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Pavement Performance Modeling for Pavement Management

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Systematic pavement management requires estimates or predictions of future pavement performance so that rational comparisons may be made among alternative courses of action. Performance models are required in two distinct contexts, depending on the pavement management level involved. At the project level, fairly detailed and specific models are required for predicting the performance expected for an individual pavement section. At the network level, general or average prediction models are required to provide estimates of the expected performance for a typical pavement or class of pavements. Accordingly, quite distinct modeling methods are indicated for these two different modeling needs. Performance-modeling requirements and data requirements for both network-level and project-level applications are discussed. An idealized experiment to collect data for performance modeling is presented. A specific performance prediction model based on stochastic concepts and treating pavement deterioration as a Markov process is presented as an example of the development of prediction models for network-level applications.

All highway agencies are faced with the problem of providing and maintaining a network of roadways to serve the public. This requires both a considerable capital investment and an adequate maintenance and rehabilitation program. During the past decade, various economic, social, and political factors have made it increasingly important that transportation agencies take every step to make the most beneficial use of their often-inadequate budgets. This has resulted in the rise of the pavement management system from a theoretical concept discussed by university professors to a practical reality under development and implementation throughout the nation.

As a result of this increasing emphasis, both the conceptual and the practical elements of systematic pavement management have been widely discussed (1-11). Great strides have been made, but significant problems have also been encountered. One such problem, which will be addressed in this paper, is the difficulty in predicting pavement performance.

Systematic pavement management is based on the idea that it is possible to determine, in a reasonably objective fashion, how best to use the public funds made available for providing pavements. Budgets are typically allocated on a one- or two-year cycle, and construction, maintenance, and rehabilitation activities are generally planned on an annual basis. Nevertheless, activities carried out (or postponed) now can have a significant impact on roadway conditions for several years or even decades. In order to make rational choices among alternative courses of action, it is therefore necessary to be able to predict or estimate the future performance of the roadway under each alternative action.

Pavement performance has in the past generally been defined as a summary or accumulation of pavement serviceability index based on objective mea-

surements of roughness and/or pavement distress. This use of the word "performance" stems from the work of Carey and Irick (12), although their original definition left considerable room for greater generality. More specifically, performance has been equated with the area under the serviceability history curve or the shape of the serviceability curve. This is the concept of performance adopted in this paper. It should be mentioned, however, that there has been no universal agreement on the definition of pavement performance. For example, in the recent literature, pavement performance is defined variously as (a) the ability of a pavement to provide an acceptable level of serviceability with a specified degree of reliability at an assumed level of maintenance (13), (b) allowable repetitions of loading prior to the functional failure of the pavement (14), and (c) the probability that a critical life of the pavement will be achieved based on the onset of critical conditions (15).

Since serviceability is almost universally measured by using a serviceability index based on roughness or riding comfort, the generally accepted use makes pavement performance a function of pavement roughness. However, many other factors, such as skid resistance, structural adequacy, and cracking, may be important in determining the overall adequacy of a pavement. The word "performance" is a natural candidate to describe this overall adequacy, so it is somewhat unfortunate that it has been defined more narrowly as a function of roughness. We are hopeful that at some future time pavement specialists can agree to reserve the word "performance" to denote this overall adequacy.

PERFORMANCE-MODEL REQUIREMENTS FOR PAVEMENT MANAGEMENT

Performance models are used in two distinct contexts as a part of pavement management, depending on the pavement management level involved. At the project level, fairly detailed and specific models are required for predicting the performance expected for an individual pavement section. At the network level, general or average prediction models are required to provide estimates of the expected performance for a typical pavement or class of pavements. Accordingly, quite distinct modeling methods are indicated for these two different modeling needs.

Project-Level Models

At the project level, considerable information will be available regarding the pavement structure, the current and expected traffic, current and past distress measurements, deflection, and so forth. The prediction model used must be able to predict specific values for the performance of the given section in an accurate and reliable fashion. Thus, a fairly accurate prediction model specific to the individual conditions appropriate to a single project is needed.

One approach to project-level modeling is based on the use of current and historical information on pavement condition to predict the future service-ability of the pavement. Such models are often termed "distress/performance relationships," and the problem of relating pavement distress to service-ability and performance has been under attack for some time (16-20). Time-dependent distress/performance relationships that are broadly applicable but that yield accurate predictions for individual sections of roadway are extremely difficult to derive. The primary reasons for this difficulty are the lack of adequate data records that cover a sufficiently long time period and the inherent variability associated with measurements of pavement condition.

A variant to the distress/performance problem involves predicting future distress and then relating distress to serviceability in a time-independent model. Distress-prediction models for various distress types are available and have been discussed and evaluated in the literature (13, 17, 21-27). At the project level, it is feasible to obtain sufficient data to provide input to one or more of these mechanistic models. The output would be a prediction of one or more future distress levels. It then only remains to relate these future distress levels to serviceability. We recently found that, given the current state of available data, it is in fact more feasible to relate distress to serviceability directly, with no time dependence, then to develop time-dependent relationships (17).

