Development and Evaluation of a Synthetically Self-Calibrating Gravity Model

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The development of an alternative to the quick-response technique of using transferable parameters in trip distribution for small- and medium-sized urban areas is discussed. The proposed quick-response procedure involves an origin-zone-specific, self-calibrating gravity model in which the only input data required are the zonal productions and attractions, a zone-to-zone travel time matrix (skim tree), and the origin-zone terminal times. A travel time distribution determined from an origin-destination survey for internal trips is not needed for calibration. Tests conducted on three separate study areas indicated that the proposed model is able to reproduce trip patterns as accurately as the traditionally calibrated gravity model procedure based on origin-destination survey data. The accuracy was achieved by synthetic calibration of the model at the origin-zone level rather than at the aggregate level of the entire study area. Development of the proposed procedure was also based on the consideration that trip distribution is critically dependent on the spatial distribution of land use activities about each of the origin zones. This consideration was incorporated in the proposed procedure through the explicit measurement of the origin-zone-specific opportunity travel time distribution. The opportunity distribution for each origin zone was represented in the model by the origin-zone average travel time, computed from a gravity model trip distribution that has constant friction factors. From this initial key variable, the final model was developed, and to this the very acceptable results can be credited.

The initiation or updating of a comprehensive transportation plan requires considerable time and money. However, the changed emphasis from traditional long-range system planning to short-range, quick-response improvement programs no longer allows for the frequently long time span between the initiation of a transportation study and the final report. In addition, the recently addressed issues of transportation impact analysis in the areas of energy conservation, air quality management, and other environmental, economic, and political issues have increased the overall scope that must be considered in a truly comprehensive transportation plan. Requests by elected and public officials for quick responses, in combination with the ever-widening scope of planning issues, necessitates that the traditional transportation analysis process be modified if it is to be relevant to the short-range planning process. Capabilities need to be developed for simplified methods in the conventional four-step estimation process (trip generation, trip distribution, mode split, and traffic assignment) and also for various impact analysis and evaluation techniques. This paper discusses only one phase of the total process, trip distribution modeling, and presents a new procedure for trip distribution gravity model calibration that is designed in the context of quick-response capabilities with limited input data requirements and without sacrificing the accuracy obtained through traditional origin and destination (O-D) survey calibration techniques (1).

CURRENT TRIP DISTRIBUTION TECHNIQUES

Six basic techniques are commonly used in trip distribution modeling. These techniques are best characterized in terms of their resource requirements of time, data, and money. A hierarchical ranking of these six techniques follows:

1. Traditional gravity model that requires a relatively large O-D survey for calibrating the travel-time-impedance function (friction-factor curve) [such a process is described in the PLANPAC/BACKPAC General Information book (2)],
2. Traditional gravity model that uses a small O-D survey for travel-time-impedance function calibration (3),
3. Traditional gravity model with a calibrated friction-factor curve from a similar urban area (4),
4. Traditional gravity model that uses standardized or default friction-factor curves or parameters based on population size (5,6),
5. Manual gravity model that has standardized or default parameters and interzonal travel times based on airline distances and other factors (6), and
6. Traditional gravity model that has constant friction factors.

The first two techniques are very similar: the only difference is the size of the O-D survey sample. Although both techniques require an O-D survey, the first technique requires a much larger commitment of time and money.

Quick-response trip-distribution techniques three and four are also similar to each other because both methods assume that similar transportation study areas exist and that these similarities can be classified. The predominant criterion for similarity has been the size of the population of the study area. Figures 1 (1,7,8) and 2 (1) illustrate the discrepancies that may occur when this assumption of similarity is used. Figure 1 shows the great variability of home-based work trip average travel time with respect to study area population size (7,8). A plot of the non-home-based trip beta (β) calibration parameter of the negative exponential friction-factor function versus the size of the population of the study area in Figure 2 also illustrates a large variability with respect to population size, especially at the lower population ranges.

The fifth technique is similar to the third and fourth in that standardized calibration parameters assume similar study areas from which these values are established. This procedure estimates travel times from airline distances and other adjustment factors. The difference in this technique is its manual instead of computerized trip distribution. It is therefore limited in terms of the number of zones and network detail because of possible time constraints.

