether or not the provision of bus service involves economies of scale.

In our numerical analysis, in some cases road capacity is held fixed. An increase in total travel, therefore, leads to an increase in arterial volume to capacity ratios. The resulting reduction in travel speeds tends to offset the increasing returns aspect (if there is any) of the bus system. For the combinations of parameter values studied in our numerical analysis, these decreasing returns aspects of the system largely outweigh its increasing returns features. That is why we observe that, in some cases, optimal bus fares generate revenues in excess of bus system costs.

However, for the congestion levels (ratio of trip volume to arterial capacity) that seem typical of urban areas at peak hours, model 1 yields socially optimal automobile tolls far higher than the roughly 3-9 cents/mile implied by the gasoline taxes and other excises imposed on automobile travel in North American cities. Therefore, if existing tolls on automobile travel are taken to be incapable of alteration, truly astronomical subsidies for buses would be required to maximize the net benefits of all trips.

These considerations suggest the desirability of adding the following pricing constraint to model 1: Wrong (i.e., inefficiently low) tolls are charged for automobile travel and bus operations must break even.

MODEL 3: OPTIMALITY CONDITIONS UNDER THE ASSUMPTION OF RESERVED BUS LANES

This model differs from the previous ones only in that it allows a fraction (η) of the total capacity of each side of the artery to be reserved for buses, and the remaining capacity [$(1 - \eta)K$] is allocated to automobiles. In our numerical work, we select the values of η that comprise an integral number of lanes of the artery. For example, in the framework of our simple model, we can allocate zero, one, or two lanes to buses (i.e., η is given the value of 0, 0.5, or 1). Under this assumption, the time required for a one-mile automobile trip is

$$ta_t = ta_0 \{1 + a[(M\, Na_t/A)/(1-\eta)K]^b\} \tag{9}$$

and in the expression for tt_t, given by Equation 6, ta_t is replaced by

$$ta_0[1 + a(\delta X_t/\eta K)^b] \tag{10}$$

To allow reserved bus lanes is, in effect, to allow the division of an artery into two separate rights-of-way and to permit allocation of the artery's total capacity so as to equalize the value of its marginal benefit on these two rights-of-way. The policy of allowing reserved bus lanes is based on the argument that the provision of bus service involves economies of scale. The validity of this argument constitutes a strong reason for permitting reserved bus lanes. Then, the equality of the marginal products of capacity calls for a lower volume to capacity ratio on the bus than on the automobile right-of-way. The resulting increase in travel speeds for buses in comparison with travel speeds for automobiles tends to encourage bus riding and, as a result, to increase the social benefits derived by travelers from traveling on the artery.

MODEL 4: OPTIMALITY CONDITIONS UNDER THE ASSUMPTION OF RESERVED BUS LANES AND PRICING CONSTRAINTS

This model differs from model 2 in the same way that

Table 1. Parameter value combinations studied for parameters that vary between peak and off-peak periods in numerical analysis.

Parameter	Peak			Off-Peak	
	Drive Alone	Shared Ride	Bus	Automobile	Bus
A = passengers per automobile	1	2.5		2	
B = hourly bus costs ($)			15		9
β = waiting time between bus headways			-[a]		1/2
ha = automobile access time (min)	4.3	14.3		3.3	
g = fixed costs of automobile trip (cents)	12.5	14.2		5.3	
ht = bus access time (min)			18.5		5.6

[a]During peak periods, the waiting time between buses is ½(1 + 0.000 001 X^2).

model 3 differs from model 1. That is, model 2 involves the assumption that automobiles and buses are uniformly distributed across the artery, and model 4 allows for reserved bus lanes.

In comparison with models 1 and 3, in model 4 as well as model 2 we maximize the total net benefits to the travelers from traveling on the artery given that bus operations must break even for the case that wrong tolls are charged for automobile travel.

BENEFIT/COST IMPLICATIONS OF THE MODELS

Pseudoempirical analysis of the models described is undertaken for the parameter value combinations indicated below for parameters that have the same values in all runs and in Table 1 (5) for parameters that vary.

