ACKNOWLEDGMENT

This paper was written while I was a faculty fellow at the Transportation Systems Center, U.S. Department of Transportation, Cambridge, Massachusetts.

REFERENCES

1. R.G. Hennes and M. Ekse. Fundamentals of Transportation Engineering. McGraw-Hill, London, 1969.
2. J.S. DeSalvo and L.B. Lave. A Statistical Engineering Approach to Estimation of Railway Cost Functions. Rand Corp., New York, NY, 1968.
3. A.F. Daughety and M.A. Turnquist. Development of Hybrid Cost Functions from Engineering and Statistical Techniques: The Case of Rail--First Year Final Report. Transportation Center, Northwestern Univ., Evanston, IL, 1979.
4. J. Marsden, D. Pingry, and A. Whinston. Engineering Foundations of Production Functions. Journal of Economic Theory, 1974, pp. 124-140.
5. H.B. Chenery. Engineering Production Functions. Quarterly Journal of Economics, Vol. 63, No. 4, Nov. 1949, pp. 507-531.
6. A.D. Schuster. An Economic Analysis of Motor Carrier Less-Than-Truckload Transportation Services. Ohio State Univ., Columbus, Ph.D. dissertation, 1977.
7. A.D. Schuster. Statistical Cost Analysis: An Engineering Approach. Logistics and Transportation Review, Vol. 14, No. 2, 1978, pp. 151-164.
8. P.A. Dierks. Applying Cost Accounting to Transit System Financing. Management Accounting, Vol. 60, No. 6, Dec. 1978, pp. 20-23.
9. W. Cherwony. Cost Centers: A New Approach to Transit Performance. Transit Journal, Vol. 3, No. 4, Fall 1977, pp. 70-80.
10. H.O. Whitten. Functional Cost Accounting for Railroads. ICC Practitioners Journal, Vol. 43, March 1976.
11. R. Young. A New Regulatory Accounting System for Railroads. ICC Practitioners Journal, Vol. 43, May 1976.
12. R.R. Braeutigam. An Analysis of Fully Distributed Cost Pricing in Regulated Industries. Bell Journal of Economics, Vol. 11, No. 1, Spring 1980, pp. 182-196.
13. R.H. Spady and A.F. Friedlaender. Econometric Estimation of Cost Functions in the Transportation Industries. Center for Transportation Studies, Massachusetts Institute of Technology, Cambridge, Sept. 1976.
14. R.H. Spady and A.F. Friedlaender. Hedonic Cost Functions for the Regulated Trucking Industry. Bell Journal of Economics, Vol. 9, No. 1, Spring 1978, pp. 159-179.
15. G.W. Wilson. On the Output Unit in Transportation. Land Economics, Vol. 35, No. 3, Aug. 1959.
16. W.G. Waters II. Statistical Costing in Transportation. Transportation Journal, Vol. 15, Spring 1976, pp. 49-62.
17. T.H. Oum. A Cross-Sectional Study of Freight Transport Demand and Rail-Truck Competition in Canada. Bell Journal of Economics, Vol. 10, No. 2, Autumn 1979, pp. 463-482.
18. R.S. Brown, D.W. Caves, and L.R. Christensen. Modeling the Structure of Cost and Production for Multiproduct Firms. Southern Economic Journal, Vol. 40, No. 1, July 1979, pp. 256-273.
19. D.W. Caves, L.R. Christensen, and J. Swanson. Productivity in U.S. Railroads, 1951-1974. Bell Journal of Economics, Vol. 11, No. 1, Spring 1980, pp. 65-83.
20. P.A. Samuelson and S. Swamy. Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis. American Economic Review, Vol. 64, 1974, pp. 566-593.
21. F.M. Fisher. Approximate Aggregation and the Leontief Conditions. Econometrica, Vol. 37, July 1969, pp. 457-469.

Publication of this paper sponsored by Task Force on Economic Analysis.

Abridgment

New Ranking Procedure and Set of Decision Rules for Method of Internal Rate of Return

MARTIN WOHL

A new ranking procedure and revised set of decision rules have been developed for the method of the internal rate of return. Their application will lead to a clear-cut and proper decision about acceptability and about the best alternative, at least as long as the minimum attractive rate of return is at least as large as the borrowing rate for capital that must be acquired outside the firm or agency.

I will not argue here about which economic analysis method (e.g., internal rate of return, net present value, or benefit/cost ratio) is preferable but instead will outline a new ranking procedure and a new set of decision rules for the method of internal rate of return in order to ensure that the decisions that result from its use are always correct and unambiguous. Of some importance, this discussion will be limited to cases in which the minimum attractive rate of return (MARR) will be at least as large as the borrowing rate (BORR) for capital that must be acquired outside the firm or agency. [For a discussion of the case in which MARR < BORR see the Discussions and Closures included with the paper by Wohl (1).]

SITUATIONS THAT CAN LEAD TO AMBIGUOUS OR INCORRECT DECISIONS

One situation that sometimes leads to incorrect or ambiguous decisions is that in which there is more than one internal rate of return for a given alternative. Specifically, whenever the net annual cash flows during the n-year analysis period (i.e.,

Table 1. Annual cash flows for bridge improvement.

