Statistical Geotechnical Properties of Lockport Clays

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A study is described in which the objective was to apply statistical and probabilistic tools to describe the variability of and relation between common geotechnical parameters. With these tools, one can select values of soil parameters that correspond to specified confidence levels, assign risk values for particular designs, and, by using regressional analysis, determine relations between soil properties. A statistical analysis of soil properties obtained during an extensive testing program on undisturbed samples of clay is presented. This program is associated with the design of the Lockport Expressway in western New York State and includes classification, oedometer, triaxial, and field vane tests. The results of the study are presented in a number of tables and charts.

To estimate reliable values of soil properties on the basis of relatively few samples is an important task in geotechnical engineering practice. Sampling disturbance and errors due to laboratory and field testing also contribute to the variability of soil properties. Moreover, the heterogeneous nature of soil makes the task of selecting reliable and representative soil properties quite formidable.

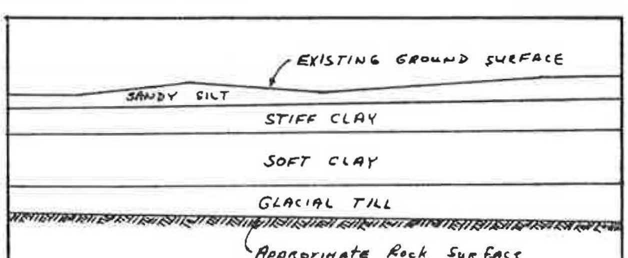
Thus, there is a need to develop a systematic and rational approach to represent the variability of soil properties. Application of probability and statistics to geotechnical problems has received considerable attention in the past decade. Various researchers (1-5) have studied soil-property variability by statistical and probabilistic concepts and have presented their findings in a series of publications. Statistics and probability allow one to make inference decisions about the properties of a specific soil deposit based on the small amount of data the engineer normally deals with.

This paper presents results obtained in the application of statistics and probability on actual test data obtained for the Lockport Expressway project in western New York State, north of Buffalo.

To design the Lockport Expressway, an extensive subsurface exploration program was undertaken. Topographically, the area is a flat glacial lake bed and the subsurface material consists mainly of lacustrine deposits. A relatively thin top layer of sandy silt is typical. A typical section of the soil profile is shown in Figure 1.

Split spoon and Shelby tube samples were obtained from numerous drill holes. The soil samples obtained were classified as inorganic silty clays of low to high plasticity based on the plasticity chart shown in Figure 2. Examination of these samples revealed the presence of a layer of stiff silty clay (9-20 blows) overlying softer silty clay (0-4 blows). A layer of compact glacial till with varying thickness underlies the softer deposit. Even though the clay samples were taken over a large area, they were uniform in composition.

Figure 1. Typical soil profile.



Laboratory and field testing of the clay was performed by the Soil Mechanics Bureau of the New York State Department of Transportation. The testing program included determinations of Atterberg limits, moisture content, wet density, and specific gravity, consolidated isotropic undrained (CIU) triaxial tests, oedometer tests, and field vane tests. Relatively few triaxial and oedometer tests were performed on the stiff material because the material was sufficiently strong to prevent sample preparation.

There were enough percentage clay size and Atterberg limits data available to determine the activity of the clay deposit (see Figure 3). The activity of the clay deposit was found to be approximately 0.39, which means that the clay material is inactive.

Data on the stress history and undrained strength of a typical undisturbed hole are summarized in Figure 4. The soft clay layer is moderately precompressed, and the stiff upper layer is heavily precompressed, a probable result of dessication.

The statistical characteristics, such as the mean, standard deviation, coefficient of variation, skewness, and kurtosis of the soil properties, are determined. Afterwards, probability distributions for a few selected soil properties are proposed and predictive models are developed between a few typical soil properties.

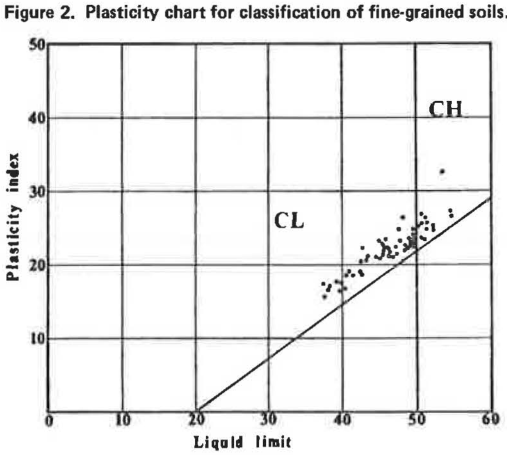
SAMPLE STATISTICS

In order to analyze the data from the project area, it is first necessary to define the population from which the data were drawn. This is extremely important, since the mixing of different populations can often lead to a misinterpretation of the results.

