Eccentrically Loaded Surface Footing on Sand Layer
Resting on Rough Rigid Base

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Laboratory model-test results are given for the ultimate bearing capacity of an eccentrically loaded rough rigid strip surface footing on sand that has a rough rigid base located at a shallow depth. For centrally loaded footing, the modified bearing-capacity factors calculated from the experimental ultimate loads are compared with the existing theory as presented by Mandel and Salencon. When the ratio of the depth of the sand layer to the width of the footing is smaller than 0.6, the experimental bearing capacity is somewhat lower than that predicted by theory by using the direct shear angle of friction. However, if the experimental variation of the bearing-capacity factor for the centrally loaded footing is assumed to be correct, Meyerhof's effective-area method may be used to estimate the bearing capacity of eccentrically loaded footing.

The ultimate bearing capacity of a shallow continuous footing in a homogeneous soil subjected to centric loading is generally expressed by the following equation:

\[ q_u = cN_c + qN_q + \frac{1}{2}yBN_y \]  

where

- \( q_u \) = ultimate load per unit area of footing,
- \( c \) = cohesion of soil,
- \( y \) = soil unit weight,
- \( B \) = width of footing, and
- \( N_c, N_q, \) and \( N_y \) = bearing-capacity factors.

For a footing placed at the surface of a cohesionless soil, \( c = 0 \) and \( q = 0 \); hence, Equation 1 will transform to the following form:

\[ q_u = \frac{1}{2}yBN_y \]  

A number of solutions for the variation of the bearing-capacity factor \( N_y \) with the angle of friction of the soil have been proposed in the past by such investigators as Terzaghi (~), Meyerhof (~), Caquot and Kerisel (~), and Lundgren and Mortensen (~).

If a rough rigid continuous surface footing that rests on a sand layer is subjected to a uniformly distributed load of magnitude \( q_u \), bearing-capacity failure will take place, and the slip lines will extend to a depth \( H_{cr} \) as shown in Figure 1a. However, if we consider a condition in which a rough rigid base is located at a shallow depth \( H \) so that \( H < H_{cr} \) (Figure 1b), the development of the slip lines at failure will be somewhat affected. Mandel and Salencon (~) have developed a theoretical solution for the ultimate bearing capacity for such cases, and it can be given by the following equation:

\[ q_u = \frac{1}{2}yBN_y' \]  

where \( N_y' \) is the modified bearing-capacity factor.

According to Mandel and Salencon (~), for \( H > H_{cr} \) the value of \( N_y' \) becomes equal to \( N_y \) as determined by Lundgren and Mortensen (~). On the other hand, for \( H < H_{cr} \) the modified bearing-capacity factor increases with the decrease of \( H/B \). The theoretical variation of \( N_y' \) with \( H/B \) is shown in Figure 2 (~, ~) for several values of the soil friction angle \( \phi \).

Experimental studies in the laboratory have been conducted by Meyerhof (~) and by Pfeifile and Das (~) to compare the experimental values of the modified bearing-capacity factor with the theoretical values presented in Figure 2. This study is not a repetition of any previous work by Dan (~).

The purpose of this paper is to present some small-scale laboratory model-test results for the ultimate bearing capacity of eccentrically loaded rough rigid surface footings on sand that have a rough rigid base located at a shallow depth as shown in Figure 3 and to evaluate whether the effective-area concept as suggested by Meyerhof (~) can be used in this case. It must be pointed out that Meyerhof's effective-area concept was originally suggested for conditions in which the rough rigid base is located at great depths (i.e., \( H/B > H_{cr}/B \)). According to the effective-area concept, if a load is applied on a strip footing that has an eccentricity \( e \) measured from the center line, it can be assumed to be equivalent to a strip footing of width \( B' = B - 2e \) that has the load applied along the center line as shown in Figure 4.

EXPERIMENTAL PROCEDURE

Model tests were conducted in a box that measured 0.915 m x 0.3048 m x 0.457 m (3 ft x 1 ft x 1.5 ft). The walls of the box were reinforced by steel channels against possible yielding during tests. The bottom of the box was a wooden plank 50.8 mm (2 in) thick. In order to make the bottom of the box rough, a sand-glue mixture was spread over a masonite panel. This was allowed to dry for several days and was then attached to the bottom of the box by means of wood screws. The sand used for the sand-glue mixture was the same sand used for the model tests.

Figure 1. Bearing-capacity failure for rough rigid surface footing on sand.

