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Analytical Descriptions of Track-Geometry Variations

A. HAMID AND T-L. YANG

Track-geometry variations can be divided into two broad categories: (a) typical variations that account for random waviness in the rail and periodic behavior at joints, and (b) isolated variations that occur occasionally but do have regular patterns. An overview of analytical descriptions of track-geometry variations found in U.S. track is given. These descriptions provide mathematical representations required for simulations and design studies. Time-series-analysis techniques were applied to develop the statistical descriptions of typical track-geometry variations. Models based on autospectral densities are presented along with the parameters of these models for the current Federal Railroad Administration track classes. Various shapes of isolated variations as found in U.S. track are identified. Mathematical functions for these shapes are given along with the values of parameters of these functions. Also discussed are the relations between track-geometry parameters. This includes correlations between gage and alignment, cross level and alignment, and cross level and profile.

Track-geometry variations are the primary dynamic inputs to rail vehicles. In order to study vehicle-track interaction, it is essential to provide quantitative descriptions of track-geometry variations. Analytical descriptions of track-geometry variations are essential in performing simulation studies for improved rail safety. Such descriptions are also needed for evaluation of track quality, vehicle performance, passenger comfort, and lading damage. Since an infinite number of track-geometry variations can occur in the railway track, a statistical approach is used in the characterization of track-geometry variations.

Track irregularities or variations in track geometry are the result of cumulative forces that have shaped the track structure during its lifetime. These variations often begin with small imperfections in materials and tolerances and errors in the manufacture of rail and other track components. Terrain variations and survey errors during the design and construction of track also add to the variations. Progressive deterioration of track geometry occurs under traffic and environmental factors. Various deformations may sometimes be induced by maintenance operation intended to correct poor geometry.

Most track can be separated into segments that are constructed and maintained in a uniform manner. These segments exhibit similar track-geometry variations that consist of random waviness with relatively large amplitudes at joints and welds. Such variations are called "typical" variations in this paper.

Track-geometry variations not covered by typical variations will be called "isolated" track-geometry variations. Isolated variations usually occur at special track work or at physical features such as switches, turnouts, crossings, and bridges. These variations occur occasionally, but they do have regular patterns.

A track-geometry data base that represents a reasonable sample of track in the United States was established for the analytical characterization of track-geometry variations. The data base consists of approximately 500 miles of track-geometry data selected from approximately 70 000 miles of data collected by Federal Railroad Administration (FRA) track-geometry cars during 1980 and 1981. The data base consists of 5-10 sections of track-geometry data in each of the current FRA track classes (FRA, Track Safety Standards, 1973). Each section varied from 5 to 10 miles in length. These sections were broadly distributed throughout the United States and reflected various types of operating conditions and maintenance practices of different railroads.

Analytical descriptions of track-geometry variations were developed under a track characterization program directed by the Transportation Systems Center in support of the FRA improved track structures program. This program was initiated in 1976 and two interim reports have been published (1,2). This paper gives an overview of the results of this program.

TYPICAL TRACK-GEOMETRY VARIATIONS

Time-series-analysis techniques (3) were applied to track-geometry data to obtain analytical representations of typical track-geometry variations. It was shown that a periodically modulated random process provided an adequate representation of typical track-geometry variations (4). This process consists of a stationary random process that accounts for the random irregularities in the rail and a periodic process that describes the regularly spaced rail joints that have a non-zero mean amplitude. The amplitude of joints varies randomly while the joint spacing stays the same.

The power spectral density (PSD) is a useful tool for analyzing the periodically modulated random process. In track-geometry PSDs, the stationary random process produces the smooth continuum and a non-zero mean in joint amplitudes (periodic process) causes spectral peaks.

Figure 1 shows a typical PSD of the profile geometry of bolted track. The power density is plotted as a function of spatial frequency (1/wavelength). Note the pronounced peaks on a relatively smooth continuum. These peaks appear at wavelengths that correspond to the rail length (about 39 ft) and its harmonics indicate the existence of a periodic component.

Stationary Random Process

The PSD continuum that represents the stationary random process can be modeled by even-powered laws as a function of break frequencies and a roughness parameter. For example, over a wavelength range of 5-1000 ft, the profile and alignment PSD can be modeled as

$$S(\phi) = [A\phi_2{}^2(\phi^2 + \phi_1{}^2)]/[\phi^4(\phi^2 + \phi_2{}^2)] \qquad (1)$$

where

$S(\phi)$ = PSD,
ϕ = spatial frequency,
A = roughness constant, and
ϕ_1, ϕ_2 = break frequencies.