The test results were grouped into two categories: (a) the relatively few results from the stiff clay deposit and (b) the results obtained from the soft clay deposit. The determination of what samples were considered stiff or soft was based on examination of blow counts, moisture contents, wet densities, and stress history. Once this was accomplished, the data obtained from classification tests, triaxial tests, oedometer tests, and field vane tests were analyzed to determine the statistical characteristics of the various soil properties.

The soil properties are considered as being

Figure 2. Plasticity chart for classification of fine-grained soils.



random variables that possess certain probability density distributions. Histograms that give an indication of the type of distribution each of the soil properties can possess are shown in Figures 5-16.

In order to quantitatively assess the variability of the soil properties, it is necessary to determine their sample statistics, such as the mean, standard deviation, and coefficient of variation. The coefficient of variation--which is the standard deviation divided by the mean--is the most commonly used measure of expressing the uncertainty associated with soil-property values. Harr (6) has tabulated coefficients of variation for various soil properties obtained by many researchers.

Classification Tests

The statistical results of the classification tests in Table 1 show that they have relatively low coefficients of variation. This is important in that it indicates that the same soil type is being analyzed. Extremely high coefficients of variation may indicate that different soil units are being combined.

Compressibility Test

Eighty-five oedometer tests were performed: 68 on the soft clay and the remainder on the stiff clay. The compression ratio and the recompression ratio are the only soil compressibility properties considered in this paper.

Table 2 gives the test results of the compression ratios when they are grouped in several ways. Sample statistics were determined for all test values and then for the test values according to whether they came from the stiff clay or the soft clay deposit. The coefficient of variation dropped when the test values were analyzed separately, which indicates that the test results should not have been combined. Furthermore, the histogram that represents the mixing of the test results (see Figure 8) exhibits a bimodal distribution (two apparent peaks). From the statistical point of view, this indicates that the sample statistics obtained from the grouping of the data were invalid (i.e., two sets of data were being combined). Statistical treatment of mixed samples may not satisfactorily reflect the actual behavior of the deposit under consideration. Lumb (7) refers to this as sample contamination in that possibly two populations were being analyzed.

Strength Tests

There were 64 CIU triaxial tests and 21 field vane tests performed on the soft clay material. So few strength tests were performed on the stiff material that a statistical analysis was not done. The strength properties exhibited the highest coefficients of variation, as would intuitively be expected since soil strength is dependent on a multitude of factors (8). The statistical results of the strength characteristics are summarized in Table 3.

REGRESSION AND CORRELATION

Regression analysis is an extremely useful tool for developing predictive models for certain soil properties. The method used in performing the regression analysis in this paper is least squares (9).

Predictive models, by use of which one can predict the value of the compression ratio and recompression ratio by knowing the natural moisture content, have been suggested for the soft clay

Figure 3. Activity of soil deposit.

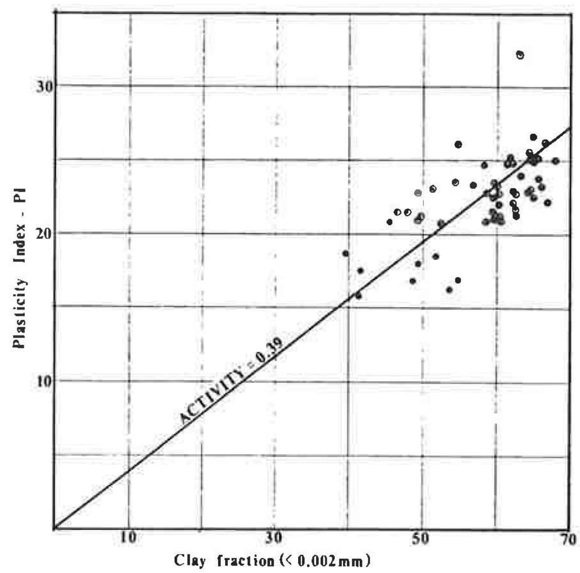


Figure 4. Data summary for typical undisturbed drill hole.