The sixth technique, constant friction factors, assumes that trip makers do not consider travel time in their destination choice process. Such an assumption can lead to very significant errors in the transportation planning modeling process. The travel time errors that result from the use of constant friction factors in the trip distribution models of two urban areas of Indiana: Lafayette (population, 100 000) and Anderson (population, 90 000), are shown in Table 1. Average travel time for all internal trips was overestimated by 30 percent for Lafayette and 19 percent for Anderson. Total link percent-root-mean-square-error (PRMSE) for a traditionally calibrated trip distribution model and its assignment for Lafayette was 12.48 percent and the total link PRMSE by using constant friction factors was 129.83 percent. Anderson's total link PRMSE was 13.28 percent for a traditionally calibrated trip distribution model and its as-

Figure 1. Home-based work average travel time versus urban area population size.

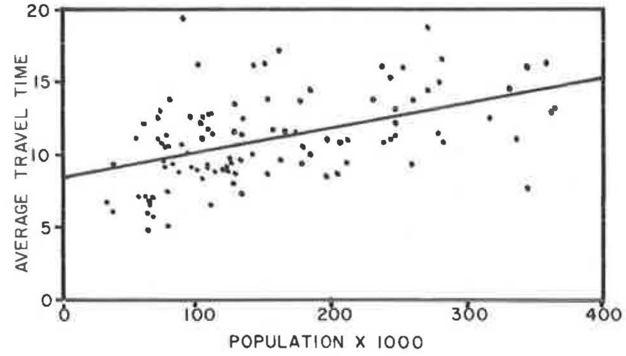


Figure 2. Variability of beta with respect to urban area population size, non-home-based trips.

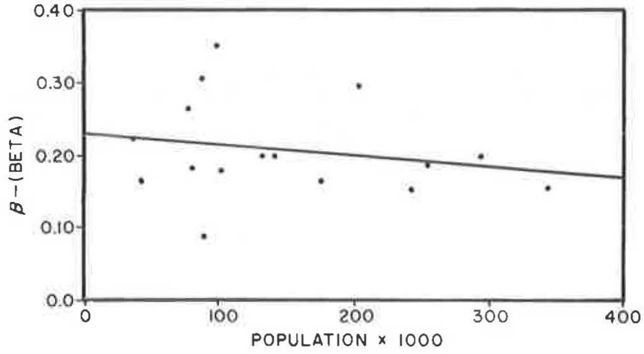


Table 1. Results of the Lafayette and Anderson gravity model with constant friction-factor values.

City	Trip Purpose	Average Travel Time		Difference (%)
		Survey	Model	
Lafayette	Home-based work	10.57	12.32	+16.56
	Home-based other	8.56	11.78	+37.62
	Nonhome based	9.12	11.42	+25.22
	Avg	9.05	11.76	+29.94
Anderson	Home-based work	11.44	12.10	+5.77
	Home-based other	10.04	12.30	+22.51
	Nonhome based	9.50	11.72	+15.58
	Avg	10.14	12.07	+19.03

signment and 86.00 percent when constant friction factors were used. The average overestimated link volumes for Lafayette and Anderson were 1760 and 771 vehicle trips, respectively. Clearly, the error introduced by using constant friction factors is unacceptable.

The above six trip distribution techniques can be further classified into three categories. Techniques one and two are sensitive to the transportation network and study area spatial land use activity distribution because of their ability to use the O-D survey data for calibration purposes. Techniques three, four, and five use borrowed or standardized friction-factor values and not derived values from O-D survey data; hence, the gravity models cannot be accurately calibrated and are therefore less sensitive to a particular study area's network and spatial land use activity distribution. Technique six is insensitive to the transportation network because it completely excludes travel time. The transportation planner thus has a choice of conducting an O-D survey for greater sensitivity and therefore greater model accuracy or using standardized, borrowed, or constant friction-factor values and probably sacrifice accuracy in return for expediency. Remember, though, that the accuracy needed is a function of the degree of precision desired by the transportation planner.

SENSITIVITY OF GRAVITY MODELS TO FRICTION-FACTOR PARAMETER ERRORS

The occurrence of some error is expected when quick-response trip distribution techniques are used; however, it would be useful to have some knowledge of the degree to which gravity models are sensitive to friction-factor parameter errors. To this end, a limited sensitivity analysis had been conducted by using the Lafayette and Anderson transportation study areas (1). The results of this analysis are summarized below.

