work diagram. Subsequently, these errors can be corrected by using the interactive data modification capability provided. This portion of the PRENET enhancement provides the user with an intuitive, direct, and efficient method to debug the NETSIM input data.

Having used NETSIM to define potential problems that relate to traffic control strategies in an urban street network, traffic engineers must gain a full understanding of how and why such problems have evolved. The ICG enhancements NETDIS and POSDIS demonstrate the feasibility and usefulness of graphically displaying link-specific MOEs as generated by the NETSIM simulation model. Such displays provide the user with more easily assimilated information with which operation of the network can be comprehended. More precisely, the lane-specific queue build-ups can be visualized, and the overall network relative performance measures can be obtained at a glance.

With respect to the development of additional graphics capabilities, work is required to generate time-space diagrams and displays of signal indications on the lane-detailed network plots.

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# Transferability and Analysis of Prediction Errors in Mode-Choice Models for Work Trips

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**Some analyses of predictive accuracy and transferability of disaggregate work-trip mode-choice models are reported. The prediction error is separated into three components: model error, aggregation error, and transfer error. The results show that the weighted root mean square of total error is between 25 and 60 percent of the predicted shares and the distribution of total error between error sources seems to depend on how well the model is transferred. The main results of the research are that (a) total forecasting errors may be large, especially if the model transfers poorly; (b) transferability between cities in which the transit shares are very different is poor; (c) market segmentation improves forecasting accuracy only marginally, if at all, and; (d) the type of level-of-service data, i.e., manually coded versus network based, used in model estimation and prediction has some bearing on forecasting accuracy, and the use of zonally averaged socioeconomic attributes appeared to be somewhat detrimental to prediction. These and other results are to be held tentative for reasons discussed in detail.**

The sources of the total error for work-trip mode-choice models are identified and their contributions are analyzed separately. In addition, citywide predictions of travel demands are also investigated. Four data sets were used in this study. These were the data from the Minneapolis-St. Paul area (collected in 1970); the two urban travel demand forecasting surveys from the San Francisco Bay area conducted before and after the introduction of Bay Area Rapid Transit (BART) service (collected in 1972 and 1975, respectively); and the Baltimore travel demand data set, a comprehensive set of information that describes travel behavior of 967 households in Baltimore, Maryland (collected in 1977).

The effect of market segmentation on forecasting accuracy is studied by using the same model specification as that for the unsegmented market. Three types of market segments were used: households that had one car versus those that had two or more; commuters bound for the central business district (CBD) versus others; and low-income versus high-income households (annual household incomes of $12 000,

\$13 000, and \$15 000 were used as dividers between high and low income for travelers in Minneapolis, San Francisco, and Baltimore metropolitan areas, respectively).

It had previously been found (1) that the model coefficients for the two income groups were different for the models that used post-BART data; except for the alternative-specific constants, the coefficients were equal for the other two market segments. In the models that used Baltimore data, only the travel-time coefficient was statistically different for all the market segments, and for the pre-BART models the alternative-specific constants were different for all the aforementioned market segments (2). An interesting question, therefore, is whether statistical inequality of coefficients means substantially different forecasts.

The results are presented in seven sections. First, the method of analysis is described. Then a discussion of model errors follows. Third, the error in prediction due to zonal averaging of the explanatory variables is analyzed. Fourth, errors due to transfer of models over time and space are discussed. In the fifth section, total prediction error is analyzed. Then citywide modal predictions are examined. Seventh, conclusions are presented.

## METHOD

Average absolute error (AAE) and the root-mean-square error (RMSE) are used as error measures. Both are expressed as a fraction of the predicted share and calculated as follows. Actual and predicted modal shares are first calculated from 50 draws (observations) selected at random from the appropriate set of data. This process is repeated 50 times and AAE and RMSE are calculated to obtain error in the predicted share of each alternative.

AAE and RMSE are defined as follows:

$$AAE_j = (1/NT) \left[ \sum_{i=1}^{NT} |P_j - A_j|/P_j \right] \tag{1}$$

$$RMSE_j = (AE_j^2 + SE_j^2)^{1/2} \tag{2}$$

where

$$AE_j = (1/NT) \left[ \sum_{i=1}^{NT} (P_j - A_j)/P_j \right] \tag{2a}$$

$$SE_j = \left( \left\{ \sum_{i=1}^{NT} \left[ (P_j - A_j)/P_j \right]^2 - NT * AE_j^2 \right\} / (NT - 1) \right)^{1/2} \tag{2b}$$

and

JT = total number of alternatives in choice set,
NT = number of times 50 random draws are repeated (i.e., 50 times),
$AE_j$ = average error as percentage of predicted share for alternative j,
$SE_j$ = standard error for alternative j,
$P_j$ = average predicted share of alternative j calculated from 50 random draws, and
$A_j$ = average observed share of alternative j calculated from 50 random draws.

The overall error measures are the weighted average absolute error (WAAE) and weighted root-mean-square error (WRMSE) (3). These are defined by the following equations:

$$WAAE = \sum_{j=1}^{JT} \left[ AAE_j * (1/NT) \left( \sum_{i=1}^{NT} P_j \right) \right] \tag{3}$$

$$WRMSE = \sum_{j=1}^{JT} \left[ RMSE_j^2 * (1/NT) \left( \sum_{i=1}^{NT} P_j \right) \right]^{1/2} \tag{4}$$

WRMSE can also be disaggregated into weighted average error (WAE) and weighted standard deviation of the error (WSDE) as follows:

$$WAE = \sum_{j=1}^{JT} \left[ AE_j * (1/NT) \left( \sum_{i=1}^{NT} P_j \right) \right] \tag{5}$$

$$WSDE = (WRMSE^2 - WAE^2)^{1/2} \tag{6}$$

### Disaggregation of Errors in Prediction

Total error in prediction may be attributed to the source of the error in the following way. Total WAAE is the sum of WAAEs contributed by each component, and total WRMSE is the sum of WRMSEs contributed by each component. They are defined as follows:

$$WAAE_T = WAAE_M + WAAE_A + WAAE_F \tag{7}$$

$$(WRMSE)_T^2 = (WRMSE)_M^2 + (WRMSE)_A^2 + (WRMSE)_F^2 \tag{8}$$

where

T = total error,
M = error in choice model,
A = error in aggregation, and
F = error in transfer.