The break frequencies are functions of the specific track-geometry parameter and do not change significantly for different track classes. For example, profile ϕ_1 is 0.0071 cycle/ft and ϕ_2 is 0.04 cycle/ft for all track classes. Therefore, the stationary random process is adequately described by the roughness parameter, A. The roughness parameter is strongly dependent on track class. This is illustrated in Figure 2 for the roughness parameter of mean profile.

Models based on PSDs were developed for all

Figure 1. Typical PSD of profile.

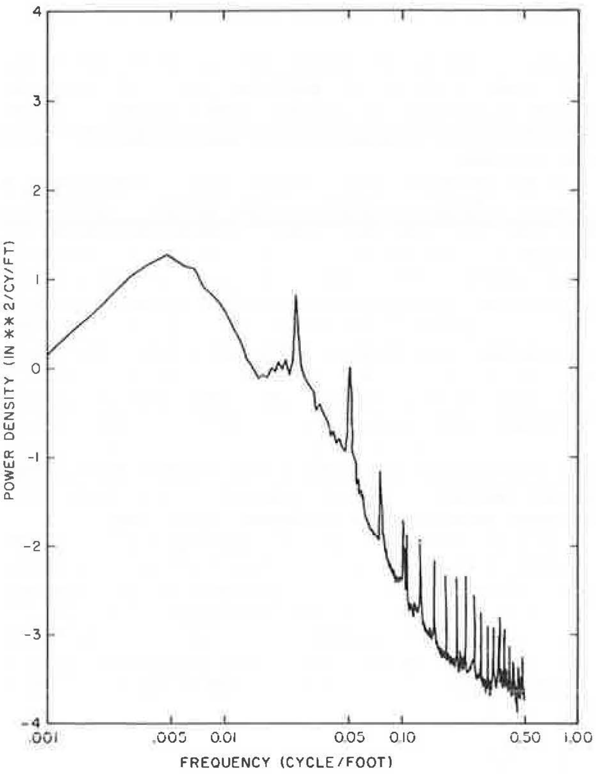


Figure 2. Profile roughness parameter versus track class.

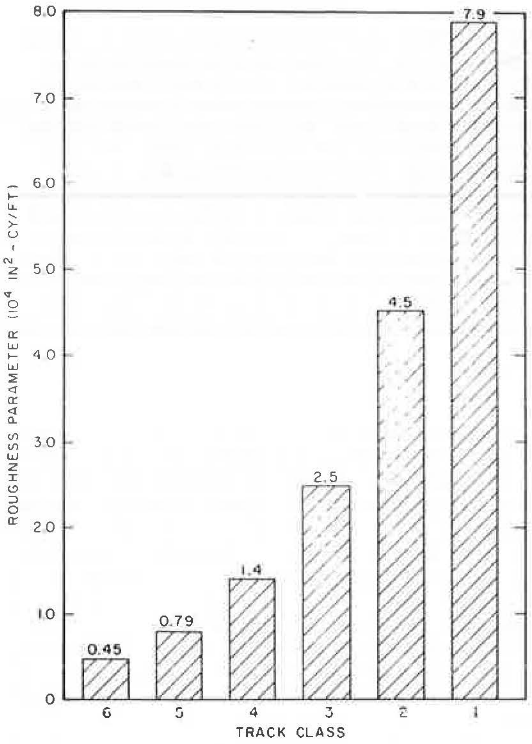


Table 1. Analytical characterization of typical track-geometry variations.

Process	Track Geometry	Model	Parameter		Values of Parameters by Track Class					
			Symbol	Unit	6	5	4	3	2	1
Stationary random	Gage	$s(\phi) = A\phi_2{}^2/[(\phi^2 + \phi_1{}^2)(\phi^2 + \phi_2{}^2)]$	A	10^4 in^2 cycle/ft	0.3	0.5	0.9	1.6	2.8	5.0
			ϕ_1	10^3 cycle/ft	8.9	8.9	8.9	8.9	8.9	8.9
			ϕ_2	10^2 cycle/ft	7.1	7.1	7.1	7.1	7.1	7.1
	Cross level	Same as gage	A	10^4 in^2 cycle/ft	0.3	0.5	0.7	1.1	1.6	2.3
			ϕ_1	10^3 cycle/ft	7.1	7.1	7.1	7.1	7.1	7.1
			ϕ_2	10^2 cycle/ft	4.0	4.0	4.0	4.0	4.0	4.0
	Profile	$S(\phi) = [A\phi_2{}^2(\phi^2 + \phi_1{}^2)] \div [\phi^4(\phi^2 + \phi_2{}^2)]$	A	10^4 in^2 cycle/ft	0.5	0.8	1.4	2.5	4.5	7.9
			ϕ_1	10^3 cycle/ft	7.1	7.1	7.1	7.1	7.1	7.1
			ϕ_2	10^2 cycle/ft	4.0	4.0	4.0	4.0	4.0	4.0
	Alignment	Same as profile	A	10^4 in^2 cycle/ft	0.3	0.5	0.9	1.6	2.8	5.0
			ϕ_1	10^3 cycle/ft	10.0	10.0	10.0	10.0	10.0	10.0
			ϕ_2	10^2 cycle/ft	5.6	5.6	5.6	5.6	5.6	5.6
Periodic	Profile	$y(x) = \overline{C}\exp(-k\lvert x\rvert)$	$\overline{C}$	in	0.11	0.14	0.19	0.25	0.33	0.45
			k	ft^{-1}	0.25	0.20	0.15	0.14	0.13	0.13
	Alignment	Same as profile	$\overline{C}$	in	0.08	0.11	0.15	0.20	0.27	0.35
			k	ft^{-1}	0.57	0.46	0.35	0.20	0.15	0.12