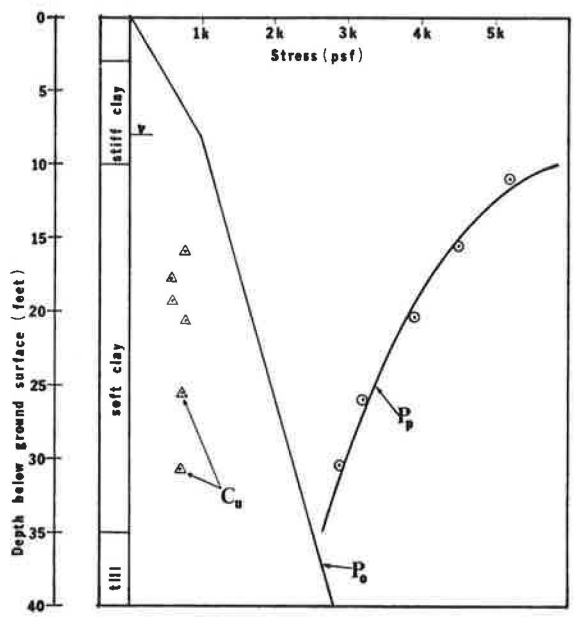


Figure 5. Frequency distribution of liquid limit.

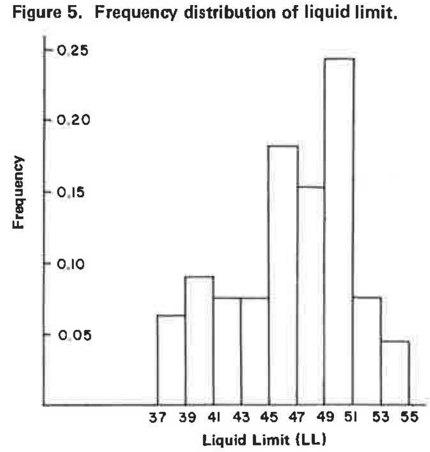


Figure 6. Frequency distribution of plastic limit.

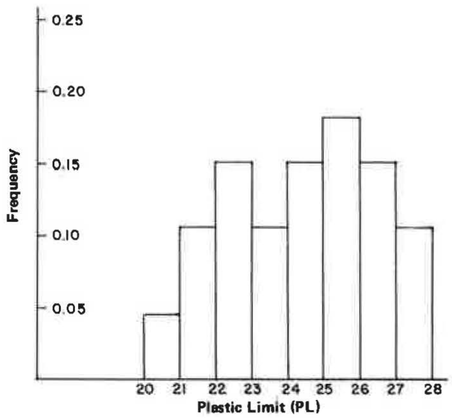


Figure 7. Frequency distribution of activity.

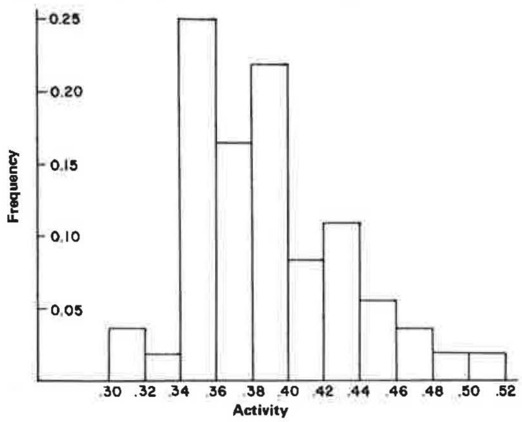
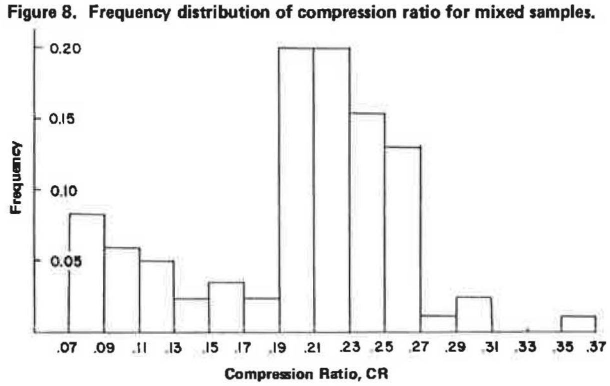


