expected supply of empty cars, and backlog of demand. Central management could also make available to the terminals the operating guidelines developed by using the model as a simulation tool. A very important by-product of operational implementation would be the daily documentation of the costs of system shortages and surpluses. This could serve as input to decisions regarding disposal or acquisition of cars in the long term. By using this information, a railroad could develop a relation between fleet size and the volume of demand that can be satisfied over a given network and flow pattern.

CONCLUSIONS

An optimization model for the management of empty freight cars has been developed. The perspective on which this research is based is that an important element of the car-distribution problem is the lack of coordination between decisions made centrally for the railroad as a whole and decisions made locally at individual terminals. The model provides a mechanism to coordinate the decisions made at these two levels. The basis of the model is the creation of internal transfer prices that reflect the opportunity costs of cars at various points in the network in each time period. The model can be used by a railroad as a simulation tool for testing and evaluating potential improvements in car-distribution practices. It can also be used operationally to obtain transfer prices for empty cars on a daily basis. This type of use would be consistent with a greater emphasis on individual terminals as profit centers for management purposes.

Tests of the model by using data from a cooperating railroad indicate that it leads to distribution decisions that reduce empty-car miles, empty trips, and empty-car days, without reducing the percentage of demand satisfied. Thus, the methods developed in this research can improve car use significantly and, consequently, enhance railroad profitability.

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REFERENCES


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Where Does the Energy Go? A Simplified Perspective on Fuel Efficiency in Rail Freight Transportation

JOHN B. HOPKINS

The potential impact of measures intended to reduce fuel use in the operation of mainline freight trains can often be bounded or approximated by relatively simple calculations. In some cases, this may be sufficient to reject options that are intuitively appealing but actually offer very small gains, thereby avoiding expensive simulation or measurement programs. When a potential conservation measure is found to have sufficient promise to warrant detailed examination, preliminary estimates can ensure that further efforts are well-founded and properly structured. In this paper, a standard train-resistance equation and industrywide aggregated data are used to assess the effect of various equipment characteristics and operating scenarios on fuel use for a baseline case (a 4700-ton, 66-car train traveling at 40 mph) in order to develop useful approximations. Emphasis is on examination of the specific physical mechanisms associated with dissipation of energy in the movement of trains. Topics considered include car weight and aerodynamics, radial trucks, locomotive idling, power-to-weight ratio, track structure, stops, operating speed and variations in speed, and coasting strategies. Quantitative estimates of fuel-consumption impacts are presented for each factor in terms of the baseline train. Extension to other cases is facilitated by presentation of data and general formulas. One noteworthy finding is that locomotive efficiency factors and train resistance account for only half of the fuel actually consumed; the major portion of the remainder appears to be dissipated in braking and, to a lesser extent, in overcoming curve resistance.

The rapid increase in fuel costs in recent years has dramatically increased the importance of minimizing
energy use in transportation. For the railroad industry, fuel costs now represent approximately 13 percent of total operating costs, as compared to 3.6 percent in 1970 (1). Evaluation of equipment and operational alternatives to reduce petroleum consumption has become a matter of great importance. However, many interacting factors affect fuel efficiency and in practice the process of assessing conservation options often leans toward one of two extremes: (a) intuitive judgments or (b) elaborate and expensive computer modeling and in-service measurements. The former approach is highly variable in accuracy and often lacks sufficient credibility to be useful. Simulations and measurement programs are generally more convincing but can be very expensive, thus limiting their use; and even then validity of the results will depend on the degree to which the basic subject is understood and all relevant effects are considered. Subtle but critical factors, such as variation of diesel locomotive efficiency with power and speed, are often not included in computer models or may be approximated in a manner insufficient in accuracy for most cases but not adequate when the conservation option focuses on those aspects. Both simulations and in-service measurements impose severe constraints on the user in terms of controlling all variables or having information sufficient to compensate for variation. For example, fuel use is strongly related to speed. Implementation of a locomotive modification (on a computer model or a real train) may produce, as a by-product, a reduction in average speed, and the improvement found may in fact be due to the slower operation rather than to the equipment modification. In the assessment of conservation options, difficulties associated with oversimplification or with excessive complexity can be greatly lessened by understanding the various physical processes by which fuel energy is transformed into useful work or dissipated as heat. Fuel can be saved only by affecting these processes. Once they are analyzed, simple back-of-envelope calculations may provide estimates of potential savings that are sufficient for decisionmaking, particularly when the conclusion is negative; they will almost always be helpful in focusing and structuring simulation studies or revenue-service testing.