1. The Lafayette and Anderson study areas differed greatly in terms of their sensitivity to parameter errors in friction-factor equations. Lafayette was more sensitive to parameter errors in terms of travel time estimation and traffic assignment link volumes. This difference in sensitivity was also suggested by the previously mentioned differences in errors that result from the use of constant friction factors. Sensitivity difference was probably due to underlying differences in urban structure.
2. When the single-parameter inverse travel time or negative exponential travel-time-impedance function is used, the gravity models and resulting traffic assignments are less sensitive to overestimates of the parameters than to underestimates.
3. Acceptable model results often occur when synthetic techniques are used because (a) friction-factor parameters used are approximately equal to the values that would have been determined through the traditional calibration process, (b) transportation models for the particular urban area are insensitive to errors (e.g., Anderson, Indiana), or (c) offsetting errors in trip purpose modeling lessen the total model error.

A synthetic technique that provides acceptable results due to good modeling and not to occasional model insensitivity or offsetting model errors would be greatly preferred, especially for those urban areas that are not typical or similar.

PROPOSED MODEL CONCEPT

A trip distribution technique is needed that is sensitive to both the transportation network and spatial land use activity system, but without the necessity of conducting even a limited O-D survey. The hypothesis on which the proposed model was developed is that there exists additional information within the network description, the zonal productions and attractions, and the gravity model itself by which the friction-factor relationship can be estimated at an origin-zone-specific level. Other researchers such as Voorhees (7), Wilson (9), and Fisk and Brown (10) have also suggested the possibility of calibrating the gravity model at the origin-zone-specific level.

Search for Calibration Relationships

In order to search for possible inherent relationships within the gravity model and its variables, the trip distribution gravity models for three Indiana study areas--Lafayette, Anderson, and Muncie (population range, 79 000 to 100 000)--were calibrated at the origin-zone level by using their respective O-D survey data, thereby the amount of information available for analysis was maximized. The travel time impedance function used to calibrate the model at the origin-zone level was the negative exponential function as given in Equation 1.

$$F_{ij} = e^{-\beta_i C_{ij}} \tag{1}$$

where

F_{ij} = value of the friction factor between origin zone i and destination zone j,
C_{ij} = travel time (cost) between zone i and j,
e = base of the natural logarithm, and
β_i = origin-zone-specific calibration parameter.

Thus, for each study area and trip purpose, the following origin-zone-specific information was available for further study: origin-zone productions, origin-zone β_i calibration parameter, origin-zone average travel time (AVE_i), and origin-zone travel-time variance.

One additional piece of information contained within a gravity model is the concept of the opportunity distribution as set forth by Voorhees and others (7). Opportunity distribution is the gravity model distribution that results from travel-time impedance factors of constant value at all time increments. As demonstrated by Voorhees and others, this distribution can be used as a measure of the spatial arrangement of land use activities, especially when used at the origin-zone level (7). Origin-zone-specific opportunity average travel time ($AVEO_i$) and travel-time variance were therefore computed and added to the data set.

Regression analysis procedures used in the search for additional variable relationships consisted of weighting each origin-zone observation by its respective trip productions. Another weighting procedure was used to balance the number of trip productions among the three study areas by trip purpose. These two weighting techniques were necessary to minimize bias. The first weighting scheme prevents an origin zone that has very few trip productions from having the same statistical weight as an origin zone that has many trip productions. The second weighting scheme prevents bias toward any one particular study area due to the relative number of trips per study area for any particular trip purpose.

Various researchers (10,11) have suggested that it may be possible to relate the β_i calibration parameter of the negative exponential friction-factor equation (Equation 1) to travel time statistics at the origin-zone level. To this end, extensive regression analysis by using the above-generated data set by trip purpose resulted in prediction equations of poor statistical fit. However, the correlation matrix revealed another possible approach to the problem.

A high correlation between the origin-zone-specific average travel time (AVE_i) and opportunity average travel time ($AVEO_i$) was observed for the home-based work, home-based other, and non-home-based trip purposes. This high correlation suggested that it may be possible to relate AVE_i as a function of $AVEO_i$. In other words, origin-zone average travel time AVE_i is a function of how the attractions (land use activities) are spatially distributed about the origin zone with respect to the interconnecting highway network as measured by $AVEO_i$.