It should be mentioned that some of the mechanistic distress models referred to above (VESYS, for example) also include serviceability predictions. Thus, at the project level, some performance-prediction models that incorporate distress/performance relationships are already available.

Models developed from data collected on small groups of similar pavements are more likely to be reliable than those developed from large data bases. Therefore, by carefully selecting several classes of similar pavements, an agency could produce time-independent distress/serviceability relationships for each class that would be reliable enough for project-level pavement management use. Such models would, of necessity, be very limited in applicability; that is, each model would apply to only a very small class of pavements, so that each agency would require several such models in order to predict performance for a variety of pavement projects. The number of pavement sections to be included in each modeling class and the number of different classes to be used will depend on the needs and resources of the agency. In the extreme case, a separate model could be used for each pavement section. A single functional form that has variable coefficients could be chosen to represent the desired relationship for a wide variety of pavements, and the coefficients could be determined separately for each section of pavement. Such an approach requires access to considerable historical information for each section.

This same sort of approach can be employed without the use of mechanistic models to predict serviceability history or performance directly. When applied to individual pavement sections, this amounts to extrapolation of established performance trends, which again requires good records of past performance from which to extrapolate. Despite this requirement, at least one state highway agency has used this method with some success in predicting performance individually for thousands of pavement sections (28).

Network-Level Models

Direct performance prediction for individual pavement sections is also viable for network-level pavement management. In fact, the agency referred to above has used the performance predictions for individual sections for programming purposes. However, this method was only recently adopted after a decade of pavement management system development, data-base organization, and data collection. Previously, a subjectively based performance-prediction technique was used $(\underline{4,29})$.

The other project-level modeling techniques discussed above are less viable at the network level. The mechanistic distress models require information of a character that is much too detailed for network-level applications. Even if such details were available, the amount of time required for the detailed analysis would be prohibitive. On the other hand, the formulation of direct, time-dependent distress/serviceability relationships is probably not feasible in the absence of a long-term data record. Thus, the development of direct distress/serviceability relationships for network-level pavement management is not likely to be feasible for a number of years for most agencies.

There is, however, an alternative approach that involves subjective modeling of pavement performance. Markovian or Bayesian techniques may be used to develop performance-prediction models that use distress/serviceability relationships only directly. Since only an average performance prediction for any pavement section is required at the network level, the lack of adequate data is not as troublesome as it is for project-level modeling. Bayesian or Markovian techniques are particularly applicable for this case, and in fact these techniques may be implemented in situations when little or no objective data are available. An example of network-level performance prediction based on purely indirect distress/serviceability relationships is presented in a subsequent section of this paper.

Data Requirements for Performance Modeling

As discussed in the previous section, the availability of pavement data records has a significant impact on pavement performance modeling. During the conduct of recent research (17), we had occasion to review selected pavement condition data records from a dozen state highway agencies, the AASHO Road Test, and the Brampton Test Road. Data from each of these sources were found to be inadequate for the development of reliable performance models for pavement management purposes. The major factors that contribute to this inadequacy are discussed below. Of course, not all data sources exhibited all the inadequacies listed below. In some cases, only a single factor was missing, whereas in others several factors contributed to the inadequacy. However, in no case did a single data source prove entirely adequate. The following major inadequacies were identified:

1. Inadequate time records: Many of the data records reviewed involve only one to three years of pavement distress and serviceability data. The pavements represented may have an average service life of 20 years, so that such a limited sample would hardly provide an adequate basis for life-cycle performance modeling.

- 2. Omission of key variables: There is very little agreement among state agencies as to which pavement variables are essential. For flexible and composite pavements, only rutting was recorded universally in the data examined. None of the rigidpavement distress variables were reported universally. In many cases, significant distress variables were lumped together or combined into a single index whose value was reported in the data record. Most data sources reported a present serviceability index (PSI) on a scale of 0-5, but some reported only roughness or bump count. Several states reported that distress and serviceability data were not available on the same data field or simply were not available for the same pavement sections at all. In addition, very limited maintenance records were included in these data.
- 3. Lack of standardization of units: The only variable universally reported in the same units was rut depth, recorded in inches. Even so, the method for determining average rut depth varied among the data sources examined. Other distress variables were recorded in a variety of units. For example, various forms of cracking were reported in units of square feet per thousand square feet of pavement surface, linear feet per thousand square feet, square feet of area affected, total length of cracking, number of cracks per section, and by distress level in terms of severity and extent.

Given the wide variety of data sources examined, it is likely that the problems encountered here are common to the majority of existing data sources. That is, data inadequacy is a widespread problem. Therefore, it is felt that some guidance should be provided for future data-collection efforts. It should be emphasized that this discussion applies specifically to data collected for modeling purposes and not to data collected for routine inventory or other purposes.

Beyond just correcting the obvious deficiencies, the only way to assure that meaningful modeling will be achieved is to design an experiment or experiments to incorporate all the relevant factors. Consequently, an ideal experiment design was developed to provide the data necessary for effective performance modeling. It is not anticipated that this particular experiment will be performed, but it is felt that the considerations discussed here will provide guidance for future data-collection efforts. The discussion deals only with flexible pavements, for purposes of an example. However, the same basic considerations carry over to rigid pavements, and a similar design could be constructed for the case of rigid pavements.

