Constitutive Equation for Permanent Strain of Sand Subjected to Cyclic Loading

RODNEY W. LENTZ AND GILBERT Y. BALADI

Numerous rational methods for flexible pavement design and rehabilitation have been proposed to overcome some of the deficiencies of current empirical methods. Most of these rational methods predict permanent deformation in subgrade materials so that a pavement structure may be selected that will limit permanent deformation under the application of traffic loads. Thus, procedures for characterizing permanent deformation of subgrade materials are required in each of these methods. This paper presents a constitutive equation for predicting accumulated permanent strain of sand subgrade material after any number of repetitions of load. The parameters needed as input to the equation are obtained from a static triaxial test. To develop the equation, duplicate samples were tested by using both static triaxial apparatus and a closed-loop electrohydraulically actuated triaxial system. The dynamic test results were normalized with respect to parameters obtained from the corresponding static triaxial test. The difference in the normalized cyclic principal stress showed a unique relationship to the normalized accumulated permanent strain. This relationship was found to be independent of moisture content, density, and confining pressure. Based on these findings, a constitutive equation for permanent strain was developed.

Most design agencies base their procedures for pavement design on empirical design nomographs coupled with empirical subgrade strength parameters, individual experience, and local environmental conditions. The trend toward ever-increasing axle loads on highway and airport pavements has revealed serious shortcomings of empirical design methods for flexible pavements. These methods lack the ability to predict the amount of deformation anticipated after a given number of load applications. When pavement load exceeds the range for which performance data are available, empirical methods fail. Since soil is known to behave in a nonlinear fashion, performance under higher axle loads cannot be extrapolated from performance at lower load levels.

Numerous rational methods for flexible pavement design and rehabilitation have been proposed to overcome this deficiency. These are usually quasi-elastic (elastic theory to predict stresses coupled with permanent strains determined by repeated load laboratory tests) (1). Some methods also use visco-elastic theory together with laboratory testing (2,3). To be useful, these methods must have the capability of predicting cumulative permanent deformations in subgrade materials so that a pavement structure may be selected that will limit permanent deformation under traffic loading to an acceptable level. This requires the development of an adequate constitutive equation for prediction of permanent strain (3,4). Further, the method used to evaluate the constants in the constitutive equation should be simple, economical, and not require new and complicated expensive equipment or testing procedure. A constitutive equation that meets these criteria is presented in this paper.

BACKGROUND

Parameters that affect the accumulation of permanent strain in cohesionless material have been reported to be number of load repetitions, stress history, confining pressure, stress level, and density (1,3,5-12). A review of these effects is given by Lentz (10).

The effect of number of load repetitions on permanent strain has been reported by several investigators to be a straight-line relationship on a semilogarithmic plot Cyclic stress versus permanent strain curves have been shown to be analogous to static stress-strain curves (6,10-12) and describable by using hyperbolic functions developed for static test results (13,14). The results of cyclic triaxial tests can be normalized by using parameters obtained from static triaxial tests performed on duplicate samples (12). The difference in normalized principal stress showed a unique relationship to the normalized accumulated permanent strain. This relationship was found to be independent of moisture content, density, and confining pressure.

Testing Procedure and Equipment

The material used in the testing program was uniform, medium sand typical of that found in the northern half of Michigan. Duplicate triaxial samples were tested by using both static triaxial apparatus and a closed-loop electrohydraulically actuated triaxial system.

Three levels of confining pressure (σ_3 = 5, 25, 50 lb/in²) and two levels of density were used (γ = 99 percent AASHTO T-99 and 99 percent AASHTO T-180). For each combination of these variables, several levels of cyclic principal stress difference (σ_d) were used. Because stress history has a large influence on permanent strain, a new sample was required for each combination of variables. Details of the testing procedure are described elsewhere (10).

