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## Analysis of Pavements with Granular Bases

## S.F. BROWN AND J.W. PAPPIN

The nonlinear stress-strain response of unbound granular materials has to be modeled satisfactorily when numerical analysis is used for pavement structures that contain significant quantities of such material. The simple model which has been used widely for the purpose of relating resilient modulus to the sum of the principal stresses, has serious limitations. This is because the triaxial tests used in its development do not cover an adequate range of stress paths when compared with the situation in the pavement. A more complex and accurate model is described that has been developed from extensive laboratory repeated load testing. It is expressed in terms of shear and volumetric stress-strain relationships. The need for better modeling of cohesive soils for analysis is also discussed and the importance of the principle of effective stress is emphasized. A new finite-element program is described and its main features are explained. It uses a secant modulus approach, has been specifically developed to deal with nonlinear materials, and is called SENOL for secant modulus nonlinear. The use of the new granular material models for the threedimensional stress system in a pavement is discussed and results from analyses with SENOL are compared with in situ measurements from a test pavement.

Most of the development work concerned with the use of theoretical analysis in flexible pavement design has concentrated on accurate modeling of the asphalt layer. Simplifying assumptions are often made concerning the mechanical properties of the unbound granular layer and the subgrade soil. This situation is a reflection of the relative states of knowledge concerning the various materials together with a need to produce design methods that are not unduly complex.

The existence of nonlinear stress-strain characteristics for granular materials and soils has been well known for many years, and numerous papers have reported experimental data, mainly from laboratory repeated load tests (e.g.,  $\underline{1}, \underline{2}$ ). The increasing use of finite-element analysis has provided a means of more accurate modeling of nonlinear materials within pavement structures, but the full potential of the method has often not been realized. Recent work at the University of Illinois (3) represents an improvement to the finite-element approach since the analysis considers failure conditions and has been used to generate data for use in simplified design charts (4). However, the nonlinear stress-strain relations that have been used are not really adequate for analysis of a three-dimensional system, even an axisymmetric one. The approach reported here describes a more-detailed method of characterizing granular materials and discusses the use of this in a finite-element, secant modulus nonlinear (SENOL) program, which has been recently developed.

## CHARACTERIZATION OF GRANULAR MATERIALS

In general, an element of material in the granular

layer of a pavement is subjected to three principal stresses:  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$ , where  $\sigma_1 > \sigma_2 > \sigma_3$ . Each stress consists of two components, a constant value due to overburden and a transient value due to the passing wheel load.

Use of the repeated load triaxial test to obtain data on the resilient characteristics of granular materials implies that two of the stresses are equal because of axial symmetry. If, however, the stresses are expressed in invariant form, this difficulty can be partly overcome (5).

Most research in this field has involved tests with a constant confining stress and a deviator stress applied repeatedly by pneumatic or servohydraulic actuators between zero and a peak value. Various peak values have been used at a range of confining stresses and a well-established relationship has emerged to relate resilient modulus  $(M_T)$ to stress level:

$$M_r = K_1 \theta^{K_2} \tag{1}$$

in which

$$M_r = q_r / \epsilon_{ar} \tag{2}$$

where  $q_r$  is a repeated stress given by  $\sigma_{ar} - \sigma_c$  and  $\epsilon_{ar}$  is resilient (recoverable) axial strain.

 $\theta = \sigma_{ar} + 2\sigma_c = 3\sigma_c + q_r \tag{3}$ 

where

 $\begin{array}{l} \sigma_{ar} = \text{peak axial stress,} \\ \sigma_{C} = \text{constant confining stress, and} \\ \text{K}_1 \text{ and } \text{K}_2 = \text{material constants.} \end{array}$ 

Confusion has often arisen over the use of Equation 1 because the factor  $K_1$  is not dimensionless. Furthermore, distinction is rarely made between total stress and effective stress. Although this is of no consequence for dry materials, it is of fundamental importance when pore water is present.

Figure 1 shows a typical stress path in triaxial stress space for a test of the type described above. The parameters used are mean normal effective stress  $[p' = (1/3) (\sigma_a' + 2\sigma_C')]$  and the deviator stress (q). The tests are assumed to have been carried out on dry material, so that no pore pressures are generated and, hence, the path in Figure 1 represents effective stress.

## Figure 1. Typical stress path for a constant confining stress test.



Figure 2. Typical repeated load stress path.



Figure 3. Stress paths applied to test specimens at one value of pm:



Figure 4. Resilient strain contours: (left) normalized shear strain and (right) volumetric strain.