IDEAL EXPERIMENT DESIGN

The first step in the design of an experiment to collect data for pavement performance modeling is to identify the dependent variables (y's) to be measured during the experiment. This list of important variables should probably include (a) distress, (b) serviceability, (c) deflection, and (d) skid resistance. Each of these basic variables may involve several subvariables. For example, distress for flexible pavements will probably involve rutting and fatigue cracking as well as possibly low-temperature cracking, bleeding, or other variables. It is desirable to limit this set of variables as much as possible without excluding important parameters.

The next step is to acknowledge the role of time as a split-plot factor and not as a dependent variable or as a covariate. This forces the investigator to obtain measurements throughout the entire experiment at fixed intervals of time for all treat-

ment combinations, which eliminates the inadvertent confounding of time with particular treatment combinations and allows investigation of the interactions of time with all factors in the experiment. This probably is the single most important concept in the experiment design. Any departure from taking observations in a regularly scheduled time sequence will have confounding effects that cannot be completely accounted for in the analyses to follow. Of course, the shortcoming for any time-dependent variables is that the errors may be correlated, and this will hinder the development of good prediction models.

Next, a list of factors and levels to consider in the ideal experiment must be developed. An example is given below:

Category	Fac	tor
Structural	A.	Surface thickness
	B.	Surface type
	C.	Base thickness
	D.	Base type
	E.	Subgrade strength
Environment	F.	Moisture
	G.	Temperature
	H.	Freeze or thaw cycle
Load	J.	Traffic or vehicle passes
	K.	Percentage of trucks or equiva-
		lent axle load
Miscellaneous	L.	Construction variability
	M.	Drainage
	N.	Maintenance, preventive
	0.	Maintenance, corrective
	P.	Geometry
		PORT 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

These factors are not supposed to be exhaustive or mandatory. However, it is felt that 15 factors would be sufficient to include all major influences on pavement performance. It may be desirable to delete some factors in this table, such as G or H, which overlap to some degree. Similarly, some factors may need further subdivision, such as M, which may require separate treatment of surface and subsurface drainage.

If only two levels were run for each of the 15 factors, a design that would allow estimation of all main effects and two-factor interactions (assuming that three-factor interactions are zero) is a 1/128 replication of the 15 factors in 8 blocks of 32 each. (Of course, 8 blocks may not be necessary, and this design simply represents an ideal estimate.) Such a design is given by the National Bureau of Standards as Plan 128.15.32 (30, pp. 70 and 72), a 1/128 replication of 15 factors in 8 blocks of 32 units each. The identity, block confounding, and blocks for this design are reproduced in Figure 1 (30). Note that an appropriate block structure must be chosen, which may require a relabeling of the factors. For example, if the interaction (ABD) within the environmental category were to be used in blocks, then moisture, temperature, and freeze or thaw cycle would be renamed A, B, and D, respectively.

The design of Figure 1 is good for two-level interactions. However, it is anticipated that at least three levels will be needed in most (if not all) factors in order to investigate curvature (deviation from straight-line behavior). If curvature is needed in all the factors, a composite design could be run that would require a total of 31 more treatment combinations. Since probably only three levels would be used for each factor, we could represent the center point as zero level and denote "low" by -1 and "high" by +1. By using this set of definitions for the levels, the 31 treatment combinations given in Figure 2 should be added to the

Figure 1. Ideal experiment design that uses 15 factors in 8 blocks.

Plan 128.15.32. 1/128 replication of 15 factors in 5 blocks of 32 units each.

Factors: A.B.C.D.E.F.G.H.J.K.L.M.N.O.P.

Factors: A,B,C,U,E,F,G,H,J,A,L,M,N,U,F.

I - ABEGN - ACEFNP - BCFGP - BEFGO - ABDFNO - ACDGNOP - BCDEOP - ADHKO

BDEGHNON - CDEFNINOP - ABECHORP - ABFRNO - FGHNN - CGHKNP

- ABCCHKP - BCHNOP - ACEGNOP - ABEFHJO - FGHNN - BCDEFGLNOP - ACFHJP

- ABGCHLP - BCHNOP - ACEGNOP - BCEFLX - ABFCJX - ABCCHLNP - ACCHNOP

- CFIKOP - BGLKO - ABLXNO - ABKLDP - ECKLNOP - BCFKLNO - ACFGKLO

- ABDEFGKLP - DFKLNP - BCDGKLN - ACDEKL - BDHLP - ADEGHLNP - ABCCHINO

- CDFGHL - BEFGHLOP - APHLNOP - ACCHNO - CCHLO - ACHKLN - BCCGHJKL

- EFHIKLP - ABFGHKLNP - ACDEFGHJKNO - BCDFHJKLO - DGHKLOP

- ABDELTKLNOP - CLDINO - ABCDEGLO - ADEFJLOP - BEGGLNPO - CFGJLN

- ABCFJL - AGJLP - BEJINP - CDGHNO - ASCDEFHJNOP - ABCFGHJNOP

- BDHJNOP - CEFHJM - ACFGHJNN - ANJNOP - BEGGNP - ACCIM - BCEJNN

- EFGJOONP - ABFJNOP - ACDEFONO - BCDFGHJNONO - ADDEFGHJNOP

- BDGGNP - ADBJP - ABCDEFON - CDFN - BEFNOP - AFGON - ABCCIM
- CGGNO - ABCHNON - ACDGROND - BCEFGKNO - ACFNON - ABDEHDONP

