strain, the effective in situ modulus is much larger than at higher levels of shear strain.

4. The amount of shear-strain development in the granular layer appears to be related to the overall pavement strength in addition to loading magnitude, which is intuitively correct.

5. Since shear strain cannot be measured easily in the field, a provisional procedure was developed and outlined for estimating the granular base modulus adjustment factor (K_1) directly from field-measured deflection results by using an empirically derived relationship from this study.

6. With the bituminous surface and granular base layers of a conventional asphalt pavement accurately characterized, elastic-layered theory can be used to compute the subgrade modulus by matching surface deflections.

From the results of this study, the possibility of the behavior of granular material being related to mobilized shear strain seems entirely plausible. We recommend that this connection be further investigated over a broader range of pavement sections to incorporate different material types and climate conditions. In recognition of all the possible sources of error that could enter into the production of the data used in this study (e.g., field deflection measurement, laboratory material testing, mathematical manipulation, and existing natural variation), the amount of scatter exhibited in the relationships involving the proposed adjustment factor is simply too small to forego additional exploration.

ACKNOWLEDGMENT

We are indebted to the Maryland State Highway Administration for providing traffic control to assist the field testing performed under this study.

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Prediction of Subgrade Moduli for Soil that Exhibits Nonlinear Behavior

JAN MOOSSAZADEH AND MATTHEW W. WITCZAK

The main objective of this report is to develop a simple and accurate procedure to predict an equivalent one-layer subgrade modulus for a soil that exhibits nonlinear behavior in a flexible highway pavement system. The analysis is predicated on developing such a modulus that would yield identical values of either vertical strain or deflection at the top of the subgrade compared with results obtained from a stress-dependent iteration technique that accounts for the stress-dependent (nonlinear) behavior of the subgrade. Use of a modified elastic layered computer program was made to determine equivalent subgrade modulus values for nearly 3900 separate layered pavement problems. By using the results obtained, multiple regression techniques were used to determine predictive equations for the equivalent subgrade modulus values as a function of the nonlinear subgrade and layered pavement properties. Use of partial model regressions techniques allowed predictive equations to be obtained that had correlation coefficients in excess of 0.95 and residual errors less than 10 percent. Both analytical and nomographic solutions are presented to demonstrate the simplicity of the approach. It was found that values of deflection-based equivalent mod-

ulus are larger than or equal to values of strain-derived modulus for a given set of load, pavement, and nonlinear subgrade properties. Although the vertical subgrade-strain-derived subgrade moduli are generally independent of the incorporation of overburden stresses, equivalent subgrade moduli based on deflection criteria, without overburden considered, were almost always larger than equivalent deflection based moduli developed with overburden incorporated into the analysis.

Until recently, the moduli of unbound flexible pavement layer materials have been treated as parameters of constant magnitude. However, recent research in dynamic materials characterization has indicated that many pavement materials are stress dependent. Due to the importance of the subgrade modulus in the overall performance of the flexible pavement, the most accurate assessment of modulus is essential to the successful design of a pavement system. Such assessment is also required in calculation of any stress, strain, or deflection in the pavement system.

Several mathematical forms have been suggested by researchers to model the nonlinear behavior of fine-grained subgrade soil. One such useful, but simple, model is

$$M_r = K_1(\sigma_d)^{K_2} \quad (1)$$

where

M_r = resilient modulus,
K_1 and K_2 = constants dependent on material type and physical soil properties, and
σ_d = deviator stress.

This relation is illustrated in Figure 1 for two different soils that show marked differences in nonlinear behavior. The most important aspect illustrated in Figure 1 is that, since the deviator stress varies with depth in the subgrade, the modulus must likewise change with depth. Clearly, the magnitude of this change is most significantly affected by changes in the slope or K_2-value. This parameter indicates, to a large degree, the nonlinearity of a given material.

STUDY OBJECTIVE

The general purpose of this research was to develop a simple and accurate method of evaluating a single (one-layer) modulus value for the entire subgrade layer that would yield identical values of selected critical parameters to those found by using an iterative multilayer systems approach to account for the nonlinear behavior. The critical parameters selected for this study were the vertical strain (ϵ) and deflection (δ) at the top of the subgrade layer. Both of these parameters are performance indicators of the pavement system. The two resulting moduli will be referred to as the equivalent strain [$E(\epsilon)$] and equivalent deflection moduli [$E(\delta)$], respectively.

This study follows a similar research effort by Smith (1) on the stress-dependent modulus of granular base material. However, there are few, if any, studies on the stress-dependent solutions for the subgrade modulus value. The closest technique is the one developed by Treybig and others (2). However, this method is only applicable for overlay analysis. Furthermore, the method mentioned only accounts for adjustments in the state of stress at the top of subgrade layer, and, therefore, the change with depth of these parameters is neglected.

In a layered pavement system, there are three major parameter groups that affect the deviator stress in the subgrade, and hence the variation of moduli within this layer. They are as follows:

1. Nonlinear subgrade properties (K_1 and K_2),
2. Pavement layer properties (E_i, h_i, and μ_i), and
3. Vehicle load properties (P, p, and spacing).

Study on Values of K_1 and K_2

In order to arrive at typical values for K_1 and K_2, available test data in the literature were used. Three sets of data were used in this study: the results of the San Diego County Test Road (3), the results of a subgrade study by the Illinois Department of Transportation (4-6), and the data from a study conducted for the Maryland State Highway Administration (7). Altogether, 137 sets of nonlinear moduli results were analyzed.