In this paper, results are presented that relate fuel consumption to a variety of rolling stock and operational characteristics. Baseline scenarios and industrywide data are used to provide simple analytical expressions and quantitative results for typical cases. In general, extrapolation to other assumptions or circumstances will be a simple exercise. Topics considered include car weight and aerodynamics, radial trucks, locomotive idling, power-to-weight ratio, track structure, stops, operating speed and variations in speed, and coasting strategies. It is hoped that the material presented here will be of value to railroads and others involved in the preliminary assessment of fuel-conservation options for rail transportation and in structuring sophisticated analysis and measurement efforts. Of course, impact on fuel use is only one of the consequences of most alternatives. Evaluation of the desirability of a particular course of action must include consideration of overall cost, safety, environmental effects, service impacts, and institutional constraints.

WHERE DOES THE ENERGY GO?

At its heart, the problem examined here is a simple one. From the physicist's point of view, the fuel consumed by a diesel locomotive in moving a train from origin to destination is used to create heat by a variety of means and, if the destination is at a different elevation than the origin, to change the gravitational potential energy of the train. The change in potential energy is not normally within our control. (In some cases, this can be significant. The work done against gravity in raising a 5000-ton freight train through a height of 1 ft is equivalent to the energy available in 1.3 gal of diesel fuel; so if the train rises 1000 ft in 500 miles, the fuel converted into gravitational potential energy is 0.5 gal/1000 gross ton miles. A descending train gets the same amount of fuel as a gift from gravity.) It is that portion of the fuel energy that is converted into heat that we must be concerned with, for it is this that we can more often seek to affect. The major processes of interest are combustion, conversion to rail horsepower, overcoming rolling resistance, and braking. Each will now be described briefly; the primary emphasis in this paper is on the last two.

Thermodynamic Efficiency

Large diesel engines, whether used for locomotives, trucks, or ships, represent a relatively efficient form of internal combustion. They typically use approximately 38 percent of the chemical energy stored in the fuel (2). (The exact value depends on engine characteristics, altitude, temperature, and other factors.) Advances in this area are primarily the province of the engine designer and manufacturer—not the user—and I will not discuss this topic further. However, note that any improvement in combustion efficiency gives an equivalent percent gain in ton miles of transportation per gallon. Thus, maintenance practices that ensure that locomotives operate at top efficiency can provide a good economic return. Combustion efficiency is affected by power level—typically it is somewhat lower in third throttle notch than in eighth, for example—but this effect is a small one.

Conversion to Rail Horsepower

The result of the combustion process is to convert a portion (approximately 38 percent) of the chemical energy stored in the fuel into mechanical (kinetic) energy at the crankshaft. To do useful work, the energy available at the crankshaft is then converted into electrical energy in a generator and back into mechanical form by the traction motors that drive the wheels through gears. These processes are highly efficient, but some energy is inevitably converted into generator and motor heat, gear losses, etc. The efficiency of the overall series of transformations is a complex function of train speed, power level, and other factors, but is typically about 90 percent (2). In addition, there are several auxiliary systems on the locomotive—fans, air compressor, water pumps, etc.—that require power to operate and vary with operating conditions and environment. The total energy loss associated with these accessories is taken here as 7 percent (3). This, too, is an area in which locomotive designers continually seek to improve efficiency; the user's primary role is assurance of fuel-efficient maintenance practices.

