

REFERENCES

1. R.G. Hicks and C.L. Monismith. Factors Influencing the Resilient Response of Granular Materials. HRB, Highway Research Record 345, 1971, pp. 15-31.
2. H.B. Seed, C.K. Chan, and C.E. Lee. Resilience Characteristics of Subgrade Soils and Their Relation to Fatigue Failures in Asphalt Pavements. Proc., International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, MI, 1962, pp. 611-636.
3. L. Raad and J.L. Figueroa. Load Response of Transportation Support Systems. Transportation Engineering Journal, ASCE, Vol. 106, No. TEL, 1980, pp. 111-128.
4. J.L. Figueroa and M.R. Thompson. Simplified Structural Analyses of Flexible Pavements for Secondary Roads Based on Illi-Pave. TRB, Transportation Research Record 766, 1980, pp. 5-10.
5. S.F. Brown. Improved Framework for Predicting Permanent Deformation in Asphalt Layers. TRB, Transportation Research Record 537, 1975, pp. 18-30.
6. A.N. Schofield and C.P. Wroth. Critical State Soil Mechanics. McGraw-Hill, New York, 1968.
7. J.R. Boyce, S.F. Brown, and P.S. Pell. The Resilient Behaviour of a Granular Material Under Repeated Loading. Proc., Australian Road Research Board, Vol. 8, 1976, pp. 8-19.
8. J.R. Boyce and S.F. Brown. Measurement of Elastic Strain in Granular Material. Geotechnique, Vol. 26, No. 4, 1976, pp. 637-640.
9. J.W. Pappin and S.F. Brown. Resilient Stress-Strain Behaviour of a Crushed Rock. Proc., International Symposium on Soils Under Cyclic and Transient Loading, Swansea, Great Britain, Vol. 1, 1980, pp. 169-177.
10. P. Shaw and S.F. Brown. Further Developments of Test Equipment for Shear Reversal on Granular Materials. Univ. of Nottingham, Nottingham, England, Rept. PS/2, Feb. 1980.
11. J.W. Pappin. Characteristics of a Granular Material for Pavement Analysis. Univ. of Nottingham, Nottingham, England, Ph.D. thesis, June 1979.
12. W.S. Smith and K. Nair. Development of Procedures for Characterization of Untreated Granular Base Course and Asphalt-Treated Base Course Materials. FHWA, FHWA-RD-74-61, 1973.
13. S.F. Brown. The Characteristics of Soils for Flexible Pavement Design. Proc., 7th European Conference on Soil Mechanics and Foundation Engineering, Brighton, England, Vol. 2, 1979, pp. 15-22.
14. G.L. Dehlen. The Effect of Nonlinear Material in the Behavior of Pavements Subjected to Traffic Loads. Univ. of California, Berkeley, Ph.D. thesis, 1969.
15. D.G. Fredlund, A.T. Bergan, and P.K. Wong. Relations Between Resilient Modulus and Stress Conditions for Cohesive Subgrade Soils. TRB, Transportation Research Record 642, 1977, pp. 73-81.
16. S.F. Brown, A.K.F. Lashine, and A.F.L. Hyde. Repeated Load Triaxial Testing of a Silty Clay. Geotechnique, Vol. 25, No. 1, 1975, pp. 95-114.
17. M.R. Thompson and Q.L. Robnett. Resilient Properties of Subgrade Soils. Transportation Engineering Journal, ASCE, Vol. 105, No. TEL, 1979, pp. 71-89.
18. S.F. Brown, J.W. Pappin, and B.V. Brodrick. Permanent Deformation of Flexible Pavements. European Research Office, U.S. Army, Univ. of Nottingham, Nottingham, England, Final Rept., 1980.
19. C. Van der Poel. A General System Describing the Visco-Elastic Properties of Bitumens and Its Relation to Routine Test Data. Journal of Applied Chemistry, No. 4, 1954, pp. 221-236.
20. W. Heukelom and A.J.G. Klomp. Road Design and Dynamic Loading. Proc., Assoc. of Asphalt Paving Technologists, Vol. 33, 1964, pp. 92-123.

Publication of this paper sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.

Comprehensive Evaluation of Laboratory Resilient Moduli Results for Granular Material

GONZALO RADA AND MATTHEW W. WITCZAK

A comprehensive evaluation of nonlinear resilient modulus test results on granular materials is presented. A total of 271 test results obtained from 10 different research agencies were used as the data base. The main objectives of the study were to (a) determine whether typical M_r relations exist for various granular materials; (b) develop a comprehensive summary of factors that affect the M_r response and determine whether predictive equations or typical relations could be stated; and (c) investigate whether a correlation exists between laboratory-measured M_r and laboratory-measured California bearing ratio (CBR) values. The results indicate that there appears to be an inverse relationship between K_1 and K_2 ($M_r = K_1 \theta^{K_2}$) for all granular materials. Six unique K_1 and K_2 relations are proposed for six different granular material types (silty sands, sand gravels, sand aggregate blends, crushed stone, limerock, and slag). Predictive equations are developed to relate the primary variables that influence the M_r response of six different aggregates (used by the Maryland State Highway Administration). The equations use bulk stress, degree of saturation, and percent-

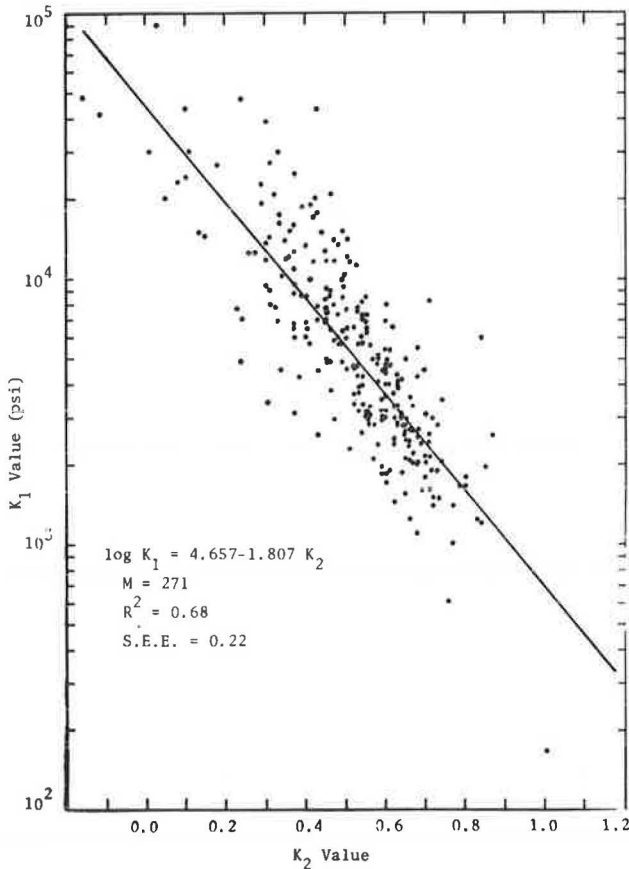
age of modified compaction. Typical M_r equations are also stated to reflect probable influences of the K_1 and K_2 values due to compaction and moisture for the Maryland aggregates. Based on an analysis of nearly 100 data pairs, a general, but variable, correlation was found between laboratory-measured M_r and CBR values. However, the constant that relates these variables is a function of stress state. For typical bulk stress values anticipated in highway pavement structures, the coefficient (constant) value is significantly lower than the 1500-value suggested by Huekelom and Foster.