Through linear regression techniques, values of K_1 and K_2 were determined for each set of data. A summary of these values is shown in Table 1. This table shows that the majority of K_1 values fall between 0 and 200 kips/in², and K_2 ranges from 0 to -1.0.

The above study also afforded some insight into the effect of soil parameters on K_1 and K_2 values. Although very general, the important findings of this study were as follows:

1. K_1 increases and K_2 decreases as the dry density increases or the moisture content decreases;
2. The value of K_1 is directly proportional to the liquid limit and plasticity index, but inversely proportional to the plastic limit; K_2 is relatively insensitive to the Atterberg limits; and
3. K_1 and K_2 are relatively independent of each other.

Arrangement of Factorial Matrix

In order to relate the equivalent subgrade modulus values to other pavement system parameters, a factorial study was conducted. A selected number of each parameter was chosen to encompass the whole range of values of such parameters encountered in design. Use was made of available data concerning the ranges of each variable, as well as the K_1-K_2 study mentioned above. Thickness values for the asphalt and base layers (h_1 and h_2) were selected from an examination of typical highway pavement sections encountered in practice. The load used in this study was a 9-kip single wheel load and a tire pressure of 70 lbf/in² to approximately model the effects of an 18-kip single-axle load. Because of this load selection, the results of this study are considered applicable only for typical highway pavements. Table 2 illustrates the values of each variable used in the factorial study. As observed, a matrix of 972 combinations was evaluated.

METHOD ANALYSIS

Equivalent Moduli

As mentioned earlier, determination of a stress-dependent subgrade modulus value with elastic layered theory involves an iterative procedure. In this study the subgrade was subdivided into four 12-in sublayers and a semi-infinite lower layer. The values of the deviator stress at the top of each sublayer were calculated by the Chevron N-layer computer program (8). The predicted values of σ_d found were then substituted in Equation 1 and a new value was calculated for each sublayer modulus. This new value of the sublayer modulus was then compared with the original value and the iteration procedure continued until a difference of ±5

Figure 1. Typical nonlinear modulus results for subgrade soils.

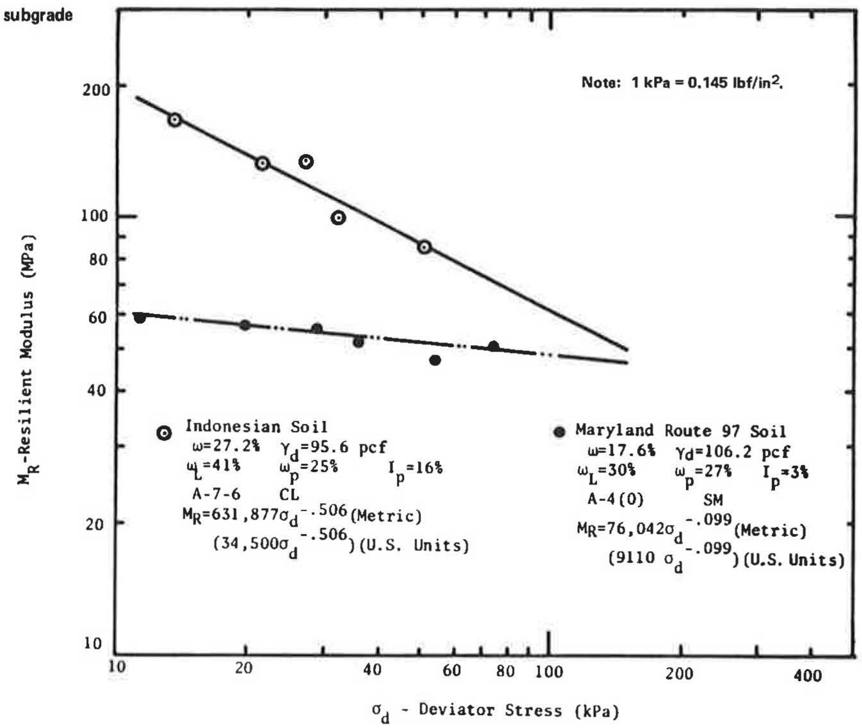


Table 1. Summary of nonlinear soil parameters.

	San Diego Study		Illinois Study		Maryland Study	
Parameter	K_1	K_2	K_1	K_2	K_1	K_2
Mean	60.6 kips/in^2	-0.37	16.5 kips/in^2	-0.42	47.7 kips/in^2	-0.51
SD	89.3 kips/in^2	0.264	8.3 kips/in^2	0.156	37.8 kips/in^2	0.290
Coefficient of variance (%)	147	71	51	37	79	57
Range	5.0 to 684 kips/in^2	-1.17 to 3.93	3.0 to 34.0 kips/in^2	-0.74 to -0.17	8.0 to 125.0 kips/in^2	-1.13 to -0.04
Total no. of samples	79		39		19	
No. of samples that have K_1 between 0 and 200 kips/in^2	76[a]		39[b]		19[b]	
No. of samples that have K_2 between 0 and -1.0	75[c]		39[b]		18[d]	

[a]This represents 96.2 percent of the samples.
[b]This represents 100 percent of the samples.
[c]This represents 94.9 percent of the samples.
[d]This represents 94.7 percent of the samples.

percent occurred between the two values. Because of the large size of the factorial matrix analyzed, we modified the Chevron N-layer program to perform this iterative sequence internally.

After the stress-dependent sublayer moduli were found, the multilayer pavement system was reduced to a three-layer system. This three-layer system had the same top two layers as did the original multilayer pavement but one subgrade layer was added that had a unique modulus value. The end product of this analysis was to determine what one-layer subgrade modulus would yield the same critical parameters as the stress-dependent sublayer moduli system.