End of Year t	B_t[a] (\$000s)	C_t[b] (\$000s)	$B_t - C_t$ (\$000s)
0	-	50	-50
1	61	55	+6
2	63	0	+63
.	.	.	.
.	.	.	.
.	.	.	.
9	77	0	+77
10	79	705	-626
11	81	610	-529
12	83	495	-412
13	85	0	+85
.	.	.	.
.	.	.	.
.	.	.	.
29	117	0	+117
30	119	0	+119

Notes: Internal rates of return r are 8.52, 18.66, and 73.57 percent. $[NPW]_{0\ percent} = +785$.
[a]Benefits during year t, net of annual maintenance and operating costs.
[b]Nonrecurring costs during year t.

Table 2. Annual cash flows for local streetcar-line extension.

End of Year t	B_t (\$000s)	C_t (\$000s)	$B_t - C_t$ (\$000s)
0	0	175	-175
1	0	1265	-1265
2	250	0	250
3	240	0	240
4	230	0	230
5	220	0	220
.	.	.	.
.	.	.	.
.	.	.	.
19	80	0	80
20	70	0	70
21	60	0	60
22	0	1900	-1900

Notes: r = 3.85 and 4.99 percent. $[NPW]_{0\ percent} = -240$.

Table 3. Annual cash flows for oil-pump alternatives.

	Alternative 1		Alternative 2		
Year	$B_{1,t}$	$C_{1,t}$	$B_{2,t}$	$C_{2,t}$	$\Delta B_t - \Delta C_t$[a]
0		100		110	-10
1	70		115		+45
2	70		30		-40

Notes: $[NPW_x]_{0\ percent} = +40$ for alternative 1, +35 for alternative 2. r_x or Δr, $r_1 = 25.69$ percent, $r_2 = 26.16$ percent, $\Delta r = 21.92$ and 228.08 percent.
[a]$\Delta B_t = B_{2,t} - B_{1,t}$ and $\Delta C_t = C_{2,t} - C_{1,t}$.

$B_t - C_t$ for $t = 0,\ldots,n$) are such that there are two or more sign changes, the possibility of multiple nonnegative internal rates of return arises. An example of this case is shown in Table 1 and might apply when a two-stage improvement program for an existing highway or bridge (e.g., minor repairs now and a major overhaul later) is analyzed. In this instance, an incorrect or ambiguous economic decision could result if the MARR were, say, 10 percent. Would one reject the project or not? Without additional information, the choice is not clear.

A second but different example of the above case is shown in Table 2. Such a situation might arise if a transit company was granted a 22-year franchise and allowed to build a streetcar line on the condition that the streetcar tracks had to be removed and the street returned to its original condition at the end of the 22-year franchise. Again, it is not clear how these two internal rates of return should be interpreted. If, for instance, the MARR were 3 percent, should we accept or reject the project?

The third situation in which difficulties can arise is that which involves incremental analysis between pairs of alternatives. Briefly, variations in benefit-accrual patterns as well as fluctuations in cost outlays can lead to multiple internal rates of return for the increments in benefit and cost between the two alternatives being compared. Fourth, erroneous decisions can result from ambiguities about how to rank alternatives that have equal initial costs.

The oil-pump example in Table 3 is the third type of situation. Briefly, the extra investment for a larger oil pump leads to an overall increase in oil production but, more importantly, permits earlier extraction of most of the remaining deposits. As a consequence, the incremental net cash flows (i.e., $\Delta B_t - \Delta C_t$ for $t = 0,1,2$) shown in Table 3 indicate two sign reversals and thus the possibility of two nonnegative incremental rates of return. In this case, there were two such rates, 21.92 percent and 228.08 percent. In turn, we must ask how to interpret the two rates. If the MARR was about 20 percent, one would probably conclude that alternative 2 (i.e., the larger oil pump) was economically preferable. That is, since $r_1 >$ MARR, alternative 1 is acceptable; since both values of Δr (the internal rates of return on the increments in cost and benefit) are greater than MARR, one presumably would regard alternative 2 as better than alternative 1 or one would regard the choice as ambiguous.

REVISED RANKING PROCEDURE

First, let me define the procedure for ranking mutually exclusive alternatives for the purpose of determining which alternative is best.

1. Determine the net present worth (NPW) for each alternative that is being analyzed at a 0 percent interest rate; that is, simply sum the net (undiscounted) annual cash flows for the n-year analysis period, or

$$[NPW_x]_{0\ percent} \sum_{t=0}^{n} (B_{x,t} - C_{x,t}) \qquad (1)$$

where $B_{x,t}$ and $C_{x,t}$ are the benefits and costs, respectively, for project x during year t of the n-year analysis period and $[NPW_x]_{0\ percent}$ is the NPW for project x at an interest rate of 0 percent.

2. Rank all alternatives in ascending order with respect to the above $[NPW_x]_{0\ percent}$ values. It is important to note that the resultant ranking can and often will differ markedly from the usual (but undesirable) ranking rule, which calls for ordering according to the initial year's costs or outlays. For instance, the above set of ranking rules would reverse the usual ranking of the alternatives as they are shown in Table 3.