track-geometry parameters, i.e., gage, alignment, cross level, and profile. These models, along with the values of the roughness constant and break frequencies for all current track classes, are given in Corbin (1) and summarized in Table 1.

Periodic Process

A cusp behavior is observed in the track geometry at joints. Analyses of track-geometry data indicate that the rail profile or alignment at a joint can be adequately represented by a cusp shape of the form:

$$y(x) = C \exp(-k|x|) \qquad (2)$$

where

x = distance along the rail,
y(x) = rail profile or alignment,
C = joint cusp amplitude, and
k = decay rate.

Thus, the shape of a joint is defined by its amplitude and its decay rate. The duration (inverse decay rates) of the joint cusp is on the order of 2-10 ft long and its amplitude for most cases falls within 0-3 in or more. Both duration and amplitude increase with degradation, which results from the structural weakness of the joint, and is accelerated by loosening of the joint-bar fastenings.

Mean joint amplitudes and decay rates were estimated from the spectral peaks. These values for all current FRA track classes are given in Corbin (1) and also included in Table 1.

Track-geometry models based on PSDs are useful to determine the sustained type of vehicle responses. The PSDs can be used to calculate mean square values of rail deviations, rail curvatures, vibration levels in the vehicle, forces at the wheel-rail interface, and relative displacements between vehicle components.

The PSD is, however, a limited analysis tool. Without detailed knowledge of the parent probability distributions that govern each input and each response mode, mean square values cannot predict peak values. Another deficiency of the PSD concerns its averaging property. Identical PSDs result from a wide variety of time histories. Therefore, isolated geometric variations are obscured by the averaging property of the PSDs. The isolated variations represent special cases that occur occasionally but do have regular patterns. These variations are often the causes of undesirable responses and are the subject of the next section.

ISOLATED TRACK-GEOMETRY VARIATIONS

This section deals with the analytical description of isolated track-geometry variations. The key signatures are first identified. The mathematical functions that can be used to describe these signatures are given along with the parameters of these functions. Typical occurrences of isolated track-geometry variations are then discussed as single events, periodic variations, and combined irregularities in track-geometry parameters.

Key Signatures

Seven key signatures have been identified in isolated track-geometry variations. These are cusp, bump, jog, plateau, trough, sinusoid, and damped sinusoid. Figure 3 gives the shapes and mathematical functions that can be used to describe these signatures.

The analytical forms of key signatures are functions of two parameters--amplitude A and a duration-related parameter k. Note that the duration of a signature is proportional to 1/k.

Table 2 gives a range of values of A and k as found in the track-geometry data analyzed in this study. Note that the values of these parameters are a function of track class, track-geometry parameter, and the signature itself. In general, the values of A and k decrease as the track class increases. However, the ranges of values overlap considerably between different track classes.

Typical Occurrences

Isolated track-geometry variations usually occur in spirals, at special track work, and other track anomalies such as soft subgrade or poor drainage areas. Isolated variations have been identified at such track features as road crossings, turnouts, interlockings, and bridges. Their frequency of occurrence depends on the number of curves and special track features.

The table below lists the typical locations where the key signatures have been seen:

Signature	Occurrence
Cusp	Joints, turnouts, interlockings, sun kinks, buffer rail, insulated joints in continuously welded rail (CWR), splice bar joint in CWR, piers at bridge
Bump	Soft spots, washouts, mud spots, fouled ballast, joints, spirals, grade crossings, bridges, overpasses, loose bolts, turnouts, interlockings
Jogs	Spirals, bridges, crossings, interlockings, fill-cut transitions
Plateau	Bridges, grade crossings, areas of spot maintenance
Trough	Soft spots, soft and unstable subgrades, spirals
Sinusoid	Spirals, soft spots, bridges
Damped sinusoid	Spirals, turnouts, localized soft spot

These signatures occur as single events, in combination with each other, and in a periodic fashion. Furthermore, isolated track-geometry defects can occur simultaneously in more than one track-geometry parameter.