Test Results

It has been shown elsewhere (10-12) that the data for each sample could be approximated by a straight line on a plot of permanent strain versus logarithm of number of load cycles. Least-squares technique was used to pass a best fit straight line through each set of data. The equation of the line is of the form

$$\epsilon_p = a + b \ln N \qquad (1)$$

where

ϵ_p = accumulated permanent strain,
N = number of load repetitions, and
a and b = regression constants from least-squares best fit.

The constant a represents the permanent strain that occurs during the first cycle of load. The constant b represents the rate of change in permanent strain with increasing number of load repetitions. The values of a and b are, of course, different for each sample, depending on density, confining pressure, and level of cyclic principal stress difference.

DISCUSSION OF RESULTS

The principal objective of this research project was to obtain a constitutive relationship that will predict the amount of permanent strain under any number of load applications at any specified stress level. This relationship should account for sample variables (e.g., density or moisture content) and

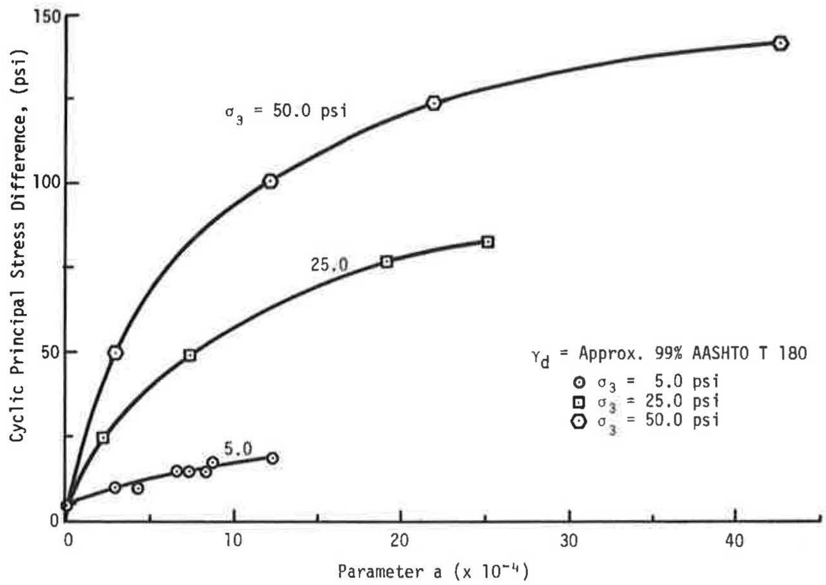


Figure 1. Relationship between cyclic principal stress difference and permanent strain during first load cycle.

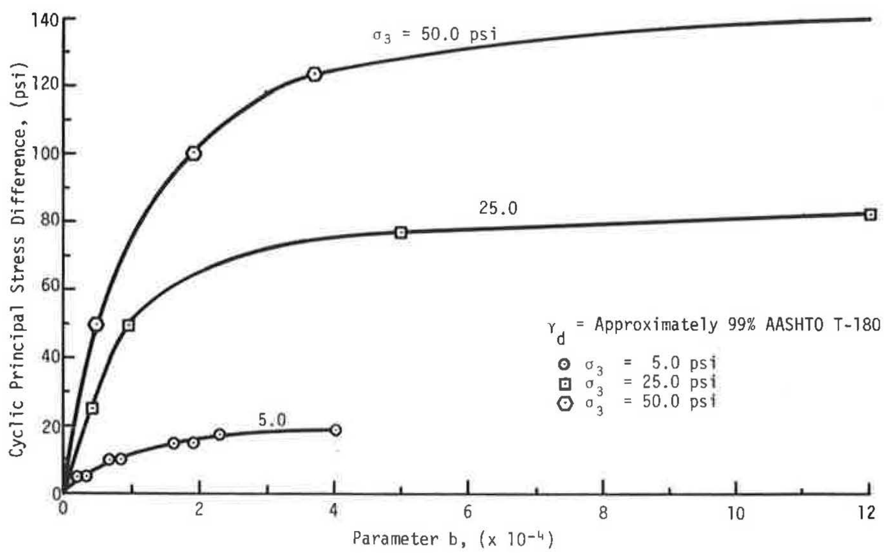


Figure 2. Relationship between cyclic principal stress difference and rate of change of permanent strain during cyclic loading.

confining pressure as well as cyclic principal stress difference and number of load applications.