As mentioned before, the two selected parameters (critical criteria) used in this study were the values of vertical strain and deflection at the top of the base-subgrade interface. These two parameters would, as expected, result in two different E_3 values [$E(\varepsilon)$ and $E(\delta)$]. In addition, the effects of both ignoring and including the pavement overburden stresses on the E_{eq} values were also studied. Thus, four separate E_{eq} parameters were investigated.

The procedure used to determine the E_{eq} value consisted of examining the effect of subgrade modulus on the desired parameter (ε or δ) for a particular three-layer pavement system. As it turned out, these relations were virtually linear on a log-log scale. The value of the equivalent subgrade modulus was then determined as that modulus yielding the same strain (or deflection) as that found from the multilayer stress-dependent iteration study. Figure 2 depicts one such solution for the $E_{eq}(\varepsilon)$ and $E_{eq}(\delta)$ value. Although the figure denotes a graphical solution, the above steps in the equivalent subgrade study were computerized to save time and increase the accuracy of the solution.

Statistical Data Analysis

For each combination of pavement and soil parameters examined (i.e., h_1, E_1, h_2, E_2, K_1, and K_2) a unique E_{eq} value for a given critical criterion was obtained. The next study step was to examine whether a relation existed between the mentioned variables and E_{eq}. The use of multiple regression techniques was employed to determine the best model (highest correlation coefficients). In this phase of the study, 10 different model combinations were analyzed. The best model (highest R^2) form was found to be a total log model. In order to reduce the number of independent variables in the prediction equations, the individual thickness and moduli parameters were combined into one unique parameter by the concept of equivalent layers. One such equivalent layer technique is that recommended by Palmer and Barber (9). In this method, it is assumed that the upper layer of thickness (h_1), modulus (E_1), and Poisson's ratio (ν_1), may be replaced by an equivalent thickness (h_1) of the base material, with E_2 and ν_2, as follows:

$$h_1' = h_1 [E_1(1 - \nu_2^2)/E_2(1 - \nu_1^2)]^{1/3} \quad (2)$$

Table 2. Summary of study parameters.

Parameter	Value Used	Levels
Tire load, single wheel	9 kips	1
Tire pressure	70 lbf/in^2	1
Asphalt layer		
Thickness (h_1)	4, 8, and 12 in	3
Modulus (E_1)	50, 100, 500, and 1000 kips/in^2	4
Density (γ_1)	145 lb/ft^3	1
Poisson's ratio (μ_1)	0.3	1
Granular base layer		
Thickness (h_2)	4, 10, and 16 in	3
Modulus (E_2)	20, 50, and 80 kips/in^2	3
Density (γ_2)	130 lb/ft^3	1
Poisson's ratio (μ_2)	0.4	1
Subgrade layer		
Thickness	Semi-infinite	1
K_1	10, 50, and 200 kips/in^2	3
K_2	-0.1, -0.3, and -1.0	3
Density (γ_3)	110 lb/ft^3	1
Poisson's ratio (μ_3)	0.45	1

Note: Total matrix size is 972 combinations.

Thus, the new equivalent layer has a modulus (E_2), Poisson's ratio (ν_2), and thickness of $h_e = h_1' + h_2$. A relative stiffness value for the equivalent layer was defined as follows:

$$D = E_2 h_e^3 \quad (3)$$

where D is termed the equivalent pavement stiffness value.

Thus, the entire pavement system could now be identified by only three parameters: D, K_1, and K_2. Further multiple regressions proved that virtually the same predictive values of E_{eq} were obtained from expressions of the following form:

$$\log E_{eq} = f(\log D, \log K_1, \log K_2) \quad (4)$$

compared with the use of log h_1, log E_1, log h_2, and log E_2 individually. The correlation coefficients of the predictive models stayed almost the same with R^2 values that ranged between 0.906 and 0.938. Finally, although not discussed in this report, the equivalent layer approach developed by Barros (equivalent moduli) (9) was also analyzed and found to yield identical solutions.

Partial Models

The regression models developed resulted in very high correlation coefficients; however, a study of residuals from Equation 4 showed some large numerical differences between the actual and regression-predicted E_{eq} values. In order to overcome this problem, separate predictive models were developed by holding constant the least-significant parameter (i.e., the variable that had the smallest correlation value). This parameter turned out to be K_2. The results were improved correlations and lower residuals. The models were of the general form:

$$\log E_o \text{ or } \log E_{eq} = f(\log D, \log K_1) \quad (5)$$

where K_2 is a constant.

Figure 2. Determination of $E_{eq}(\delta)$ and $E_{eq}(\epsilon)$ for specific problem input.