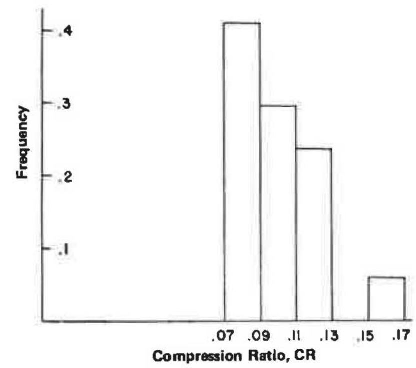
Figure 8. Frequency distribution of compression ratio for mixed samples.



deposit in the project area. These models are shown in Figures 17 and 18. Several other regression models have been suggested (2,10) for relating compression ratio to moisture content for similar soil deposits. These relations are shown in Figure 19 so as to make a comparison with the prediction model just developed. As can be seen, all of the models fall within a narrow band.

Correlation coefficients (r) between various soil properties are of interest because they measure how well the two properties under investigation follow a linear relation. The values of r obtained between various soil properties are as follows:

Figure 9. Frequency distribution of compression ratio for stiff silty clay.



Soil Properties	Material	r
Liquid limit versus plastic limit		0.755
Liquid limit versus plasticity index		0.899
Plastic limit versus plasticity index		0.348
Plasticity index versus percentage finer than 0.002 mm		0.633
Moisture content versus compression ratio	Soft and stiff silty clay	0.898
	Soft silty clay	0.635
	Stiff silty clay	0.783
Moisture content versus recompression ratio	Soft silty clay	0.417
Recompression ratio versus compression ratio	Soft silty clay	0.266
Compression ratio versus preconsolidation pressure	Soft silty clay	0.049
Moisture content versus preconsolidation pressure	Soft silty clay	-0.508
Wet density versus cohesion		0.486
Moisture content versus cohesion		-0.341
Cohesion versus undrained friction angle		-0.162

The r for compression ratio versus moisture content was 0.635, and that for recompression ratio versus moisture content was 0.417. The results are indicative of positive correlation, as would be expected. A correlation value close to zero for compression ratio versus preconsolidation pressure (P_p) was computed, which indicates no correlation between the two properties.

The use of this type of analysis would prove to be very beneficial in geotechnical practice, and the study of the relations between various soil properties should be extended.

PROBABILITY DENSITY FUNCTIONS

The most commonly used probability density function (pdf) for modeling variability in soil properties is the normal (Gaussian) distribution (3-5). The major

Figure 10. Frequency distribution of compression ratio for soft silty clay.

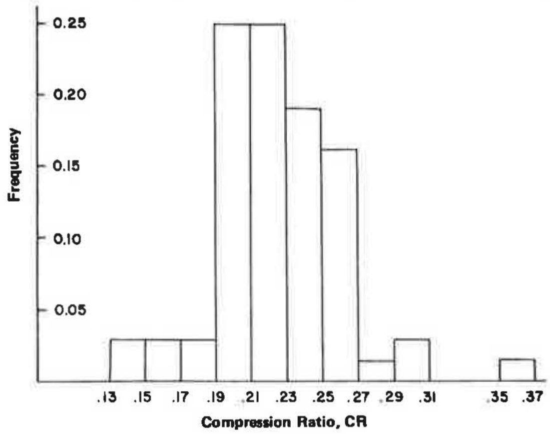


Figure 11. Frequency distribution of recompression ratio for soft silty clay.

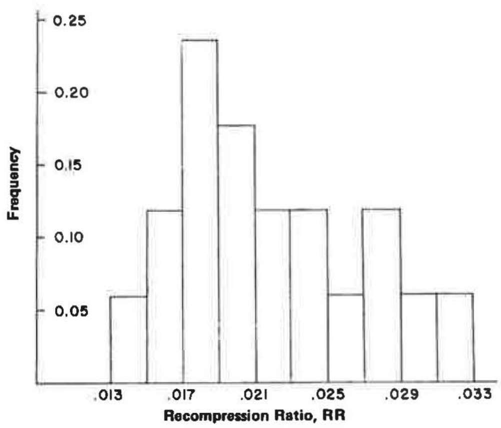


Figure 12. Frequency distribution of undrained shear strength from CIU tests.