- DFGHOW - BCDHOM - ACDGROND - BCEFGKNO - ACFNON - ABBEHDONP

- DFGHOW - BCDHOM - ACDGROND - ABCDGHKNOP - CDEHJKNOP

- BDGGNOP - BCDHOM - ACDGROND - ABCDGHKNOP - BHJKLNOP

- BDGFGLIKNN - ADFHIKLN - ABCEFILIKNOP - CFGLIKNOP - BHJKLNOP DEGINOP = SCHNOL = ACDEGINON = ABCOGLIKUMP = CDELIKUMP BEDEGIKUMD = ADFILIKUM = ABCEPILIKUMP = CFGRIKUMON = STALIMO = BUXILIMO AEGIKUMO = BCGIJIMOP = AEEIJIMON = FIZIMO = STALIMO = SCDEJI ACDEGIJIMOP = ABDIJIM = DEGIJIM = ADGKUMON = BDEKIMO = CDEFKLMOP ABCDEKLMOP = AEEKIMO = BEGKIM = CKIMOP = ABCGKLMOP = GHUMO ABEHUM = ACEFGHUMP = BCFHLMOP = DEFHLMON = ABDEGILMO = ACDHUMOP BCDEGHIJMOP.

Block confounding: ABD, ACF, BCDF, ABCE, CDE, BEE, ADEF Blocks only: All two-factor interactions are measurable.

> Blocks 1

(1)	bcdfgjmo		abfhjkmo	acdgjl	abflmo	hjkl	bedfghklmo
cefhjklmn	bdcghklno		abcelno	adcfghkmn	abcehjkno	cefun	bdegjno
adefhjlop	abceghlmp		bdeklmp	cefghop	bdehjmp	adcfkop	abcegjkmp
acdkmnop	abfgjknp		bcdfhjnp	gjklmnop	bcdjklnp	acdhjlunop	abfghlnp
2	3	4	5		7	8	

Figure 2. Additional treatment combinations needed to investigate curvature in ideal experiment.

							F	ACTOR								
	A	<u>B</u>	<u>c</u>	D	E	<u>F</u>	<u>c</u>	<u> </u>	J	K	<u>L</u>	<u>H</u>	N	<u>o</u>	Ē	
1.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
2.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	
3.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	
4.	0	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	
5.	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	
6.	0	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	
7.	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	
в.	0	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	
9.	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	
10.	0	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	
11.	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	
12.	0	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	
13.	0	O	0	0	0	0	0	0	0	-1	0	0	0	0	0	
14.	0	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	
15.	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	
16.	0	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	
17.	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	
18.	0	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	
19.	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	
20.	0	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	
21.	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	
22.	0	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	
23.	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	
24.	0	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	
25.	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	
26.	0	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	
27.	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	
28.	0	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	
29.	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	
30.	+1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
31.	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

256 in the original design, which would make 287 combinations. Of course, there would need to be repeats of these 287 treatment combinations. It would be ideal to have a complete replicate of the whole experiment, but one may conceive of 13 repeats if engineering information were available on the experimental error and the 13 were used only to check the error. This would yield a total of 300 treatment combinations.

Analysis

Let us assume that all measurements of servicedeflection (y_2) , skid resistance (y_4) , and fatigue cracking (y_5) ability (y_1) , (y3), rutting have been taken over the time intervals desired for all 300 treatment combinations. There may be as many time intervals as desired, but an ideal experiment should encompass a reasonably large fraction of the estimated service life of the pavements.

Analysis at Each Time Period

One could run an analysis of variance (ANOVA) on the 256 treatment combinations plus the 13 repeats for pure error at each time period for each of the five y's (assuming that appropriate transformations were made to make all variables normally distributed). The ANOVA would involve the sources and degrees of freedom (df) identified below:

Source	df
Blocks	7
δ	0
Main effects (ME)	15
Two-factor	
interactions (2fi)	105
Residual	128
Pure error	13
Total	268

After finding out which two-factor interactions are significant, one could use all 300 observations and run a multiple regression on each dependent variable, y_i (i = 1,2,...,5), as follows:

$$y_i = \beta_0 + \beta_1 x_1 + ... + \beta_{15} x_{15} + \beta_{1,1} x_1^2 + ... + \beta_{15,15} x_{15}^2$$

+ all two-factor interactions significant in ANOVA for

Analysis over Time

Here time is a split-plot factor. Run an ANOVA on each y, say, over 11 time periods, to get a number to show in the ANOVA (the number of time periods could be greater or smaller). The sources and df's of this ANOVA are shown below:

Source	đf
Blocks	7
δη	0
ME (A-P)	15
2fi (A-P)	105
Residual	128
Pure error	13
82	0
Time (T)	10
T x blocks	70
T x ME	150
T x 2fi	1050
T x residual	1280
T x pure error	130

The most important part of the ANOVA shown above is to find out whether the interpretation of T x pure-error mean square is of the same order of magnitude as pure-error mean square. This concept has been covered by Anderson and McLean (31, Chapter 7). The next important part, given that the first one shows that these errors are the same size, is T x residual versus residual, followed by (T x 2fi) versus two-factor interactions, and finally (T x ME) versus main effects.