Because travel time is composed of three distinct time segments (origin-zone terminal time, link travel time, and destination-zone terminal time), origin-zone-opportunity average travel time was separated into two parts before regression analysis was performed. The first part consisted only of the origin-zone terminal time ($TERM_i$), which is a constant for any particular origin-zone-based trip. The second part consists of that part of the travel time over which the trip maker can make choices as to trip length; that is, the combined link travel time and destination-zone terminal time or ($AVEO_i$ - $TERM_i$). This separation of travel time into these two parts allows the regression-analysis procedure to reveal the relative importances of these two variables with respect to trip purpose.

Regression analysis of AVE_i on ($AVEO_i$ - $TERM_i$) and $TERM_i$ was accomplished by weighting the origin-zone-specific data, as previously described, in order to properly balance the data by trip productions (origins) and by population of the study area for each trip purpose. Equations 2, 3, and 4 present the results of the regression analysis for the home-based work, home-based other, and non-home-based trips, respectively. Total R^2 and the change in R^2 for the independent variables are also given. (Because of the weighting by productions in the regression procedure, t- or F-statistics are meaningless and are therefore not given.)

Home-Based Work

$$AVE_i = -0.4328 + 1.2761\ TERM_i\ [\Delta R^2 = 0.114] + 0.8921(AVEO_i - TERM_i)\ [\Delta R^2 = 0.751] \tag{2}$$
$$R^2 = 0.865$$

Home-Based Other

$$AVE_i = -0.7499 + 1.3548\ TERM_i\ [\Delta R^2 = 0.187] + 0.7558(AVEO_i - TERM_i)\ [\Delta R^2 = 0.661] \tag{3}$$
$$R^2 = 0.848$$

Non-Home-Based

$$AVE_i = 0.5245 + 1.3416\ TERM_1\ [\Delta R^2 = 0.423] + 0.6073\ (AVEO_i - TERM_i)\ [\Delta R^2 = 0.296] \tag{4}$$
$$R^2 = 0.719$$

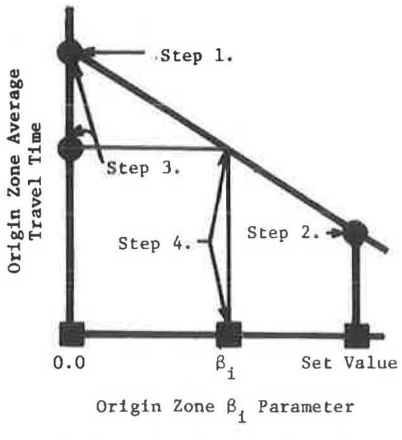
The strong statistical fit of the equations clearly shows the importance of $AVEO_i$, the origin-zone-specific opportunity average travel time, on the destination-choice process. This, in turn, demonstrates the large effect that the location of the origin zone and the distribution of land use activities about the origin zone have on the destination-choice process.

Relative importance of travel time by trip purpose in the destination-choice process is indicated by the coefficients of the $AVEO_i$ variable. In general, home-based work trips have one or few choices of destination, and the non-home-based trips have many choices of destination. Consequently, the equations show that the importance of travel time in terms of choice is the lowest for home-based work, highest for non-home-based, and of some intermediate magnitude for home-based other trips.

In addition, variable $TERM_i$, the origin-zone terminal time, becomes a greater proportion of total travel time as trip purpose changes from home-based work to home-based other or non-home-based, as is evidenced by the change in R^2. This is because $TERM_i$ is an origin-zone-specific constant and cannot be reduced by destination choice, unlike the link travel times and destination-zone terminal times.

Also of interest are the coefficients of the origin-zone terminal times. It was first expected

Figure 3. Flow chart of synthetically self-calibrated, origin-zone-specific gravity model as applied to a typical zone.



Step 1 - Average at β_i equal zero, $AVEO_i$

Step 2 - Average at β_i equal to a set value.

Step 3 - Average from regression equation, AVE_i

Step 4 - Interpolate for new β_i

Table 2. Origin-zone-specific, synthetically self-calibrating gravity model travel time statistics.