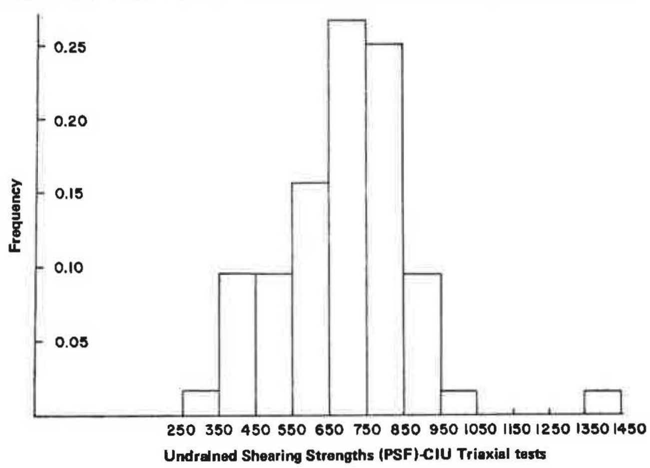


Figure 13. Frequency distribution of undrained shear strength from vane tests.

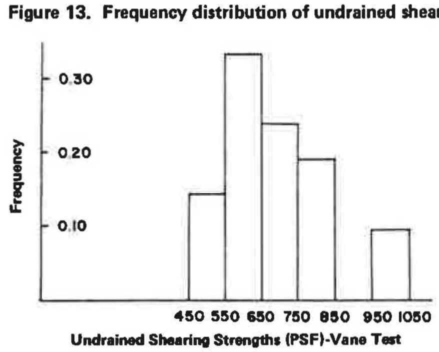


Figure 14. Frequency distribution of remolded strengths from vane tests.

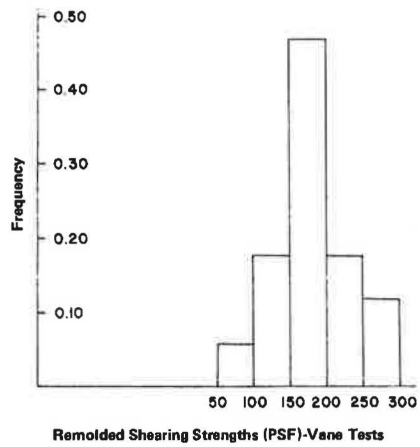


Figure 15. Frequency distribution of t = tan ϕ.

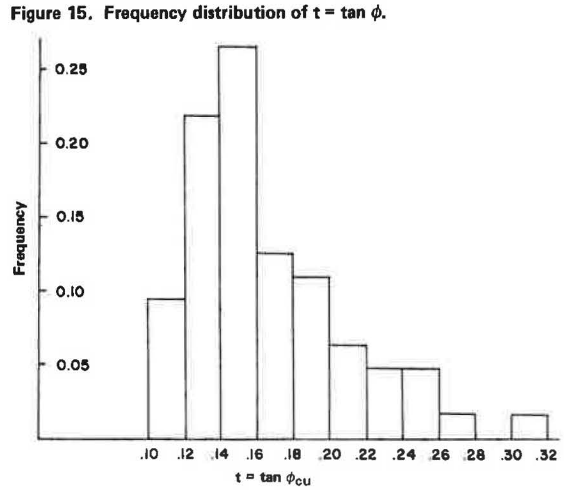
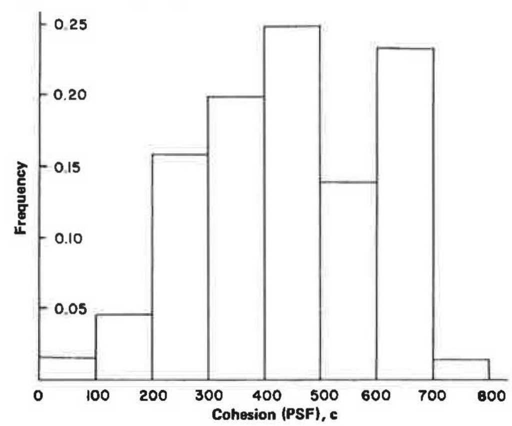


Figure 16. Frequency distribution of cohesive intercept for soft silty clay.



characteristics of the normal distribution are that it is symmetric about its mean, the tails tend toward infinity, and it can take on negative values. In reality, soil-property values are positive and bounded and their distributional characteristics are very rarely symmetric. As can be seen by looking at the histograms of the soil properties investigated (Figures 5-16), all exhibit a certain degree of skewness. Therefore, from the physical

Table 1. Variability of classification parameters.