Train Resistance

The heart of railroad fuel consumption is the need to overcome various types of friction and other forces that resist movement of the train. The friction of steel wheels rolling on steel rails is very low (this is the heart of the relatively high fuel efficiency attainable by trains) but is not zero. A
variety of physical mechanisms is involved in over-
all train resistance, but few of them are understood
in a cogent, quantified, direct wheel and
rail friction, flange resistance, and "pumping" of
energy into the roadbed structure are direct inter-
actions with the track. The car bearings and bear-
ing seals convert a small but still significant
amount of energy into heat. The entire car (or
locomotive) body is subject to aerodynamic drag.
All of these factors are potentially relevant to
improvement of fuel efficiency.

Braking

The basic principle normally embodied in stopping
any land vehicle is conversion of the kinetic energy
associated with its movement into heat energy in the
brakes, which is then dissipated into the air.
Thus, train braking practices and specification of
speed profiles are operational factors that can have
strong impact on fuel efficiency. An added compli-
cation (not addressed in this paper) is the critical
relation between braking practices and safety.
Under some circumstances, braking practices that
minimize fuel consumption might conceivably lead to
dangerously high dynamic forces and impacts within
the train. This is an important constraint on cer-
tain operational fuel-efficiency options.

BASIC DATA, ASSUMPTIONS, AND SCENARIOS

The starting point in the estimation of freight
train fuel consumption is the train-resistance equa-
tion that expresses the force required to move a
freight car or locomotive in terms of car weight,
speed, and an aerodynamic factor related to frontal
area. A commonly used resistance equation in the
United States is the "modified Davis equation" (4)
that has the same functional form as the much ear-
lier equation of Davis (5) but assumes different
coefficient values. Both equations are based on
fitting curves to a limited quantity of experimental
measurements of actual train resistance; the modi-
ified form, dating from the 1950s, is thought to be
more appropriate to modern rolling stock than the
original Davis values. It has recently been found to
give the better fit to experimental measurements
of fuel consumption (6). For standard freight cars,
the modified Davis equation is as follows:

\[ R = 0.6W + 80 + 0.01Wv + 0.07v^2 \]  

(1)

where

- \( W \) = car weight (tons),
- \( V \) = speed (mph), and
- \( R \) = train resistance (pounds of force).

Although the equation is basically the result of
curve fitting, it is customary to associate each
term with a particular physical mechanism, and the
subsequent analysis here is based on these assump-
tions. The meaning ascribed to each term is as
follows: 0.6W = rolling friction, 80 = bearing fric-
tion (assumes 4 axles), 0.01Wv = flange fric-
tion, and 0.07v^2 = aerodynamic drag.

The increase in resistance on curves, primarily
arising from the non-zero angle between wheel and
rail for a rigid 4-axle truck, is included by adding
the term 0.6W, where C is the track curvature in
degrees. For ascending or descending grades, an
additional term is added—20gW, with g being the
gradient expressed in percent. This term is simply
the component of gravitational force that acts
parallel to the track. A similar equation is used
for locomotives, the only change being a larger
coefficient for the aerodynamic term for the first
locomotive.

This equation permits immediate calculation of
useful information. For example, it says that the
force (tractive effort) to move a 65-ton freight car
at 40 mph is 257 lb. (This is often expressed in
newtons, lb force as 1158 lbf/ton.) A 4-car train of
65-ton cars, the force needed is 25 700 lb, and
the horsepower required (power = force x speed)
is 2741 hp. Locomotives typically provide about
23-hp per gallon of fuel, so by this back-of-envelope calculation it is found that 119
gal should be needed to overcome train resistance in
moving this 6500-ton train for 1 h, or for 40 miles;
this corresponds to fuel use of 2184 gross ton
miles/gal.

However, the above calculation ignores locomotive
losses, grades, curves, and braking, which will be
shown later to be very important. For example, at
40 mph each degree of track curvature increases the
resistance of a 65-ton car (and its fuel consump-
tion) by 20 percent. The gravity term is 20 lb/ton
per percent gradient, which completely overshadows
the other component (4 lb/ton in the example just
given) for any significant grade. Energy put into
moving the train up a hill may be partially returned
when coming down the other side, but only to the
degree that the train is allowed to coast. This
element requires careful examination in any case.
(Certain aspects of operation on grades are dis-
cussed in a later section.)