REVISED DECISION RULES FOR DETERMINING ACCEPTABILITY

1. Determine $[NPW_x]_{0\ percent}$ as indicated in Equation 1.

2. Determine r_x, the internal rate of return for alternative x (i.e., determine the discount rate or rates at which the discounted benefits just equal the discounted costs over the n-year planning horizon). If there are multiple rates of return, list them in ascending order, as follows: r_x', r_x'', r_x''', r_x'''', ...; however, exclude all nonpositive rates.

3. When $[NPW_x]_{0\ percent} \neq 0$, accept or re-

ject alternative x according to the following rules:

Condition	Slope $[NPW_x]_{0\ percent} > 0$	$[NPW_x]_{0\ percent} < 0$
MARR < r_x or MARR < r_x'	Accept	Reject
r_x' < MARR < r_x''	Reject	Accept
r_x'' < MARR < r_x'''	Accept	Reject
r_x''' < MARR < r_x''''	Reject	Accept
Etc.	Etc.	Etc.

4. When $[NPW_x]_{0\ percent} = 0$, first determine the slope of the NPW function for an interest rate of 0 percent, as follows:

$$\text{Slope of } [NPW_x]_{0\ percent} = -\sum_{t=1}^{n} t(B_{x,t} - C_{x,t}) \quad (2)$$

In turn, accept or reject alternative x according to the following rules when there are multiple rates of return:

Condition	Slope Greater Than Zero	Less Than Zero
MARR < r_x'	Accept	Reject
r_x' < MARR < r_x''	Reject	Accept
r_x'' < MARR < r_x'''	Accept	Reject
r_x''' < MARR < r_x''''	Reject	Accept
Etc.	Etc.	Etc.

Also, when there is only one internal rate of return and thus r_x is equal to 0 percent, accept the alternative when the slope is positive and reject the alternative when the slope is negative.

5 Whenever all internal rates of return are negative or indeterminate, accept alternative x if the $[NPW_x]_{0\ percent}$ is nonnegative (i.e., ≥ 0) and reject it if the $[NPW_x]_{0\ percent}$ is negative.

REVISED RULES FOR DETERMINING BEST ALTERNATIVE

1. Determine $[NPW_x]_{0\ percent}$ for all mutually exclusive alternatives.
2. Rank the alternatives in ascending order with respect to the NPW at 0 percent (i.e., $[NPW_x]_{0\ percent}$ for all x). However, if the NPW values for two or more alternatives are equal, determine the slope of the NPW function at 0 percent (as shown in Equation 2) and rank them in ascending order with respect to the algebraic value of the slopes; that is, the alternative that has the most-positive (or least-negative) slope will be the highest-ranked al- ternative.
3. Determine the incremental internal rate of return for increments in benefits and costs between the lowest-ranked pair of alternatives or Δr_{1-2}; if there are multiple incremental rates of return, list them in ascending order as follows: $\Delta r_{1-2}'$, $\Delta r_{1-2}''$, $\Delta r_{1-2}'''$,...; however, exclude all nonpositive rates.
4. Accept or reject the higher-ranked alternative of the two being compared according to the following rules:

Condition	Rule
MARR < Δr_{1-2} or MARR < $\Delta r_{1-2}'$	Accept
$\Delta r_{1-2}'$ < MARR < $\Delta r_{1-2}''$	Reject
$\Delta r_{1-2}''$ < MARR < $\Delta r_{1-2}'''$	Accept
$\Delta r_{1-2}'''$ < MARR < $\Delta r_{1-2}''''$	Reject
Etc.	Etc.

5. Apply the above acceptability test to successively higher-ranked alternatives until the highest-ranked alternative is found that is more acceptable than lower-ranked ones. That is, if alternative 2 is more acceptable than alternative 1, then apply the test to determine whether alternative 3 is more acceptable than alternative 2, and so forth. But if alternative 2 is rejected in favor of alternative 1, then compare alternatives 1 and 3 to determine whether alternative 3 is more acceptable than alternative 1.
6. Whenever all incremental internal rates of return are negative or indeterminate, the higher-ranked alternative (according to the rule cited above in Equation 2) will always be preferable.

SUMMARY

The new ranking procedure and set of decision rules for applying the method of the internal rate of return to the evaluation of mutually exclusive alternatives has been described in some detail. Its use will ensure that the economic decisions resulting therefrom (about acceptability and which project is best) will be identical to those that will prevail from use of either the benefit-cost-ratio or net-present-value methods.

ACKNOWLEDGMENT

This paper was written while I was on leave from Carnegie-Mellon University. I am grateful to my colleague, Tung Au, for bringing the modifications noted in this paper to my attention. Were it not for his careful and painstaking review, I would have overlooked the cases when roots are negative or indeterminate.

REFERENCE

1. M. Wohl. Common Misunderstandings About the Internal-Rate-of-Return and Net Present Value Economic Analysis Methods. TRB, Transportation Research Record 731, 1979, pp. 1-19.

Publication of this paper sponsored by Task Force on Economic Analysis.