- M = trip length (miles) = 5,
- Y = allowable stops per mile = 8,
- K = capacity of artery (vehicles/h) = 625 number of lanes (WID),
- δ = Congestion equivalent of a bus = 3 automobiles,
- ta_0 = 2.0 min/mile,
- a = 2.62 min/mile,
- b = 5,
- Ca = cents per automobile mile = 10.565 - 5.706(1/ta),
- § = added time per bus stop made = 0.3 min,
- ε = passenger boarding or unloading time = 0.03 min, and
- Cr = costs of arterial street expansion (cents) = $(100/720)\{[0.06/(1 - e^{-2.1})] + 0.06(0.342)\}WID^{1.0304}$ $[\exp(12.767) + \{2917\ WID + 0.000\ 45[360(5w_1N_1 + 5w_2N_2)]\}$

The net travelers' welfare losses per trip shown in Tables 2 and 3 reflect the net welfare-maximizing levels for travelers of bus service, bus fare, automobile tolls, and road capacity, given the parameter values shown in the tables. However, these optimization problems are solved under the following two constraints: (a) the decision variables cannot have a negative value and (b) the levels of bus service and road capacity cannot be noninteger.

Benefit/Cost Implications of Model 1

Tables 2 and 3 reveal that the scale economies associated with bus operation yield only a modest reduction in system net travelers' welfare losses with increases in the scale of the system. Naturally enough, the net travelers' welfare maximizing share of bus trips (Nt/N) declines with reductions in MN/K, the volume to capacity ratio that would prevail if all trips were taken in single-occupant automobiles. With MN/K equal to 3.2, the maximization of net travelers' welfare would call for 59.4 percent of all trips to be taken by bus (Nt/N = 59.4) and 18.7 percent to be taken by shared-ride (Ns/N = 18.7) but, depending on system scale, a share of bus trips (Nt/N) in the 28.9 to 33.0 per-

Table 2. Welfare losses per trip as a function of congestion level and travel demand for peak cost conditions.

Travel Rate	Number of Lanes	Welfare Losses per Trip	Modal Split			Optimal Automobile Tolls (cents/mile)	Value of Marginal Product of Capacity[a] (cents)
			Drive Alone	Shared-Ride	Bus		
400	4	1.74	0.775	0.225	0.000	3	0.2
400	3	2.08	0.773	0.227	0.000	3	1.0
400	2	3.17	0.420	0.250	0.330	63	7.9
800	4	3.00	0.488	0.223	0.280	40	8.1
800	2	3.36	0.219	0.187	0.594	93	26.8

Note: Length of peak period = 2 h, peak-hour bus costs = $15.
[a]Cost per number of cars/h along a given mile.

Table 3. Welfare losses per trip as a function of congestion level and travel demand for off-peak cost conditions.

Travel Rate	Number of Lanes	Welfare Losses per Trip	Modal Split				Optimal Automobile Tolls (cents/mile)	Value of Marginal Product of Capacity[a] (cents)
			Shopping		Social and Recreational			
			Automobile	Bus	Automobile	Bus		
200	4	1.75	1.00	0.00	1.00	0.00	2	0.0
200	3	1.76	1.00	0.00	1.00	0.00	2	0.0
200	2	1.79	1.00	0.00	1.00	0.00	2	0.3
400	4	1.79	1.00	0.00	1.00	0.00	2	0.3
400	2	1.86	0.778	0.222	0.496	0.504	9	1.5

Note: Length of off-peak period = 10 h and off-peak hour bus costs = $9.
[a]Cost per number of cars/h along a given mile.

cent range would be called for with an MN/K value of 1.6. When MN/K values are high, the provision of an optimal level of bus service would reduce net welfare losses for travelers on the system below the levels that would result with only automobile travel. When MN/K = 1.6, however, the benefits of bus service are modest. When MN/K $\leq$ 1.2, the constrained optimal level of bus service would result in net welfare losses for travelers either identical to or higher than welfare losses with only automobile travel.

The potential benefit of an optimal level of bus service when the level of congestion is high on a traffic artery could reflect the inherent superiority of mass transit vehicles for urban travel. On the other hand, these benefits could result from an inefficiently low level of capacity on that artery. Which of these possibilities is true for a particular case depends on just how costly it would be to add to the capacity of the artery. Tables 4 and 5 show that, when the congestion level is high on a traffic artery and specifically when automobile tolls are constrained to the level of gasoline taxes, the potential benefits of an optimal level of bus service result mainly from an inefficiently low level of capacity on the artery. The optimal level of arterial capacity when MN/K values are high (i.e., MN/K = 3.2) is about four 10-ft lanes. When MN/K = 3.2, an optimal level of arterial capacity would result in welfare losses of 10.7 percent less than those with the present arterial capacity levels.