Equation 1 expresses the cumulative permanent strain as a function of the number of load repetitions. Parameters a and b of the equation were thought to represent characteristics of the sample behavior under the particular testing conditions. Thus, it was convenient to develop the constitutive relationship by starting from Equation 1. Recall that the value of parameter a represents the permanent strain due to the first-load application. The value of parameter b indicates the rate at which permanent strain accumulates with increasing number of load repetitions. Therefore, if parameters a and b are expressed as functions of the sample variables, the testing conditions may provide the desired constitutive relationship.

It was shown (12) that the static stress-strain results of a sample of sand can be used to predict the cumulative permanent strain of an identical sample tested under cyclic loading conditions. Changes in material or testing conditions are reflected by changes in static stress-strain behavior and in parameters a and b. Thus, normalization of the cyclic stress and strain with respect to static stress and strain will eliminate or reduce the effect of these variables. Note that the static stress-strain data should be obtained from a sample identical in every respect to the dynamically tested specimens. This required one static test for each combination of density, water content, and confining pressure used in the dynamic testing program.

Figures 1 and 2 show plots of cyclic principal stress difference versus parameters a and b, respectively, for three different confining pressures. The values of cyclic principal stress difference plotted in Figure 1 were normalized by dividing by the peak static strength (S_d) from the corresponding static triaxial test. The values of parameter a were likewise normalized by dividing by the static strain that corresponds to a stress equal to 95 percent of S_d. The procedure for determining this normalizing strain ($\epsilon_{0.95S_d}$) has been described elsewhere (12). After the above normalizing procedure was applied, the data from Figure 1 were replotted in Figure 3. Examination of this figure indicates that the data from all tests collapsed together to form a single curve. By inspection, it was found that this curve could be represented by the following function:

$$a/\epsilon_{0.95S_d} = \ln(1 - \sigma_d/S_d)^{-0.15} \quad (2)$$

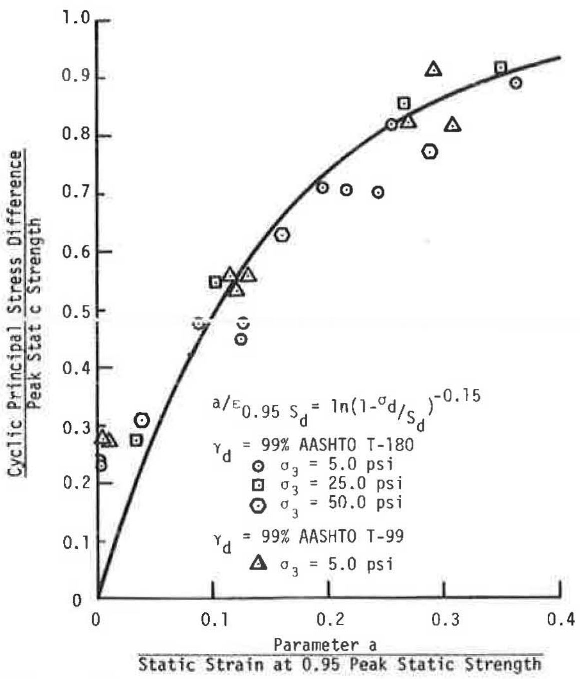
where

- a = regression parameter from Equation 1,
- $\epsilon_{0.95S_d}$ = static strain at 95 percent of static strength,
- σ_d = cyclic principal stress difference, and
- S_d = static strength.

This can be rewritten to give

$$a = \epsilon_{0.95S_d} \ln(1 - \sigma_d/S_d)^{-0.15} \quad (3)$$

Figure 3. Relationship between normalized cyclic principal stress difference and normalized parameter a.