In order to estimate the amount of fuel that is
consumed by the average train to overcome curve
resistance and braking losses, it is necessary to
examine aggregated data. In 1979, the nation's
freight railroads consumed 4.07 billion gal of fuel
(1). The primary fuel uses, in addition to movement
of trains, are idling of line-haul locomotives and
operation of yard-switching locomotives. It can be
calculated that the 23 000 line-haul locomotives in
the United States, which idle approximately 12 h/day
(2), have a fuel-consumption rate at idle that
averages 4.9 gal/h for the fleet. This implies a
total use of 490 million gal. Extrapolation of data
from a recent classification yard study (3) indi-
cates that 390 million gal are consumed by yard
and switching engines. Fuel leakage, spillage, and
theft have been estimated as high as 10 percent, but
recent interest in conservation has probably reduced
these losses. A value of 2 percent (80 million gal)
is used in this analysis. Finally, 40 percent (40
million gal) is assigned to work trains (2). Under
these assumptions, the total amount used for
revenue-service train movement in 1979 was approxi-
mately 3.07 billion gal. In the same year, freight
train miles totaled 447 million, with an average
train consisting of 66 cars. The average car was
loaded 58 percent of the time, and the mileage-
weighted average load was 58 tons (1). For an
assumed average empty-car weight of 30 tons, the
average train weight then becomes \((66 x (0.58 x 58 +
30)) = 4200\) tons. A reasonable power consist for a
4200-ton train would be three 2000-hp locomotives,
each weighing 175 tons. For a total average train
weight of 4725 tons. Gross ton miles in 1979 then
total 2112 billion \((4725 x 447 000 000)\), for an
implied overall specific fuel consumption of 1.45
gal/1000 gross ton miles. This corresponds to an
overall industry average of 1688 gross ton miles/gal
or, by using the above average-load figures, 320 net
ton miles/gal. This result is based only on fuel
used in actual movement of revenue-service freight
trains.

These industrywide aggregated figures can now be
combined with the train-resistance equation dis-
calculated for the other components used, as follows:  

- **Yards and switching**
  - Gallons (000 000s): 390
  - Percentage of Total: 9.6

- **Idling**
  - Gallons (000 000s): 490
  - Percentage of Total: 12.0

- **Spillage, work trains, etc.**
  - Gallons (000 000s): 120
  - Percentage of Total: 3.0

### Line-Haul Service

- **Locomotive losses**
  - Gallons (000 000s): 522
  - Percentage of Total: 12.8
- **Rolling friction**
  - Gallons (000 000s): 163
  - Percentage of Total: 4.0
- **Flange resistance**
  - Gallons (000 000s): 347
  - Percentage of Total: 8.5
- **Bearing losses**
  - Gallons (000 000s): 110
  - Percentage of Total: 2.7
- **Aerodynamic drag**
  - Gallons (000 000s): 485
  - Percentage of Total: 11.9
- **Curve resistance**
  - Gallons (000 000s): 215
  - Percentage of Total: 5.3
- **Braking**
  - Gallons (000 000s): 1228
  - Percentage of Total: 30.2

**Total**: 4070

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The sensitivity of fuel use to various freight car characteristics that affect train resistance is now considered. This discussion is based on the average train described earlier: number of cars, 66; empty-car weight, 30 tons; average age and load, 3.5 tons; total power, 6000 hp; locomotive weight, 525 tons; gross weight, 4725 tons; net weight, 2220 tons; power-to-weight ratio, 1.27; and gallons per 1000 gross ton miles at 40 mph, 1.45.

The scenario assumed here is a car lifetime of 500,000 miles. Since a car with reduced resistance to movement requires less horsepower for the same speed, on average there is a secondary benefit to lower resistance through decreased locomotive requirements. This could have significant economic effects. However, I shall confine this discussion to energy benefits, where the only secondary advantage is reduction (on average) of locomotive losses and weight due to reduced power needs. This factor is incorporated by taking the value of 1.45 gal/1000 gross ton miles as associated entirely with cars, distributed among the various resistance factors, in proportion to the magnitude of each. The estimated normalized fuel consumption and expected fuel use over the 500,000-mile life of each car are shown in Table 2, based on this assumption.