### Mode-Choice Models Used in Analysis

The work-trip mode-choice models used in the analysis are a five-mode-choice model (drive alone, local bus, express bus, rail, shared ride) developed by using post-BART data (1); a three-mode-choice model (drive alone, bus, shared ride) developed by using Baltimore data (2); and a four-mode-choice model (drive alone, bus with walk access, bus with car access, and shared ride) that used pre-BART data. The model specification and coefficients are given in Tables 1 and 2. The same specification was also used for travelers in different market segments. [A

**Table 1. Model specification and coefficients: Baltimore data.**

| Variable | Alternative Entered[a] | With WORKRS Coefficient | With WORKRS t-Value | Without WORKRS Coefficient | Without WORKRS t-Value |
|---|---|---|---|---|---|
| TTIME | 1-3 | −0.008 56 | 3.10 | −0.008 65 | 3.30 |
| COST/INC | 1-3 | −29.091 | 1.97 | −24.881 | 1.70 |
| CARS | 1,3 | 0.421 | 3.10 | 0.365 | 2.80 |
| WACCESS | 2 | 0.350 | 0.84 | 0.292 | 0.73 |
| CBD | 1,3 | −1.114 | 2.24 | −0.892 | 1.87 |
| WORKRS | 2 | 0.403 | 5.61 | - | - |
| INC | 1,3 | 0.000 031 2 | 2.35 | 0.000 019 9 | 1.70 |
| CONST | 1 | 0.779 | 1.44 | −0.102 | 0.20 |
| CONST | 3 | −0.495 | 0.94 | −1.359 | 2.80 |

[a]Alternatives: 1 = drive alone, 2 = bus, 3 = shared ride (≥2 occupants). Model used is multinomial logit fitted by maximum-likelihood method.

note on the nomenclature is in order. Two types of coefficients or post-BART models are used in the analyses. The first set of coefficients, called "true coefficients," were estimated by using the individual socioeconomic attributes and observed (manually coded) level-of-service attributes (TRUE-LOS). The second set of coefficients, called "network coefficients," were estimated by using the individual socioeconomic attributes and zone-to-zone network level-of-service attributes (NET-LOS). In addition, two specifications (coefficients) for Baltimore and pre-BART models are used. In the first, the number of workers in the household (WORKRS) is included; in the second, it is excluded from the model.] For detailed information regarding model specification and estimated coefficients of market-specific models, see the papers by Dehghani and Talvitie (1,2) and by Dehghani (4). The explanatory variables are defined below:

| Variable | Definition |
|---|---|
| INVT | In-vehicle time or time spent inside a vehicle when traveling from origin to destination, door to door (round-trip time) (min) |
| WKT | Walk time to and from bus stop or rail station, in transfer, or to and from car's parking place (min) |
| WT | Sum of wait times of all transit vehicles, normally one half of first headway plus transfer wait (min) |
| TTIME | Sum of in-vehicle time, walk time, and wait time (min) |
| COST | Out-of-pocket travel cost (cents) |
| INC | Household income (dollars per year) |
| WORKRS | Number of workers in household |
| CARS | Number of cars owned |
| EMPD | Employment density in neighborhood (employees per acre) |
| CBD | Dummy variable constructed to differentiate trips destined to CBD from those destined to other locations (takes value of 1.0 if trips are destined to CBD, zero otherwise) |
| WACCESS | Walk access to transit facility (takes value of 1.0) |
| CONST | Constant (takes value of 1.0 for specified alternative, zero otherwise) |

The supporting statistics are given below (the success index is the weighted average of differences in correct prediction between the full model and the model that has only alternative-specific constants). For Table 1:

| Statistic | With WORKRS | Without WORKRS |
|---|---|---|
| L*(0) | -564.802 | -564.802 |
| L*(β) | -445.632 | -463.02 |
| Percent right (maximum utility classification) | 64.89 | 62.5 |
| Sample size | 544 | 544 |
| Success index | 0.124 | 0.104 |
| Proportion successfully predicted (expected value) | 0.513 | 0.493 |
| Proportion of prediction success due to variables other than alternative-specific constants | 0.124/0.513 = 0.24 | 0.104/0.493 = 0.21 |

and for Table 2:

| Statistic | With WORKRS | Without WORKRS |
|---|---|---|
| L*(0) | -1197.66 | -1198.35 |
| L*(β) | -776.98 | -791.90 |
| Percent right | 66.33 | 65.67 |
| Sample size | 906 | 906 |
| Success index | 0.115 | 0.107 |
| Proportion successfully predicted | 0.523 | 0.514 |
| Proportion of prediction success due to variables other than alternative-specific constants | 0.22 | 0.21 |

MODEL ERROR

Definition

The model error captures errors caused by several factors. These are the specification error due to omitted variables and the model form (i.e., logit); the sampling errors in the model parameters; and the sampling errors in the estimated shares. Except for the sampling errors in the estimated shares, the other components of the model error are present in the forecasting situation also; they are an inherent part of the model. The possible sampling errors in estimated shares can be easily quantified and subtracted out of the model error. For example, SD (sampling error) for an alternative that has a sample size of 50 and an estimated share of 10 percent is $\delta_p = [0.10 \quad (1 - 0.10)/50.0]^{1/2} = 0.0424$, and the sampling error as a percentage of the estimated share is $(\delta_p/p) = (0.0424/0.10) = 0.424$. But in reality the error in estimated shares always exists, now as well as in the future. For this reason, the logic of subtracting it out of the model error is not self-evident.

The model error is calculated by using Equations 1 through 4 and the appropriate data set for this calculation in the estimation sample itself. The results are presented in Table 3.