A plot of normalized principal stress difference versus parameter b of Equation 1 is shown in Figure 4. Study of the figure shows that the effect of sample density vanishes, but each value of confining pressure yielded a distinctly separate curve. Note that the shapes of these curves are similar to static stress-strain curves.

Hyperbolic functions can be used to approximate static stress-strain curves (13,14). The applicability of hyperbolic function to describe the relationship between cumulative permanent strain and cyclic stress at a specific number of load applications has been demonstrated for unstabilized base material (6) and for fine-grained subgrade soil (15). Since the curves in Figure 4 appear similar in shape to static stress-strain curves, it seems reasonable to extend the use of the hyperbolic relationship to describe them. The hyperbolic relationship used by others (15) to describe cyclic principal stress difference versus permanent strain is of the form

$$\sigma_d = \epsilon_p/(n + m\epsilon_p) \quad (4)$$

where

- n and m = regression constants,
- ϵ_p = cumulative permanent strain, and
- σ_d = cyclic principal stress difference.

Modification of Equation 4 so that it applies to the relationship between normalized cyclic principal stress versus parameter b (shown in Figure 4) yields the following:

$$\sigma_d/S_d = b/(n + mb) \quad (5)$$

This can be rewritten in linear form to give

$$b/(\sigma_d/S_d) = n + mb \quad (6)$$

Least-squares technique was used to obtain the best-fit straight lines represented by Equation 6 for the data at each confining pressure. The values of n and m that were determined are shown in Figure 4 along with plots of the resulting hyperbolic

Figure 4. Relationship between normalized cyclic principal stress difference and parameter b.

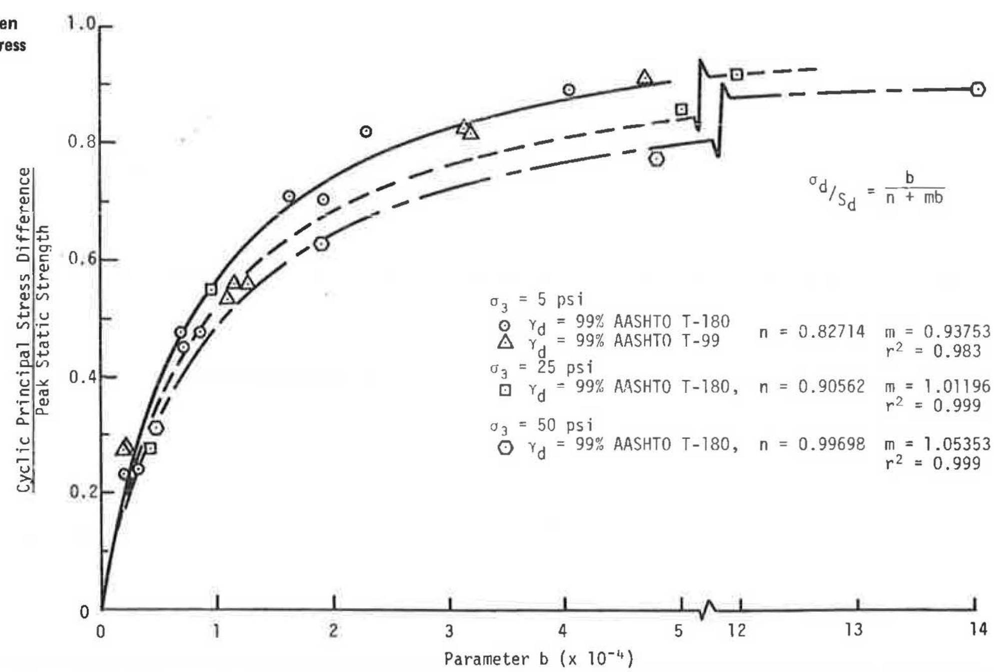


Figure 5. Relationship between confining pressure and regression constants n and m.