The resilient modulus test of unbound granular materials has been used for several years as a means of evaluating the response of granular material in the laboratory. The modulus (M_r) is a dynamic test response defined as the ratio of repeated axial

Table 1. Summary of M_r test results evaluated.

Research Agency	Reference	No. of Tests	Material
Asphalt Institute	(1-6)	36	Sands, silty sand, sand gravels, gravels, crushed gravel, crushed stone, and limerock
University of California at Berkeley	(7-9)	41	Sand gravels, gravels, and crushed gravels
Georgia Institute of Technology	(10)	19	Sand gravels, soil-aggregate blends, and crushed stone
National Crushed Stone Association	(11)	6	Crushed stone
University of Illinois	(12)	18	Crushed stone and gravels
Pennsylvania State University	(13)	2	Crushed stone
Florida Department of Transportation	(14)	5	Limerock and sands
U.S. Army Corps of Engineers	(12, 15-17)	11	Silty sand, sand gravels, and crushed stone
Woodward Clyde	(18, 19)	6	Sand gravels, crushed gravels, and crushed stone
University of Maryland, previous studies	(20)	26	Sands, soil-aggregate blends, sand gravels, crushed stone, limerock, and slag
Subtotal		170	
University of Maryland, MSHA project	MSHA study	101	Sand gravels, soil-aggregate blends, crushed stone, and slag
Total		271	

Figure 1. K_1 - K_2 relation for all aggregate M_r results.



deviatoric stress (σ_d) to the recoverable or resilient axial strain (ϵ_r) or

$$M_r = \sigma_d / \epsilon_r \quad (1)$$

In general, the test is conducted in a triaxial cell that is equipped to monitor repetitive load conditions (pneumatic or electrohydraulic loading system) on cylindrical samples, usually 4-6 in in diameter. Both dynamic load and deformations [through linear variable differential transformers (LVDTs)] are measured continuously during the test. Previous studies have indicated that resilient deformations generally stabilize after 200 repetitions of load and, as such, the M_r value is usually computed at this level of repetition.

Because of the known nonlinear (stress-dependent)

properties of most granular materials, the test is conducted at combinations of confining pressure (σ_c and σ_1/σ_3) ratios. Results of a single test are usually presented in a mathematical form that directly incorporates the stress sensitivity of the M_r value in terms of either σ_c or bulk stress (first stress invariant) (θ) by

$$M_r = K_1 \theta^{K_2} = K_1' \sigma_c^{K_2'} \quad (2)$$

The constants (K_1' , K_2' or K_1 , K_2) are obtained from regression analysis of the test results and depend on the type of material and physical properties of the specimen during the test.

STUDY OBJECTIVES

Since the development of the M_r test, numerous individual research studies have investigated the dynamic response of granular materials. However, in general, each study was directed toward a specific type of material or a particular parameter on the M_r response for a given material. In view of this, an extensive laboratory study was initiated in 1978 at the University of Maryland for the Maryland State Highway Administration (MSHA) to develop M_r responses for typical base and subbase materials used in pavement systems. The combined evaluation of previous results and the Maryland study afforded the opportunity to assess various questions not previously addressed by other researchers.

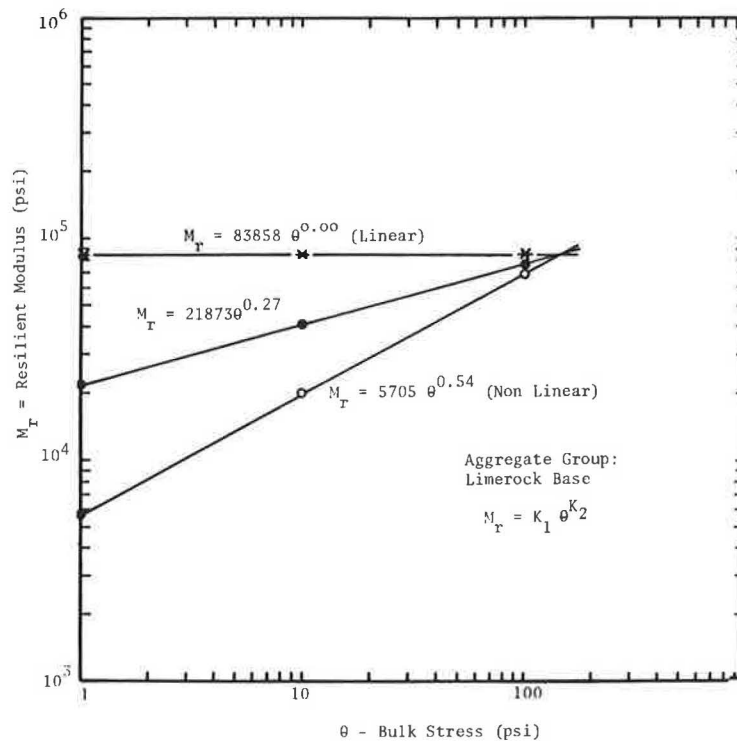
Based on the relatively large data base collected, the following specific objectives were studied in this project:

1. Investigate whether typical limits of M_r relations exist for various granular material classes (types),
2. Investigate the feasibility of developing predictive M_r equations from physical properties of the granular materials,
3. Develop a comprehensive evaluation of factors that affect the M_r response of granular materials, and
4. Investigate whether accurate correlations between M_r and the California bearing ratio (CBR) exist for granular materials.

Data Sources

As noted, a comprehensive literature review of the M_r results of granular material reported by other researchers was conducted. Table 1 is a summary of the agencies, number of tests, materials studied, and references obtained from this literature review. As noted, a total of 170 individual M_r tests was found in the literature from 10 different

Figure 2. Effects of K_1 and K_2 on the resilient modulus.



agencies. The results obtained were summarized for only the form that used the bulk stress (θ) equation (i.e., K_1 and K_2 values).

University of Maryland Study

Also shown in Table 1 is information regarding the M_r study on MSHA base and subbase materials. As noted, 101 separate M_r tests were conducted in this project. Thus, the combined number of individual granular material M_r results used in this report was 271.

Specific Testing Program

Although 271 combined M_r test results were available, not all of the previous studies contained information that could be used in all phases of the study objectives. The testing program of the MSHA study was thus developed to fill information gaps concerning the M_r response of various materials.

The study for MSHA involved testing six different aggregate types [two limestones that meet MSHA and dense-graded aggregate (DGA) specifications; a crushed stone and slag that meet MSHA crusher run (CR) 6 specifications; a bank-run gravel that meets MSHA specifications; and a sand-aggregate subbase blend]. For each aggregate investigated, three hand-blended gradations were used. On each aggregate-gradation combination, three compaction energies (low, standard, and modified) were used to develop moisture-density relations. In addition to developing compaction curves, as-molded and soaked CBR tests (588) were conducted.

The resilient modulus phase of the test program involved the testing of 18 specimens per aggregate. In general, for each aggregate-gradation combination, three M_r tests were conducted at modified compaction effort (optimum and ± 2 percent optimum moisture), two tests at standard compaction effort (optimum and $+2$ percent optimum moisture), and one at optimum for a low (2200 pound \cdot ft/ft 3) compaction effort. The above M_r test program theoretic-

cally should have yielded 108 M_r data points, but 7 specimens failed or were unable to be tested.

TYPICAL M_r RESULTS FOR AGGREGATES

The analysis of all 271 individual M_r test results (from all agencies) led to an interesting observation relative to the overall behavior of all granular materials as a group. Figure 1 shows the relation between K_1 and K_2 for the entire data set. Observe that a definite correlation exists between increasing K_1 and decreasing K_2 for granular materials. Although the R^2 value of 0.68 is not exceptionally high from a statistical viewpoint, recognize that the results shown represent extremely variable physical properties as well as aggregates that range from fine silty sands to slag and lime-rock materials.