It appears that the variable WORKRS, in spite of its statistical significance, does not reduce the

Table 2. Model specification and coefficients: pre-BART data.

| Variable | Alternative Entered[a] | With WORKRS Coefficient | With WORKRS t-Value | Without WORKRS Coefficient | Without WORKRS t-Value |
|---|---|---|---|---|---|
| TTIME | 1-4 | -0.015 7 | 5.98 | -0.015 3 | 5.96 |
| COST/INC | 1-4 | -16.154 | 2.70 | -16.251 | 2.70 |
| CARS | 1,4 | 1.326 | 7.50 | 0.928 | 6.0 |
| EMPD | 2,3 | 0.002 34 | 4.0 | 0.002 16 | 3.82 |
| EMPD | 1 | -0.003 73 | 5.50 | -0.003 76 | 5.50 |
| WORKRS | 2,3 | 0.925 | 5.31 | - | - |
| CONST | 1 | 0.955 | 2.99 | -0.014 4 | 0.06 |
| CONST | 3 | -1.437 | -7.30 | -1.441 | 7.35 |
| CONST | 4 | -0.354 | 1.14 | -1.338 | 5.48 |

[a]Alternatives: 1 = drive alone, 2 = bus and walk, 3 = bus and car, 4 = shared ride. Model used is multinomial logit fitted by the maximum-likelihood method.

**Table 3. Prediction error due to model specification.**

| Market Segment | Model Coefficient | Baltimore Data | | | | Pre-BART Data | | | | Post-BART Data | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | With WORKRS | | Without WORKRS | | With WORKRS | | Without WORKRS | | NET-LOS Coefficient | | TRUE-LOS Coefficient | |
| | | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE |
| Total | C | 12.1 | 16.3 | 14.0 | 18.3 | 15.1 | 22.8 | 15.3 | 22.6 | 17.8 | 25.8 | 14.0 | 22.0 |
| High-income | C | 15.4 | 20.8 | 17.6 | 23.1 | 16.0 | 29.6 | 16.0 | 29.6 | 37.0 | 27.8 | 18.0 | 27.8 |
| | MS | 17.1 | 22.7 | 14.4 | 20.4 | 19.7 | 52.5 | 19.7 | 52.5 | 13.0 | 21.2 | 13.0 | 20.0 |
| Low-income | C | 17.2 | 20.7 | 24.4 | 28.2 | 14.6 | 19.7 | 14.6 | 19.7 | 16.0 | 28.0 | 18.0 | 23.7 |
| | MS | 11.8 | 15.1 | 12.1 | 15.4 | 34.6 | 30.2 | 24.6 | 30.2 | 16.0 | 25.8 | 14.0 | 22.5 |
| CBD | C | NA | NA | NA | NA | 13.4 | 18.7 | 12.9 | 18.3 | 19.0 | 24.5 | 16.0 | 21.7 |
| | MS | NA | NA | NA | NA | 13.6 | 22.5 | 13.1 | 21.8 | 16.0 | 20.9 | 15.0 | 19.8 |
| Non-CBD | C | NA | NA | NA | NA | 17.7 | 38.7 | 18.5 | 36.8 | 17.0 | 35.9 | 14.0 | 27.9 |
| | MS | NA | NA | NA | NA | 10.9 | 17.0 | 10.9 | 17.0 | 15.0 | 29.6 | 13.0 | 26.6 |
| One-car household | C | 20.1 | 29.6 | 20.4 | 29.4 | 16.7 | 23.3 | 17.1 | 23.3 | 15.5 | 30.7 | 13.7 | 21.9 |
| | MS | 12.5 | 16.7 | 12.9 | 17.4 | 17.1 | 26.2 | 24.8 | 37.9 | 16.0 | 25.5 | 14.0 | 22.3 |
| Household with two or more cars | C | 14.6 | 19.9 | 15.0 | 20.3 | 16.8 | 40.6 | 16.2 | 41.6 | 17.8 | 38.0 | 14.0 | 29.1 |
| | MS | 10.5 | 15.8 | 10.5 | 15.6 | 14.2 | 24.5 | 24.5 | 32.8 | 14.0 | 27.3 | 13.0 | 23.8 |

Notes: NA = not applicable due to small sample size.
Model coefficients: C = common, MS = market-specific.

model error in either the Baltimore model or the pre-BART model. [Note that the variable WORKRS was selected not only for its statistical significance but for its contribution to improving the model's summary prediction success indices as well. Other specifications were also studied.] The post-BART model with TRUE-LOS (i.e., true coefficients) does have a smaller model error than the NET-LOS-based model does, but only marginally. The ranges of WAAE and WRMSE are 12.1-17.8 and 16.3-25.8, respectively. The median values of WAAE and WRMSE are 15.1 and 22.8. [Note that error measures, i.e., WAAE and WRMSE, are expressed as the percentage of predicted values and include the sampling error.]

The magnitude of WRMSE had been observed previously by Koppelman in his study of error analysis by using disaggregate choice models (3). WRMSE was 15.9 percent when the average sampling error was subtracted and 25.9 percent otherwise. Note that Koppelman performed the analysis by using 50 observations, on the average, per prediction group.

### Market Segmentation

Examination of Table 3 by market segment shows that, again, the variable WORKRS does not reduce the model error except in a few isolated cases. The post-BART model that uses TRUE-LOS is slightly better than its network-based counterpart.

Overall, the grouping of travelers into population segments does not always reduce the model error. Market segmentation by car ownership but not by income does substantially reduce the model error, yet the resulting error is no less than the overall error of the common-market model. These results lead to one important inference: Statistical inequality of coefficients does not necessarily mean gross dissimilarity in the overall accuracy of predictions. It may be recalled that the model coefficients for the two income groups in the post-BART and Baltimore models were statistically unequal, yet the market-segment-specific models appear to perform only marginally better than the common-market model does. For the car-ownership groups the Baltimore model's travel-time coefficients and the coefficients as a group were statistically different, and in post-BART models the coefficients were statistically equal. Yet both models show smaller model error for the market-segment-specific model than for the common-market model. These contradictory results cannot be easily explained, and no attempt is made to do so.

Another observation is that the accuracy of the market-segment-specific model is often worse than the overall accuracy of the common-market model. This result and the previous results may be obtained from the reduction in sample size to roughly one-half for estimating the market-segment-specific models and reduces the precision of the coefficients. This increase in sampling error of the parameters offsets the decrease in taste variation presumably gained by market segmetation.