From the definitions of  $p^{\, t}$  and q it is apparent that:

$$p' = \sigma_{c}' + (1/3)q$$
 (4)

Hence the stress path in Figure 1 has an intercept on the p' axis equal to the effective confining stress and has a slope of 3. Figure 1 also shows the position of the failure line for which q/p'equals M, a constant. This is related to the angle of shearing resistance ( $\phi'$ ) (<u>6</u>) by

$$A = 6\sin\phi'/(3 - \sin\phi') \tag{5}$$

Experiments have shown  $(\underline{3})$  that the expression for resilient modulus (Equation 1) applies even for stress paths that approach failure, but most tests have involved stress paths well short of this condition so that specimens are not subjected to significant permanent strains.

Tests of the type represented in Figure 1 are restrictive because they do not investigate the influence of the mean value of deviator stress nor do they allow any variation in slope of the stress path. Both of these factors are likely to occur in the pavement situation and can be reproduced with triaxial test equipment.

Figure 2 shows a more general stress path that requires facilities to cycle the confining stress and operate from a nonzero minimum stress. The path is defined by the mean and peak to peak values of p' and q (i.e.,  $p_m'$ ,  $q_m$  and  $p_r'$ ,  $q_r$ , respectively).

Servohydraulic equipment to apply these general stress paths has been developed at Nottingham in recent years  $(\underline{7},\underline{8})$  and used to investigate the behavior of crushed limestone. In these tests, large numbers of stress paths were applied both in triaxial compression and extension (negative deviator stress), as shown in Figure 3 for one value of pm'. Results for well-graded material have been presented by Pappin and Brown ( $\underline{9}$ ) and for uniformly graded specimens by Shaw and Brown ( $\underline{10}$ ). A general pattern of behavior has emerged both from these tests and from similar ones conducted by Mayhew at the Transport and Road Research Laboratory.

Figure 4 shows the way in which the compression results were expressed in terms of contours of resilient shear strain ( $\epsilon_r$ ) (Figure 4, left) and volumetric strain ( $v_r$ ) (Figure 4, right),

$$\epsilon_{\rm r} = (2/3)(\epsilon_{\rm ar} - \epsilon_{\rm rr}) \tag{6}$$

(7)

and

$$v_r = e_{ar} + 2e_{rr}$$

where  $\varepsilon_{\text{rr}}$  is the resilient radial strain.

For a particular material, Figure 4 (right) can be used directly to determine the resilient volumetric strain for a stress path simply by subtracting the contour value for one end of the path from that at the other. The contour values  $v_{\rm rc}$  are given by an equation of the form:

$$v_{rc} = (p'/A)^{m} [1 - B(q/p')^{n}]$$
(8)

where A, B, m, and n are material constants and p' and q are the coordinates of the stress point concerned.

For shear strain, the situation is slightly more complicated because the results were dependent on the stress path (9). In practice, this meant that the resilient shear strain not only depended on the end points of the stress path but also on its length. The pattern of contours shown in Figure 4 (left) is for normalized values of shear strain  $(\varepsilon_{\rm rn})$ ; a correction was introduced to allow for the path length [see Pappin and Brown (9)].

The contour value is given by an equation of the form:

$$\epsilon_{\rm rn} = Cq/(p' + D) \tag{9}$$

where C and D are material constants and D is the negative value of p' through which all the contours pass. The actual resilient strain for a stress path between the points  $(p_1',q_1)$  and  $(p_2',q_2)$  is calculated from:

$$\epsilon_{\rm r} = C\{[q_1/(p_1'+D)] - [q_2/(p_2'+D)]\} (\ell_{\rm r}/p_{\rm m}')^{\rm r}$$
(10)

where  $\ell_r$  is path length equals  $\sqrt{p'r^2 + q_r^2}$  and r is a material constant.

Values of the various constants for two gradings of crushed limestone are given in Table 1.

Pappin (<u>11</u>) performed some drained and undrained tests on saturated specimens of well-graded crushed limestone. He showed that the model established for dry material was applicable provided that effective stresses were used. Tests on partially saturated specimens, which required estimates to be made of the negative pore pressures, confirmed this conclusion as did the data of Smith and Nair (12).

This approach to the characterization of resilient behavior for granular materials is more flexible and potentially more accurate, though more complex, than the frequently used combination of Equation 1 for stress-dependent resilient modulus and a constant Poisson's ratio of, typically, 0.3.

In pavement analysis, a wide range of stress paths is encountered and, hence, the simple model does not provide a satisfactory basis for computations. Similarly, the full model, which was only developed for axisymmetric triaxial stress conditions, may be inapplicable to the general threedimensional system that occurs in a pavement layer under wheel loading. The results from triaxial tests carried out in extension provided a basis for dealing with the general case.