The significance of this relation may also be used to denote the relative degree of material nonlinearity. Recall the form of equation 2: As K_2 approaches a value of $K_2 = 0$, the material is truly linear, whereas larger K_2 values imply a greater degree of nonlinearity. This is illustrated in Figure 2, which shows M_r equations developed on a single limerock material tested under three differing sets of physical properties. The relative changes in the K_1 and K_2 values and their respective influence on the degree of nonlinearity can be observed.

Aggregate Class Analysis

In order to determine whether the scatter of points shown in Figure 1 could be reduced, a study was undertaken to determine whether relatively unique K_1 - K_2 relations existed among various types (classes) of granular materials. Although not presented in this report, groupings of data based on both American Association of State Highway and Transportation Officials (AASHTO) and Unified Soil Classification System (USCS) systems were evaluated and found to yield poor correlations. By far, the

Table 2. Summary of K_1 and K_2 statistics by aggregate class.

Aggregate Class	No. of Data Points	K_1 Parameter			K_2 Parameter		
		Mean	SD	Range	Mean	SD	Range
Silty sands	8	1 620	780	710 to 3830	0.62	0.13	0.36 to 0.80
Sand gravel	37	4 480	4 300	860 to 12 840	0.53	0.17	0.24 to 0.80
Sand-aggregate blends	78	4 350	2 630	1880 to 11 070	0.59	0.13	0.23 to 0.82
Crushed stone	115	7 210	7 490	1705 to 56 670	0.45	0.23	-0.16 to 0.86
Limerock	13	14 030	10 240	5700 to 83 860	0.40	0.11	0.00 to 0.54
Slag	20	24 250	19 910	9300 to 92 360	0.37	0.13	0.00 to 0.52
All data	271	9 240	11 225	710 to 92 360	0.52	0.17	-0.16 to 0.86

Table 3. K_1 - K_2 regression models by aggregate class.

Aggregate Class	Regression Constants of Form: $\log K_1 = A_0 + A_1 K_2$		R^2	Standard Error of Estimate
	A_0	A_1		
Silty sands	4.183	-1.666	0.75	0.14
Sand gravel	4.613	-2.100	0.82	0.17
Sand-aggregate blends	4.345	-1.308	0.56	0.15
Crushed stone	4.515	-1.492	0.68	0.19
Limerock	4.924	-2.162	0.92	0.08
Slag	4.965	-1.917	0.50	0.26
All data	4.657	-1.807	0.68	0.22

best grouping of the 271 data points occurred when the granular materials were divided qualitatively into the following six categories:

1. Silty sands;
2. Sand gravels;
3. Sand-aggregate blends, including shell-stabilized sands and crushed gravels;
4. Crushed stones;
5. Limerocks; and
6. Slags.

The resulting population of K_1 and K_2 values for a given group was analyzed to determine the typical limits associated with each group. Table 2 summarizes the results of this study by aggregate class. Regression studies between K_1 and K_2 for each class were conducted and the results summarized in Table 3. Figure 3 illustrates the resulting K_1 and K_2 relations found in the analysis. The limits of each line shown in this figure represent the actual limits of test results found.

Based on these results, the following observation can be made. Each class of aggregates appears to have its own relatively unique K_1 - K_2 relation that distinguishes it from other groups. Although the R^2 values range from 0.50 to 0.92, much of the variation is due to material variations within a class (e.g., crushed limestone versus crushed gneiss) as well as the physical properties of each specimen during the test. The overall mean values, for all granular materials, of $K_1 = 9240$ and $K_2 = 0.52$ are generally in excellent agreement with typical values assumed in many design situations.

However, although this may be true from a global viewpoint, the range of K_1 (and hence K_2) within a given class appears to be significant. The largest range shown occurs for the crushed stone group, which has a range of K_1 values from 1700 to 57 000. Because of this, the influence of the specific type and moisture-density conditions are critical to accurate evaluation of the proper K_1 - K_2 value to be used in design. In fact, the use of typical K_1 - K_2 values to represent a specific granular material such as crushed stone or sand gravels may be more misleading in design.

Finally, observe from Table 2 that the average K_1 and K_2 values by aggregate class appear reasonable. In general, the order in which the aggregate classes are stated coincides with increasing K_1 order (decreasing K_2). This indicates that the overall degree of linearity increases from the silty sand category to the slag group. The order shown also corresponds generally to what one would associate with increasing shear strength behavior by the aggregate classes noted.

Loading Conditions

The effect of loading conditions in the M_r test for granular materials is generally well understood from previous research investigations. The most-significant loading factor that affects the modulus is the stress level (8,9,11,12,19,21-25). In general, it is customary to relate either θ and σ_c to the modulus. In this study, the M_r relation that uses θ was used because of its ease of adaptation into nonlinear solutions in layered pavement systems.

Other load factors, such as stress duration, stress frequency, sequence of load, and number of stress repetitions necessary to reach an equilibrium-resilient strain response have been covered adequately in the literature (9,11,12,21,23,24). In general, these factors have little, if any, effect on the M_r response. In the testing at the University of Maryland, a haversine pulse load (0.1-s load duration) at 30 repetitions/min was used. Modulus values were computed with resilient strains after 200 load repetitions.

Degree of Saturation

Based on previous studies (9,12,19,21,26), the degree of saturation (for a given aggregate) plays a major role in the M_r response. Repeated load triaxial tests, conducted by Haynes and Yoder (12,19,26) on gravels and crushed stone, indicated that there was a critical degree of saturation near 80-85 percent, above which granular materials became unstable and deteriorated rapidly under repeated loading. Seed and others (12,19,26) found that, for well-graded gravels, K_1 was reduced and K_2 unchanged with increasing saturation (S_r) values. This was also substantiated in work conducted by Kallas, Riley, Shifley, Hicks, and Finn (12,19,26).

Figure 4 shows the general effect of S_r on the K_1 and M_r values (at $\theta = 10$ lbf/in²) for 159 separate test results on a variety of aggregates. From this global viewpoint, the marked reduction in the K_1 value and modulus with increasing saturation is clearly evident. In general, the effects of moisture can change the typical K_1 values from 30 000 (dry) to 1000 (saturation), with resultant changes in modulus from 40 000 to 10 000 lbf/in² or less.

Although the above is true in a general sense, the exact influence of saturation appears to be

Figure 3. Typical K_1 - K_2 relations by aggregate class.