If the errors (pure and residual) can all be pooled, then an overall regression analysis may be

run for each y; as follows:

$$y_i = \beta_1 x_1 + ... + \beta_{15} x_{15} + \beta_{1,1} x_{15}^2 + ... + \beta_{15,15} x_{15}^2$$

- + significant two-factor interactions of the 15 factors
- + significant time and interactions of time effects

If it turns out that the errors (pure and T x pure) cannot be pooled, there may be correlation of the errors. To examine the effects of this condition in the factorial part, one may use the procedure given by Anderson and McLean (31, p. 166). In this case, one calculates the sums of squares as given above, but the df's for the sources are used as listed below:

So	T x blocks T x ME T x 2fi	<u>df</u>	
\mathbf{T}			1
\mathbf{T}	x	blocks	7
\mathbf{T}	x	ME	15
\mathbf{T}	x	2fi	105
T	x	residual	128
T	x	error	13

If the results of all the F-tests are the same as for the previous tests by using 10 times the df's, one need not be concerned about correlated errors. If, however, there are major differences, care must be taken in the interpretation and use of the variables in the regression equations. There is no clear-cut way to obtain ideally all the information due to time if the errors are too highly correlated.

Other Design Approaches

There are many types of designed experiments that could be used for this problem. However, the most efficient one seems to be the one discussed above.

If it is necessary to investigate three-factor interactions, an entirely different design must be made, which requires many more treatment combinations than the design presented here. If curvature must be examined for all combinations of the 15 factors, a fractional factorial of 3¹⁵ may be needed. The number of treatment combinations required for this type of design is quite large.

Other designs could involve fewer factors if, for example, a state agency felt that some of the factors listed earlier were not needed to represent conditions that faced the agency adequately. However, the primary problem must still be faced: In order to develop models capable of accurately predicting the performance of a given section of pavement, considerable data must be available for that section (or similar sections) over a fairly long period of time. This data requirement may be partly circumvented through the use of subjective data or expert opinion. Subjective models based on Bayesian theory may be used for several years until adequate objective data can be acquired (32). In this approach the requirement for objective data is replaced by a similar requirement for experts who have had considerable experience in the performance of pavements over a long period of time. Most highway agencies have such knowledgeable people to draw on, so this approach offers great promise in future modeling applications.

STOCHASTIC SERVICEABILITY DETERIORATION MODEL FOR RIGID PAVEMENTS

Network-level pavement management requires performance predictions that are reliable on the average. That is, specific predictions for individual sec-

tions may deviate considerably from actual performance as long as random fluctuations are involved. Thus, stochastic models are particularly suited for network-level applications such as rehabilitation programming.

The development of such a performance model is discussed in this section. The model presented is quite simple in scope and concept, and it is not expected to be universally implemented. Rather, it is hoped that this discussion may provide interested agencies with guidance in the development of probabilistic performance models. The techniques employed have been discussed in the literature (27, 2, 32-38).

Development of Model

As part of a recent modeling effort (17), we examined serviceability histories for rigid-pavement sections from loops 3, 4, 5, and 6 of the AASHO Road Test. It was observed that roughly 7 of every 10 sections that reached terminal serviceability during the road test exhibited a characteristic serviceability pattern--a long period of nearly constant serviceability followed by a precipitous drop near the end of the service life. This pattern is evident in Figure 3, which illustrates serviceability plotted against service life for 20 of the rigidpavement sections that failed during the road test. Service life is defined here as the length of time between the beginning of the test and the time at which a particular section reached terminal serviceability. Thus, the service-life scale used in the figure is a time scale normalized by the total length of time that each pavement section was in service.

The pattern illustrated in Figure 3 was found for pavement sections that had a slab thickness that ranged from 2.5 in to 11 in, applied axle loads that ranged from 6-kip single axles to 48-kip tandem axles, pumping scores from 500 to 60 000, and a similar range of other parameters. Thus, it was felt that this pattern could serve as the basis for a fairly general, widely applicable performance model.

There are a number of functional forms that could be used to reproduce the general shape illustrated in Figure 3. In the hope of obtaining a model that could be adapted to a variety of pavement types and structures, the following general form was chosen:

$$\overline{PSI}(T) = C_1 + (C_2 / \{ \exp[\beta(T/\tau - 1)] + 1 \})$$
(3)

where

PSI (T) = average predicted serviceability at time T:

 c_1 , c_2 = constants determined from initial and terminal serviceabilities;

- β = parameter, presumably dependent on pavement structure, load, and environment, that determines the shape of the predicted serviceability history curve; and
- τ = expected service life of pavement (time in years from beginning of traffic to terminal serviceability).