A second set of observations can be made with regard to model specification and type of data. (It may be recalled that the variable WORKRS was one of the variables that did improve the model's summary prediction-success indices. Also, the issue of model complexity is still under study.) It may be seen that the WORKRS variable, in spite of its statistical significance, does not systematically reduce the model error in either the pre-BART or the Baltimore models. The post-BART model that has true coefficients (i.e., observed rather than network LOS variables) has only marginally smaller model error than the network-based model error (it may be recalled that some of the alternative-specific constants were statistically different in these two post-BART models also). Without documentation it is also mentioned that the use of total travel time in place of excess and line-haul travel times with separate coefficients did not increase the model error; the values of WRMSE were 22.8 and 24.8 for the models that used observed and network data, respectively.

In conclusion, then, it may be said that market segmentation by car ownership seems somewhat promising in at least curtailing the model error, if not reducing it, but that minor improvements in model specification or type of data have no appreciable impact on the model error. It is to be noted, however, that the model error is only one component of the error; the other two components--aggregation error and transfer error--cannot be ignored when one attempts to assess the total error.

## ERROR IN PREDICTION DUE TO ZONAL AVERAGING OF VARIABLES

The error in prediction due to zonal averaging of variables (aggregation error) was calculated from Equations 7 and 8 by comparing the predicted shares from the model by using observed LOS and/or socioeconomic data with the predictions from the same model by using zonally averaged LOS and/or socioeconomic attributes. The results can be seen in Table 4. Note that the values that appear in Table 4 are the net contributions of predicted errors due to averaging of these attributes. They are calculated as the differences in the WAAEs and WRMSEs due to the model alone (e.g., 15.3 and 22.6, respectively, from Table 3 for the unsegmented market of the pre-BART model) and the model error by using average values of the socioeconomic attributes (WAAE and WRMSE of 18.5 and 26.4 for the unsegmented market, respectively). For example,

$$WAAE_A = WAAE_{M+A} - WAAE_M = 18.5 - 15.3 = 3.2 \tag{9}$$

$$WRMSE_A = (WRMSE^2_{M+A} - WRMSE_M{}^2)^{1/2} = (26.4^2 - 22.6^2)^{1/2} = 13.5 \tag{10}$$

WAAE and WRMSE are 3.2 and 13.5 percent of the predicted share, respectively. Note that the aggregation errors presented in Table 4 for Baltimore and pre-BART models are due only to the zonally averaged values of socioeconomic attributes because of the lack of non-network-based values for the LOS attributes. Note also that the zonal averages were calculated from the sample for the BART data but provided externally for the Baltimore sample.

In their analysis of prediction error, Talvitie, Dehghani, and Anderson (5) found that the overall prediction errors due to the use of zonally averaged values of the LOS and socioeconomic attributes were each about the same magnitude and had WRMSFs of 9.30 and 10.30, respectively. However, when the results were examined by market segment, the average LOS attributes sometimes reduced and sometimes increased the aggregation error of the models. One plausible explanation was the existence of a strong correlation between the true values of LOS attributes and socioeconomic attributes, such as wait time and car ownership. The use of zonal averages for socioeconomic data caused no error for many market segments. It was also noted that the calculation of the averages from the sample itself (which often contained only a few data points) and also sampling errors due to lightly used modes might have prevented the detection of the effect of averaging socioeconomic variables in that study.

The error committed by the use of zonally averaged values of independent variables is often referred to as "aggregation error by naive procedure." This aggregation error appears to vary for each data set used in this study. For overall predictions the ranges of WAAE and WRMSE were found (Table 4) to be from 3.2 to 9.6 and from 0.0 to 20.5; the median values were 4.0 and 12.7, respectively. Koppelman (3) obtained an error of 8.0 percent for the "naive" method of aggregation. It is worth noting that, except for the Baltimore data, the average socioeconomic variables are computed from the sample; they are not true zonal averages. Koppelman's study used network (i.e., average) LOS attributes.

The examination of error values given in Table 4 reveals that market segmentation does not necessarily reduce the aggregation error. There are some cases in which market segmentation has substantially increased the aggregation error. Visual examination of Tables 3 and 4 suggests that there is an inverse dependency between model error and aggregation error. If market segmentation reduces model error, it increases aggregation error, and vice versa. It appears that, for some reason, market segmentation by income is the most desirable if the objective is to reduce aggregation error only.

The most interesting result in Table 4 concerns the size of the aggregation error for the "true versus network" coefficients. It is seen that, nearly uniformly, the use of network coefficients results in a lower aggregation error and that furthermore this aggregation is often zero. This result is not totally unanticipated. Talvitie (6) showed that unbiased forecasts are possible even with (biased) network coefficients, provided that the curvature of the logit model is not too large and out-of-range forecasts are not required. Which types of coefficients result in more-accurate forecast can be studied only by examining model-transfer errors. The magnitude of these transfer errors is examined next.

## ERRORS DUE TO MODEL TRANSFER

The transfer error is calculated by applying the mode-choice models presented in Tables 1 and 2 to predict modal shares that have values of LOS and socioeconomic attributes by using Twin Cities, pre-BART, post-BART, and Baltimore data from 1970, 1972, 1975, and 1980, respectively. It is noted that the transferability being studied concerns transferability over both space and time (at most for eight years). The results are presented in Tables 5 and 6.

It can be seen from Tables 5 and 6 that the overall magnitude of transfer error in predictions is large when the models' coefficients are applied to Twin Cities data. The values of WAAEs and WRMSEs