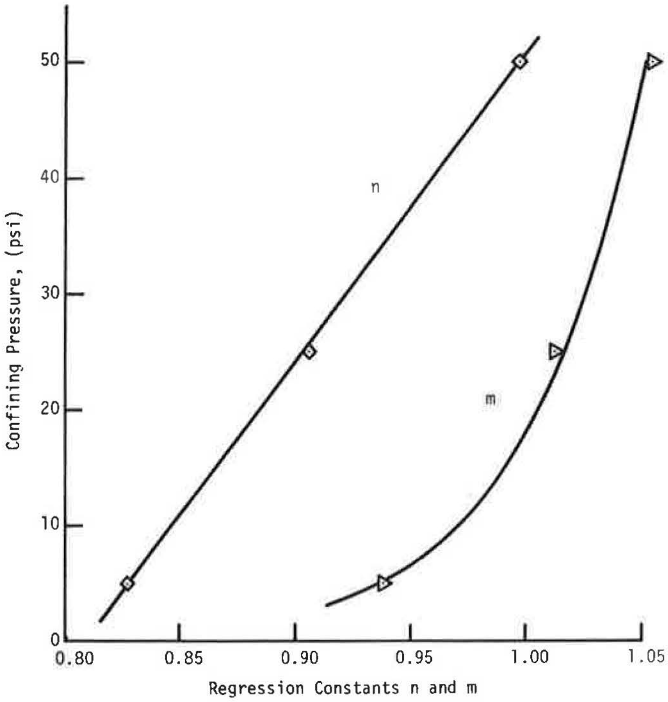
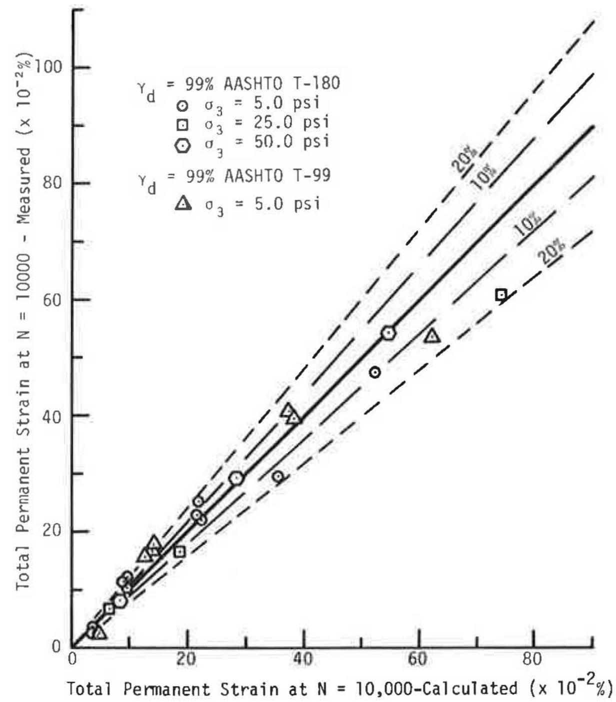


Figure 6. Comparison of measured and calculated permanent strain at N = 10 000.



curves and coefficients of correlation. Inspection of the figure indicates that the hyperbolic relationship fits the data rather well. Rearrangement of Equation 6 and solving for b yields

$$b = (\sigma_d/S_d)n/[1 - m(\sigma_d/S_d)] \quad (7)$$

Note that the coefficients n and m in Equation 7 have different values for each confining pressure. Thus, to complete the constitutive equation, n and m should be expressed as functions of confining pressure. The values of n and m were plotted against the confining pressure in Figure 5. Examination of the figure indicated that n can be related to confining pressure by a linear function. The relationship for m was found to be logarithmic. Equations 8 and 9 were obtained by using least-square fitting technique to represent these functional relationships:

$$n = (0.809\,399 + 0.003\,769\sigma_3) \times 10^{-4} \quad (8)$$

$$m = 0.856\,355 + 0.049\,650 \ln \sigma_3 \quad (9)$$

where σ_3 is the confining pressure in pounds per square inch.