One conservation measure being applied extensively to automobiles is vehicle weight reduction. Let us examine that strategy for freight cars in the light of the fuel-use components in Table 2 (except bearing losses and aerodynamic drag) are proportional to car weight, so the total weight-sensitive amount during the car life is 35,400 gal. For the baseline case, the empty-car weight is 47 percent of the average weight in service; so the 30-ton empty weight utility costs 16,700 gal of fuel. This implies that over a car lifetime, every ton by which empty weight is reduced saves 560 gal—about 1 quart of fuel/lb. Several warnings go with this calculation and with subsequent results of the same type. It is based on numerous assumptions and estimates that may either be inaccurate or be inapplicable in a particular case. Further, the economic benefits and costs of any change should be analyzed in terms of discounted net present value, based on actual cash flows. Most equipment modifications impose the cost at the beginning, whereas the benefits accrue slowly over several decades.

These results indicate that about three-eighths of rail freight fuel consumption is associated with empty-car weight. However, achieving the required structural strength at lower weight by substitution of lighter materials would imply significantly higher material costs, so the economics of that approach are unlikely to be favorable. Instead, improvements are likely to arise from advances in structural design. Indeed, major structural and design changes are just what is observed in the recent industry development of lightweight flatcars for containers and trailers.
Reduction of aerodynamic losses has been a major factor in motivating changes in the design of rolling stock for trailer-on-flatcar (TOFC) service. For the average boxcar considered in the baseline case, air drag accounts for about 20 percent of the fuel consumed over the service life of the car. For the TOFC case, with a much larger cross section (when loaded) and operation typically at higher speeds, drag is much more important. The aerodynamic coefficient used in the modified Davis equation for train resistance is customarily increased from the box-hopper car value of 0.07 to a value between 0.16 and 0.20 for TOFC. For a value of 0.20, a speed of 60 mph, and a total loaded weight of 85 tons, the conventional TOFC car has more than 2.5 times the resistance per ton (and fuel consumption per ton mile) of a baseline boxcar. Indeed, for this high-speed case, the aerodynamic drag represents 80 percent of the total resistance and would be responsible for use of 46 400 gal of fuel in 500 000 miles of loaded operation. Actual car use might be substantially different than assumed above, but the finding that a 10 percent reduction in drag could be worth several thousand gallons of fuel (4640 gal in this example) explains the interest that both government and industry have had in the subject. For most other rolling stock, the reduction in the coefficient of streaming is much less (860 gal for a 10 percent drag reduction in the baseline case, but it is possible that some improvements in this area may be accomplished relatively easily for specific cases once their importance is realized.

The final area considered in looking at freight car fuel use is in the curve resistance component. This term is primarily associated with an inherent property of the conventional rigid three-piece freight car truck. Both axles are held parallel to each other so that only one at most can be aligned along the radius of the curve. Freight car trucks going around curves are usually cocked at a slight angle so that neither axle is perfectly radial, which results in friction and wear at the wheel-rail contact points. A number of radial truck designs have recently come into use, all with the objective of allowing both axles simultaneously to align themselves radially so that the wheels can roll smoothly around curves. Although reduction of costly wheel and rail wear is a major motivation for this development, energy considerations can also be significant. As shown in Table 2, the car-life fuel cost associated with curve resistance for the baseline TOFC car was estimated to be almost 4000 gal, or 2000 gal/truck. An additional benefit in reduced flange resistance is also possible.