However, these results are sensitive to changes in demand and, in particular, to the cost conditions that prevail in Table 4. For example, as the constant parameter (b) of the automobile travel time function (Equation 4) increases, the sensitivity with respect to a high level of MN/K increases and the sensitivity with respect to a low level of MN/K decreases. Then for a different value of b, our results may be different. In addition, in Table 4 we estimate the costs of arterial street expansion given a 6 percent interest rate and 35 years of effective lifetime for the road. As Table 5 shows, a change in the interest rate or the effective lifetime of the road changes the costs of street expansion and, as a result, the optimal level of capacity.

Benefit/Cost Implications of Model 3

Tables 6 and 7 deal with the potential benefits of reserved bus lanes when socially optimal automobile tolls and bus fares are charged and when automobile tolls are restricted to those implicit in current gasoline taxes (e.g., 45 cents/five-mile trip in peak hours and 30 cents/five-mile trip in off-peak hours) and deficit constraints.

Parameter values for Table 6 are those that seem most representative of peak-hour travel (MN/K = 1.6); off-peak values are the basis for Table 7 (MN/K = 0.8).

Perhaps the most important generalization suggested by these tables is that, even if socially optimal bus fares and automobile tolls could be charged, the reserved bus lane would result in some benefits (i.e., a reduction in welfare losses of about 31.0 percent). An optimum allocation of capacity would result in a 31.0 percent reduction in welfare losses under peak conditions (Table 6), and

Table 4. Comparison of system operation with and without reserved bus lanes under different pricing and financial constraints: base peak-hour case.

	Welfare Losses per Trip		Optimal Bus Fare (cents)		Buses per Hour		Bus Operating Cost per Passenger (cents)		Bus Share of Capacity (%)		Marginal Benefit per Subsidy Dollar		Value of Marginal Product of Capacity[a] (cents)
Constraint	Without	With	Without	With	Without	With	Without	With	Without	With	Without	With	Without
Social welfare maximizing	3.74	2.09	0	25.0	23	58	22.0	25.0	6.1	100			39.9
Pricing constraint													
Automobile toll = \$0.09/mile	67.47	4.35	60	3.5	11	8	4187.6	3.5	1.1	50			3208.0
Automobile toll = \$0.09/mile and bus operating subsidies = \$0.0	99.43				0				0		7.2		624.7
Automobile toll = \$0.12/mile	64.39	4.88	60	0	11	3	3808.5	2.0	1.1	50	7.9	14.2	3050.9
Automobile toll = \$0.12/mile and bus operating subsidies = \$0.0	97.25	5.02		1.4	0	3		1.4	0	50			610.4

Note: Length of peak period = 2 h, number of lanes = 2, travel rate = 800, and peak-hour bus costs = \$40.

[a]Cost per number of cars/h along a given mile.

Table 5. Average daily capacity costs per mile of the arterial street as a function of number of lanes, interest rate, flow of vehicle trips over the street, and effective lifetime of the street.

	Average Daily Capacity Cost per Mile (cents)					
	Flow of Vehicles = 400 peak and 200 nonpeak			Flow of Vehicles = 800 peak and 400 nonpeak		
Interest Rate (r) and Effective Lifetime (L)	Two Lanes	Three Lanes	Four Lanes	Two Lanes	Three Lanes	Four Lanes
r = 6 percent, L = 35 years	9 964.439	14 954.143	19 991.593	10 279.439	15 269.143	20 306.593
r = 12 percent, L = 35 years	54 050.59	81 906.248	110 047.79	54 365.59	82 221.248	110 362.79
r = 6 percent, L = 30 years	10 313.775	15 484.669	20 705.194	10 628.775	15 799.669	21 020.194
r = 12 percent, L = 30 years	54 204.133	82 190.013	110 361.43	54 519.133	82 505.013	110 676.43
r = 6 percent, L = 40 years	9 727.454	14 594.243	19 507.497	10 042.454	14 909.243	19 822.497
r = 12 percent, L = 40 years	53 967.948	81 780.743	109 878.97	54 282.948	82 095.743	110 193.97

Table 6. Comparison of system operation with and without reserved bus lanes under different pricing and financial constraints: base peak-hour case.