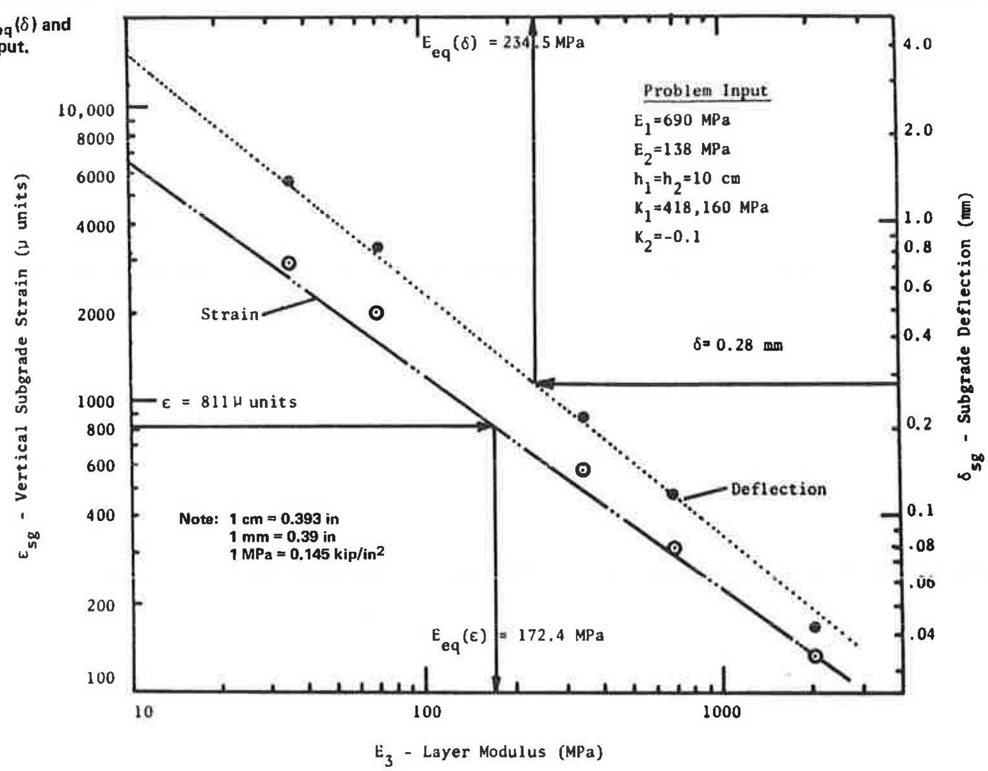


Table 3. Summary of regression models.

Equation	Total Model (K_2 included) $\log E_{eq(or\ o)} = A_0 + A_1 \log D + A_2 \log K_1 + A_3 \log \lvert K_2 \rvert$: A_0	A_1	A_2	A_3	R^2	Partial Model (K_2 = constant) $\log E_{eq(or\ o)} = B_0 + B_1 \log D + B_2 \log K_1$: K_2	B_0	B_1	B_2	R^2
$E_{eq}(\delta)$	-1.3262	+0.2011	+0.8316	-0.3211	0.9350	-0.1	-0.4986	+0.0592	+0.9823	0.9971
						-0.3	-1.0069	-0.1543	+0.8965	0.9967
						-1.0	-1.9840	+0.3897	+0.6160	0.9941
$E_o(\delta)$	-0.9897	+0.1029	+0.9012	-0.5035	0.9723	-0.1	-0.3944	+0.0373	+0.9955	0.9977
						-0.3	-0.7043	+0.0887	+0.9389	0.9974
						-1.0	-1.1036	+0.1827	+0.7692	0.9853
$E_{eq}(\epsilon)$	-1.3058	+0.1563	+0.8224	-0.6111	0.9074	-0.1	-0.9361	+0.0464	+1.0708	0.9841
						-0.3	-1.1224	+0.1373	+0.9026	0.9794
						-1.0	-0.9284	+0.2852	+0.4937	0.9702
$E_o(\epsilon)$	-1.2189	+0.1372	+0.8437	-0.5955	0.9129	-0.1	-0.9447	+0.0423	+1.0816	0.9842
						-0.3	-1.0862	+0.1213	+0.9267	0.9779
						-1.0	-0.7188	+0.2479	+0.5229	0.9592

DEVELOPMENT OF PREDICTIVE MODELS

Table 3 summarizes the regression models developed for prediction of E_{eq} (without overburden) or E_o (with overburden). Both total and partial regression equations for $E_{eq}(\epsilon)$ and $E_{eq}(\delta)$ parameters, with and without overburden stress, are shown. An examination of the tabulated R^2 values in the table clearly supports the accuracy of all model combinations--the lowest R^2 value found was 0.9074.

The increase in accuracy provided by using the partial models (k_2 = constant) can be observed to yield regression equations that have R^2 values of 0.96-0.99. A study of the residual errors between the total and partial models also greatly supports the partial model approach. The total models resulted in residuals in excess of 100 percent of the actual values for some cases, and the partial models reduced the maximum residuals to less than 20 percent; the majority of points were within the 1-10 percent limit.

Example Solution

The mechanics of using any of the models in Table 3 is quite direct. Units for the input values are as follows:

D in inch-pounds (h_e = inches and E_2 = pounds per square inch),
K_1 in pounds per square inch, and
K_2 is dimensionless.

The resulting E_{eq} value is then expressed directly in units of pounds per square inch. For example, consider if the E_{eq} value is desired for the conditions noted below.

For the total model with overburden, equivalent deflection criteria are as follows:

h_1 = 4 in, E_1 = 100 000 lbf/in², and v_1 = 0.3;
h_2 = 8 in, E_2 = 40 000 lbf/in², and v_2 = 0.4; and
$h_3 = \infty$ and $E_3(M_r) = 25\ 000\ \sigma_d^{-0.4}$.