**LOCOMOTIVES**

It is estimated above that in 1979 the 23 000 line-haul locomotives in the United States consumed approximately 3.07 billion gal of fuel annually in mainline service, with conversion, transmission losses, and power for auxiliaries responsible for about 17 percent of this, or 22 700 gal/year. This overall average implies use of 133 500 gal/year per locomotive. However, it is clear that this aggregated figure is strongly affected by the inclusion of many units used in branch-line and industrial switching operations. Based on standard fuel rates and throttle-notch duty cycles, it has been estimated that locomotives in mainline freight operation burn 320 000 to 400 000 gal/year (4). The 17 percent losses within the locomotive then imply 54 000 to 68 000 gal/year. This loss is probably declining because of major efforts by locomotive manufacturers to improve the efficiency of their products. Nonetheless, as noted earlier, it is important that maintenance practices be such that the full efficiency potential of the locomotive is realized; a 5 percent degradation would cost more than 1100 gal/year per locomotive on average, and as much as 3400 gal/year for units in mainline service.

The previously noted expenditure of an estimated 450 million gal of fuel per year used during idling for the 23 000 line-haul locomotives implies a use of 21 300 gal/unit. Recent developments may have reduced this significantly for newer models (3). There are several technical and operational obstacles to temporary shut-down of locomotive diesel engines, but the attention focused on this subject since the first oil shortage in 1973 has undoubtedly made the point that, where practical, reduction of idling time can be a significant cost saving. A 5 percent improvement moves almost 1100 gal/year.

The subject of optimal power-to-weight ratios often comes up when rail freight fuel efficiency is discussed. The importance of this factor is sometimes exaggerated. It must be remembered that we use only the horsepower—and hence fuel—that is needed to move the train, regardless of how much power is available. Only when the engines operate at very low throttle settings—second or third notch (10 percent of rated power)—is there any significant fall-off in efficiency, and even then it is only about a 10 percent drop. The other basic energy cost of a high power-to-weight ratio is due to the additional train resistance associated with the extra locomotives. Typical 3000- to 3600-hp locomotives deliver approximately 18 hp/ton of locomotive weight. Simple algebra then yields the relationship

$$W_p = \frac{18}{(18 - P_L)} W_t$$

where $W_p$ is the total train weight, $W_t$ is the total weight of the cars, and $P_L$ is the power-to-weight ratio. It is a straightforward matter to use the train-resistance equation to calculate the ratio of the fuel used for a specified $P_L$ to the fuel required for the baseline train. That ratio is slightly less than unity for $P_L = 1$, rising to 1.3 at $P_L = 5$. Results would differ for any other baseline train or assumed speed, but the gradual nature of the variation and the magnitude of these values are representative.

There are several technical and operational factors that dominate the choice of power-to-weight ratios, including grades, speed, reliability, and locomotive availability. However, it is increasingly relevant to include fuel costs in the decision process.

**TRACK**

A subject not frequently mentioned is the relation of track condition and parameters to train resistance, and hence to fuel use. Yet, the figures shown previously in Table 2 imply a direct relevance. Rolling friction, flange resistance, and curve resistance are all likely to be affected to some degree by track characteristics. For the baseline case, track-related factors total 0.41 gal/1000 ton miles, or about 28 percent of total fuel use. Perhaps more important, and not directly represented in the train-resistance equations, are the effects of poor substructure, mismatched joints, etc. Most defects in track structure will in some degree facilitate transfer of energy from the train into the earth below, thereby wasting fuel. If, for example, in a severe case these factors lead to an increased fuel use of 0.20 gal/1000 ton miles (a 50 percent increase over the baseline), the effect would be significant. That would imply an annual consumption of an unnecessary 200 gal/mile of track for every million gross tons (MGT) hauled over that mile. To place this in perspective, note that in 1979 the industry spent approximately $4.6 billion on maintenance-of-way and structures while moving
approximately 1600 billion gross ton miles of freight (1), or $2500/mile per MGT. Although the relation between track structure and rolling resistance is not well understood, this finding suggests potential relevance of this consideration to research and maintenance decisions.