City	Trip Purpose	Average Travel Time: O-D Survey	Average Travel Time: Model	Difference (%)
Lafayette	Home-based work	10.57	10.97	+3.78
	Home-based other	8.56	8.56	+1.05
	Nonhome based	9.12	8.93	-2.08
	Avg	9.05	9.09	+0.71
Anderson	Home-based work	11.44	11.27	-1.49
	Home-based other	10.04	10.02	-0.20
	Nonhome based	9.50	9.64	+1.47
	Avg	10.14	10.14	0.00
Muncie	Home-based work	9.07	8.82	-2.79
	Home-based other	7.48	7.37	-1.46
	Nonhome based	6.93	7.04	+1.63
	Avg	7.52	7.46	-0.80

that the coefficients would have a magnitude very close to one. The coefficients were found to be greater than one. No explanation could be rationalized. Further investigation would be necessary to account for this result.

After estimating equations were developed by trip purpose for the origin-zone-specific average travel time (AVE_i) based on the origin-zone-specific opportunity average travel time ($AVEO_i$), a procedure was then formulated to incorporate these equations into a gravity model.

Synthetic Gravity Model Formulation

One method of calibrating the gravity model would be to first distribute all trips with a constant friction factor, thereby allowing for the computation of $AVEO_i$. AVE_i could then be computed by using the regression equations discussed in the previous section. These estimated origin-zone-specific average travel times then could be used to repeatedly adjust the origin-zone-specific β_i parameters until the gravity-model origin-zone-specific average travel times match the estimated values. However, this type of calibration procedure would require many iterations before convergence of β_i for all the origin zones would occur. The many iterations required for convergence would consume a large amount of computer time. This would be an undesirable characteristic for a synthetic model to possess.

Consequently, another method was developed for eventual adoption for synthetic calibration and was based on three assumptions regarding the interaction of origin-zone average travel time and the origin-zone β_i parameter. The first assumption was that the origin-zone average travel time varies linearly with the origin-zone β_i parameters (12). The second was that origin-zone average travel time is independent of the β_i parameters of other zones. Third, origin-zone average travel time is independent of the attraction-constraint iteration procedure. These assumptions, however, do not always hold true for some of the individual zones when the origin-zone β_i parameters are allowed to vary independently, as in the proposed model. This aspect was not demonstrated until after the model was developed and tested, but as the final results indicated, aggregation of the origin-zone results up to the entire study area, as evidenced by the acceptable traffic assignment, nullified these minor errors. This is typical of other transportation planning procedures.

The estimating equations of AVE_i and the three assumptions listed above were incorporated into a gravity model procedure that can best be described in conjunction with Figure 3. First, trips are distributed with all origin-zone-specific β_i parameters equal to 0.0 (constant friction factor) and then the resulting origin-zone-opportunity average times ($AVEO_i$) are computed. Second, trips are distributed with all origin-zone-specific β_i parameters equal to a single preselected value and the resulting origin-zone average travel times are computed. This step provides a linear relationship between each of the origin-zone-specific β_i's and average travel times. Third, by using the estimating equation for the appropriate trip purpose and the $AVEO_i$ values computed in step one, AVE_i is determined. Fourth, by using the data points from steps one and three and then estimating AVE_i, the origin-zone-specific β_i parameters are obtained through interpolation. All trips are then distributed by using the interpolated origin-zone-specific β_i parameters.

EVALUATION OF THE PROPOSED MODEL

The gravity model procedure was used to distribute the trips for Lafayette, Anderson, and Muncie, Indiana. The resulting average travel time statistics by trip purpose and for all internal trips are given in Table 2. Comparison of these statistics to the corresponding O-D survey values is also shown. Lafayette's total internal trip average travel time was only 0.71 percent greater than the O-D survey value; Anderson's total internal average travel time was identical to the O-D survey value. Muncie's total internal average travel time was 0.80 percent lower than the O-D survey results.

The average travel times for each trip purpose also indicate very acceptable results; the largest percentage error among the area's individual trip purposes was 3.78 percent, this being the only one outside of Federal Highway Administration's (FHWA) suggested ±3 percent error range when survey data

Figure 4. Volume group link comparison for Lafayette all or nothing traffic assignment: origin-zone-specific, synthetically self-calibrating gravity model versus O-D survey.