**Table 4. Prediction error due to aggregation.**

| | | Baltimore Data[a] | | | | Post-BART Data[b] | | | | Pre-BART Data[a,b] | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | With WORKRS | | Without WORKRS | | NET-LOS Coefficient | | TRUE-LOS Coefficient | | Without WORKRS | |
| Market Segment | Model Coefficient | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE | WAAE | WRMSE |
| Total | C | 7.0 | 16.5 | 9.6 | 20.5 | 5.0 | 0.0 | 4.0 | 12.7 | 3.2 | 13.5 |
| High-income | C | 1.5 | 0.0 | 4.9 | 0.0 | 25.0 | 0.0 | 10.0 | 8.9 | 0.0 | 0.0 |
| | MS | 5.0 | 0.0 | 4.03 | 12.8 | 2.0 | 7.5 | 15.0 | 27.6 | 0.6 | 0.0 |
| Low-income | C | 34.4 | 50.6 | 25.7 | 46.1 | 6.0 | 15.5 | 0.0 | 14.3 | 3.3 | 14.3 |
| | MS | 25.9 | 36.6 | 19.9 | 31.7 | 0.0 | 0.0 | 4.0 | 14.0 | 1.6 | 8.9 |
| CBD | C | NA | NA | NA | NA | 1.0 | 8.7 | 9.0 | 25.8 | 6.9 | 19.7 |
| | MS | NA | NA | NA | NA | 1.0 | 4.1 | 6.0 | 17.0 | 5.5 | 19.0 |
| Non-CBD | C | NA | NA | NA | NA | 2.0 | 0.0 | 5.0 | 0.0 | 1.4 | 6.1 |
| | MS | NA | NA | NA | NA | 1.0 | 16.3 | 4.0 | 9.9 | 1.5 | 8.5 |
| One-car household | C | 3.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.8 | 9.8 | 0.7 | 9.3 |
| | MS | 19.3 | 36.0 | 17.8 | 34.6 | 2.0 | 11.6 | 8.0 | 21.7 | 2.0 | 0.0 |
| Household with two or more cars | C | 0.0 | 0.0 | 8.9 | 20.0 | 0 0 | 0.0 | 6.8 | 6.9 | 0.8 | 0.0 |
| | MS | 19.6 | 33.0 | 17.6 | 28.9 | 1.0 | 0.0 | 7.0 | 6.6 | 6.3 | 23.5 |

Note: NA = not applicable due to small sample size.
[a]LOS variables were from the networks and already are zonal averages; aggregation error is due to averaging of socioeconomic attributes only.
[b]Socioeconomic averages from sample.

**Table 5. Prediction error due to model transfer: Baltimore model.**

| Market Segment | Model Coefficient | Pre-BART Data: With WORKRS WAAE | WRMSE | Pre-BART Data: Without WORKRS WAAE | WRMSE | Post-BART Data: NET-LOS Coefficient WAAE | WRMSE | Post-BART Data: TRUE-LOS Coefficient WAAE | WRMSE | Twin Cities Data WAAE | WRMSE |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Total | C | 4.2 | 16.5 | 8.5 | 22.2 | 16.1 | 32.6 | 6.0 | 25.6 | 44.5 | 57.9 |
| High-income | C | 4.7 | 19.8 | 7.5 | 25.6 | 16.6 | 33.6 | 0.4 | 18.8 | 29.9 | 44.6 |
| | MS | 16.7 | 33.3 | 7.4 | 22.3 | 17.8 | 34.3 | 0.7 | 16.8 | 30.6 | 43.8 |
| Low-income | C | 2.0 | 8.3 | 5.5 | 0.0 | 2.2 | 0.0 | 8.4 | 0.0 | 29.6 | 49.7 |
| | MS | 5.7 | 15.9 | 14.8 | 26.8 | 19.5 | 33.8 | 10.0 | 22.3 | 44.8 | 57.6 |
| One-car household | C | 0.0 | 0.0 | 0.0 | 18.4 | 19.4 | 28.3 | 6.1 | 17.9 | 43.2 | 58.42 |
| | MS | 6.3 | 19.3 | 7.0 | 17.5 | 43.7 | 51.4 | 16.9 | 29.4 | 40.5 | 52.4 |
| Household with two or more cars | C | 1.3 | 18.1 | 11.4 | 31.8 | 12.0 | 6.2 | 3.6 | 12.6 | 38.1 | 51.33 |
| | MS | 10.3 | 22.8 | 16.8 | 31.7 | 42.4 | 55.0 | 10.1 | 27.1 | 25.6 | 37.8 |

**Table 6. Prediction error due to model transfer: pre-BART model.**

| Market Segment | Model Coef-ficient | Baltimore Data with WORKRS: TRUE SE Variable WAAE | WRMSE | Baltimore Data with WORKRS: Avg SE Variable WAAE | WRMSE | Baltimore Data Without WORKRS: TRUE SE Variable WAAE | WRMSE | Baltimore Data Without WORKRS: Avg SE Variable WAAE | WRMSE | Post-BART Data: NET-LOS Coefficient WAAE | WRMSE | Post-BART Data: TRUE-LOS Coefficient WAAE | WRMSE | Twin Cities Data WAAE | WRMSE |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Total | C | 0.6 | 0.0 | 18.3 | 38.6 | 5.4 | 18.4 | 15.9 | 32.9 | 2.1 | 17.4 | 2.9 | 23.6 | 30.5 | 43.7 |
| High-income | C | 2.0 | 0.0 | 0 0 | 0.0 | 6.4 | 17.9 | 0.0 | 0.0 | 4.6 | 17.5 | 2.8 | 19.2 | 3.9 | 12.2 |
| | MS | 1.6 | 0.0 | 0.0 | 0.0 | 3.7 | 0.0 | 0.0 | 0.0 | 4.9 | 19.8 | 2.6 | 4.6 | 6.8 | 0.0 |
| Low-income | C | 2.5 | 0.0 | 27.2 | 47.6 | 4.9 | 11.8 | 34.9 | 56.2 | 1.7 | 12.4 | 1.6 | 10.3 | 21.8 | 40.7 |
| | MS | 10.8 | 0.0 | 19.6 | 48.9 | 2.6 | 18.9 | 22.2 | 53.0 | 12.7 | 28.9 | 8.7 | 28.2 | 15.9 | 39.6 |
| CBD | C | NA | NA | NA | NA | NA | NA | NA | NA | 18.1 | 33.8 | 15.1 | 30.9 | 21.2 | 33.4 |
| | MS | NA | NA | NA | NA | NA | NA | NA | NA | 30.2 | 60.6 | 28.4 | 54.6 | 28.2 | 38.9 |
| Non-CBD | C | NA | NA | NA | NA | NA | NA | NA | NA | 4.2 | 0.0 | 5.4 | 0.0 | 26.2 | 31.7 |
| | MS | NA | NA | NA | NA | NA | NA | NA | NA | 11.4 | 38.4 | 5.0 | 23.3 | 42.4 | 53.3 |
| One-car household | C | 9.4 | 27.2 | 7.3 | 16.2 | 8.7 | 20.0 | 5.8 | 14.0 | 2.6 | 13.8 | 0.0 | 29.4 | 36.1 | 50.8 |
| | MS | 7.5 | 17.6 | 5.8 | 10.4 | 4.7 | 5.1 | 0.0 | 6.1 | 0.1 | 0.0 | 10.0 | 0.0 | 42.2 | 57.0 |
| Household with two or more cars | C | 1.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.3 | 21.2 | 17.1 | 0.0 |
| | MS | 3.3 | 11.6 | 13.1 | 60.3 | 11.9 | 45.1 | 10.3 | 42.8 | 10.3 | 9.0 | 10.3 | 0.0 | 9.4 | 20.0 |

are 44.5 and 57.9 and 30.5 and 43.7 by using Baltimore and pre-BART models, respectively. In general, the median value of error for the overall prediction due to model transfer is about the same magnitude as model misspecification with WRMSE of 23.6 percent, or about the same as the model error.