OPERATING PRACTICES

The principal operating variable that affects fuel use is train speed. First, I use the modified Davis equation to assess the basic sensitivity of fuel consumption to speed. For the baseline train used in this analysis, the fuel used per car is approximately linearly with speed in the range from 30 to 50 mph, with the percentage change in fuel consumption per mile very nearly equal to the percentage change in speed. Thus, a 10 percent increase in speed (from 40 to 44 mph) will produce approximately a 10 percent saving in the fuel needed to overcome train resistance. It is also likely that the effect on the fuel associated with braking will be at least the same magnitude, since the kinetic energy to be dissipated increases as the square of the speed. (The effect of braking will be discussed further.) Some other cases may also be of interest. The same calculations for a train of 100-ton cars show that, in the range from 15 to 35 mph, the relative fuel-consumption change is only about half as great as the relative speed change. On the other hand, for a typical TOFC train with 85-ton cars, fuel use in the vicinity of 60 mph increases about 1.6 times faster than velocity; consumption is 75 percent greater for 70 mph than 50 mph, a speed change of only 40 percent.

The fuel loss associated with slowing down and stopping—dissipation of energy as heat in the brakes and wheels—can be significant if such events are frequent. This loss can be expressed in terms of the miles a train could travel at its nominal speed on the fuel dissipated in a single stop. This quantity can be estimated simply. The energy lost in a stop is the train's kinetic energy, \( \frac{l}{2}mv^2 \), with \( m \) the mass of the train. If this energy had instead been used to move the train, it could have done work equal to \( RXD \), where \( R \) is the train resistance and \( D \) is the distance the train could have moved. When appropriate unit conversions are made, the resulting expression for \( D \) is simply

\[
D = \frac{1}{2} m \frac{v^4}{R}.
\]

In miles per hour, \( D \) is

\[
\frac{1}{2} \frac{m}{R} \left(\frac{v}{20}\right)^4.
\]

For slowdowns, \( v' \) is replaced by \( v' - v'' \), where \( v' \) and \( v'' \) are the initial and final speeds, respectively. For example, for a 40-mph baseline train, \( R' \) is 4.0, which yields a distance \( D \) of 5.1 miles. A 70-mph TOFC train (85-ton cars) can gain about 4.5 miles on the fuel used in a full stop, and a 20-mph train of 100-ton cars in one full stop uses enough fuel to go 2.7 miles. The ratio of these distances to the average distance between stops is a good measure of the total fuel impact of the stop.

It is unusual for a freight train to go from origin to destination at a constant speed. Even on flat terrain, there are typically varying speed limits for a variety of reasons: track conditions, traffic, track maintenance, rail-highway crossings, etc. To some degree, it may be possible for a railroad to control or modify factors such as these or at least minimize their impact. It is therefore relevant to seek a simple means of estimating the effect of speed variations. One can gain insight into this topic by considering a particularly simple scenario: alternation between two speeds \( v_0 + v' \) and \( v_0 - v'' \), with the distances traveled at the two speeds such that the overall average speed is \( v_0 \). This will be true if the fraction of the trip at the lower speed is \( (v_0 - v')/2v_0 \), and the fraction at the higher speed is \( (v_0 + v')/2v_0 \). Under these circumstances, an expression is easily obtained for the factor by which the amount of fuel consumed for the varying-speed scenario exceeds that which would be used at constant speed. This factor turns out to be

\[
\left[1 + \frac{r(v' - v')^2}{v_0^2}\right],
\]

where \( r \) is the ratio of relative change in fuel consumption to relative change in speed; \( r \) was calculated above for several cases in the discussion of fuel use as a function of speed. (For the baseline case, \( r = 1.0; \) for low-speed 100-ton-cars, \( r = 0.5; \) for high-speed TOFC, \( r = 1.6. \)) Thus, if the baseline train achieves an average speed of 40 mph by actually traveling part of the trip at 30 mph and part at 50 mph, \( v' = 10, v_0 = 40, \) and fuel consumption is greater by 6 percent than for operation at a constant 40 mph.

Insight into the fuel implications of operations in rolling terrain can be gained through a simple analysis of the two scenarios. A train could be operated at a constant velocity (with braking) down a descending grade and up the following ascent (with potential energy excluded). Alternatively, the train could be allowed to accelerate under gravity on the descending grade, and then coast part or all of the way up the subsequent hill. In the constant-velocity case, the energy per ton necessary to overcome train resistance (\( R' \)) for the up-down sequence (including the gravity term) is calculated from ascending train resistance multiplied by the ascent distance only, since no energy need be supplied on the descent. If both grades are of distance \( D \) (miles) and gradient \( g \) (percent) with train velocity \( V \), the energy (per ton) is given by

\[
(R' + 20g)D,
\]

where \( R' \) a function of \( V \).