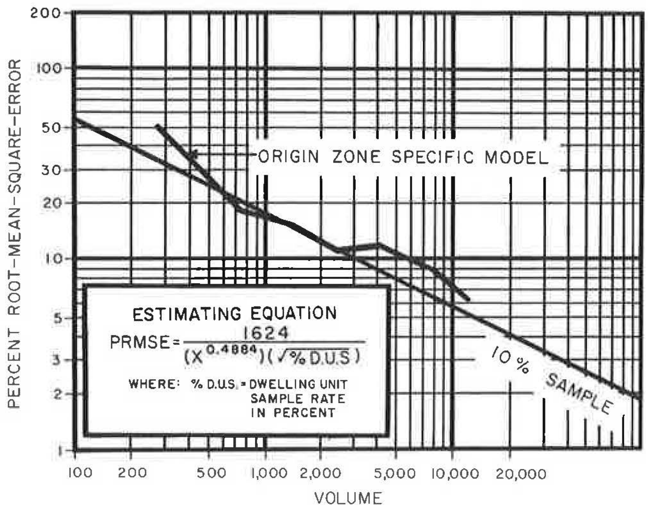
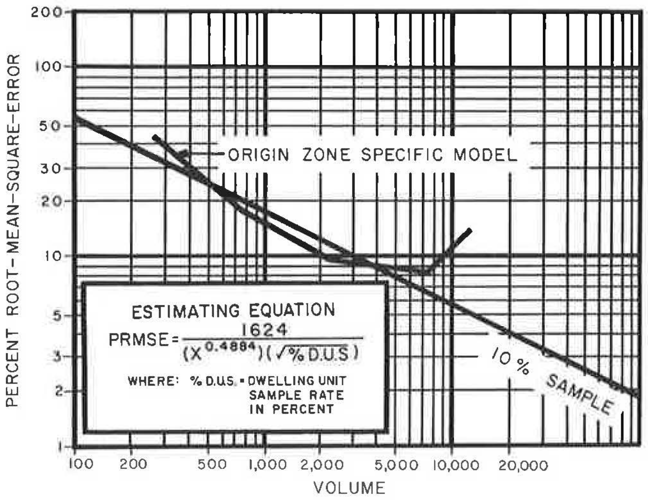


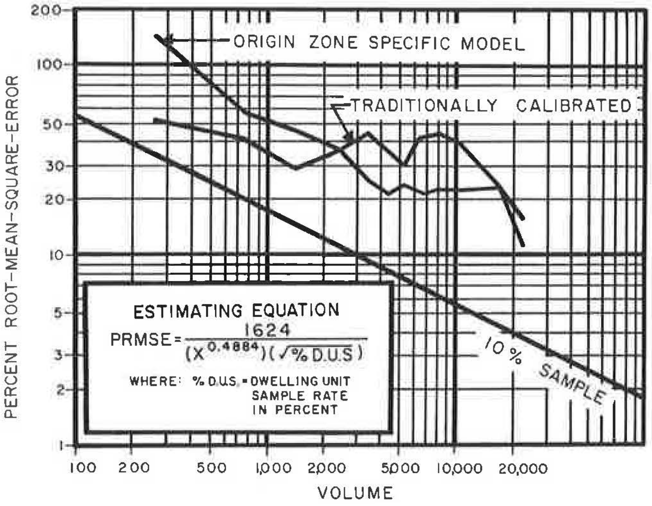
Figure 5. Volume group link comparison for Anderson all or nothing traffic assignment: origin-zone-specific, synthetically self-calibrating gravity model versus O-D survey.



are used for calibration (13).

Volume group link comparisons of the three synthetic gravity models' all-or-nothing traffic assignments were performed by using their corresponding O-D survey traffic assignments as the base for comparison. An all-or-nothing traffic assignment was used because only an analysis of gravity model accuracy was desired; errors due to capacity-restraint differences and ground count errors are eliminated. Total link trip PRMSE for Lafayette was computed to be 12.67 percent. This value was practically identical to the corresponding value (12.48) obtained from the gravity model calibrated by the traditional procedure. Total link trip PRMSE was calculated to be 14.55 percent as compared with the traditionally calibrated gravity model value of 13.28 percent for Anderson. A surprising result occurred for the Muncie study area, where total link trip PRMSE was 47.7 percent--a major improvement compared with the traditional calibrated gravity model result of 67.8 percent. Volume group link comparisons are shown graphically in Figures 4, 5, and 6. Also shown on the figures are the expected PRMSE values associated with a 10 percent dwelling unit sampling rate for the O-D home interview survey. In addition, Figure 6 also shows the PRMSE statistics for the traditionally calibrated gravity model. The PRMSE values for Lafayette, Anderson, and Muncie were all observed to be acceptable.