The inclusion of the variable WORKRS in model specification does not consistently reduce the transfer error. However, it does reduce it often and independent of whether true socioeconomic variables or zonal averages are used. It is noted that it is the latter type of variable that is a standard in transportation studies.

Interesting results are obtained with respect to the use of zonal averages of the socioeconomic and LOS attributes. It can be seen from Table 6 that WAAEs and WRMSEs for model transfer obtained by using the Baltimore data and zonally averaged socioeconomic attributes are about the same and even smaller than the counterpart values obtained by using disaggregate data. For example, the WAAEs and WRMSEs for the common-market pre-BART model to the Baltimore data for the above two cases are 5.4 and 18.4 and 15.9 and 32.9, respectively, when the number of workers is excluded from the model. Thus, the use of zonally averaged socioeconomic attributes seems to result in forecasts that are somewhat worse than those that use disaggregate values. It can also be seen from Tables 5 and 6 that the use of true LOS attributes results in more-accurate forecasts that the use of network-based values does for the Baltimore model but not for the pre-BART model. This section can be concluded with a remark about the market segmentation. Taken together, Tables 5 and 6 suggest that the application of the pre-BART model favors market segmentation by car ownership but the application of the Baltimore model does not support such market segmentation in order to reduce the transfer error.

## TOTAL PREDICTION ERROR

As interesting as the examination of different error sources is, the question uppermost in a practitioner's mind is the total prediction error and whether it can be reduced by market segmentation or other means. So far, we have obtained conflicting information. Market segmentation by car ownership or income may reduce the model specification error somewhat. This was suggested by both the statistical tests of the equality of model parameters and the RMSEs in Table 3. Aggregation error, on the other hand, seems to increase as a result of market segmentation by car ownership. Finally, market segmentation sometimes increases and sometimes decreases the transfer error.

The distribution of the total error among the three error sources is also unsystematic. Table 7 shows the total error for the entire travel market and its distribution among the three sources by using the common-market model. If one wants to glean an average from Table 7, one would assign 40 percent of the total error to model and transfer error and assign the remaining 20 percent to aggregation error. With market-segment-specific models the distribution of the total error shifts 10 percent of

**Table 7. Total prediction error.**

| | Baltimore Model | | | | | Pre-BART Model | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Pre-BART Data | | Post-BART Data | | | Baltimore Data Without WORKRS | | Post-BART Data | | |
| Error Type | With WORKRS | Without WORKRS | NET-LOS | TRUE-LOS | Twin Cities | TRUE SE Variable | Avg SE Variable | NET-LOS | TRUE-LOS | Twin Cities |
| Total Market | | | | | | | | | | |
| WAAE | 23.8 | 32.1 | 39.7 | 29.6 | 68.1 | 23.9 | 34.4 | 20.6 | 21.4 | 49.0 |
| WRMSE | 28.5 | 35.3 | 42.6 | 37.6 | 64.0 | 32.0 | 42.13 | 31.6 | 35.3 | 51.0 |
| Model Error (%) | | | | | | | | | | |
| WAAE | 51.0 | 44.0 | 35.0 | 47.0 | 20.0 | 25.0 | 45.0 | 74.0 | 70.0 | 31.0 |
| WRMSE | 33.0 | 27.0 | 18.0 | 24.0 | 8.0 | 50.0 | 29.0 | 51.0 | 41.0 | 20.0 |
| Aggregation Error (%) | | | | | | | | | | |
| WAAE | 32.0 | 30.0 | 24.0 | 32.0 | 14.0 | 13.0 | 9.0 | 16.0 | 15.0 | 7.0 |
| WRMSE | 33.5 | 34.0 | 23.0 | 30.0 | 10.0 | 18.0 | 10.0 | 18.0 | 15.0 | 7.0 |
| Transfer Error (%) | | | | | | | | | | |
| WAAE | 17.0 | 26.0 | 41.0 | 21.0 | 66.0 | 22.0 | 46.0 | 10.0 | 15.0 | 62.0 |
| WRMSE | 33.5 | 39.0 | 59.0 | 46.0 | 82.0 | 32.0 | 61.0 | 31.0 | 44.0 | 73.0 |

the model error to the transfer error.

One has the feeling that the numbers tell two things. First, and most conspicuous, if the model transfers poorly due to the overwhelming influence of the alternative-specific constants (that is, drastically different model shares between the estimation data and the transfer data), then the transfer error is dominant. Second, if there is substantial within-zone variation in the LOS data, then the aggregation error is large. This is often the case for low-income or CBD-bound travel in which the number of transit users, and hence great variance in excess time components, exists.

In general, the message is that the total prediction error is large and little is gained by market segmentation and complex specification. Good judgment in model application and careful preparation of data are keys to forecasting success; even then the forecasts are marked with uncertainty.

## CITYWIDE PREDICTIONS BY MODE

It would be inappropriate to conclude this paper without taking a brief look at the predictions (transferability) of the models by mode of travel between different cities. The more complex calculations of RMSE include the variations in individual predictions and provide a convenient one-number measure of forecasting accuracy. On the other hand, such a one-number measure seems to hide information and prevent drawing useful conclusions. The simple share predictions are easy to calculate and comprehend and provide results that seem to be in accordance with statistical tests of coefficient equality or inequality. Results are given in Table 8 (Baltimore and pre-BART models) for the unsegmented market. The results in Table 8 tell us that the WORKRS variable does make a contribution to the model accuracy, especially for the transit share. The two post-BART experiments show that the observed (true) LOS attributes make an important contribution to model accuracy. It may be recalled that the post-BART network coding was found to be quite different from the manual coding and, by inference, faulty. The results here confirm this inference.