Single events provide transitory input to the vehicle and can cause severe dynamic interaction. Large-amplitude single events can appear in any track-geometry parameter at isolated locations. Figure 4 shows some examples of these events as seen in the track-geometry data.

The key signatures that occur in succession are defined as periodic track-geometry variations. The amplitude of these signatures may vary; however, the wavelength remains more or less constant over several cycles. The periodic variations can cause severe vehicle-track dynamic interaction. Large-amplitude vehicle response results when the frequency of these variations coincides with the natural frequency of the vehicles.

The periodic variations have been observed in the form of cusp, bump, jog, and sinusoid signatures. The periodic behavior was not observed for other signatures in the track-geometry data analyzed in this study.

Perhaps the most familiar example of vehicle response to periodic behavior is the rock-and-roll phenomenon due to consecutive low joints. The consecutive low joints appear as periodic cusps in the cross-level traces. Severe low joints can give

Figure 3. Key signatures of isolated variations.

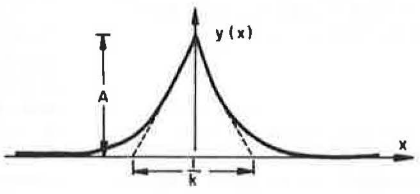
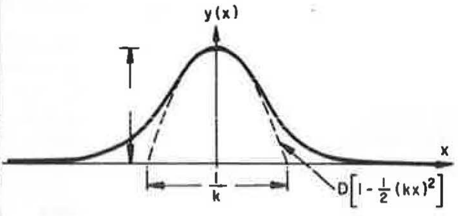
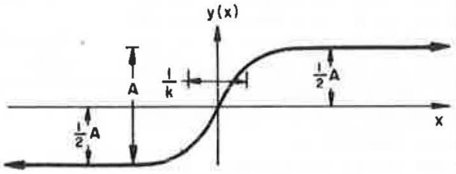
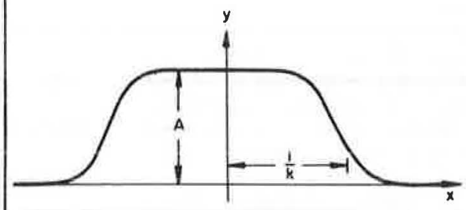
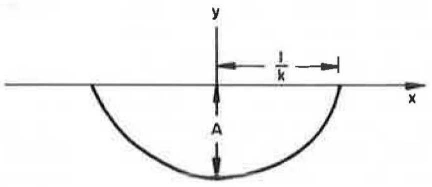
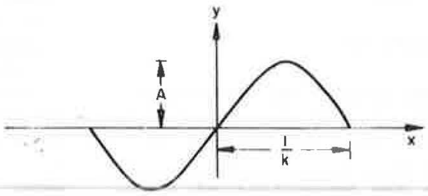
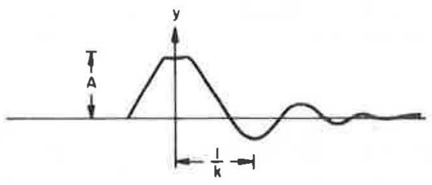
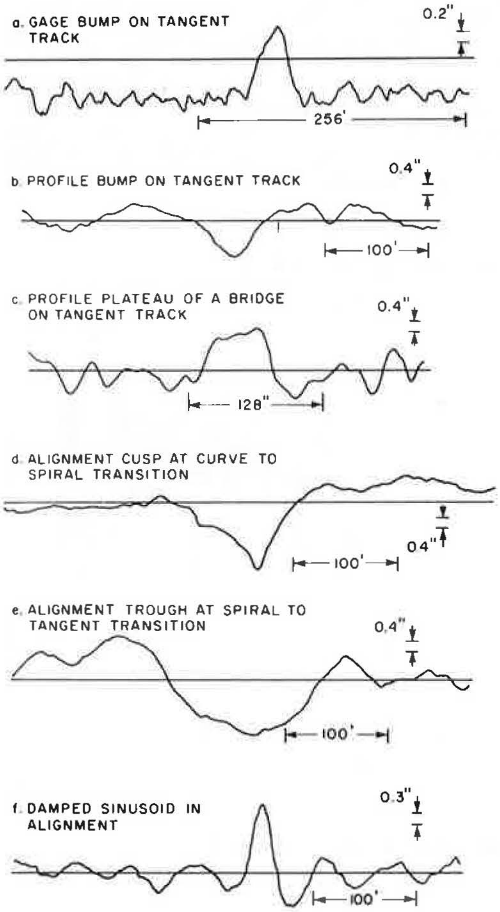
Signature	Shape	Analytical Representation
Cusp		$y = Ae^{-k\lvert x\rvert}$
Bump		$y = Ae^{-\frac{1}{2}(kx)^2}$
Jog		$y = \frac{Akx}{\sqrt{1 + 4k^2x^2}}$
Plateau		$y = \sqrt{\frac{A^2}{1 + (kx)^8}}$
Trough		$y = Ak \quad \frac{1}{k}^{2} - x^2$
Sinusoid		$y = A \sin \pi kx$
Damped Sinusoid		$y = Ae^{-kx} \cos \pi kx$