The coasting mode, on the other hand, requires sufficient power to overcome train resistance at all times on both segments (down and up) while the gravitational energy is merely transformed through acceleration and deceleration from potential energy at the top to kinetic energy at the bottom and back to potential energy again. For the assumed symmetric situation, the gravity component cancels out insofar as the power requirements are concerned. (In a more realistic model no power would be applied on the descent, with some potential energy going not into increased kinetic energy, but into coming train resistance. However, an equal amount of energy would then have to be supplied on the ascent, so the situation is nearly equivalent.)

The average velocity, \( V \), is well approximated by

\[
V = \frac{V' + V''}{2},
\]

where \( V' \) and \( V'' \) the speeds at the top and bottom of the grades. The energy for each segment is the integral of force (the train resistance \( R' \)) and distance. While suitable approximations, it can be shown that the ratio of the energy (fuel) required for the constant-velocity case to the energy for the coasting strategy can be expressed as

\[
\frac{0.5 + 10g}{R'}\frac{D}{V'},
\]

with \( g \) in percent and \( R' \) in pounds per ton. The difference between the two cases is basically the energy lost in downgrade braking in the constant-velocity mode. \( R' \) is typically in the range of 4-8 lb/ton, so for a 1 percent grade the constant-velocity case will require about 1.7 times as much fuel. For a 0.5 percent grade the differential is 1.1 times better. This very simple analysis does not include the idling fuel consumed on the downgrade for the second scenario, which would produce a fuel ratio lower (closer to unity) than the energy ratio determined above. On the other hand, the constant-velocity case may use dynamic brake, which also entails a significant fuel penalty. Relatively simple model-
Use of the coasting mode is limited by the acceptable minimum and maximum speeds \( V' \) and \( V'' \). Simple recourse to the law of conservation of energy and appropriate conversion of units yields the result that \( \frac{(V'')^2 - (V')^2}{1627\ \text{Dg}} = 1 \). This permits calculation of the maximum distance over which coasting can be applied without violating the speed constraints. For example, if \( V'' = 50 \) and \( V' = 60 \), \( \text{Dg} = 0.55 \), and \( D \) will be 1.1 miles for a 0.5 percent grade. In a more extreme case, if \( V'' \) is allowed to drop to 35 mph and \( V' \) to reach 65 mph, \( D \) would be 3.7 miles for a 0.5 percent grade and 1.84 miles for a 1 percent grade. (Recall that \( D \) is half the total descent-ascent distance.)

CONCLUSION

It is the purpose of this paper to demonstrate that many of the questions that arise in considering various fuel-efficiency measures in railroad freight transportation can be clarified by simple back-of-envelope calculations. Occasionally, rough approximations of this nature will yield sufficient understanding to support a final decision. More often, such preliminary estimates will be of value in determining the parameters and required accuracy of a measurement program or in establishing the model sophistication and scenarios to be used for computer simulation. The simple formulations presented here have generally been applied to a stated baseline case, but these approximate expressions are readily adapted to other cases. This discussion is also intended to encourage a broad awareness of many of the factors that bear on fuel efficiency, so that experimental or computer-based evaluations of options can be conducted with sensitivity to potential pitfalls and confounding factors.

In conclusion, it must be emphasized that these findings are to be used with care. They consider fuel efficiency only, whereas real-world decisions must encompass considerations of overall cost, safety, service, labor agreements, institutional constraints, etc. In addition, the expressions and calculations shown here are based on a high degree of simplification and apply to the specific cases considered. This paper is intended to provide insight and a realistic sense of the magnitude of the effects involved. It should not be seen as presenting precise quantitative results or as supporting or advocating any particular course of action.

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REFERENCES