The completed constitutive equation can be obtained by substituting the expressions for parameters a (Equation 3) and b (Equation 7) into Equation 1. These substitutions yield

$$\epsilon_p = \epsilon_{0.95S_d} \ln(1 - \sigma_d/S_d)^{-0.15} + (\sigma_d/S_d)n/[1 - m(\sigma_d/S_d)] \ln N \quad (10)$$

where n and m are given by Equations 8 and 9, respectively.

To check the reliability of Equation 10, values of accumulated permanent strain at the end of 10 000 cycles were calculated by using Equation 10. These calculated values were plotted in Figure 6 versus permanent strain measured during the cyclic triaxial testing program. Note that the points plotted in Figure 6 represent samples tested at three different confining pressures and two different densities. Perfect correspondence between calculated and measured permanent strain would result in the plots of all points being along a 45° diagonal line. This diagonal line, along with lines that indicate a deviation of ±10 percent and ±20 percent from it are also shown in the figure. Inspection of Figure 6 shows that most of the data points fall close to the 45° line. This indicates a good correspondence between measured and calculated values. Note, however, that the data used to derive the constitutive equation are from the same samples used for the measured permanent strain plotted in Figure 6. Thus, the figure indicates how well the equation fits the data it was derived from rather than its predictive capability.

To examine the predictive capability of Equation 10, three additional samples were prepared at a density of 99 percent of AASHTO T-99. One of these samples was tested in a static triaxial test to obtain the parameters (S_d and $\epsilon_{0.95S_d}$) that are needed in Equation 10. These values were used to calculate the cumulative permanent strains for the other two samples at two different stress levels. These calculated values were then compared with measured permanent strain. This comparison yielded results similar to those presented in Figure 6. Note that the only parameters needed in Equation 10 can be obtained from a static triaxial test conducted at the expected density and confining pressure.

CONCLUSION

This paper has presented a constitutive equation that will predict the amount of permanent strain that will occur under any number of load applications at any specified stress level. The equation accounts for sample and test variables and requires only the results of a static triaxial test. The development of this equation was based on results from a single sand subgrade material. Hence, additional research is needed to extend the usefulness of the equation to a wider range of subgrade materials. Once additional research has verified or modified these findings, use of this constitutive equation for predicting permanent strain should result in significant saving of laboratory time and equipment because only static triaxial test results are required for its use. Also, rational methods of pavement design, which require characterization of permanent strain behavior, will be more likely to gain quick acceptance by practicing engineers if they have available such a simple means of predicting permanent strain.

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Abridgment

Evaluation of In Situ Elastic Moduli from Road-Rater Deflection Basin

M.C. WANG AND B.A. ANANI

This paper presents a computer method for evaluating the in situ modulus of pavement layers from road-rater deflection basins. The method that is developed on the basis of the results of a theoretical analysis uses the bitumen-structures-analysis-in-roads (BISAR) computer program and the procedure of successive approximation. The method was used to evaluate the in situ modulus of experimental pavements at the Pennsylvania Transportation Research Facility. The computed modulus values were analyzed statistically to determine the factors that most significantly influence the in situ modulus of each pavement layer. Results indicate that, for the bituminous concrete surface and base materials, the sum of pavement surface temperature and the average five-day air temperature prior to the deflection measurements is the most significant among the factors analyzed. For the subbase material, no single influential factor is identified as significant. The subgrade modulus is influenced most by the subgrade water content, as expected.

One major difficulty in response analysis of pavement structure is to determine the elastic moduli of pavement constituent layers. Two methods are currently available for modulus determination. One method is by means of laboratory testing on specimens either compacted in the laboratory or extracted from the pavement structure; the other method is by nondestructive testing on the pavement surface. Because of its relative ease in data collection in addition to the advantage of nondestruction to the pavement structure, the method of using surface-deflection basins to determine elastic modulus is preferred. Further, of the various instruments available for surface-deflection measurement, the road rater has received increased use due to its relatively high degree of mobility. For these reasons, this paper presents a method for evaluating the in situ elastic modulus from road-rater surface-deflection basins.