By adjusting the values of coefficients τ and β , Equation 3 can be made to reproduce the shape of a wide variety of serviceability patterns for flexible or rigid pavements. The values chosen here to reproduce the behavior of Figure 3 are shown in the following equation:

$$\overline{PSI}(T) = -1.5 + (6.0/\{\exp[10(T/\tau - 1)] + 1\})$$
(4)

Figure 3. Serviceability history (normalized service-life time scale) for AASHO Road Test rigid pavements.

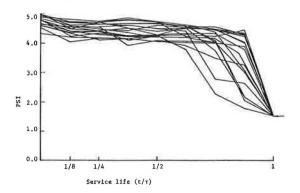
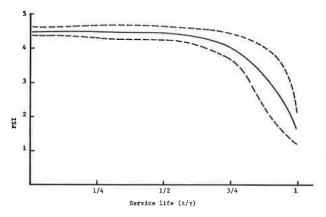


Figure 4. Predicted serviceability history based on stochastic version of Equation 4.



This equation is plotted as the solid curve in Figure 4. Note that the parameter τ need not be specified in order to compute serviceability at any fraction of the expected service life. However, the value of τ must be fixed in order to translate this fraction of service life into an actual elapsed time in years.

Equation 4 represents a very simple serviceability prediction model. Of course, some variability is observed in Figure 3 for the serviceability of individual pavement sections. In order to account for this, a stochastic feature was added to the prediction given by Equation 4. In this approach, the PSI predicted by Equation 4 is to be interpreted as a mean serviceability index for the pavements in question. Variations about this mean are incorporated by defining an artificial variance or standard deviation. Estimated values for such a standard deviation were derived from the magnitude of the variation observed in Figure 3. These values were used to construct the dashed lines in Figure 4, which represent the mean value plus or minus twice the artificial standard deviation.

In order to make practical use of this stochastic feature, the predicted serviceability history of Equation 4 and Figure 4 was incorporated in a Markovian framework. In such an approach, the pavement is described as being in a certain state at any given time, and the probability that the pavement will undergo a transition to each other possible state within a fixed short period of time is specified. Such a model is conveniently expressed in matrix notation—transition probabilities are arrayed in a square matrix, and possible pavement

states are listed in a single-column matrix. In this example, pavement states are specified in terms of pavement serviceability, but other significant variables may be incorporated in the description of pavement states (37, 38).

Example

Twenty possible pavement states, or serviceability values, were selected for use in this example. These are listed below:

State	Nominal PSI	State	Nominal PSI
1	5.0	11	3.8
2	4.9	12	3.6
3	4.8	13	3.4
4	4.7	14	3.2
5	4.6	15	3.0
6	4.5	16	2.7
7	4.4	17	2.4
8	4.3	18	2.1
9	4.2	19	1.8
10	4.0	20	1.5

Nominal serviceability values are specified for each state. Once these states have been specified, transition probabilities between states may be calculated by using Equation 4 and Figure 4. A set of transition probabilities that effectively reproduce the behavior illustrated in Figure 4 are shown in Figure 5. In developing this transition matrix, the time interval between transitions was fixed at 1/100 of the expected service life of the pavement. Thus, for a service life of 20 years, five transitions per year are incorporated in Figure 5. At each transition, the pavement may remain in its current state or enter another state (improve or deteriorate).

Predictions of pavement serviceability are carried out in the following manner. First, the initial state of the pavement is specified. This is done in terms of the probability that the pavement has a specific serviceability level at the initial time. If the pavement is known to have a PSI of 4.5 exactly, then the probability that the pavement exists in state number 6 given above is 1.0, and the probability for all the other states is 0. However, in the general case, the serviceability of the pavement can be specified only within some limit, say, a mean serviceability of 4.5 and standard deviation The initial state specification for this case is given in Table 1. Such a state is called a mixed state. The probability values in this case may be thought of as expressing the likelihood that a repeat measurement of PSI would yield the nominal PSI value asociated with each state in the table.

The state of the pavement at future times is calculated by multiplying the initial state by the transition matrix. The state of the pavement after one transition is obtained by multiplying the initial state by the transition matrix once. For two transitions, the multiplication is carried out twice, and so forth. In this example, the state of the pavement at the midpoint of its service life would require 50 such multiplications. Thus, in actual practice, it might be advisable to use fewer transitions per year, perform the calculations on a computer, or both.

In this approach, the procedure for obtaining predictions for future PSI values is fixed: Multiply the existing pavement state by the transition matrix. However, the formalism allows modification of pavement states and transition probabilities to account for such effects as resurfacing, accelerated pavement distress, increased traffic, and so forth. If the observed state of the pavement is found to

Figure 5. Transition probability matrix for Marcov sample problem.