Table 2. Parameters of analytical representation of isolated variations.

	Range of Values							
	Gage		Alignment		Cross Level		Profile	
Signature	A (in)	k (ft^{-1})	A (in)	k (ft^{-1})	A (in)	k (ft^{-1})	A (in)	k (ft^{-1})
Cusp	0.8-1.4	0.016-0.061	0.5-3.0	0.011-0.103	0.9-3.0	0.031-0.095	0.9-3.0	0.016-0.095
Bump	0.8-1.4	0.031-0.040	0.5-2.8	0.009-0.083	1.0-3.0	0.017-0.031	0.5-4.0	0.013-0.065
Jog	–[a]	–[a]	0.5-3.3	0.006-0.025	1.6-2.8	0.020-0.050	0.5-5.0	0.008-0.045
Plateau	0.8-1.3	0.029-0.08	1.2-1.6	0.025-0.027	0.6-1.0	0.026-0.04	0.9-3.0	0.009-0.033
Trough	–[a]	–[a]	1.4-2.2	0.013-0.029	–[a]	–[a]	0.7-2.0	0.020-0.025
Sinusoid	–[a]	–[a]	0.8-1.2	0.033-0.020	–[a]	–[a]	1.0-1.5	0.020-0.025
Damped sinusoid	0.5-1.0	–[a]	1.0-2.2	0.013-0.015	0.9-1.2	0.051-0.061	–[a]	–[a]

[a] Signature not observed in the data.

Figure 4. Examples of signatures of isolated track-geometry variations.



the appearance of saw-tooth cross level. Periodic bumps and sinusoids commonly appear in mean alignment on bridges and spirals. A periodic cuspy-type behavior is also commonly observed in gage and single rail alignment in curves. The mean profile can also develop quasi-periodic bumps at mud spots and periodic jogs in spirals. Figure 5 shows some examples of periodic signatures.

For the purpose of this discussion, the combined track-geometry variations are defined as the ones that occur simultaneously in more than one track-geometry parameter. Some of the track-geometry parameters such as gage and alignment and cross level and profile are closely related with each other. However, large-amplitude isolated variations may also exist simultaneously in other pairs of track-geometry parameters. Such combined variations may cause a severe vehicle-track dynamic interaction.

Figure 6 is an example of combined track-geometry variations in a compound curve. A plateau in the cross-level deviations is evident at the transition point. Both profile and alignment show bump signatures at the same point. Furthermore, the gage shows a cuspy periodic behavior throughout the curve.

RELATIONS BETWEEN TRACK-GEOMETRY PARAMETERS

A vehicle receives simultaneous input from gage, line, and surface irregularities. In order to provide reasonable experimental and analytic simulations of actual railroad operating conditions, it is therefore necessary to investigate the relations between track-geometry parameters.

Track-geometry data typical of U.S. track were analyzed to determine the linear relations between track-geometry parameters. These analyses were conducted in the frequency domain by generating auto-spectral densities, cross-spectral densities, coherence functions, and transfer functions (3).

Gage and Alignment

Analyses were conducted to determine the relations between gage and mean alignment and gage and single-rail alignment (left alignment or right alignment). No significant correlation was found between gage and mean alignment variations. However, gage and single-rail alignment were found to have a significant linear relation.

Figure 7 is an example of coherence between gage and single-rail alignment. The coherence here is defined as

$$\hat{\gamma}^2(f) = [|\bar{G}_{xy}(f)|^2]/[\bar{G}_x(f)\bar{G}_y(f)] \qquad (3)$$

where

$\hat{\gamma}^2(f)$ = coherence function,
$\bar{G}_{xy}$ = average cross-spectral density between gage and alignment,
$\bar{G}_x(f)$ = average auto-spectral density of gage, and
$\bar{G}_y(f)$ = average auto-spectral density of alignment.

Note that many authors define ordinary coherence as the square root of $\hat{\gamma}^2(f)$. However, $\hat{\gamma}^2(f)$ will simply be called coherence here since it has a direct interpretation. The values of $\hat{\gamma}^2(f)$ lie between zero and one. A value of zero indicates no linear relation between gage and alignment. On the other hand, a value of unity indicates a perfect linear relation. An intermediate value such as 0.75 means that 75 percent of the variations in gage are explained by the linear relation between gage and alignment.