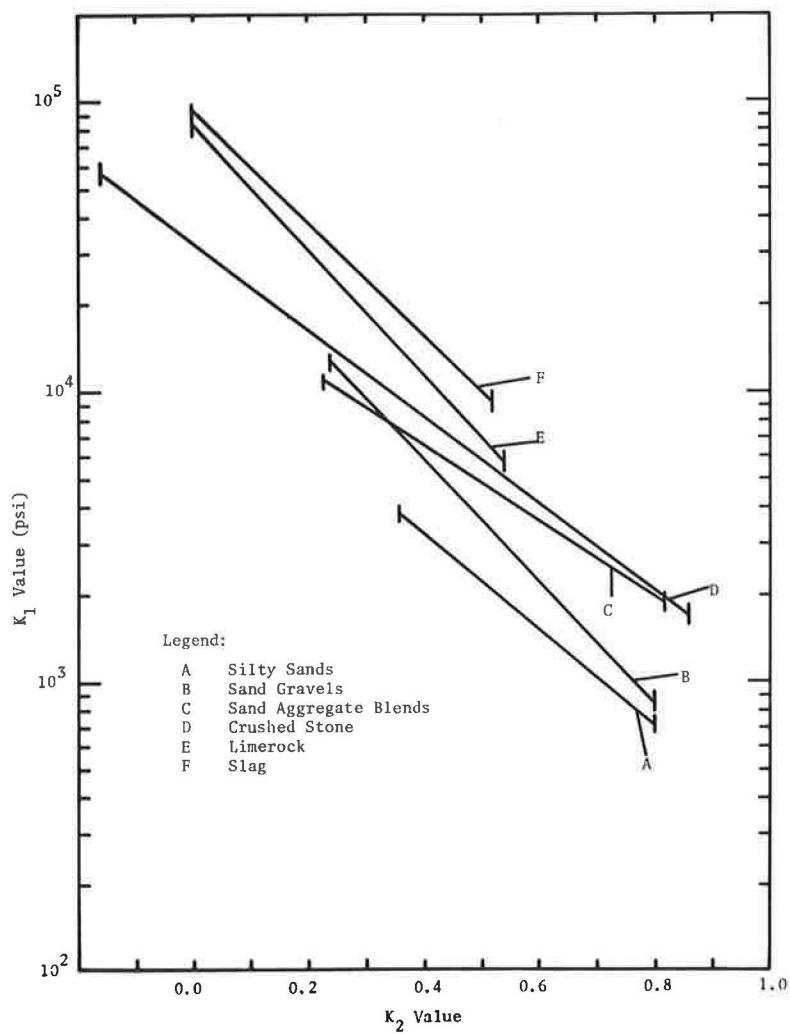


Figure 4. Effect of degree of saturation on K_1 and M_r .

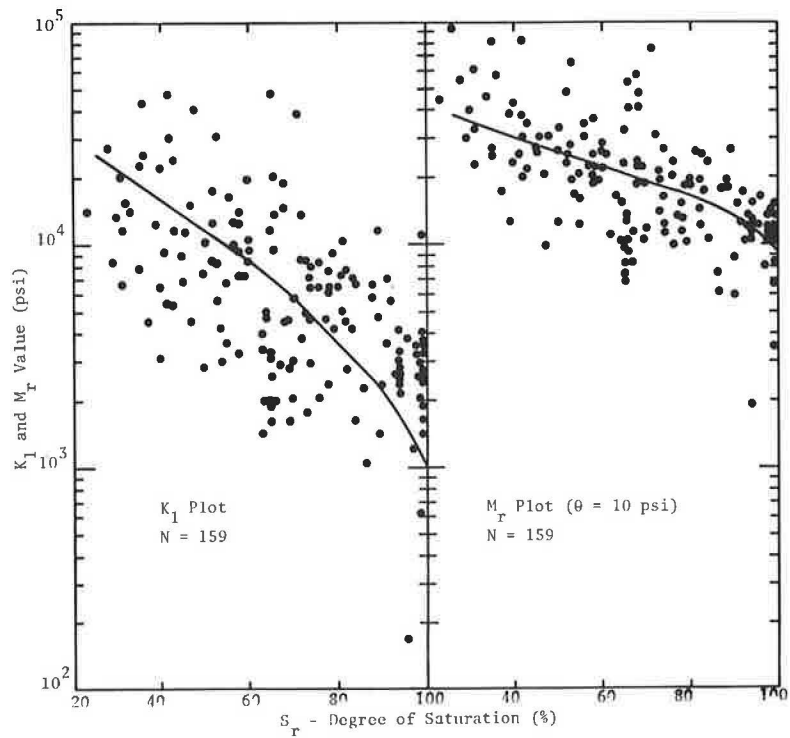
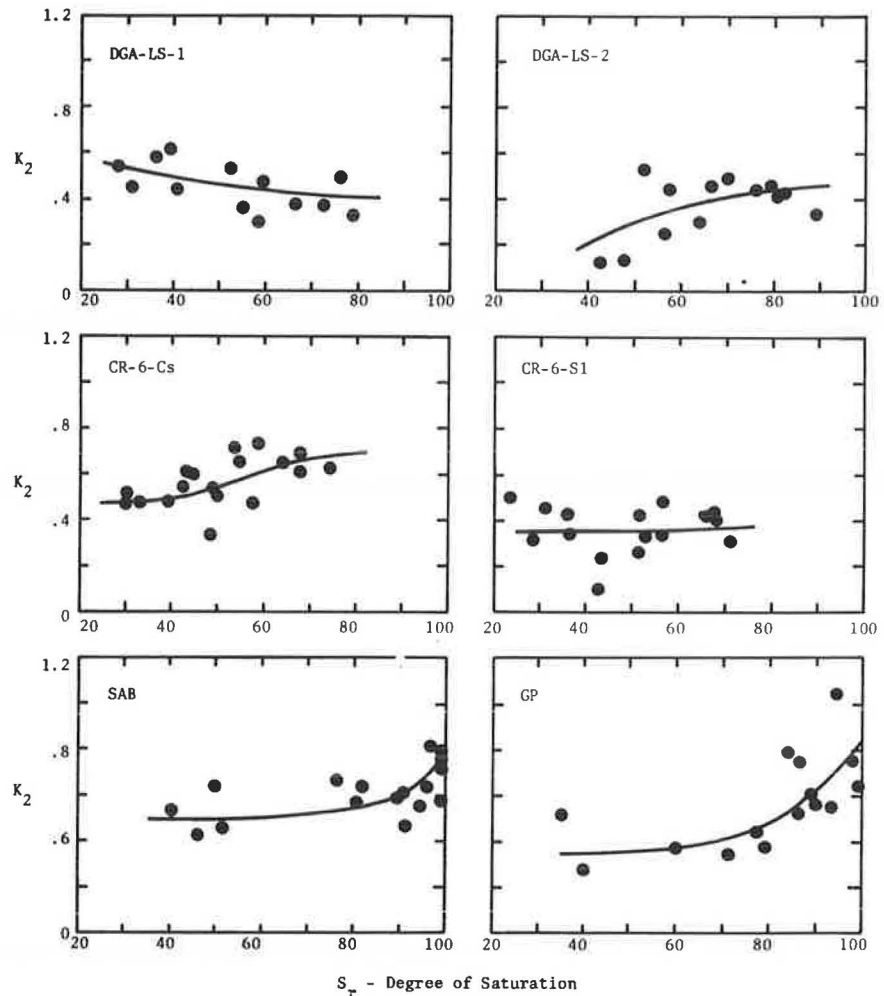


Figure 5. Effect of degree of saturation on K_2 .

dependent on the aggregate type. The following acronyms for the aggregate are used in Figures 5-9:

LS--limestone,
 CS--crushed stone,
 Sl--slag,
 SAB--sand-aggregate blend, and
 GP--bank-run gravel.

Figure 5 illustrates the influence of S_r on K_2 for the six aggregates investigated in the MSHA study. Observe that, although changes in K_2 may not be large, various trends are recognizable from the plots. There is, however, no uniform trend among all aggregates. For example, a comparison of the K_2 values for the two different crushed limestone materials (DGA-limestone-1 and DGA-limestone-2) show decreasing and increasing K_2 trends with increased S_r , respectively. However, in general, the increase in K_2 for S_r values near saturation is very pronounced for the subbase materials (sand-aggregate blend and bank-run gravel). For these materials, a definite increase in K_2 at high saturations is clearly evident. Finally, the analysis of K_2 and S_r showed no distinctive pattern relative to the influence of compactive effort and aggregate gradation on the results.