Figure 5 shows a view down the space diagonal  $(\sigma_1' = \sigma_2' = \sigma_3')$  in a plot that has orthogonal axes for the three principal stresses. All triaxial tests take place on the vertical line  $F_e-0-F_C$ , with compression tests above zero and extension tests below. The resilient-response model was developed for stress paths such as AB in Figure 5. The extension tests (9) showed that a path A'B' could be defined that gave the same resilient strains. The relationship between these corresponding points was based on the Mohr-Coulomb failure criterion, which states that:

For a particular value of p', corresponding failure values of q can be obtained in both compression and extension, shown in Figure 5 as  $F_c$  and  $F_e$ , respectively. The same procedure was used to relate stress conditions below failure in compression and extension.

Since the three principal stress axes are interchangeable, all the arguments concerning the vertical line on Figure 5 can be equally applied to the other principal axes. This results in the familiar hexagonal shape for the Mohr-Coulomb failure surface in this view. It also means that points that correspond to A and B can be identified, as shown, on the corners of similar hexagons. A general threedimensional stress path such as XY, for instance, can be related back to points in the compression region such as AB and appropriate resilient strains can be predicted by using the material model.

Typical data for both the simple and the new model are given in Table 1 for two gradings of a crushed limestone.

## CHARACTERIZATION OF COHESIVE SOILS

A simple nonlinear model, analogous to that for granular materials, has been used for fine-grained soils. It is based on tests similar to that illustrated by the stress path in Figure 1, except that the tests were generally carried out undrained, the materials were usually partly saturated, and hence the pore pressure was not known so that only the total stress path was defined. However, it has become clear from tests on both saturated and partly saturated soils (13) that it is the initial effective stress that influences resilient characteristics (i.e.,  $\sigma_c$ ' in Figure 1). This stress is the one to which elements of soil are subjected prior to the application of wheel loading. The mag-

## Table 1. Material constants for different gradings of crushed limestone.

| Constant                    | 3-mm Uniform<br>Grading  | Continuous<br>Grading 40-mm<br>Maximum Size  |
|-----------------------------|--|--|
| A(kPa)                      | 12.3 billion   | 1.9 trillion   |
| В                           | 0.033  | 0.08   |
| m                           | 0.5  | 0.33   |
| n                           | 3.5  | 2.0  |
| Shear strain C<br>D(kPa)    | 0.000 55   | 0.000 24   |
|                             | 130  | 13   |
| r                           | 0.45   | 0.4  |
| K <sub>1</sub> <sup>a</sup> | 19 454   | 8634   |
| K <sub>2</sub> <sup>a</sup> | 0.5  | 0.69   |
|                             | Constant<br>A(kPa)<br>B<br>m<br>n<br>C<br>D(kPa)<br>r<br>K <sub>1</sub> <sup>a</sup><br>K <sub>2</sub> | 3-mm Uniform<br>Grading           A(kPa)         12.3 billion           B         0.033           m         0.5           n         3.5           C         0.000 55           D(kPa)         130           r         0.45           K <sub>1</sub> <sup>a</sup> 19 454           K <sub>2</sub> 0.5 |

<sup>a</sup>Use of these values in Equation 1 gives M<sub>r</sub> in kilopascals.

Figure 5. View down the space diagonal in three-dimensional stress space.



nitude of this stress depends on the position of the water table and the depth of the element. For deep water tables, high suctions will develop and the magnitude of suction may be taken as equal to the effective stress. Dehlen (14) and others (e.g., 15) have shown that resilient modulus is related to soil suction. For high water tables, it is possible to estimate the negative pore pressure and hence effective stress in the held water zone. Several workers have noted (2, 15) that confining stress has little influence on the resilient modulus of cohesive soils. This results from tests where the soil suction is high in relation to the range of confining stresses and hence the effective stress, which controls the behavior, is relatively unaffected. Tests with saturated soils by using a range of stress histories, and hence initial effective stresses, have illustrated this point (16).

The simple models for cohesive soils relate resilient modulus, as defined by Equation 2, to repeated deviator stress, which shows a stresssoftening effect. A more-detailed model that shows subsequent stiffening at higher deviator stresses has been developed at the University of Illinois  $(\underline{17})$ .

The relation developed at Nottingham takes the form:

 $M_r = K(p_0'/q_{max})^8$ 

where  $p_O'$  is initial effective stress, equals  $\sigma_C',$  and K and s are material constants.