From Table 3, the appropriate A_i coefficients for the total model $-E_o(\delta)$ are shown. Thus,

$$E_o(\delta) = -0.9897 + 0.1029 + \log D + 0.9012 \log K_1 - 0.5035 \log \lvert K_2 \rvert$$

$$h_1' = h_1\ [E_1(1 - v_2^2)/E_2(1 - v_1^2)]^{1/3}$$

$$= 4[(100\ 000/40\ 000)\ (1 - 0.16)/(1 - 0.09)]^{1/3} = 5.29 \text{ in}$$

$$h_e = h_1' + h_2 = 5.29 + 8 = 13.29 \text{ in}$$

$$D = E_2 h_e^3 = 40\ 000(13.29)^3 = 9.38\text{x}10^7 \text{ in} \cdot \text{lb}$$

for

$$D = 9.38\text{x}10^7, K_1 = 25\ 000, \text{and } K_2 = -0.4$$

$$\log E_o(\delta) = -0.9897 + 0.8203 + 3.9634 + 0.2004 = 3.9944$$

$$E_o(\delta) = 9872 \text{ lbf/in}^2$$

Nomographic Solutions

The simplicity afforded by nomographic solutions in practical analysis problems is appealing. Moossazadeh (10), presents nomographs (and their specifications for construction) for solutions of D as well as all 16 equations shown in Table 3. Figures 3 and 4 are solutions for the total model equations, with and without overburden effects considered for both deflection and strain-based equivalent modulus. If the more accurate partial model solutions are desired, refer to Moossazadeh (10). The use of the partial model equations or nomographs requires that an interpolation procedure be employed for the actual $\lvert K_2 \rvert$ value of the subgrade soil. The accuracy of the nomograph can be easily verified with the example problem previously presented.

DISCUSSION OF RESULTS

Effect of Parameters

An examination of the equations in Table 3 indicates that the value of E_{eq} is directly dependent on all pertinent parameters. These parameters, in order of importance, were K_1, K_2, h_1, E_1, and h_2. The value of E_{eq} was almost independent of E_2. The results confirm the logic that, as the value of h_1, h_2, E_1, or E_2 increases (an increase in D), the pavement system approaches the condition of a rigid slab, which in turn, results in a decrease of σ_d due to the external loads within the subgrade layer. This decrease, in turn, results in an increase in E_{eq}.

The value of K_1, however, had the strongest effect on E_{eq}. In almost all cases, the variation in K_1 contributed to more than 80 percent of the variation in E_{eq}, as measured by the correlation coefficient.

Comparison of Equivalent Subgrade Modulus Types

The values of $E_{eq}(\epsilon)$ versus $E_{eq}(\delta)$ are presented in Figure 5. As seen, all values of $E_{eq}(\delta)$ are larger than or equal to the corresponding values of $E_{eq}(\epsilon)$. The influence of

Figure 3. Nomographs for $E_{eq}(\delta)$ and $E_{eq}(\epsilon)$ without overburden for total model.

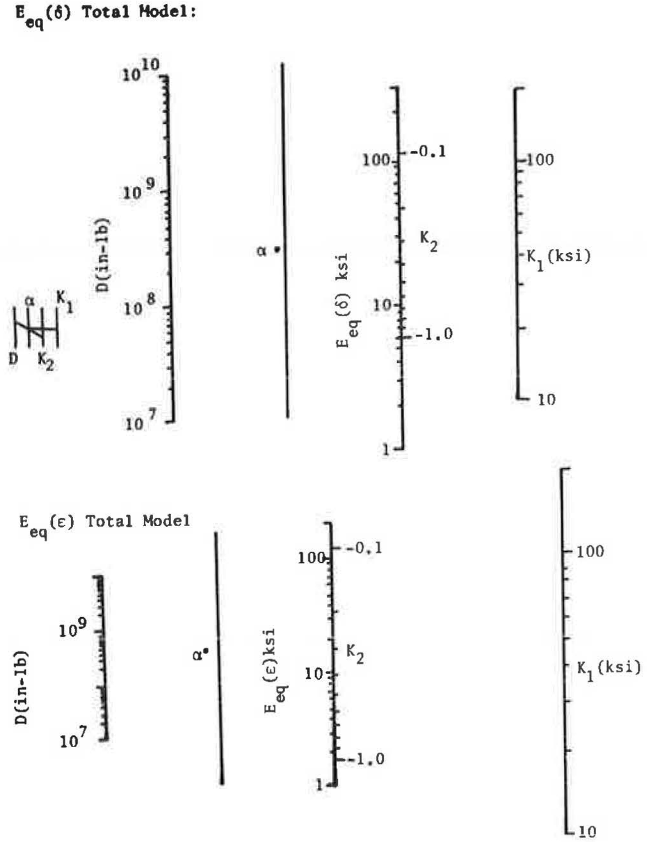


Figure 4. Nomographs for $E_o(\delta)$ and $E_o(\epsilon)$ with overburden for total model.