In contrast to the relatively minor influence of S_r on K_2 , the influence of S_r on the K_1 values may be significant. Figure 6 shows the results for three of the six aggregates investigated in the MSHA study (two crushed limestone-DGA and the bank-run gravel). For the two crushed stone mate-

rials, the influence of S_r on the K_1 value is seen to vary even within a common aggregate class. The DGA-limestone-1 material is a relatively hard, dense (low-abrasion) limestone and appears to exhibit less of a sensitivity in K_1 to the effects of moisture. This contrasts to the softer (high-abrasion) DGA-limestone-2 material that yields a greater influence of K_1 to S_r .

Although the two crushed limestones may exhibit differing degrees of moisture sensitivity, we can also see that they exhibit less sensitivity compared with the bank-run gravel results, especially near the critical 80-85 percent level suggested by Haynes and Yoder. The resulting decrease in modulus for these materials, with increasing S_r values, is clearly shown in Figure 6.

Percentage Compaction

Several studies have been conducted by previous researchers into the effects of density on the M_r response of granular material (8,9,12,21-23,26). These studies have indicated that, although increase in density results in an increase in modulus, the effect is relatively small compared with changes caused by stress level and moisture. The results of the MSHA study generally confirm this over a wide range of aggregate types.

Figure 7 is a plot of the average K_1 and K_2 values (by aggregate) as a function of the unit volumetric compaction energy (E_c). As previously noted, three levels of compaction energy were studied. The low compactive effort was $E_c = 2200$

Figure 6. Influence of degree of saturation on K_1 and M_r for several aggregates.

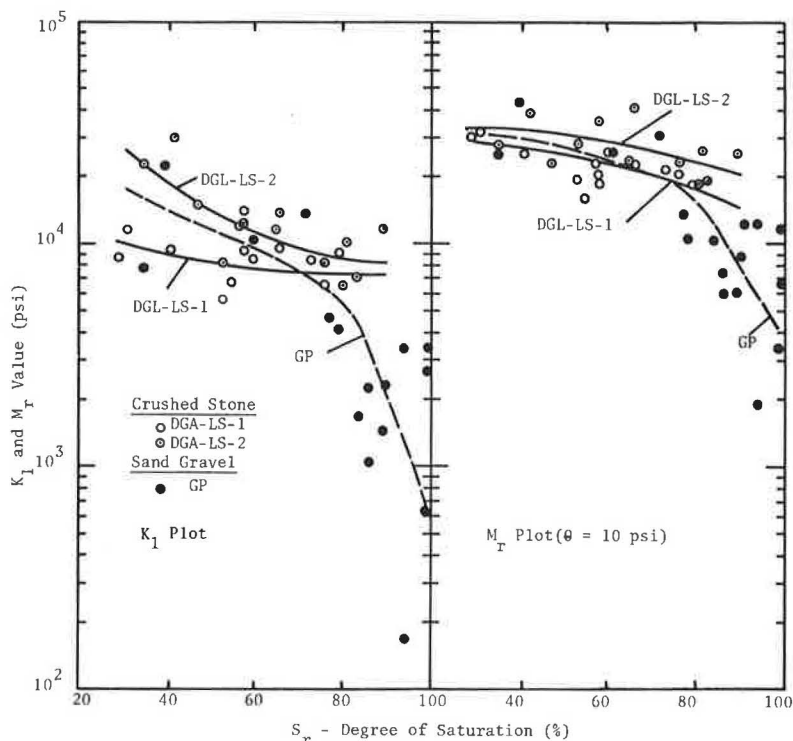
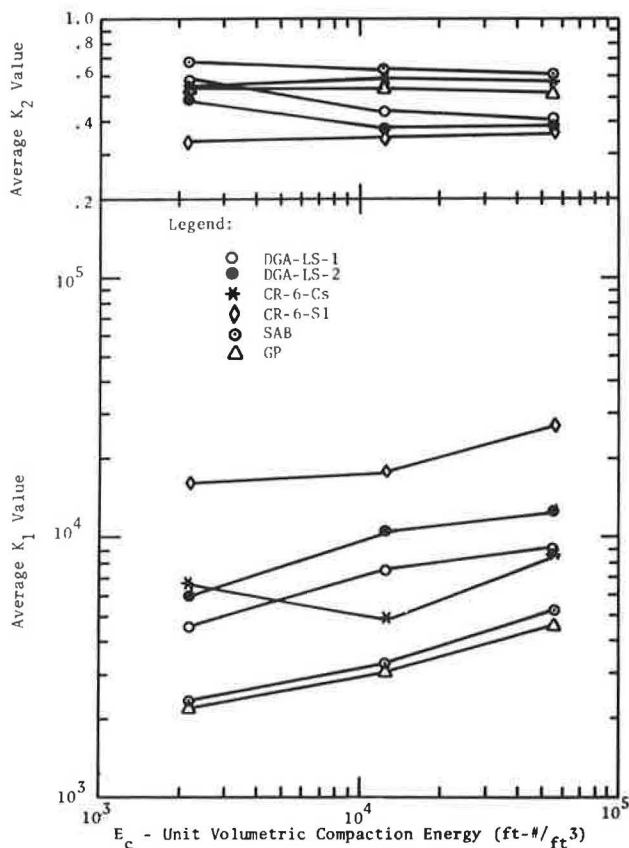


Figure 7. Influence of compaction energy on K_1 and K_2 .



ft·lb/ft³, although the remaining two levels were standard and modified (12 300 and 56 200 ft·lb/ft³, respectively). From this figure, observe that K_1 increases gradually with increas-

ing compaction and the K_2 value remains essentially constant. The average increase in K_1 , from standard to modified compaction energy, was nearly 48 percent: the DGA aggregates averaged 25 percent, CR-6 averaged 62 percent, and the subbase materials (sand-aggregate blends and bank-run gravel) averaged 58 percent. Based on these results, the influence of compaction in improving the modulus (K_1) cannot be ignored.

Aggregate Gradation

Previous research investigations of the effect of aggregate gradation have indicated no general trend regarding the influence of fines (percentage that passes no. 200) on the M_r response (8,9,12,21-23, 26). In general, the degree of influence of this parameter appears to be related to the aggregate investigated.

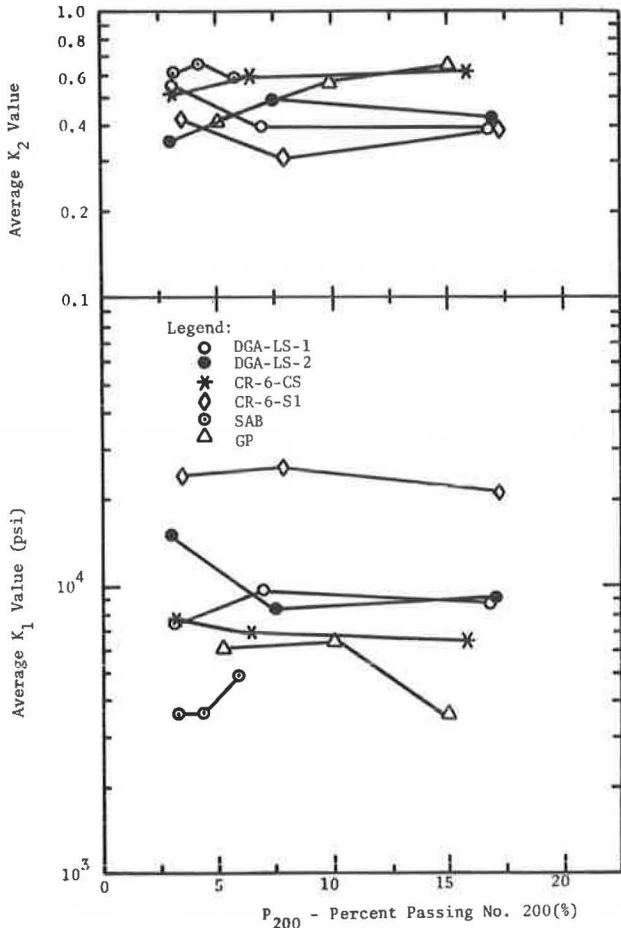
In the MSHA study, hand-blended gradations were used to investigate the effect of the p_{200} variable. All of the six aggregates studied had a $D_{max} = 1.0$ in. The four base materials (DGA and CR-6) were graded according to