The Twin Cities predictions are the worst of all. The reasoning is that, because the actual shares in Twin Cities are so different from the shares in the estimation sample and because the alternative-specific constants account for the bulk of the model power, that alone renders the estimated model nontransferable to cities that have vastly different modal use.

The same type of results can be read from Table 8 for the pre-BART model. Again, the WORKRS variable makes a contribution to the forecasting accuracy, as is evident from the first two rows. The next two rows provide a partial contradiction; if zonal averages are used for socioeconomic variables, the advantage of the better model specification is lost. The behavior of the model is exactly according to the theory; small shares (<0.50) are predicted as being even smaller and large shares (>0.50) are predicted as being even larger than they really are.

**Table 8. Citywide predictions by mode.**

| | Mode | | | |
|---|---|---|---|---|
| Data Source | Drive Alone | Transit | Shared Ride | N |
| Baltimore Model | | | | |
| Predicted (with WORKRS) | 0.53 | 0.28 | 0.19 | 900 |
| Predicted (without WORKRS) | 0.51 | 0.32 | 0.17 | 900 |
| Actual | 0.55 | 0.24 | 0.21 | |
| Post-BART (NET-LOS) | | | | |
| Predicted | 0.48 | 0.35[a] | 0.17 | 623 |
| Actual | 0.57 | 0.22[a] | 0.21 | |
| Post-BART (TRUE-LOS) | | | | |
| Predicted | 0.54 | 0.28[b] | 0.18 | 565 |
| Actual | 0.55 | 0.25[b] | 0.20 | |
| Twin Cities | | | | |
| Predicted | 0.56 | 0.27 | 0.17 | 665 |
| Actual | 0.86 | 0.05 | 0.09 | |
| Pre-BART Model | | | | |
| Baltimore | | | | |
| Predicted (with WORKRS and TRUE SE) | 0.50 | 0.33 | 0.17 | 544 |
| Predicted (without WORKRS and TRUE SE) | 0.57 | 0.21 | 0.22 | 544 |
| Actual | 0.51 | 0.29 | 0.20 | |
| Baltimore | | | | |
| Predicted (with WORKRS and avg SE) | 0.64 | 0.17 | 0.19 | 561 |
| Predicted (without WORKRS and avg SE) | 0.63 | 0.19 | 0.18 | 561 |
| Actual | 0.51 | 0.29 | 0.20 | |
| Post-BART (NET-LOS) | | | | |
| Predicted | 0.55 | 0.24[c] | 0.21 | 623 |
| Actual | 0.56 | 0.23[c] | 0.21 | |
| Post-BART (TRUE-LOS) | | | | |
| Predicted | 0.56 | 0.23[d] | 0.21 | 565 |
| Actual | 0.55 | 0.25[d] | 0.20 | |
| Twin Cities | | | | |
| Predicted | 0.64 | 0.17 | 0.19 | 665 |
| Actual | 0.87 | 0.05 | 0.08 | |

[a]The predicted and actual BART shares are 0.12 and 0.08, respectively.
[b]The predicted and actual BART shares are 0.09 and 0.10, respectively.
[c]The predicted and actual BART shares are 0.08 and 0.084, respectively.
[d]The predicted and actual BART shares are 0.07 and 0.098, respectively.

The predictions for the post-BART situation are excellent and, again, contradictory to the results obtained by using the Baltimore model. An assumption can be made that the extra mode-specific constant available in the pre-BART model is very helpful. The prediction of the Twin Cities modal shares is done as poorly as the case with the Baltimore model. The comment made then applies now, too. Constants were assigned as follows:

1. Alternatives in estimation samples:
   a. Drive alone, shared ride, bus
   b. Drive alone, shared ride, local bus, express bus
2. Model whose modal constant was assigned to rail or express bus, or both:
   a. Shared ride
   b. Express bus

An interesting complement to these predictions is provided by the market-segment-specific models. The results of the application of the Baltimore and the pre-BART models are shown in Table 9 for the case in which the modal shares for the one-car households and the households that have two or more cars are aggregated. Note that no-car households are not included in this table. Examination of the data in Table 9 shows that market-segment-specific (Baltimore) models are better predictors only occasionally; generally the common-market models are better or at least consistent. Two other things also stand out: The more-accurate true LOS variables yield much better predictions, and the Twin Cities' predictions remain very poor.

The comments made above apply here for both types of pre-BART models as well. To repeat, the market-segment-specific models are not better, the observed LOS variables are better, and the Twin Cities predictions are poor.

The fact that the use of zonal averages has such a drastic detrimental impact on forecasting accuracy, especially on the drive-alone and the bus predictions, would merit serious study. However, to do so would require the assessment of the errors and differences in the extraneous zonal averages versus those calculated from the sample itself. This was felt to be outside the scope of the present study. At any rate, zonal averages and market segmentation do not mix.

## CONCLUSIONS

The results presented here are complicated, but the following conclusions can be drawn on the basis of the predictions. Data accuracy is clearly important; this is shown by the clear superiority of the true LOS data over the often glaringly erroneous coding of the post-BART network.

The use of zonal averages in predictions seems to be a source of serious concern. Unfortunately, it is not known to what extent data error rather than the model or aggregation error is responsible for the results. The fact that the Baltimore sample is a stratified random sample should also be factored in, and this was not done here. An interesting thought is to add up the drive-alone and shared-ride percentages and use a car-occupancy model to convert travelers to vehicles. Of course, this could not be done in all applications.

Finally, there is the fact that the common-market model performed very well; the Twin Cities data were the only (occasional) exception. This result argues in favor of aggregated total-market forecasts.

Much remains to be done to ensure accuracy in travel forecasts. Foremost among these is the updating of modal constants to apply in cases when the modal shares in the estimation sample are very different from those likely to be experienced in the city in which the model is to be transferred. The second item that needs constant attention is the accuracy of both the socioeconomic and LOS data. Data must be carefully prepared if forecasting errors are to be avoided. These are the first steps.