Figure 6. Volume group link comparison for Muncie all or nothing traffic assignment: origin-zone-specific, synthetically self-calibrating gravity model versus O-D survey.



Of special interest in Figure 6 is the result that PRMSE values for the proposed model for the higher volume links are significantly lower than results obtained with the traditional model. Also, note that a probable explanation for the high PRMSE values for Muncie by using both the traditional and proposed Muncie gravity models may be that the Muncie study area included a relatively high number of zones and links for an urban area of its population size. The zones and links are proportionally much greater than in either Lafayette or Anderson, which suggests that the Muncie zone and network system were probably too detailed for the number of trips produced. For example, the Muncie data included 1249 out of 2698 links that had an average daily traffic of less than 500 vehicles/day; peak-hour traffic on these links would probably not exceed 50-60 vehicles/h.

Hence, the trip distribution and traffic assignment results of the three test areas indicate that the proposed origin-zone-specific, synthetically self-calibrating gravity model is capable of distributing trips at least as accurately as the traditionally calibrated gravity model without the need for an O-D survey.

Revision of the Calibration Equation

Because of the inability to give an interpretation to the constant terms in Equations 2, 3, and 4 and the fact that travel time is composed of only terminal times and link travel time, a regression analysis was performed to force the regression through the origin and therefore eliminate the constant term in the equations. The result of this forced regression is given in Equations 5, 6, and 7.

For home-based work trips,

$$\mathrm{AVE}_i = 1.1910\ \mathrm{TERM}_i + 0.8638(\mathrm{AVEO}_i - \mathrm{TERM}_i) \tag{5}$$

For home-based other trips,

$$\mathrm{AVE}_i = 1.122\,34\ \mathrm{TERM}_i + 0.7033(\mathrm{AVEO}_i - \mathrm{TERM}_i) \tag{6}$$

For non-home-based trips,

$$AVE_i = 1.2524\, TERM_i + 0.6856(AVEO_i - TERM_i) \qquad (7)$$

Trial runs of the model with the above equations indicated no significant change in the model results compared with the equations that have the constant term.

Computer Availability and Program Requirements

The model was initially coded as optional user code in the Urban Transportation Planning System (UTPS) UMODEL program. A similar version was also coded for the PLANPAC package. A FORTRAN IV program is also available for incorporation into other packages.

For the 271 internal zone Muncie study area, 180 s of central processing unit (CPU) time were required to execute the model for the home-based other trip purpose on a CDC 6500. The same Muncie home-based other trips required 14 min of CPU time on an IBM 370/148 by using the UTPS UMODEL program. CPU time for the UMODEL version is highly variable, depending on UMODEL options and reports selected.

Test Case Application

The Indiana State Highway Commission will use the origin-zone-specific, synthetically self-calibrating gravity model during the initial transportation plan development study for Bloomington, Indiana, during the summer and fall of 1981. This test case application will provide a real-world test of the model.

FUTURE RESEARCH

It is hoped that this model can be tested by using the data from a much larger urban area in the 750 000 to 1 000 000 population range. Such an urban area would provide a large data base for testing the model and for comparing the characteristics of a large urban area with those of the three smaller study areas used in the development of the model.

SUMMARY AND CONCLUSIONS

Because of the need to reduce time, money, and effort involved in the transportation planning process, as well as to address emerging planning issues, many new techniques need to be developed and evaluated. One such technique is the proposed synthetic trip-distribution procedure, which consists of an origin-zone-specific, synthetically self-calibrating gravity model in which the only input data required are the zonal productions and attractions, a zone-to-zone travel-time matrix (skim tree), and the origin-zone terminal times. A travel-time distribution is no longer needed for calibration, thereby eliminating the necessity for an O-D survey for internal-internal trips.

Tests conducted on three separate study areas indicated that the proposed model is able to reproduce trip patterns as accurately as the traditionally calibrated gravity model procedure that is calibrated on O-D survey data. The accuracy was achieved by synthetically calibrating the model at the origin-zone level rather than at the aggregate level of the entire study area. Development of the proposed procedure was also based on the consideration that the trip distribution is critically dependent on the special distribution of land use activities about the origin zone. This consideration was incorporated in the proposed procedure through the explicit measurement of the origin-zone-specific opportunity travel-time distribution.

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