TRANSITIONS TO THESE PSI STATES

	5.0	4.9	4.8	4.7	4.6	4.5	4.4	4.3	4.2	4.0	3.0	3,6	3.4	3,2	3.0	2.7	2.4	2.1	1.8	1.5
5.0			1.0																	
4.9			1.0																	
4.8			.010	.030	.100	. 220	. 280	. 220	.120	.020										
4.7			.002	.026	.153	.448	.210	.095	.053	.013	.001									
4.6			.002	.024	.145	.421	. 202	.099	.074	.030	.004									
4.5			.002	.023	.137	. 399	.193	.097	.081	.049	.017	.003	.001							
4.4			.002	.023	.137	. 399	.193	.097	.081	.049	.017	.003	.001							
4.3			.002	.021	.127	. 371	.179	.091	.079	.058	.036	.021	.011	.004	.001					
4.2			.002	.021	.119	.272	.184	.108	.098	.074	.048	.034	.022	.012	.006	,001				
4.0			.003	.012	.042	.097	.139	.138	.150	.120	.082	.063	.052	.038	.032	.019	.008	.003	.001	
3.8				.002	.011	.030	.061	.088	.153	.159	.116	.090	.078	.061	.059	.045	.026	.013	.006	.003
						.002	.006	.017	.055	.134	.148	.128	.112	.091						.016
3.4						.002	.005	.014	.045	.108	.119	.103	.092	.077	.084	.084	.073	.060	.053	.085
3.2								.003	.015	.056	.096	.111	.107	.091	.100	.100			.060	. 102
								.001	.006	.024	.056	.089	.109	.106	.117					.119
2.7									.001	.011	.029	.058	.091	.107	.130	.137	.119	.098	.082	. 139
2.4										.005	.015	.032	.066	.089	.130	.153	, 139	.115	.096	.162
2.1										.002	.008	.017	.044	.069	, 112	.157	.156	.134	.112	.190
1.8										.002	.008	.017	.044	.069	.112	.157	.156	.134	.112	.190
1.5											.002	.007	.024	.045	.090	.147	.164	.154	.133	.233
	4.9 4.8 4.7 4.6 4.5 4.4 4.3 4.0 3.8 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 4.9 4.8 4.7 4.6 4.5 4.4 4.3 4.2 4.0 3.8 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 4.9 4.8 4.7 4.6 4.5 4.4 4.3 4.2 4.0 3.8 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 4.7 .002 4.6 .002 4.5 .002 4.4 .002 4.3 .002 4.2 .002 4.0 .003 3.8 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 .030 4.7 .002 .026 4.6 .002 .024 4.5 .002 .023 4.4 .002 .021 4.2 .002 .021 4.0 .003 .012 3.8 .002 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 .030 .100 4.7 .002 .026 .153 4.6 .002 .024 .145 4.5 .002 .023 .137 4.4 .002 .023 .137 4.2 .002 .021 .127 4.2 .002 .021 .119 4.0 .003 .012 .042 3.8 .002 .011 3.6 3.4 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 4.7 .002 .026 .153 .448 4.6 .002 .024 .145 .421 4.5 .002 .023 .137 .399 4.4 .002 .023 .137 .399 4.3 .002 .021 .127 .371 4.2 .002 .021 .119 .272 4.0 .003 .012 .042 .097 3.8 .002 .011 .030 3.6 .002 3.4 .002 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 4.7 .002 .026 .153 .448 .210 4.6 .002 .024 .145 .421 .202 4.5 .002 .023 .137 .399 .193 4.4 .002 .023 .137 .399 .193 4.3 .002 .021 .127 .371 .179 4.2 .002 .021 .119 .272 .184 4.0 .003 .012 .042 .097 .139 3.8 .002 .011 .030 .061 3.4 .002 .005 3.2 3.0 2.7 2.4 2.1 1.8	5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 4.7 .002 .026 .153 .448 .210 .095 4.6 .002 .024 .145 .421 .202 .099 4.5 .002 .023 .137 .399 .193 .097 4.4 .002 .023 .137 .399 .193 .097 4.3 .002 .021 .127 .371 .179 .091 4.2 .002 .021 .119 .272 .184 .108 4.0 .003 .012 .042 .097 .139 .138 3.8 .002 .011 .030 .061 .088 3.6 .002 .001 .000 .061 .088 3.2 .003 .001 .002 .005 .014 3.2 .003 .001 .002 .005 .001 2.7 .2.4 .2.1 .11 .002 <td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 4.7 .002 .026 .153 .448 .210 .095 .053 4.6 .002 .024 .145 .421 .202 .099 .074 4.5 .002 .023 .137 .399 .193 .097 .081 4.4 .002 .023 .137 .399 .193 .097 .081 4.3 .002 .021 .127 .371 .179 .091 .079 4.2 .002 .021 .119 .272 .184 .108 .098 4.0 .003 .012 .042 .097 .139 .136 .150 3.8 .002 .011 .030 .061 .088 .153 3.6 .002 .005 .014 .045 3.2 .003 .006 .017 .055 3.0 .001 .006 .001 .006</td> <td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 4.6 .002 .024 .145 .421 .202 .099 .074 .030 4.5 .002 .023 .137 .399 .193 .097 .081 .049 4.4 .002 .023 .137 .399 .193 .097 .081 .049 4.3 .002 .021 .127 .371 .179 .091 .079 .058 4.2 .002 .021 .119 .272 .184 .108 .098 .074 4.0 .003 .012 .042 .097 .139 .138 .150 .120 3.8 .002 .011 .030 .061 .088 .153 .134 3.4 .002 .006 .017 .055 .134 3</td> <td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 .001 4.6 .002 .024 .145 .421 .202 .099 .074 .030 .004 4.5 .002 .023 .137 .399 .193 .097 .081 .049 .017 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 4.3 .002 .021 .127 .371 .179 .091 .079 .058 .036 4.2 .002 .021 .119 .272 .184 .108 .098 .074 .048 4.0 .003 .012 .042 .097 .139 .138 .150 .120 .082 3.8 .002 .011 .030 .061 .088 .153 .159 .116 <td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 .001 4.6 .002 .024 .145 .421 .202 .099 .074 .030 .004 4.5 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 4.3 .002 .021 .127 .371 .179 .091 .079 .058 .036 .021 4.2 .002 .021 .119 .272 .184 .108 .098 .074 .048 .034 4.0 .003 .012 .042 .097 .139 .138 .150 .120 .082 .063 3.8 .002 .011 .030 <t< td=""><td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 .001 4.6 .002 .024 .145 .421 .202 .099 .074 .030 .004 4.5 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.3 .002 .021 .127 .371 .179 .091 .079 .058 .036 .021 .011 4.2 .002 .021 .119 .272 .184 .108 .098 .074 .048 .034 .022 4.0 .003 .012 .042 .097 .139 .138 .150 .120 .082 .063 .052 <!--</td--><td>5.0 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.091 .079 .058 .036 .021 .011 .004 4.2 .002 .021 .119 .272 .184 .108 .098 .074 .048 .034 .022 .012 4.0 .003 .012 .042 .097 .139 .138 .150 .120 .082</td><td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 .001 4.6 .002 .024 .145 .421 .202 .099 .074 .030 .004 4.5 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.3 .002 .021 .127 .371 .179 .091 .079 .058 .036 .021 .011 .004 .001 4.2 .002 .021 .119 .272 .184 .108 .098 .074 .048 .014 .022 .012 .006 4.0 .003 .012 .042 .097 .139 .138 .150</td><td>5.0 1.0 4.9 1.0 4.8 .010 .030 .100 .220 .280 .220 .120 .020 4.7 .002 .026 .153 .448 .210 .095 .053 .013 .001 4.6 .002 .024 .145 .421 .202 .099 .074 .030 .004 4.5 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.4 .002 .023 .137 .399 .193 .097 .081 .049 .017 .003 .001 4.3 .002 .021 .119 .272 .184 .108 .098 .074 .048 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Table 1. Initial state specification for PSI = 4.5, SD = 0.1.