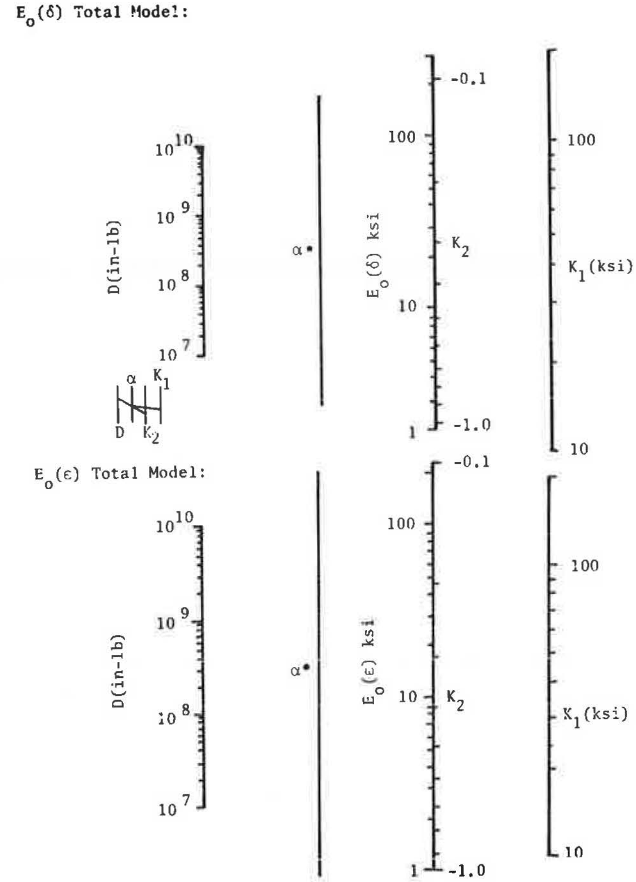


Figure 5. Relation between $E_{eq}(\delta)$ and $E_{eq}(\epsilon)$ without overburden.

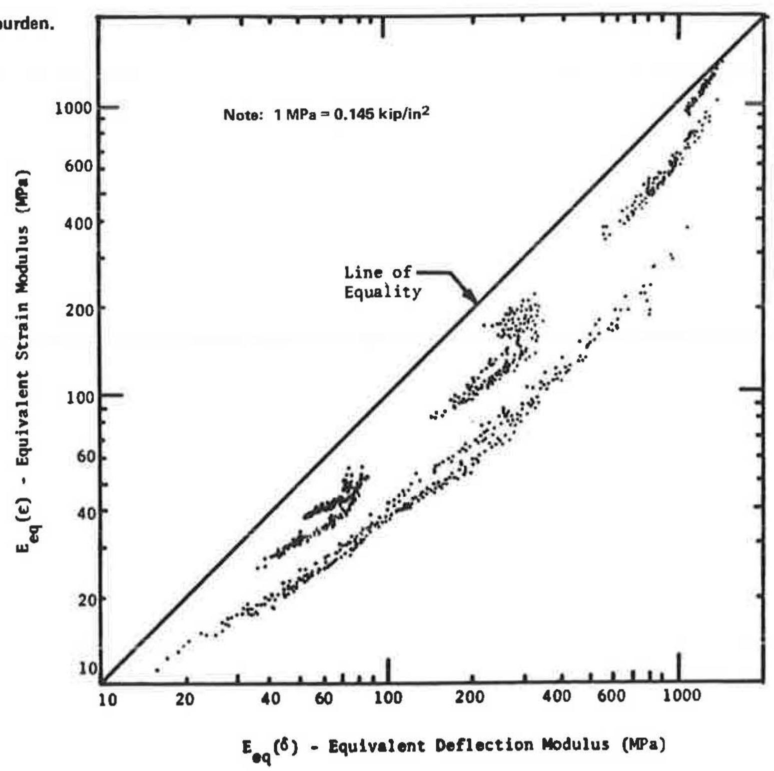
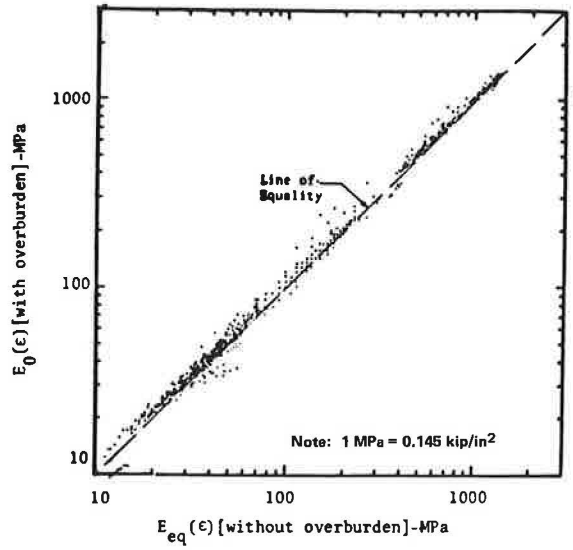
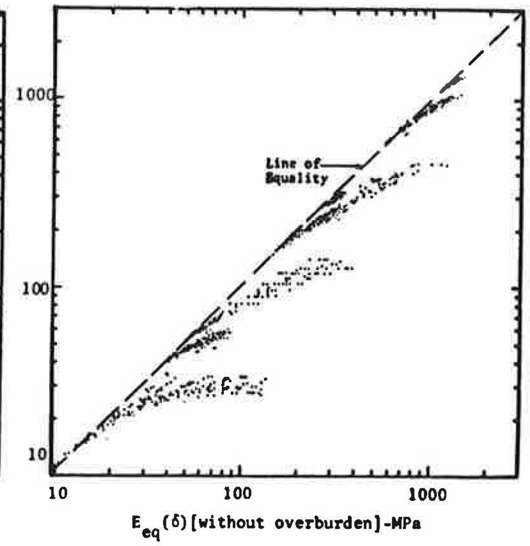


Figure 6. Comparison of equivalent one-layer subgrade modulus with and without overburden.





K_1-K_2 combinations are reflected by the banding effect on the plot. Notice that, in general, there is close agreement between the two types of equivalent modulus for small values of K_1 and K_2. However, as the absolute values of K_1 and K_2 increase, the difference between the two groups becomes larger. This can be explained by recognizing that, for small values of K_1 and $|K_2|$, the subgrade modulus is less sensitive to changes in deviator stress (more linear) and thus does not vary greatly with depth. On the other hand, large combinations of K_1 and $|K_2|$ are indicative of highly sensitive (nonlinear) subgrade modulus.