Since all triaxial tests to date have pulsed q from zero to a peak, it is not clear whether the deviator-stress parameter in Equation 12 should be  $q_{max}$  or  $q_r$  (i.e., the peak value or the repeated value). Brown (<u>13</u>) has demonstrated that the basic relation of Equation 12 seems to apply to a range of soils and moisture conditions but that further testing is needed to properly establish it.

#### SENOL PROGRAM

A new finite-element program named SENOL has been developed specifically to analyze pavements made of nonlinear materials, though it has a range of other capabilities. A detailed description of the program is available elsewhere (<u>18</u>) and it is only necessary to summarize here its major characteristics.

Either plane strain or axisymmetric analysis may be performed, the latter being used for resilient response of pavements. Figure 6 shows a typical arrangement of elements and boundary conditions. In order to use the nonlinear resilient-response model developed for granular materials, it was found advantageous to use a secant modulus approach.

An important part of the overall analysis was the initial computation of stresses due to overburden, which provides a correct starting point with appropriate values of modulus before superposition of the wheel loading. This was applied in 10 gradually increasing steps until the full load was applied in the last one. At each step the element stresses were computed and new values of modulus determined. An iterative process was then carried out by using a variable damping factor, if necessary, to assist convergence. The procedure is summarized in the flow diagram of Figure 7. The convergence error was calculated from

 $Error = \Sigma (E_n - E_0)^2 / \Sigma E_n^2$ 

where  $E_0$  is the current value of Young's modulus and  $E_n$  is that from the previous iteration. The summation took place over all nonlinear elements and convergence was considered satisfactory when the error was below 0.002.

The values of secant modulus for the granular material were determined from the models described in the previous section of the paper. The bulk modulus (K) at any point in p',q stress space was calculated from the contour value of volumetric strain (Equation 8) by using

$$K = p'/v_{rc}$$
(13)

#### Figure 6. Typical finite-element layout.



(12)

Figure 7. Flow diagram of SENOL finite-element program.



Shear modulus (G) was more difficult to determine because the contours in Figure 4 (left) were based on normalized values of shear strain and hence the zero in that figure does not correspond to zero strain. The procedure, therefore, was to estimate an initial value of shear strain  $(\varepsilon_1)$  and deviator stress  $(q_1)$  based on overburden stress conditions  $(p_1',q_1)$ . Then:

$$G_1 = q_1/3\epsilon_1 \tag{14}$$

For a particular stress path from  $(p_1',q_1)$  to  $(p_2',q_2),$  the value of  $\varepsilon_r$  was obtained from Equation 10 so that

$$\epsilon_2 = \epsilon_1 + \epsilon_r \tag{15}$$

and

$$G_2 = q_2/3\epsilon_2 \tag{16}$$

A similar procedure was adopted for all successive stress paths.

Since the model for fine-grained soils was less detailed than that for the granular material, certain approximations were used while basically following the procedures outlined above for shear modulus.

In the absence of radial strain measurements on silty clay material, an assumption had to be made so that the shear strain could be determined. For  $v \simeq 0.5$ , the shear strain is equal to the axial strain in the triaxial test because

$$\epsilon_{\mathbf{r}} = (2/3)(\epsilon_{\mathbf{ar}} - \epsilon_{\mathbf{rr}}) = (2/3)(\epsilon_{\mathbf{ar}} + 0.5\epsilon_{\mathbf{ar}}) = \epsilon_{\mathbf{ar}}$$
(17)

By using the expression for the tangent resilient modulus (Equation 12),

$$\epsilon_{\mathbf{r}} = \epsilon_{\mathbf{ar}} = (q_{\mathbf{r}}/M_{\mathbf{r}}) = (q_{\mathbf{r}}^{\prime}/K)(q_{\max}/p_{\mathbf{o}}^{\prime})^{s}$$
(18)

This expression was then used in the same way as Equation 10 for the granular material by following Equations 14-16.

The bulk modulus was calculated from an assumed constant Poisson's ratio and the correct value of G by using:

$$\mathbf{K} = [2(1+\nu)/3(1-2\nu)]\mathbf{G}$$
(19)

Clearly, K equals  $\infty$  if v equals 0.5. The asphalt layer has been treated as linear elastic in all analyses to date, the value of Young's modulus was taken as the dynamic stiffness (<u>19,20</u>), and Poisson's ratio was 0.4.

No special provisions are made in the SENOL program to prevent elements from going beyond a failure condition. A stress-adjustment procedure was used by Raad and Figueroa (3) to ensure that no element exceeded the Mohr-Coulomb failure condition. By using the detailed granular material model in SENOL, the need for a failure correction is less necessary as large strains will develop when stresses approach failure. This results in an automatic redistribution of stresses. This will not be the case when the simple model adopted by Raad and Figueroa is used and, hence, their correction procedure is necessary. The fact that elements in some analyses by SENOL did go slightly beyond failure in practice is a reflection of the inaccuracy of the resilient model at high stress levels and an indication of a weak pavement. Ideally, the model should be of a form that can deal with all stress conditions and cope with the onset of failure.