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**Table 9. Predication of modal shares by using common and market-segment-specific coefficients: households with one and with two or more cars.**

| Data Source | Drive Alone C | Drive Alone MS | Drive Alone Actual | Transit C | Transit MS | Transit Actual | Shared Ride C | Shared Ride MS | Shared Ride Actual |
|---|---|---|---|---|---|---|---|---|---|
| **Baltimore Model** | | | | | | | | | |
| Pre-BART (with WORKRS) | 0.55 | 0.51 | 0.59 | 0.28 | 0.27 | 0.19 | 0.17 | 0.22 | 0.22 |
| Pre-BART (without WORKRS) | 0.55 | 0.51 | 0.59 | 0.28 | 0.27 | 0.19 | 0.17 | 0.22 | 0.22 |
| Post-BART | - | - | 0.59 | - | - | $0.19^a$ | - | - | 0.22 |
| NET-LOS | 0.51 | 0.37 | - | $0.32^b$ | $0.43^c$ | - | 0.17 | 0.20 | - |
| Obs-LOS | 0.58 | 0.49 | - | 0.23 | 0.27 | - | 0.19 | 0.24 | - |
| Twin Cities | 0.57 | 0.64 | 0.87 | 0.24 | 0.13 | 0.05 | 0.19 | 0.23 | 0.08 |
| **Pre-BART Model** | | | | | | | | | |
| Baltimore | - | - | 0.59 | - | - | 0.21 | - | - | 0.20 |
| With WORKRS | 0.54 | 0.62 | - | 0.30 | 0.22 | - | 0.16 | 0.16 | - |
| Without WORKRS | 0.61 | 0.70 | - | 0.22 | 0.13 | - | 0.17 | 0.17 | - |
| Post-BART | - | - | 0.60 | - | - | $0.19^d$ | - | - | 0.21 |
| NET-LOS | 0.59 | 0.51 | - | $0.18^e$ | $0.29^f$ | - | 0.23 | 0.20 | - |
| Obs-LOS | 0.63 | 0.60 | - | 0.14 | 0.19 | - | 0.23 | 0.21 | - |
| Twin Cities | 0.65 | 0.61 | 0.87 | 0.16 | 0.21 | 0.05 | 0.19 | 0.18 | 0.08 |

[a] The actual BART share is 0.09.
[b] The predicted NET-LOS and Obs-LOS BART shares are 0.11 and 0.08, respectively.
[c] The predicted NET-LOS and Obs-LOS BART shares are 0.24 and 0.14, respectively.
[d] The actual BART share is 0.09.
[e] The predicted NET-LOS and Obs-LOS BART shares are 0.04 and 0.04, respectively.
[f] The predicted NET-LOS and Obs-LOS BART shares are 0.14 and 0.11, respectively.

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# Equilibrium Model for Carpools on an Urban Network

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**Traffic equilibrium methods are presented in which the population of motorists consists of individuals who are minimizers of a linear combination of cost and travel time. The relative importance of travel time versus cost varies across the population, but fairly mild conditions for the existence and uniqueness of the equilibrium can nevertheless be identified. The paradigm is of particular interest for carpooling studies because the occupants of carpools can divide the cost among themselves but they cannot do the same with the travel time. Thus, vehicles that have different occupancy levels will have different relative values of travel time and cost. The model is specially well suited to the analysis of how vehicles that have different occupancies compete for segments of the roads that are crowded or have tolls. It is therefore very useful to predict the impacts of special carpooling lanes, lower tolls for high-occupancy vehicles, and other transportation-system-management strategies on the distribution of traffic over an urban network.**

Current traffic-assignment practice takes two principal forms, which are applicable to congested and uncongested networks. Stochastic traffic-assignment models (1-5) ignore congestion but do not allocate all the traffic from an origin-destination (O-D) pair to the shortest route. Instead, they spread it over the network as if travel time was perceived with some random noise by a motorist population of travel-time minimizers.

Deterministic-equilibrium models assume that motorists are accurate minimizers of travel time but that travel time depends on the traffic flow because of congestion. Textbook-level treatments of deterministic equilibrium models can be found (6-9). The equilibrium condition for these models was stated by Wardrop (10). It can be paraphrased as follows: at equilibrium (a) routes that have flow are the shortest routes, or (b) no user can improve route travel time by unilaterally changing routes, or (c) links that have flow for a given destination are on a shortest path to the destination. Since a problem that is more closely related to deterministic-equilibrim models than to stochastic-assignment models will be addressed here, the discussion of the former is expanded below. A question that arises immediately is that of the existence and uniqueness of an equilibrium-flow pattern that satisfies all three equilibrium conditions.

Beckmann, McGuire, and Winsten (6); Netter (11); and Smith (12) have provided progressively more general existence results. It is currently known that if travel time on every link of the network is a continuously differentiable positive function of the link flows, Brouwer's fixed-point theorem guarantees the existence of the equilibrium flows.

Uniqueness was first studied for networks in which the travel time on a link depends only on its own flow (6). In this case and if travel time increases with flow for all links, the equilibrium exists and the resulting link-flow pattern is unique. This is because the equilibrium problem admits a formulation as the minimization of a strictly convex function subject to linear constraints. This formulation can be expressed in terms of link flows as follows:

$$\text{(MP)} \min \sum_i \int_0^{x_i} c_i(w)dw$$

subject to

$$\sum_{i\epsilon I(r)} x_i^s - \sum_{i\epsilon E(r)} x_i^s = q^{rs} \quad \forall r \neq s, \forall s$$

$$\sum_s x_i^s = x_i \quad \forall i$$

$$x_i^s \geq 0 \quad \forall i, s$$

In this program, the letters r and s represent nodes, and the letter i represents a link. I(r) represents the set of links that point to node r; E(r), the set of links that point out of node r; and $c_i(\cdot)$, the link-cost function that relates the flow on link $x_i$ to the link travel time $c_i$. In addition, $x_i^s$ is the total number of trips that have final destinations s and that use link i, and $q^{rs}$ is the total number of trips that go from origin r to destination s.

In order to write equilibrium problems more succinctly, the set of feasible link-flow patterns is denoted by X; thus, program (MP) is written as follows:

$$\text{(MP)} \min_{x\epsilon X} \sum_i \int_0^{x_i} c_i(w)dw$$

Link flows that are optimal for (MP) are equilib-