Constraint	Welfare Losses per Trip		Optimal Bus Fare (cents)		Buses per Hour		Bus Operating Cost per Passenger (cents)		Bus Share of Capacity (%)		Marginal Benefit per Subsidy Dollar		Value of Marginal Product of Capacity Without (cents)[a]
	Without	With	Without	With	Without	With	Without	With	Without	With	Without	With	
Social welfare maximizing	3.46	2.35	20	0	11	23	14.7	12.7	1.7	25			4.8
Pricing constraint													
Automobile toll = $0.09/mile	3.76	2.40	0	0	11	23	115.2	17.2	1.1	25	7.0	3.3	42.6
Automobile toll = $0.09/mile and bus operating subsidies = $0.0	4.85	2.65		10	0	23		10.4	0	75			9.8
Automobile toll = $0.12/mile	3.77	2.44	10	0	11	23	112.1	17.1	1.1	25	7.7	2.8	41.5
Automobile toll = $0.12/mile and bus operating subsidies = $0.0	4.84	2.66		10	0	23		10.0	0	75			9.6

Note: Length of peak period = 2 h, number of lanes = 4, travel rate = 800, and peak-hour bus costs = $40.

[a]Cost per number of cars/h along a given mile.

Table 7. Comparison of system operation with and without reserved bus lanes under different pricing and financial constraints: base off-peak-hour case.

Constraint	Welfare Losses per Trip		Optimal Bus Fare (cents)		Buses per Hour		Bus Operating Cost per Passenger (cents)		Bus Share of Capacity (%)		Value of Marginal Product of Capacity Without (cents)[a]
	Without	With	Without	With	Without	With	Without	With	Without	With	
Social welfare maximizing	2.29	2.29			0	0	0	0	0	0	0.3
Pricing constraint											
Automobile toll = $0.06/mile	2.49	2.48	0	0	2	2	1.1	1.1	1.8	50	0.0
Automobile toll = $0.08/mile and bus operating subsidies = $0.0	2.54	2.54			0	0			0	0	0.3
Automobile toll = $0.08/mile	2.58	2.56	0	0	2	2	1.0	1.1	2.1	50	0.0

Note: Length of off-peak period = 10 h, number of lanes = 4, travel rate = 400, and off-peak hour bus costs = $25.

[a]Cost per number of cars/h along a given mile.

Table 8. Comparison of system operation with and without reserved bus lanes under different pricing and financial constraints: base peak-hour case.

Constraint	Welfare Losses per Trip		Optimal Bus Fare (cents)		Buses per Hour		Bus Operating Cost per Passenger (cents)		Bus Share of Capacity (%)		Modal Split Without Reserved Bus Lanes			Marginal Benefit per Subsidy Dollar Without
	Without	With	Without	With	Without	With	Without	With	Without	With	Drive Alone	Shared Ride	Bus	
Social welfare maximizing	3.21	2.45	30	25.0	15	29	29.7	25.0	5.0	100	0.325	0.253	0.422	
Pricing constraint														
Automobile toll = $0.09/mile	3.98		0		11		213.0		2.2		0.655	0.210	0.135	3.0
Automobile toll = $0.09/mile and bus operating subsidies = $0.0	4.85				0				0		0.757	0.243	0.00	
Automobile toll = $0.12/mile	4.06		0		11		219.8		2.2		0.640	0.203	0.157	2.4
Automobile toll = $0.12/mile and bus operating subsidies = $0.0	4.84				0				0		0.750	0.250	0.00	

Note: Length of peak period = 2 h, number of lanes = 2, travel rate = 400, and peak-hour bus costs = $40.

a 0 percent reduction in off-peak hours (Table 7). Achievement of the optimal volume to capacity ratio in peak hours requires the allocation of one lane out of four to buses, given socially optimal automobile tolls, bus fares, and bus service. In contrast, achievement of the optimal volume to capacity ratio in off-peak hours requires no allocation of lanes to buses.

As Table 8 shows, for a two-lane artery that has the same congestion level as prevails in Table 6 (i.e., MN/K = 1.6), the optimum allocation of capacity requires allocation of both lanes to buses, and this would result in a decrease in welfare losses of about 23.9 percent under peak conditions. However, the allocation of one lane out of two to buses would result in an increase in social costs of about 59 percent.