The use of stress invariants to specify stress conditions avoids the problems encountered in other finite-element analyses due to tensile stress in a particular direction. So-called tension corrections have been applied, perhaps unnecessarily, since failure of granular material depends on the relative values of p' and q and hence on the complete stress system at a point. If at any stage in the computation procedure of SENOL a tensile value of p' occurred, then a very low modulus (E = 100 kPa) was assigned to the element concerned. This only occurred under very severe loading conditions when, in most cases, a convergent solution was unobtainable, which indicates pavement failure.

The SENOL program has been used in computing the response of test pavements (18) to moving wheel loads. Typical results are given in Figures 8 and 9. The structure comprised 50 mm of hot rolled asphalt and 170 mm of well-graded crushed limestone over a silty-clay subgrade. A moving wheel load of 8 kN was applied at a contact pressure of 530 kPa and a speed of 14.5 km/h. The test temperature was 25°C. The resilient strains (Figure 8) were generally better predicted than the transient stresses (Figure 9). Note that the check against measured values is thorough and involves vertical, radial, and tangential directions at different depths in the three-layered structure and includes variations in radial distance from the wheel-load center. The horizontal axis of each plot in Figures 8 and 9 is located at the relevant depth in the structure determined from the left-hand vertical scale.

We have also used the program in an extensive parameter study to examine the effects of thickness and stiffness of the asphalt layer, thickness of the granular layer, and stiffness of the subgrade on the stresses and strains of interest for pavement design. These data are proving extremely useful in formulating simplified procedures for use in design.

## CONCLUSIONS

1. The simple nonlinear model  $(M_r = K_{10} K^2)$ , frequently used for the characterization of granular materials in pavement analysis, is likely to lead to inaccurate results because it has been established from data that used a very limited range of stress paths;

2. A more complex model that gives a better description of resilient response has been developed for crushed rock and can be used in numerical analyses that involve three-dimensional stress systems;

3. Whenever possible, characteristics of soil and granular material should be expressed in terms of effective stresses;

4. More research is required on the resilient properties of fine-grained soils in order to establish reliable stress-strain relationships based on effective stress; and

5. A finite-element program, called SENOL, has been developed for the analysis of problems that involve nonlinear materials.

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Figure 8. Comparison of resilient strains in microstrain with values computed by SENOL.

Figure 9. Comparison of transient stresses in kilopascals with values computed by SENOL.



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# Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Material

## **GONZALO RADA AND MATTHEW W. WITCZAK**

A comprehensive evaluation of nonlinear resilient modulus test results on granular materials is presented. A total of 271 test results obtained from 10 different research agencies were used as the data base. The main objectives of the study were to (a) determine whether typical M<sub>r</sub> relations exist for various granular materials; (b) develop a comprehensive summary of factors that affect the M<sub>r</sub> response and determine whether predictive equations or typical relations could be stated; and (c) investigate whether a correlation exists between laboratory-measured M<sub>r</sub> and laboratory-measured California bearing ratio (CBR) values. The results indicate that there appears to be an inverse relationship between K<sub>1</sub> and K<sub>2</sub> (M<sub>r</sub> = K<sub>1</sub>  $\theta^{K_2}$ ) for all granular materials. Six unique K<sub>1</sub> and K<sub>2</sub> relations are developed to relate the primary variables that influence the M<sub>r</sub> response of six different aggregates (used by the Maryland State High:way Adresson of the site of the state of the second state of the sponse of six different uses that state and the material types (site the sponse of the site different aggregates (used by the Maryland State High:way Adresson of the state). The equations use bulk stress, degree of saturation, and percent-

age of modified compaction. Typical M<sub>r</sub> equations are also stated to reflect probable influences of the K<sub>1</sub> and K<sub>2</sub> values due to compaction and moisture for the Maryland aggregates. Based on an analysis of nearly 100 data pairs, a general, but variable, correlation was found between laboratory-measured M<sub>r</sub> and CBR values. However, the constant that relates these variables is a function of stress state. For typical bulk stress values anticipated in highway pavement structures, the coefficient (constant) value is significantly lower than the 1500-value suggested by Huekelom and Foster.

The resilient modulus test of unbound granular materials has been used for several years as a means of evaluating the response of granular material in the laboratory. The modulus  $(M_r)$  is a dynamic test response defined as the ratio of repeated axial