The value of the vertical strain at the top of the subgrade is highly sensitive to the value of the subgrade modulus near the top (first sublayer) and not so much on the subsequent sublayers. However, subgrade deflection is a function of the moduli of the total subgrade layer. Thus, the value of E_{eq} (δ) will generally always be larger than E_{eq} (ε), especially when the subgrade is highly nonlinear.

Effect of Overburden Stress

As previously noted, equivalent subgrades were analyzed under two sets of assumed deviator stress conditions (with and without overburden stresses). Obviously, the effect of including overburden resulted in changes in regression coefficients as shown in Table 3. However, all of the relative comparisons and results of the E_{eq} models (without overburden) were identical to those of the E_o models (with overburden). For example, the K_1 factor had the highest influence and E_2 the least on the regression; partial regression models greatly decreased the residual error; and the deflection-based equivalent subgrade modulus [$E_o(\delta)$] was always greater than the strain-based modulus [$E_o(\varepsilon)$].

Figure 6 shows the comparison between equivalent modulus for models with and without overburden considered. A comparison of E_{eq} and corresponding E_o values showed that $E_{eq}(\delta)$ was always larger than or equal to $E_o(\delta)$. The values of $E_{eq}(\varepsilon)$ and $E_o(\varepsilon)$, on the other hand, were virtually equal all the time. The reason is that, at shallow depths, the value of overburden stress is small enough so that the difference in the σ_d value used by the two methods is quite small. Therefore, the values of $E_3(\varepsilon)$, which are heavily influenced by the state of the top layers, stay close to each other.

At deep locations in the subgrade the inclusion of the overburden stress causes the deviator stress not to vary as much. This, in turn, prevents the subgrade modulus from increasing as rapidly. This fact causes the $E_o(\delta)$ value, which is a function of the total subgrade layer, to be smaller than $E_{eq}(\delta)$.

RECOMMENDATIONS FOR IMPLEMENTATION

Based on the previous discussion, general recommendations regarding the type model to use (δ or ε, with or without overburden) can be made. One of the more important implications of this study is that the subgrade moduli may be a function of its intended use. If the analysis is to be used in design situations with limiting subgrade strain as a distress criteria, the equivalent strain model should be used. However, if the models are to be used with in situ deflection tests (e.g., nondestructive testing) to determine in situ subgrade response, the deflection-based model should be used. This appears to be critical, especially if highly nonlinear soils are anticipated (large $|K_2|$).

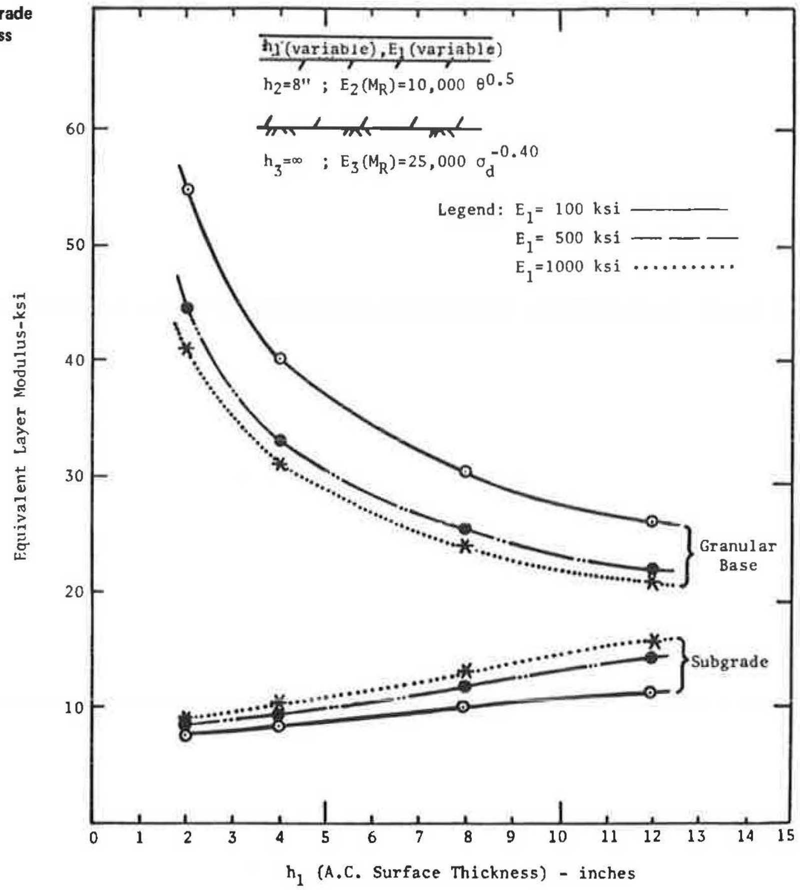
In either case, the most probable and realistic estimate of modulus can be obtained with the overburden effect included. Recall that, for the equivalent strain models, the $E_o(\varepsilon)$ was approximately equal to the $E_{eq}(\varepsilon)$, and the $E_o(\delta)$ was less than the $E_{eq}(\delta)$ for the equivalent deflection model. Thus, inclusion of the overburden effect will result in either equivalent moduli or values slightly conservative for practice regardless of the criterion selected.

SUMMARY

This study has presented a highly accurate and simple technique to determine the equivalent subgrade moduli for soils that exhibit nonlinear dynamic behavior. There are several major advantages to this approach.