These findings are subject to a very important qualification. As Tables 2-3 indicate, under all but the least-congested travel conditions, the socially optimum automobile tolls are far higher than those implicit in gasoline taxes and other automobile user charges. A proposal to levy the tolls of 40-93 cents per vehicle mile listed in Table 2 would almost certainly generate overwhelming opposition from the public and politicians (especially in North America). If attention is restricted to automobile tolls in the neighborhood of those currently charged in North American cities (for example, 45 cents/five-mile automobile trip in peak hours and 30 cents/five-mile trip in off-peak hours), reserved bus lanes appear to have considerable merit, at least during periods of high traffic flow (peak hours). That is, reserved bus lanes would result in a reduction of 36.1-45.3 percent in social costs when automobile tolls are restricted to those implicit in current gasoline taxes, for example, 45 cents/five-mile automobile trip at peak hours.

Under the conditions given in Table 6, given that automobile tolls are set equal to gasoline taxes, the maximization of net travelers' welfare would require free bus service and would result in average welfare losses per trip of $3.76 in the absence of reserved bus lanes, 8.7 percent greater than the losses achievable with socially optimal tolls. If this pricing constraint is accompanied by the allocation of one lane out of four to buses, however, minimum average welfare losses per trip work out to $2.4--2.2 percent greater than the welfare losses attainable with reserved bus lanes when socially optimal tolls are charged.

As Table 6 shows, the maximization of net travelers' welfare requires free bus service when automobile tolls are set equal to gasoline taxes. However, for the following reasons opposition would also be likely to the provision of free bus service:

1. The setting of a zero bus fare may indeed increase the elasticity of demand for trips and
2. The optimal cost subsidies required under Table 6 conditions to maximize net travelers' welfare when automobile tolls are constrained to the level of gasoline taxes are $124.4/number of cars/h along a given mile of bus service without reserved bus lanes and $62.1 with reserved lanes. Although considerably lower than the deficits that would be required to match reductions in automobile tolls and bus fares, these subsidies, in the absence of reserved bus lanes, are still substantial. In the following section we study a deficit constraint model. This model is designed to test the welfare loss implications of a zero percent subsidy level.

Benefit/Cost Implications of Model 2

In the absence of reserved bus lanes, the elimination of bus operating subsidies would result in substantial increases in welfare losses. As Table 6 indicates, the increase in average welfare losses per trip that would result from lowering bus operating subsidies from $124.4 to 0 per number of cars/h along a given mile (i.e., from the value required to maximize net travelers' welfare subject to the gasoline tax toll constraint to zero) produces a marginal benefit per subsidy dollar of $0.01 (i.e., a marginal benefit of $7.05, when benefit is measured in terms of the decrease in total welfare losses of all trips).

Table 8 shows the effect of eliminating bus operating subsidies on welfare losses for a different level of travel demand. If we compare Tables 6 and 8, we see that, when we cut both travel demand and capacity by half, the elimination of bus operating subsidies would result in moderate welfare loss increases. (This result is consistent with the statement that provision of bus service involves economies of scale.) As Table 8 shows, the increase in average welfare losses per trip that would result from lowering subsidies from $115.0 to 0 per number of cars/h along a given mile works out to a marginal benefit per subsidy dollar of $0.01 (i.e., a marginal benefit of $3.00, when benefit is measured in terms of the decrease in total welfare losses of all trips).

Table 6 also presents the effects of an increase in the level of gasoline tax tolls on the operation of the system. It shows that increasing the gasoline tax tolls from 45 to 60 cents/five-mile automobile trip at peak hours would result in a slight reduction (0.3 percent) in welfare losses per trip. The maximization of net travelers' welfare subject to a gasoline tax toll constraint of 60 cents/five-mile trip does not require free bus service. But as Table 6 shows, the elimination of bus operating subsidies would result in substantial increases in welfare losses. When we maximize net travelers' welfare subject to the 60-cent gasoline tax toll constraint, the elimination of subsidies would result in an increase of $854.4 in total welfare losses of all trips. This increase works out to a marginal benefit per subsidy dollar of $7.7.

Comparison of Tables 4 and 6 shows that, for a higher level of congestion, increasing the gasoline tax tolls from 45 to 60 cents/five-mile automobile trip at peak hours would result in a greater reduction (2.3-4.5 percent) in welfare losses per trip.