Soil Property	No. of Samples	Mean	Coefficient of Variation (%)	Range
Specific gravity	145	2.77	0.60	2.73-2.80
Percentage clay size	57	58.07	12.50	39.80-68.80
c/p ratio[a]	46	0.217	10.42	0.180-0.275
Liquid limit	66	46.59	9.12	37.7-54.6
Plastic limit	66	24.37	8.29	20.3-27.7
Plasticity index	66	22.21	13.61	15.7-32.3
Moisture content				
Soft silty clay	132	45.43	9.03	36.07-55.53
Stiff silty clay	25	29.66	12.01	24.08-35.39
Wet density				
Soft silty clay	132	111.94	2.40	106.25-119.24
Stiff silty clay	25	124.38	1.90	119.98-127.38

[a] Defined as ratio of undrained shear strength (for a normally consolidated soil) to consolidation pressure.

Table 2. Variability of compressibility parameters.

Soil Property	No. of Samples	Mean	Coefficient of Variation (%)	Range
Compression ratio				
Mixed samples	85	0.201	30.00	0.076-0.354
Soft silty clay	68	0.226	16.04	0.131-0.354
Stiff silty clay	17	0.102	22.63	0.076-0.153
Recompression ratio for soft silty clay	19	0.022	22.30	0.014-0.031

Table 3. Variability of strength parameters.

Soil Property	No. of Samples	Mean	Coefficient of Variation (%)	Range
Undrained strength				
CIU tests	64	677.34	25.55	300-1400
Field vane tests	21	687.24	19.20	486.0-973
Remolded strength from field vane tests	17	179.35	23.98	94.0-266.0
Cohesion for soft silty clay ($-\phi$ cu)	64	431.41	35.90	50-790
t = tan ϕ cu for soft silty clay	64	0.1653	26.74	0.105-0.315

Figure 17. Regression line of compression ratio versus natural moisture content for soft silty clay.

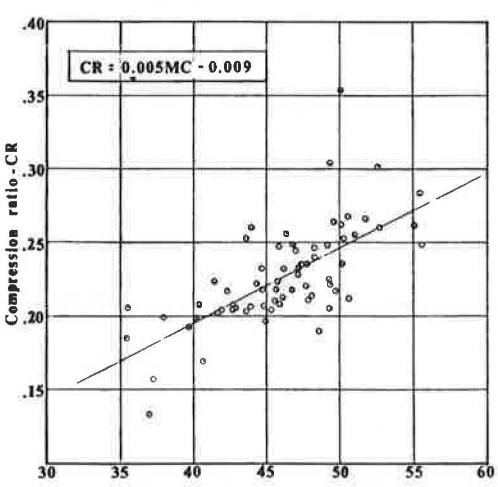


Figure 18. Regression line of recompression ratio versus natural moisture content for soft silty clay.

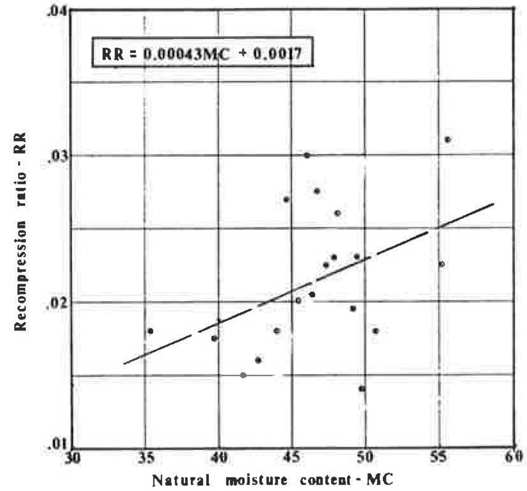
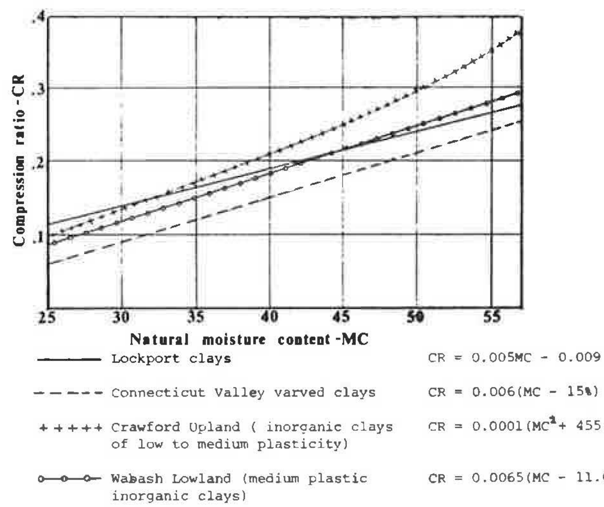


Figure 19. Regression lines of compression ratio versus natural moisture content for similar soil deposits.



point of view, the normal distribution is not a realistic model to use. The major reason for using the normal distribution to model variability in soil properties is its mathematical simplicity.