$$p = (d/D_{max})^n \tag{3}$$

with n values of 0.30, 0.45, and 0.6. The selection of these values resulted in gradations that progressed from dirty (17 percent), dense (7 percent), to lean or open (3 percent) type. Because of difficulties involved in obtaining enough fines (P_{200}) for the subbase materials, the sand-aggregate blend used three gradations that had P_{200} values of 3.2, 4.3, and 5.8 percent, and the bank-run gravel material used P_{200} values of 5.1, 10, and 15 percent.

Figure 8 illustrates the results of this study phase. We can see that there is no general or uniform trend applicable for all aggregate types, although several observations can be made. For the angular base materials (DGA and CR-6 aggregates), there appears to be little change in either K_1 or K_2 for P_{200} values in the dense to dirty range

Figure 8. Influence of percentage fines on K_1 and K_2 .



(around 7-17 percent). This is not the case, however, for the bank-run gravel. For this material, an optimum K_1 value is apparent near the dense condition and a marked decrease in K_1 occurs as the P_{200} value increases. In contrast, the K_2 value for the bank-run gravel series appears to increase with an increase in fines. Thus, just like the results for S_r effect, the relative sensitivity for the sand-gravel material is more pronounced than for crushed, angular aggregate. Finally, although no pronounced changes occur in K_1 and K_2 for the base materials, it would be intuitively obvious that increases in P_{200} beyond the 16-18 percent range would eventually have pronounced changes in the M_r response for these materials.

M_r Predictive Equations

The previous discussion has focused on the relative influence of loading and physical properties on the M_r response of various aggregates. In general, the most-significant factors found to affect M_r are the stress state, degree of saturation, and compactive effort (density). In addition, the general changes in the K_1 and K_2 values were noted with changes in several variables.

Based on the results of the MSHA study, predictive equations were investigated to develop accurate M_r predictions from the significant variables. Several forms and variables were considered in this analysis. The most accurate (highest R^2 and lowest standard error) model was found to be of the form shown in Table 4. Also shown in the table are regression constants for each aggregate type and for the model by using all aggregate types combined. The form of the model used implies that no interaction effect of the primary variables are considered for the K_2 value (note that $C_3 = K_2$). Further work is being conducted to determine whether increased accuracy will result if C_3 has various functions of the input values.

The major input variables are bulk stress (θ), degree of saturation (S_r), and percentage compaction relative to modified compactive effort (PC). Table 5 shows the influence of variables in the predictive equations studied as measured by the R^2 value. Observe that the addition of the P_{200} variables added little to the predictive accuracy; however, the elimination of the PC term lowered the R^2 values significantly for all aggregates except DGA-limestone-1 and sand-aggregate blend.

Typical Relations

Based on the MSHA study, typical $M_r = f(\theta)$ relations for the six aggregate types studied have been developed and are summarized in Table 6. This classification scheme reflects the relative influence of the significant variables on the K_1 - K_2 values.

Modulus-CBR Comparison

Although recent research has focused on the use of laboratory determined moduli in design procedures, empirical correlations to existing routine tests are frequently used. One of the most-widely used correlations is the one developed by Huekelom and Foster (25), which relates dynamic modulus to CBR. The relation established

$$E = 1500 \text{ CBR (E in lbf/in}^2\text{)}$$

was found from extensive dynamic (wave propagation)

Table 4. Summary of M_r predictive equations, physical properties.

Aggregate	No. of Data Points	Regression Constants in $\log M_r = C_0 + C_1 S_r + C_2 PC + C_3 \log \theta$					R^2	Standard Error
		C_0	C_1	C_2	C_3			
DGA-limestone-1	14	3.4060	-0.005 289	0.011 94	0.004 843	0.79	0.13	
DGA-limestone-2	17	-0.3017	-0.005 851	0.050 54	0.004 445	0.60	0.21	
CR-6-crushed stone	18	1.0666	-0.003 106	0.035 56	0.006 469	0.81	0.15	
CR-6-slag	17	3.2698	-0.003 999	0.016 63	0.003 840	0.59	0.18	
Sand-aggregate blend	18	4.1888	-0.003 312	0.021 38	0.006 785	0.83	0.15	
Bank-run gravel	17	0.9529	-0.012 07	0.041 17	0.006 035	0.84	0.17	
All data	101	4.022	-0.006 832	0.007 055	0.005 516	0.61	0.23	

Note: S_r = degree of saturation (%), PC = percentage compaction relative to modified density (%), and θ = bulk stress or first stress invariant (lbf/in²).

field tests and not laboratory resilient moduli. Although the equation is important, note that their data showed a wide range of scatter and had a band width of roughly ± 2 . Although the equation has been used frequently for predicting subgrade moduli for materials that have CBR of 20 or less, the original Heukelom and Foster data uniformly covered a range of CBR from 2 to 200.

In the MSHA study, recall that complete moisture-density strength curves (CBR, as molded and

soaked) were established for each aggregate type (six), gradation (three), and compactive energy (three) combination. Thus, for each compacted M_r test specimen studied, it was possible to establish the laboratory measured CBR value. This allowed for the comparison of laboratory derived M_r values to laboratory measured CBR values for each modulus specimen.

Because the M_r value from the resilient tests is stress dependent, there cannot be a unique relation between M_r and CBR. Figure 9 is a summary plot that compares CBR to M_r values at bulk stress levels of 10 and 100 lbf/in². As can be seen from both diagrams, a general correlation between modulus and CBR does exist. However, the results indicate that a wide range of scatter exists for the relation. In general, the scatter bands shown on each diagram ($\pm 1\sigma$) are roughly equivalent to a 50 percent relative difference in residual error. This band is also very similar to the ± 2 band width defined by Heukelom and Foster.

The results of this study were analyzed by using the relation:

$$M_r(\theta) = F(\text{CBR}) \tag{4}$$

Table 5. Influence of variables on R² values for M_r predictive equations.

Aggregate	No. of Data Points	S_r, P_{200}, PC, θ	S_r, PC, θ	S_r, θ
DGA-limestone-1	14	0.79	0.79	0.78
DGA-limestone-2	17	0.61	0.60	0.41
CR-6-crushed stone	18	0.82	0.81	0.46
CR-6-slag	17	0.59	0.59	0.46
Sand-aggregate blend	18	0.85	0.83	0.83
Bank-run gravel	17	0.84	0.84	0.76
All data	101	0.61	0.61	0.60

Table 6. Typical M_r relations for MSHA aggregates.