Figure 7 indicates strong coherence (≈0.71) for wavelengths shorter than 100 ft. This implies that there is a significant linear relation between gage and single-rail alignment.

Figure 8 shows typical coherence between the left and right alignment variations. The squared coherence for wavelengths longer than 100 ft is close to unity for most cases. The wavelengths shorter than 100 ft show a decrease in coherence. This would indicate that variations of both left and right alignment variations of the two rails become more or less independent as the wavelength decreases.

Figure 5. Examples of periodic track-geometry variations.

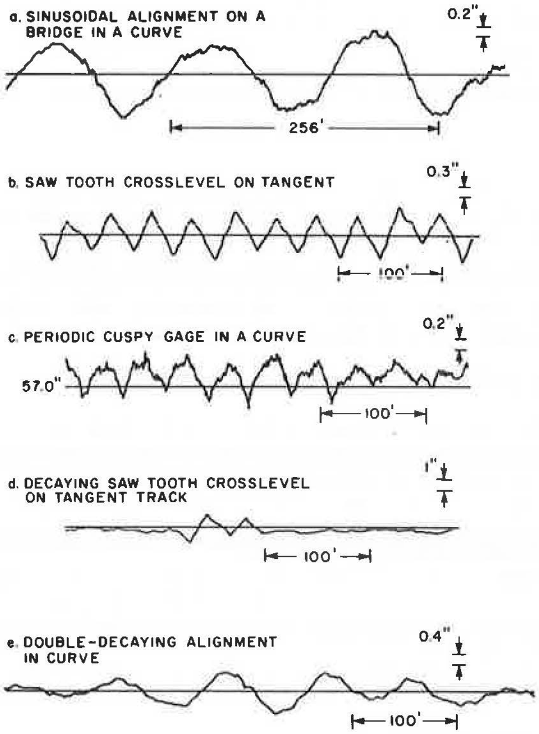
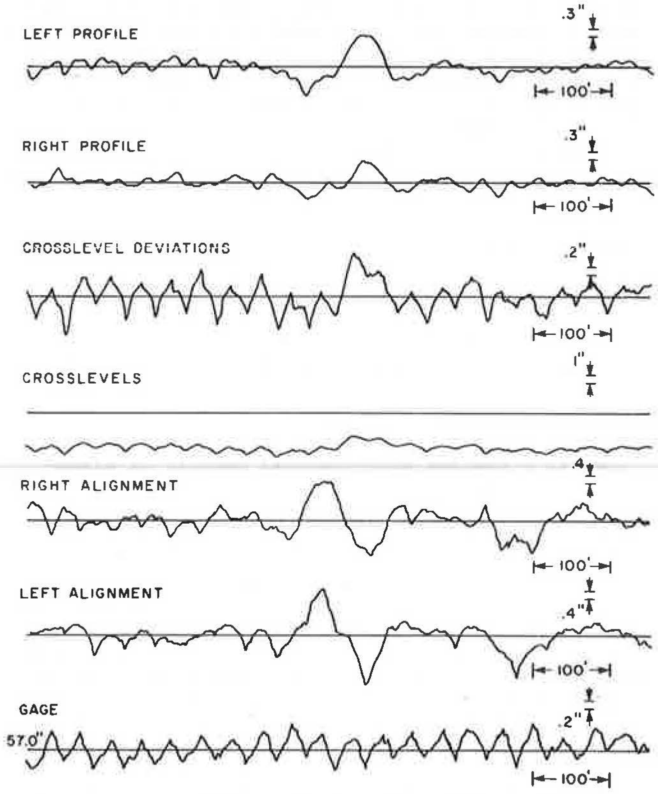


Figure 6. Combined track-geometry variations.



Cross Level and Profile

Cross level showed almost zero coherence with mean profile. However, interesting results were obtained for cross level and single-rail profile. Figure 9 shows the typical coherence between cross level and the single-rail profile for bolted track. The coherence is not very significant except at certain discrete wavelengths. The most noticeable is the peak at the 39-ft wavelength (equal to the rail length). This is attributed to relatively severe surface variations at joints.

Figure 10 is an example of typical coherence between the left and the right rail profile. A significant coherence is shown for wavelengths longer than 20 ft. However, the wavelength of 39 ft shows a decrease in coherence. For many sections of class 2 and 3 bolted track, the coherence was almost zero at this wavelength. However, there was a

Figure 7. Coherence between gage and single-rail alignment.