A distributional function known as the beta has been suggested (6,11) as being a more appropriate model to use in the study of soil-property variability. The beta distribution is so versatile that, in fact, it can be used to model the variability of most soil properties. The major advantage is that it can assume various shapes. The beta pdf, where a and b are the lower and upper limits of the soil property, is presented below:

$$f(x) = [1/(b - a)B(\alpha + 1, \beta + 1)] [(x - a)/(b - a)]^{\alpha} [(b - x)/(b - a)]^{\beta} \quad (1)$$

where $a < x < b$, $\beta > -1$, $\alpha > -1$, and

$$B(\alpha + 1, \beta + 1) = \Gamma(\alpha + 1)\Gamma(\beta + 1)/\Gamma(\alpha + \beta + 2) \quad (2)$$

If a, b, $\bar{x}$, and S_x are known, then α and β can be obtained:

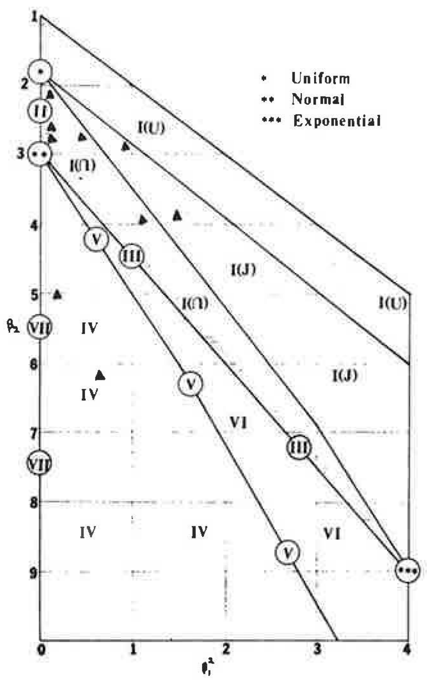
$$\alpha = (X^2/V^2)(1 - X) - (1 + x) \quad (3)$$

$$\beta[(\alpha + 1)/X] - (\alpha + 2) \quad (4)$$

where $x = (\bar{x} - a)/(b - a)$ and $V = S_x/(b - a)^2$. The α and β parameters are known as the shape parameters in that their values will determine whether the beta distribution will be U-shaped, J-shaped, symmetric, etc.

A statistician named Pearson (12) developed a chart (see Figure 20) that relates the coefficient of skewness and coefficient of kurtosis to distributions in such a way as to aid in the selection of a distribution that would model the test results. The Pearson system was used in the selection of pdf's for some of the properties investigated in this study. As can be seen in Figure 20 and Table 4, most of the soil properties can be modeled by the Pearson Type I curve, which is a beta distribution.

Figure 20. The Pearson system.



One of the limitations of using the Pearson system is that β^1 and β^2 are extremely sensitive to the amount of data used to arrive at the coefficients. One must also remember that the use of the Pearson system does not necessarily mean that a perfect fit will be assured but that a reasonable distributional model has been selected.

CONFIDENCE LIMITS

In geotechnical engineering design, the amount of information available to the designers is often limited. This is due to the fact that, in a subsurface exploration program, samples obtained from a particular soil deposit represent only a very small portion of the entire soil deposit. Yet the engineer is asked to assign to the soil deposit soil-property values that he or she believes to be reliable and representative based on only a small amount of data. Confidence limits are a means by which the engineer can make estimates of population parameters based on small sample statistics and associate with these estimates the probability of making a correct decision.

The approach used in this investigation was to establish confidence limits of the population means for a few selected soil properties, based on the t-statistic. It is then possible to state with a specified level of confidence that the true population mean lies between some lower and upper limit.

Table 5 gives the intervals at the 90, 95, and 99 percent confidence levels for a few selected soil properties. These values would enable the practicing engineer who is not familiar with the Lockport clays to make certain estimates of soil properties in that particular area.