Aggregate	Dry, $S_r < 60$ Percent				Wet, $S_r > 85$ Percent			
	Standard CE		Modified CE		Standard CE		Modified CE	
	K_1	K_2	K_1	K_2	K_1	K_2	K_1	K_2
DGA-limestone-1 ^a	8 500	0.5	10 500	0.5	7000	0.4	9000	0.4
DGA-limestone-2 ^a	11 500	0.3	15 000	0.3	6000	0.5	7500	0.5
CR-6-crushed stone ^a	6 000	0.5	9 000	0.5	3500	0.7	5000	0.7
CR-6-slag ^a	12 500	0.35	20 000	0.35	5600	0.35	9000	0.35
Sand-aggregate blends ^b	3 800	0.5	6 000	0.5	1900	0.7	3000	0.7
Bank-run gravel ^b	5 000	0.4	8 000	0.4	1250	0.7	2000	0.7

^a K_1 - K_2 values are typical for fines percentage (no. 200) of less than 15-18 percent.

^b K_1 value should be decreased and K_2 value increased if fines percentage is greater than 10 percent.

Figure 9. Comparison of laboratory-measured M_r values to laboratory-measured CBR values.

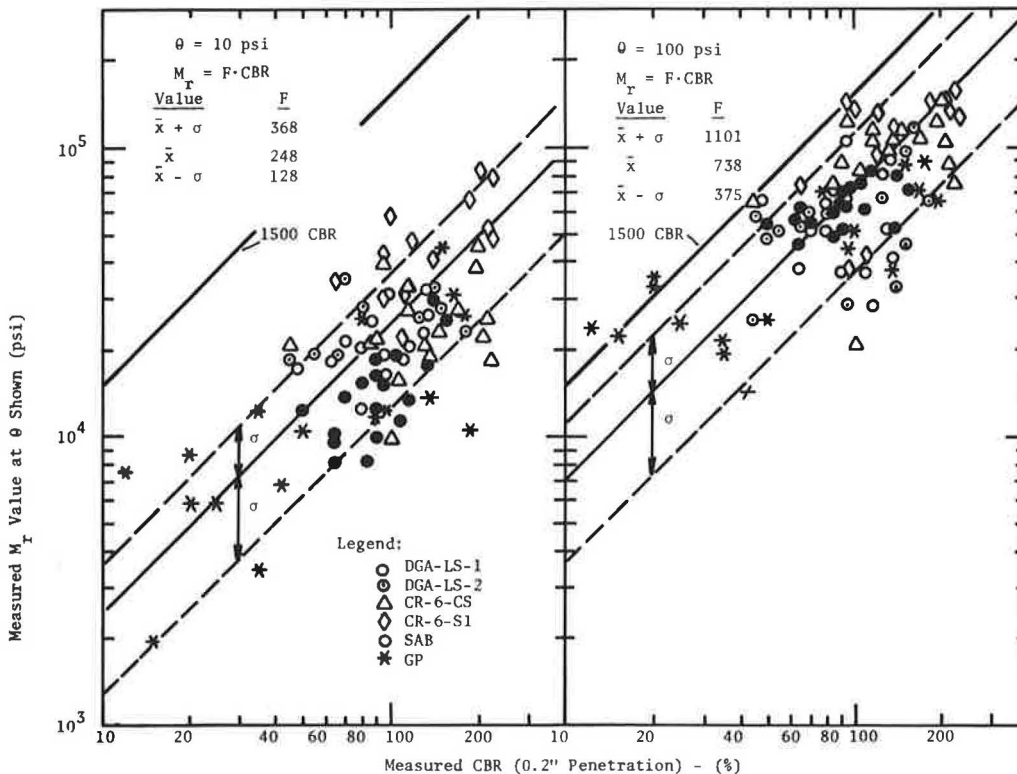


Table 7. F-value summary statistics:
 $M_r = F(\text{CBR})$.

Aggregate	$\theta = 10 \text{ lbf/in}^2$				$\theta = 100 \text{ lbf/in}^2$			
	\bar{F}	σ_F	CV(%)	90 Percent Confidence Range	\bar{F}	σ_F	CV(%)	90 Percent Confidence Range
DGA-limestone-1	240	68	28.1	± 111	727	296	40.7	± 487
DGA-limestone-2	318	170	53.4	± 279	635	373	58.7	± 613
CR-6-crushed stone	201	121	60.3	± 200	772	322	41.7	± 530
CR-6-slag	358	120	33.5	± 197	837	343	41.0	± 565
Sand-aggregate blends	165	43	25.9	± 70	699	171	24.5	± 281
Bank-run gravel	232	146	62.9	± 240	822	559	68.0	± 920
All data	248	120	48.6	± 198	738	363	49.2	± 597

where F is the adjustment factor necessary for equivalence (comparable to the 1500 value of Huekelom and Foster). Table 7 is a summary of the F-values by type of aggregate studied. It can be seen that, even within an aggregate type, the variation (σ_F) may be quite large. However, note that not all of the deviations shown in Figure 9 can be related to variations in the M_r value. This is because unavoidable errors in determining the correct CBR value may occur due to the extremely high sensitivity of moisture to CBR for most granular materials.

As shown in both Figure 9 and Table 7, the average F-values for $\theta = 10 \text{ lbf/in}^2$ and 100 lbf/in^2 were $F = 248$ and $F = 738$, respectively. By using a linear relation between F and $\log \theta$, the following general relation that relates M_r to θ and CBR is

$$M_r = (490 \log \theta - 243) \text{CBR} \quad (5)$$

Finally, because most highway design structures result in bulk stress values near $\theta = 10 \text{ lbf/in}^2$ (for subbase) and $\theta = 20\text{--}40 \text{ lbf/in}^2$ (for bases), the resulting correlations developed in this study differ significantly from that established by Huekelom and Foster. In order for the F-value to be equal to 1500, a $\theta = 350\text{--}400 \text{ lbf/in}^2$ would be required. Although this discrepancy cannot be fully explained until further research is conducted, we hypothesize that this difference can be attributed to shear strain differences caused by the dynamic wave propagation field tests of Huekelom and Foster and those found in laboratory specimens that undergo M_r testing.

SUMMARY

This study was based on resilient modulus test results of granular materials obtained from both a comprehensive literature review and extensive laboratory study. Based on this analysis, the following conclusions were obtained.

An analysis of more than 270 separate M_r test results on granular materials indicated that an inverse correlation between K_1 and K_2 (constants in $M_r = K_1 \theta^{K_2}$) exists for the global class of all granular materials. Relatively unique $K_1\text{--}K_2$ relations were found to exist for six different aggregate classes or types. These categories were silty sands, sand gravels, sand-aggregate blends, crushed stones, limerocks, and slags.

An analysis of the results indicated that the primary variables that influence the M_r response of granular materials are the stress state, degree of saturation, and degree of compaction. For crushed, angular materials, an increase in moisture leads to a small to moderate decrease in the K_1 value and relatively minor changes in the K_2 magnitude. These results contrast with those of sand gravels, which showed a marked decrease in K_1 and increase in K_2 with increasing moisture.

The influence of compaction (density) on all granular materials results in an increase in the K_1 term, and the K_2 value remains essentially constant. The gradation and its influence on the $K_1\text{--}K_2$ parameters are dependent on the type of material considered. For crushed, angular materials there was little, if any, change in either parameter over a range of 3 to 17 percent that passes the no. 200 sieve. However, for sand-gravel material, the K_1 parameter had a maximum value near optimum fines and then a marked decrease in K_1 with increasing fines content. This change concurrently led to an increase in the K_2 value with increasing fines.

By using the results of the M_r -physical property study, predictive equations that relate the specimen properties to the M_r -value were developed for six different aggregate types used in the laboratory study. Typical $K_1\text{--}K_2$ relations were stated for these aggregate types to reflect the relative influence of moisture, compaction, and gradation on their M_r response.