Benefit/Cost Implications of Model 4

Subsidy restriction is less costly when reserved bus lanes are permitted. As Table 6 shows, elimination of subsidies would result in moderate increases in welfare losses. Lowering of the bus operating subsidy from $62.1 to 0 per number of cars/h along a given mile adds 25.4 cents (10.5 percent) to the average welfare losses of a trip. That is, the increase in the total welfare losses of all trips that would result from lowering subsidies from the value required to maximize net travelers' welfare subject to the gasoline tax toll constraint ($62.1) to 0 per number of cars/h along a given mile works out to a marginal benefit per subsidy dollar of $3.3.

Under both peak and off-peak conditions, when the congestion level is low, the provision of the bus system not only does not decrease but may even increase the average welfare losses per trip. As Table 7 indicates, regardless of whether reserved bus lanes are allowed, the provision of bus service itself causes some losses when MN/K is assumed to be less than one. Under the conditions given in Table 7, when reserved bus lanes are not permitted and when MN/K is set equal to 0.8, the change from the

Table 9. Comparison of system operation with and without reserved bus lanes under different pricing and financial constraints: base off-peak-hour case.

Constraint	Welfare Losses per Trip		Optimal Bus Fare (cents)		Buses per Hour		Bus Operating Cost per Passenger (cents)		Bus Share of Capacity (%)		Modal Split Without Reserved Bus Lanes		Value of Marginal Product of Capacity Without (cents)[a]
	Without	With	Without	With	Without	With	Without	With	Without	With	Auto-mobile	Bus	
Social welfare maximizing	2.29	2.29	10.8	10.8	29	29	10.8	10.8	100	100	0.000	1.000	0.0
Pricing constraint													
Automobile toll = $0.06/mile	2.48		0		2		1.1		1.7		0.340	0.660	0.1
Automobile toll = $0.06/mile and bus operating subsidies = $0.0	3.21				0				0		1.000	0.000	11.4
Automobile toll = $0.08/mile	2.56		0		2		1.0		2.0		0.288	0.712	0.1
Automobile toll = $0.08/mile and bus operating subsidies = $0.0	3.46				0				0		1.000	0.000	11.4

Note: Length of off-peak period = 10 h, number of lanes = 2, travel rate = 400, and off-peak-hour bus costs = $25.
[a]Cost per number of cars/h along a given mile.

provision of bus service to an all-automobile travel pattern decreases average welfare losses per trip by 18.9 cents (i.e., by 7.5 percent). Therefore, under off-peak conditions, when MN/K is set equal to 0.8, the maximization of net travelers' welfare leads to minimum average losses per trip of $2.29 and requires no bus service. However, when MN/K is set equal to 1.6, as Table 9 indicates, the provision of an optimal level of bus service would reduce the average welfare losses per trip to considerably below the levels that would result with only automobile travel (i.e., a reduction of 91.8 cents--28.6 percent).

CONCLUSION

If the numerical analyses presented can be accepted as valid, they have the following policy implications:

1. Given the demand and cost conditions that these analyses are based on, the imposition of a net travelers' welfare maximizing level of decision variables (i.e., bus fares, automobile tolls, and bus service) is the best short-run solution for the traffic congestion problem. As the analyses show, the optimal level of automobile tolls at peak hours is much higher than the gasoline tax currently imposed in most North American cities.
2. The demand and cost conditions, which promise significant benefits from the imposition of optimum levels of bus fares and automobile tolls and the provision of optimum level of bus service, imply as well that road expansion would yield substantial benefits. However, the optimal level of arterial capacity changes as demand and cost conditions change.
3. Given that the imposition of optimal tolls is regarded as impracticable for technological or political reasons, the numerical analyses show that reserved bus lanes appear capable of substantially reducing current welfare losses of travel in peak hours. In other words, the numerical analyses allow us to conclude that the provision of reserved bus lanes constitutes a good solution for the traffic congestion problem.
4. Given that automobile tolls are restricted to the gasoline tax, which is much lower than the optimal level of tolls recommended for peak hours (63-93 cents/mile of automobile trip), the numerical analyses presented in this paper reveal that the optimal provision of bus service implies mass bus operating subsidies.

The above findings are based on the demand and cost conditions posited here. Our limited sensitivity analyses highlight the dependency between the results and the demand and cost conditions assumed in the study. Therefore, further sensitivity analyses are essential.

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