State Nominal PSI		Probability	State	Nominal PSI	Probability
1	5.0		11	3.8	
2	4.9	0	12	3.6	
3	4.8	0.006	13	3.4	
4	4.7	0.061	14	3.2	
5	4.6	0.241	15	3.0	
6	4.5	0.383	16	2,7	
7	4.4	0.241	17	2.4	
8	4.3	0.061	18	2.1	
9	4.2	0.007	19	1.8	
10	4.0	0	20	1.5	

differ from the predicted state, then the observed state may be substituted into the matrix multiplication process. Such a difference in observation and prediction could occur, for example, if the pavement were resurfaced after, say, two-thirds of the expected service life.

Of course, resurfacing, the occurrence of accelerated distress, or a dramatic increase in traffic volume could affect the expected rate of pavement deterioration as well as the current state of the pavement. These effects may also be incorporated into this formalism by adjusting the transition probabilities or by replacing Figure 5 with a transition matrix calculated on the basis of a faster or slower rate of deterioration. Thus, one transition matrix could be specified for the original pavement, another for overlaid pavements, etc. This is the approach taken by Smith (38). several different transition matrices are required for each pavement section to be studied, quite a large number of calculations would be required. However, the behavior illustrated in Figure 3 indicates that a wide variety of pavements may be represented by a single matrix. Thus, an agency could have one transition matrix, say, for each functional class of new pavement. An additional matrix could be specified for each functional class for overlaid pavements. If the agency must deal with pavements in widely differing environments, then a different set of matrices could be required for each different region. Thus, there is a reasonable expectation that 20 or 30 transition matrices could be sufficient to provide serviceability predictions for most or all of the pavements for which an agency is concerned. Such predictions would of necessity represent the average expected serviceability pattern for the pavement functional class, environment, etc., rather than the best estimates for an individual pavement section. Hence, such an approach is expected to be most useful for network-level pavement management applications.

SUMMARY

Pavement performance modeling is an essential part of good pavement management, and at the same time it is a very complex task. In general, the development of good performance models requires a good long-term data base, and the ideal experiment suggested here can provide guidance to agencies that wish to acquire such a data base. However, the acquisition of relevant data is of necessity a long-term operation, so it is important to realize that useful performance models may be developed for more immediate application.

The best approach to development of short-term performance models depends on the intended use of the model. At the project level, the use of existing mechanistic pavement distress models along with time-independent distress/performance correlations developed for small groups of similar pavements may provide acceptable performance models. At the network level, less detail is desired, and models based on probabilistic concepts and expert opinion may be acceptable. The simple stochastic model presented here is an example of the application of probabilistic concepts to network-level performance modeling that may provide guidance for agencies that wish to take such an approach.

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