The solution technique offers the potential for a significant savings in time and cost by not having to conduct detailed iterative nonlinear elastic layered computerized solutions. The solution also

Figure 7. Summary of equivalent granular base and subgrade modulus as functions of asphalt concrete surface thickness and modulus.



provides a basis for quick determination of single subgrade modulus values for nonlinear soils for use in pavement design charts such as found in the Shell Oil, the Asphalt Institute, or even American Association of State Highway and Transportation Officials (AASHTO) design schemes. The results of this study can also be used to evaluate the influence of various layer pavement parameters (h_i and E_i) on the equivalent subgrade modulus.

This study on nonlinear subgrade soils followed the study concepts used by Smith (1) to predict equivalent granular base moduli. The results of the two studies were intended to allow the combination of their predictive equations to quickly and accurately determine the simultaneous solution of both unbound granular and subgrade soil nonlinear moduli problems. Figure 7 shows the results of such a sample solution by using nonlinear moduli properties for both the granular base and subgrade from predictive equations developed in this study and Smith's work (1). Note that the values shown in the figure were computed manually for the sample pavement cross section noted.

Finally, the solution presented allows examination of the probable levels of the subgrade deviatoric stress for any given pavement structure. The technique for this is best illustrated by referring to the example problem previously worked. For a K_1 = 25 000 and K_2 = -0.4, the $E_o(\delta)$ was found to be 9872 lbf/in². Because

$$M_r = E_o(\delta) = K_1 \sigma_d^{K_2} \qquad (6)$$

the deviator stress (σ_d) can be easily computed to be σ_d = 10.2 lbf/in².

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Analysis of Pavements with Granular Bases

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The nonlinear stress-strain response of unbound granular materials has to be modeled satisfactorily when numerical analysis is used for pavement structures that contain significant quantities of such material. The simple model, which has been used widely for the purpose of relating resilient modulus to the sum of the principal stresses, has serious limitations. This is because the triaxial tests used in its development do not cover an adequate range of stress paths when compared with the situation in the pavement. A more complex and accurate model is described that has been developed from extensive laboratory repeated load testing. It is expressed in terms of shear and volumetric stress-strain relationships. The need for better modeling of cohesive soils for analysis is also discussed and the importance of the principle of effective stress is emphasized. A new finite-element program is described and its main features are explained. It uses a secant modulus approach, has been specifically developed to deal with nonlinear materials, and is called SENOL for secant modulus nonlinear. The use of the new granular material models for the three-dimensional stress system in a pavement is discussed and results from analyses with SENOL are compared with in situ measurements from a test pavement.

Most of the development work concerned with the use of theoretical analysis in flexible pavement design has concentrated on accurate modeling of the asphalt layer. Simplifying assumptions are often made concerning the mechanical properties of the unbound granular layer and the subgrade soil. This situation is a reflection of the relative states of knowledge concerning the various materials together with a need to produce design methods that are not unduly complex.

The existence of nonlinear stress-strain characteristics for granular materials and soils has been well known for many years, and numerous papers have reported experimental data, mainly from laboratory repeated load tests (e.g., 1,2). The increasing use of finite-element analysis has provided a means of more accurate modeling of nonlinear materials within pavement structures, but the full potential of the method has often not been realized. Recent work at the University of Illinois (3) represents an improvement to the finite-element approach since the analysis considers failure conditions and has been used to generate data for use in simplified design charts (4). However, the nonlinear stress-strain relations that have been used are not really adequate for analysis of a three-dimensional system, even an axisymmetric one. The approach reported here describes a more-detailed method of characterizing granular materials and discusses the use of this in a finite-element, secant modulus nonlinear (SENOL) program, which has been recently developed.

CHARACTERIZATION OF GRANULAR MATERIALS

In general, an element of material in the granular layer of a pavement is subjected to three principal stresses: σ_1, σ_2, and σ_3, where $\sigma_1 > \sigma_2 > \sigma_3$. Each stress consists of two components, a constant value due to overburden and a transient value due to the passing wheel load.

Use of the repeated load triaxial test to obtain data on the resilient characteristics of granular materials implies that two of the stresses are equal because of axial symmetry. If, however, the stresses are expressed in invariant form, this difficulty can be partly overcome (5).

Most research in this field has involved tests with a constant confining stress and a deviator stress applied repeatedly by pneumatic or servohydraulic actuators between zero and a peak value. Various peak values have been used at a range of confining stresses and a well-established relationship has emerged to relate resilient modulus (M_r) to stress level:

$$M_r = K_1\theta^{K_2} \quad (1)$$

in which

$$M_r = q_r/\epsilon_{ar} \quad (2)$$

where q_r is a repeated stress given by $\sigma_{ar} - \sigma_c$ and ϵ_{ar} is resilient (recoverable) axial strain.

$$\theta = \sigma_{ar} + 2\sigma_c = 3\sigma_c + q_r \quad (3)$$

where

σ_{ar} = peak axial stress,
σ_c = constant confining stress, and
K_1 and K_2 = material constants.

Confusion has often arisen over the use of Equation 1 because the factor K_1 is not dimensionless. Furthermore, distinction is rarely made between total stress and effective stress. Although this is of no consequence for dry materials, it is of fundamental importance when pore water is present.

Figure 1 shows a typical stress path in triaxial stress space for a test of the type described above. The parameters used are mean normal effective stress [$p' = (1/3)(\sigma_a' + 2\sigma_c')$] and the deviator stress (q). The tests are assumed to have been carried out on dry material, so that no pore pressures are generated and, hence, the path in Figure 1 represents effective stress.