NEED FOR STATISTICAL ANALYSIS

Designing geotechnical structures from a probabilistic viewpoint requires knowledge of the variability of soil properties. In conventional design, soil properties such as shearing strength or compressibility are treated as single-valued quantities when in fact they can exhibit a great deal of variability. A probabilistic approach accounts for

Table 4. Probability distributions as determined by the Pearson system.

Soil Property	Material	No. of Samples	Mean	SD	Beta Values B_1	Beta Values B_2	Type of Distribution	α	β
Undrained cohesion	Soft silty clay	64	431.41	155.05	0.331	2.692	I	1.417	1.272
Undrained friction angle ϕ cu	Soft silty clay	64	9.37	2.45	1.222	3.827	I	0.048	1.528
t = tan ϕ cu	Soft silty clay	64	0.1653	0.0442	1.091	3.969	I	0.038	1.585
Undrained strength									
From CIU tests	Soft silty clay	64	677.34	173.076	0.762	6.1936	IV	0	-
From field vane tests	Soft silty clay	21	687.35	131.98	0.622	2.707	I	-0.049	0.350
Remolded strength from field vane tests	Soft silty clay	17	179.35	43.01	0.224	2.744	I	0.487	0.510
Compression ratio	Soft silty clay	68	0.226	0.0359	0.420	5.030	IV	0	-
	Stiff silty clay	17	0.102	0.0223	0.955	2.8386	I	-0.436	0.105
Recompression ratio	Soft silty clay	19	0.022	0.0048	0.396	2.1662	I	-0.093	0.142

Table 5. Confidence intervals for various population parameters at different confidence levels.

Soil Parameter	90 Percent Lower Limit	90 Percent Upper Limit	95 Percent Lower Limit	95 Percent Upper Limit	99 Percent Lower Limit	99 Percent Upper Limit
Undrained strength						
CIU tests	641.21	713.47	634.09	720.59	619.84	734.84
Field vane tests	637.64	737.06	627.24	747.46	605.37	769.33
Compression ratio for soft silty clay	0.2187	0.2333	0.2173	0.2347	0.2144	0.2376
Recompression ratio for soft silty clay	0.0207	0.0233	0.0198	0.0243	0.0188	0.0252

soil-property variability while the statistical analysis quantifies it.

CONCLUSIONS

On the basis of results obtained in this study, the following conclusions can be drawn:

1. The statistical approach is useful in systematically organizing data.
2. In the sorting of data, histograms that exhibit more than one peak (multimodal) can indicate whether one or more populations are present.
3. Shear-strength characteristics exhibited the most variability. This is in agreement with other research.
4. Low coefficients of variation for the classification parameters may indicate whether one is dealing with the same soil type.
5. The beta distribution was found to model most soil properties investigated in this paper. In fact, due to its versatility, it could be expected to model most soil properties.
6. Whenever large amounts of data are available for a particular soil unit, a statistical treatment may provide better insights into the interrelations of the various soil properties and help the engineer to reduce the amount of judgment necessary in the selection of design parameters.
7. It is important to note that the statistical results presented in this paper apply only to clay material. If one were dealing with material similar in geologic origin and stress history, the results presented here could be of value.

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Use of Point Estimates for Probability Moments in Geotechnical Engineering

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In probabilistic geotechnical engineering, it is often necessary to obtain estimates of the mean and standard deviation of a function of one or more random variables. For this purpose, Rosenblueth first proposed the method of using point estimates for approximating probability moments. This method is advantageous in that it requires neither extensive computer capabilities nor complex mathematical derivations. The point-estimate method is described and compared with existing methods, and its usefulness is illustrated with examples of its application to common geotechnical functions.

Analytic expressions are available and can be used to evaluate the statistical values (mean, variance, higher moments) of soil properties with random variation, such as plasticity index, compression ratio, and undrained shear strength. Moreover, of equal importance in geotechnical practice is the determination of the statistical values of functions of soil properties.

As an example, consider the commonly used settlement equation expressed in the following form:

$$S = H \times CR \times \log(P_f/P_o) \quad (1)$$

where

S = total settlement within a soil layer,
H = thickness of the layer,
CR = compression ratio of the layer,
P_o = initial vertical stress within the soil layer, and
P_f = final vertical stress within the soil layer.