(a) Centrically loaded footing

(b) Eccentrically loaded footing
The sand used for the test had 100 percent passing through a no. 10 sieve, 52 percent passing through a no. 40 sieve, and 4 percent passing through a no. 200 sieve. For the model tests, the sand was compacted in small layers in the test box to the required depth to an average unit weight of 16.0 kN/m³ (101.8 lbf/ft³). This yielded an average relative density of compaction of 80 percent. At this compaction, the triaxial angle of friction was determined to be 39° and the direct shear angle of friction was 42°.

The model footing was made of a 12.7-mm (0.5-in) steel plate that measured 101.6 mm x 304.8 mm (4 in x 12 in). The bottom of the model footing was made rough by spreading a similar sand-glue mixture as that used for the bottom of the box and allowing it to dry.

Vertical load was applied to the model footing by a hydraulic jack through a steel shaft 38.1 mm (1.5 in) in diameter. The bottom of the shaft would just fit were cut on the top of the footing. This allowed free rotation of the footing during failure. These grooves were cut parallel to the center line of the footing at distances of e = 0, 12.7 mm (0.5 in), and 19.05 mm (0.75 in). During the model tests, the load was measured by a proving ring. The deflection along the center line of the footing was measured by a dial gauge. Tests were conducted at various values of e/B and H/B. Some grain crushing occurred during the tests at lower values of H/B.

MODEL-TEST RESULTS

Diagrams of typical load per unit length versus settlement of footing for the model footing at H/B = 0.375 determined from the laboratory tests are shown in Figure 5. The ultimate load per unit length of the footing at failure Q_u was determined from these diagrams. Figure 6 shows a plot of Q_u versus H/B for all the tests conducted in this program.

For tests in which the loads on the footing were centrally applied (i.e., e/B = 0), the modified bearing-capacity factor can be given by

\[ N'_y = \frac{Q_u(B)}{l/2 y B^2 N'_y} \text{ (Equation 3), or} \]

\[ N'_y = \frac{Q_u}{0.5 y B^2} \]  

(4)

By using the experimental values of Q_u (for the tests in which e/B = 0) given in Figure 6, the values of N'_y were determined and are shown in Figure 7. For comparison purposes, the theoretical values of N'_y (from Figure 2) for \( \phi = 39°, 42°, \) and 43° have been plotted in Figure 7. A comparison of the experimental and theoretical values shows the following:

1. The experimental values of N'_y are higher than those presented by theory with \( \phi = 39° \), which is the triaxial angle of friction. However, for tests of continuous footing, the plane strain friction angle should be used. The experimental results of N'_y are fairly close to those predicted by using friction angles determined from direct shear tests up to a value of H/B > about 0.6. In the region of H/B between 0.6 and 0.4, the experimental values are lower than those of the theory (by using direct shear angle of friction).

2. The experimental values of N'_y for H/B < 0.4 are lower than those predicted by theory using the triaxial friction angle of 39°. There can be several factors that would cause the type of experimental results obtained here and their derivations from the theory for the zone of H/B < 0.6. They are as follows: (a) the sand-placement technique makes the sand anisotropic; (b) although a
quantitative evaluation was not made, some grain crushing did occur during the tests; and (c) there was curvilinearity of the Mohr-Coulomb failure envelope at very high pressures.

However, an evaluation of the model-test results shows that, if the experimental variation of $N'_y$ versus $H/B$ is assumed to be correct, the ultimate bearing capacity of the eccentrically loaded footing can be evaluated approximately by using the effective-area concept presented by Meyerhof (8) for footings in which $H/B > H_{cr}/B$. Hence,

$$B' = B - 2e$$

where $B'$ is the effective width.

For surface footings, the ultimate load per unit length can be expressed as follows:

$$Q_u = \frac{1}{2\gamma}(N'_y(B')^2) = \frac{1}{2\gamma}B'^2N'_y$$

$$N'_y = Q_u/(0.5\gamma(B - 2e)^2)$$

(6)

Note that $N'_y$ is now a function of $H/B'$. For centrally loaded footings (i.e., $e/B = 0$), the experimental modified bearing-capacity factor $N'_y$ compares reasonably well with the theory (with the assumption that the direct shear angle of friction is valid) only in the range of $H/B > 0.6$. For $H/B < 0.4$, the experimental values of $N'_y$ are lower than those predicted by theory, even by using the triaxial friction angle. Grain crushing, curvilinearity of the Mohr-Coulomb failure envelope at high normal stress, and possible anisotropy in sand due to the placement technique may be responsible for such results.

2. If the experimental variation of $N'_y$ with $H/B'$ for centrally loaded footings is assumed to be correct, the ultimate bearing capacity of eccentrically loaded surface footings can be reasonably estimated by using the concept of effective area for continuous footing, $Q_u = 1/2\gamma(B - e)^2N'_y$.

REFERENCES


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