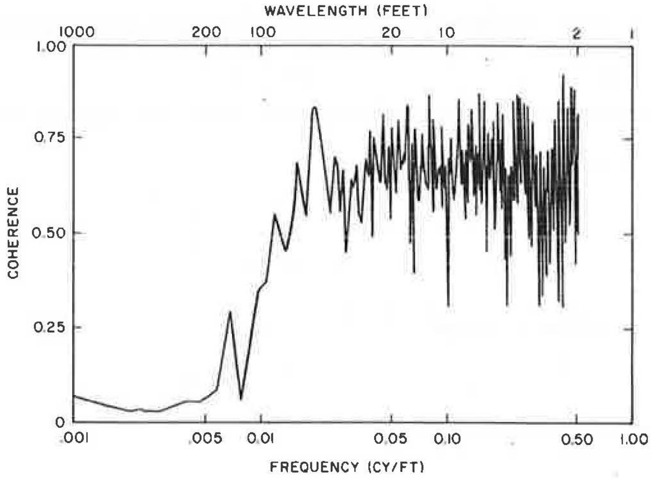


Figure 8. Coherence between left and right alignment.

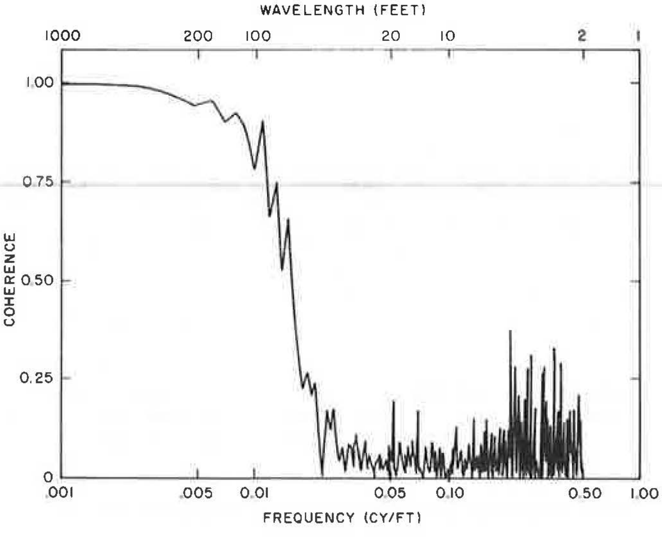


Figure 9. Coherence between cross level and left profile.

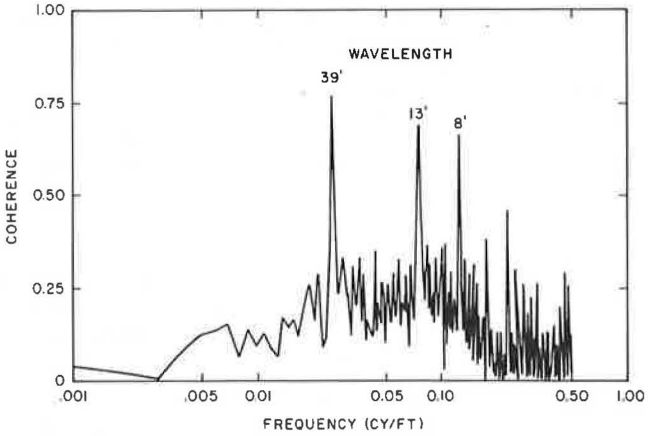


Figure 10. Coherence between left and right profile.

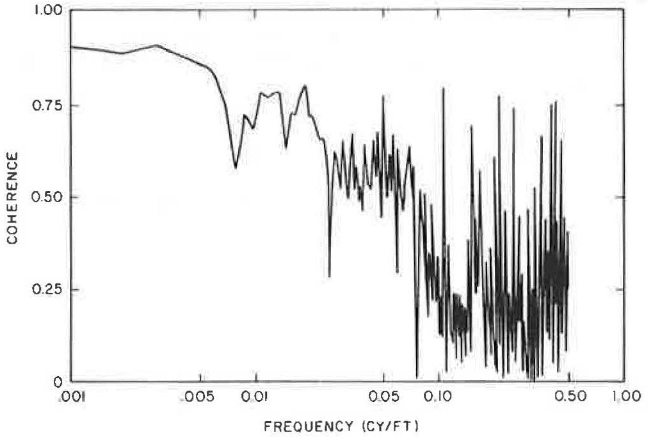


Figure 11. Coherence between cross level and right alignment.