The final phase of the study dealt with investigating whether general correlations exist between laboratory measured resilient modulus and laboratory measured CBR results. An analysis of nearly 100 data sets indicated that a very general, but variable, correlation does exist. However, because the M_r test is stress dependent, the coefficient that relates M_r to CBR must be stress dependent and not a unique or constant value such as has been noted by previous investigators. The average coefficient values found at a bulk stress of 10 and 100 lbf/in^2 ($F = 248$ and $F = 738$) is significantly lower than the 1500 value proposed by Huekelom and Foster.

REFERENCES

1. M.P. Jones. Analysis of the Subgrade Modulus and Pavement Fatigue on the San Diego Test Road. Univ. of Maryland, College Park, M.S. thesis, Dec. 1975.
2. B.F. Kallas and J.C. Riley. Mechanical Properties of Asphalt Materials. Proc., 2nd International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, MI, Aug. 1967.
3. B.F. Kallas and J.F. Shook. San Diego Experimental Base Project. Asphalt Institute, College Park, MD, Final Rept. 77-1 (RR-77-1), Nov. 1977.
4. R.I. Kinghan and B.F. Kallas. Laboratory Fatigue and Its Relationship to Pavement Performance. Proc., 3rd International Conference on the Structural Design of Asphalt Pavements, London, England, Sept. 1972.
5. M.W. Witczak. Design Analysis--Asphalt Concrete Overlay Requirements for Runway 18-36, Washington National Airport. Asphalt Institute, College Park, MD, Research Rept., Dec. 1971.
6. M.W. Witczak. Asphalt Pavement Performance at Baltimore-Washington International Airport. Asphalt Institute, College Park, MD, Research

- Rept. 74-2(RR-74-2), 1974.
7. R.G. Hicks and C.L. Monismith. Prediction of the Resilient Response of Pavements Containing Granular Layers Using Non-Linear Elastic Theory. Proc., 3rd International Conference on the Structural Design of Asphalt Pavements, London, England, Sept. 1976.
 8. C.L. Monismith, H.B. Seed, F.G. Mitry, and C.K. Chan. Prediction of Pavement Deflections from Laboratory Tests. Proc., 2nd International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, MI, Aug. 1967.
 9. C.L. Monismith, R.G. Hicks, and Y.M. Salam. Basic Properties of Pavement Components. Federal Highway Administration, Berkeley, CA, Final Rept. FHWA-RD-72-19, Sept. 1971.
 10. R.D. Barksdale. Laboratory Evaluation of Rutting in Base Course Materials. Proc., 3rd International Conference on the Structural Design of Asphalt Pavements, London, England, Sept. 1972.
 11. I.V. Kalcheff. Rational Characterization of Granular Bases. Proc., 1st Federally Coordinated Program on Research Progress Review, San Francisco, CA, Sept. 1973.
 12. Y.T. Chou. Engineering Behavior of Pavement Materials--State of the Art. Federal Aviation Administration; U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS, Final Rept. FAA-RD-77-37, Feb. 1977.
 13. M.G. Sharma and W.J. Kenis. Evaluation of Flexible Pavement Design Methodology. Proc., Association of Asphalt Paving Technologists, Technical Sessions, Phoenix, AZ, Feb. 1975.
 14. J. Sharma, L.L. Smith, and B.E. Ruth. Implementation and Verification of Flexible Pavement Design Methodology. Proc., 4th International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, MI, Aug. 1977.
 15. W.R. Barker and R.C. Gunkel. Structural Evaluation of Open-Graded Bases for Highway Pavements. U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS, Final Rept., Miscellaneous Paper GL-79, May 1979.
 16. E.E. Chisolm and F.C. Townsend. Behavioral Characteristics of Gravelly Sand and Crushed Limestone for Pavement Design. Federal Aviation Administration; U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS, Final Rept. FAA-RD-75-177, Sept. 1976.
 17. M.W. Witczak. A Study of the Deflection Behavior of the Ikalanian Sand Winchendon Test Section. U.S. Army Corps of Engineers, Cold Regions Research and Engineering Laboratory, Hanover, NH, Jan. 1980.
 18. F. Finn, C. Saraf, B. Kulkarni, K. Nau, W. Smith, and A. Abdullah. The Use of Distress Prediction Subsystems for the Design of Pavement Structures. Proc., 4th International Conference on the Structural Design of Asphalt Pavements, Ann Arbor, MI, Aug. 1977.
 19. W.S. Smith and K. Nair. Development of Procedures for Characterization of Untreated Granular Base Course and Asphalt-Treated Base Course Materials. Federal Highway Administration, Oakland, CA, Final Rept. FHWA-RD-74-61, Oct. 1973.
 20. M.W. Witczak and K.R. Bell. Determination of Remaining Flexible Pavement Life--Predicted Flexible Pavement Remaining Life Interpreted from Various Analytical Procedures. Maryland Department of Transportation, College Park, Res. Rept. FHWA-MD-R-79-3, Vol. 3, Oct. 1978.
 21. E.J. Barenberg. Response of Subgrade Soils to Repeated Dynamic Loads--State of the Art. Proc., Allerton Park Conference on Systems Approach to Airfield Pavements, Champaign, IL, March 1970.
 22. Y.T. Chou. Evaluation of Nonlinear Resilient Module of Unbound Granular Materials from Accelerated Traffic Test Data. U.S. Army Engineer Waterways Experiment Station, Vicksburg, MS, Final Tech. Rept. S-76-12, Aug. 1976.
 23. I.V. Kalcheff. Characteristics of Graded Aggregates As Related to Their Behavior Under Varying Loads and Environments. Proc., Conference on Utilization of Graded Aggregate Base Materials in Flexible Pavements, Oak Brook, IL, March 1974.
 24. I.V. Kalcheff and R.G. Hicks. A Test Procedure for Determining the Resilient Properties of Granular Materials. Journal of Testing and Evaluation, ASTM, Vol. 1, No. 6, Nov. 1973.
 25. E.J. Yoder and M.W. Witczak. Principles of Pavement Design, 2nd ed. Wiley, New York, 1975.
 26. T.C. Johnson. Is Graded Aggregate Base the Solution in Frost Areas? Proc., Conference on Utilization of Graded Aggregate Base Materials in Flexible Pavements, Oak Brook, IL, March 1974.

Publication of this paper sponsored by Committee on Strength and Deformation Characteristics of Pavement Sections.

Use of Pressuremeter Test to Predict Modulus and Strength of Pavement Layers

JEAN-LOUIS BRIAUD AND DONALD H. SHIELDS

Most airport pavements are designed or evaluated on the basis of a plate test. This article shows how a pressuremeter test can replace plate tests to advantage. A 1-in diameter hole is made through the pavement and pressuremeter tests are run every foot down to a depth of 6 ft. Each test yields a deformation modulus, so that a modulus profile is obtained at each hole location. The results of a comparison between 93 pavement pressuremeter tests and 11 plate tests performed at two airport sites indicate that the pavement pressuremeter can be used for pavement design; the plate tests performed were standard for the design of airport pavements in Canada. A chart for the design of flexible

airport pavements by using the results of pavement pressuremeter tests is presented. A procedure based on multilayer elastic theory is also discussed. This procedure uses the pressuremeter modulus as the elastic modulus of each layer. The multilayer elastic procedure makes more thorough use of the pressuremeter test results than does the chart procedure.

Most airport pavements are designed or evaluated on the basis of a plate test. This article shows how a