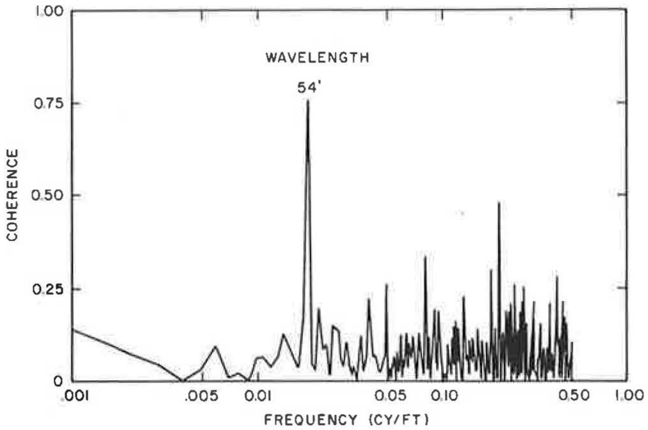
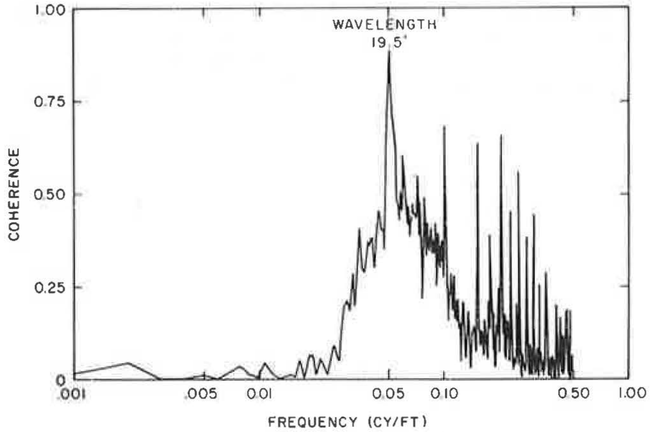


Figure 12. Coherence between gage and mean profile.



signif t coherence peak at the 20-ft wavelength. Th ienomena are due to regularly spaced half- ced joints.

ss Level and Alignment

In general, cross level showed insignificant correlation with alignment. However, there were exceptions for some track sections. Such an example is shown in Figure 11, which shows a coherence peak at a 54-ft wavelength. This was especially true for some welded track sections of class 4 or better track. In many cases, the most pronounced wavelength was 78 ft where the coherence in some cases peaked from 0.7 to 1.0. The exact cause for this is not known at this time. This can possibly be attributed to combined cross-level and alignment variations due to certain local structural, traffic, or maintenance characteristics.

Other Track-Geometry Parameters

Typically, there is no correlation between gage and cross-level variations. This, in general, is also true for gage and profile variations as well as profile and alignment variations. However, simultaneous degradation of track-geometry parameters may result in significant coherence at certain wavelengths. The bolted-track sections analyzed in this study exhibited strong coherence between gage and profile and between profile and alignment at a wavelength equal to one-half the rail length.

Figure 12 shows an example of coherence between gage and mean profile. There is an increase in coherence for wavelengths between 13 and 39 ft with a peak at 19.5 ft. This is believed to be due to the simultaneous degradation of track-geometry parameters at joints. Note that the degradation that corresponds to a joint is encountered every half-rail length on the half-staggered bolted track. This results in a significant correlation between gage and profile and profile and alignment variations at a wavelength equal to one-half the rail length.

SUMMARY AND CONCLUSIONS

This paper gives an overview of analytical descriptions of track-geometry variations. These descriptions are useful for design and simulation studies intended to improve track quality.

Track-geometry variations can be divided into two broad categories: typical and isolated variations. Typical variations can be described by a stationary random process that accounts for random waviness in the rail and a periodic process that describes relatively severe variations at joints. PSD models can be used to characterize the stationary random process. These models are functions of a roughness parameter and break frequencies. The break frequencies do not change with track class. However, the roughness parameter is strongly related to the current track class. A joint can be characterized by an exponential model with an amplitude parameter and a decay-rate parameter. Both parameters increase with track degradation.

The isolated track-geometry variations usually occur in spirals and at special track features such as bridges, road crossings, and turnouts. Most of the isolated track-geometry variations can be characterized by one of seven key signatures: cusp, bump, jog, plateau, trough, sinusoid, and damped sinusoid. These key signatures can be modeled as a function of amplitude and duration. The key signatures can occur as single events, in periodic forms, and simultaneously in more than one track-geometry parameter.

There is a significant linear relation between gage and single-rail alignment. In general, there is an insignificant correlation between any pair of gage, cross-level, mean profile, and mean alignment values. However, long wavelength combined with variations in cross level and alignment may result in strong correlations at certain discrete wavelengths between 50 and 90 ft. Furthermore, simultaneous degradation of track geometry at joints may result in significant correlation between any pair of gage, mean profile, and mean alignment values at a wavelength equal to one-half the rail length.

ACKNOWLEDGMENT

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