provement of day-to-day system reliability, rendezvous of scheduled and nonscheduled vehicles, and response to emergency situations. Although the day-to-day service improvements were quite difficult to separate from normal background variation, it seems clear that, at an absolute minimum, those operations did not degrade with AVM in place and positive results were obtained in several instances.

No attempt was made to place an absolute dollar value on the benefits noted, as several aspects are quite qualitative and their consideration beyond the scope of this evaluation; e.g., quantifying the value of increased rider confidence in on-time, or at least more predictable, transit performance. Although some of the benefits may be partly quantifiable in the long term (e.g., if increased confidence leads to increased patronage of the system), such data are not currently available or are subject to substantial error in approximation.

It is anticipated that future analysis will provide a more complete picture of the impact of day-to-day improvements that might be expected through exercising AVM control. For example, during the latter stages of the data-collection effort, dispatcher use of system capabilities became more consistent. Thus, subsequent analysis and comparison of results during this time period with those obtained more than a year ago will have the advantage of both the consistent use of AVM capabilities by the dispatchers and the opportunity to ignore some of the temporal (seasonal) variations.

It is also anticipated that as the SCRTD staff becomes more familiar with the types of data available from the system for route scheduling and so forth, greater advantage will be taken of those opportunities in a straightforward fashion (i.e., better scheduling for the instrumented lines) as well as in using the four lines for testing new strategies for controlling day-to-day operations that might be transferable to non-AVM lines.

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Publication of this paper sponsored by Committee on Bus Transit Systems.

Comparing Fixed-Route and Flexible-Route Strategies for Intraurban Bus Transit

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The usual fixed-route strategy is not the only possible strategy for operating intraurban bus transit. Among the alternatives are flexible-route strategies. This paper focuses on the problem of choosing between fixed-route and flexible-route strategies in order to optimize operations. A mathematical model is used to determine the optimum quantity of service that should be provided under each strategy so as to minimize the costs to operators and users. The quantity of service is characterized by the headways between buses and is given as a function of the average ridership rates, unit costs, and travel times. By comparing the optimum states for the two strategies, the conditions under which one strategy performs better than the other are derived. Findings from the latter are then used to derive a general methodology for comparing the strategies. The highlight of the proposed methodology is that the two strategies must be compared at the extremes of a typical day's ridership levels before one can ascertain whether operating exclusively with either strategy or jointly with both strategies will give the best results. The present study is, however, limited to very small service areas.

The current methodology for comparing fixed-route and flexible-route operating strategies of intraurban bus transit consists essentially of using cost-effectiveness curves (1,2). The cost of providing a preselected level of service is determined as a function of demand for each strategy. That level of demand for which the service cost is equal for the two strategies is referred to as the critical ridership rate. If the design ridership rate is less than the critical ridership rate, then a flexible-route strategy is considered to be more suitable than a fixed-route strategy, while a fixed-route strategy is more suitable where the design ridership rate exceeds the critical ridership rate.

However, the above method ignores the time-variant nature of transit ridership. Because only design demand is considered in the analysis, one cannot be certain that whichever strategy is chosen is actually superior to the other over all ranges of demand encountered on a typical day. Also, a preselected level of service will not always ensure optimality.

In this paper, a new method that seeks to correct the above-mentioned deficiencies is proposed. By using a simple mathematical model of an intraurban bus transit system, the complications of choosing between the strategies are investigated. The proposed method compares the strategies on the basis of their capabilities to minimize costs to operators and users.

The physical setting assumed in developing the mathematical model is one in which the service area is given, but the transit operator wishes to develop optimal operating policies given that only scheduled fixed-route and/or scheduled flexible-route operating strategies will be considered. In order to simplify the analysis, a simple situation that consists of a very small service area that requires only a single fixed-route or a single service zone for flexible-route service is considered. It is hoped that when the mathematical model is extended to cover larger areas, a general comparison method for areawide as well as a corridor-by-corridor analysis would emerge.

The paper consists of two parts. The formulation of the mathematical model, which constitutes the foundation of the comparison method, is presented first. The second part contains relevant deductions and theorems as well as the derivation of the method. Table 1 contains the nomenclature that will be used throughout the paper.

Table 1. List of notations used.

Notation	Definition
γ	Average bus operating cost per unit time
γ_{iv}	Unit value of passenger's travel time spent inside a bus
γ_{ov}	Unit value of passenger's travel time spent outside a bus
λ	Fraction of total passengers who require a communication device to register their demands under flexible-route strategy
σ_2	Fraction of bus trip time for which a typical passenger remains on bus under fixed-route strategy
σ_2'	Fraction of bus trip time for which a typical passenger remains on bus under flexible-route strategy
σ_3	Average increase in the variance of bus trip time per passenger under flexible-route strategy
Ψ	Average communication cost per passenger
C_o	Operators' total cost per unit time
C_u	Users' average cost per unit time
F	Number of buses used for service
T	Time scheduled for a bus trip
T_{rd}	Actual driving time during a bus trip
Q	Average number of passenger trips per unit time
Q_{max}	Maximum value of Q on a typical day
Q_{min}	Minimum value of Q on a typical day
$Q_{c,0}$	Value of Q at and beyond which bus capacity is constrained under fixed-route strategy
$Q_{c,1}$	Value of Q at and beyond which optimum headways are constrained by bus capacity under flexible-route strategy
Z	Total system cost
z_0^*	Minimum value of Z for fixed-route strategy divided by Q
z_0^{**}	Value of z_0^* when optimum headways are constrained by vehicle capacity
z_1^*	Minimum value of Z for flexible-route strategy divided by Q
z_1^{**}	Value of z_1^* when optimum headways are constrained by vehicle capacity
h	Time headway between buses
h_0^*	Theoretically optimum headway under the fixed-route strategy
h_1^*	Theoretically optimum headway under the flexible-route strategy
h_1'	Optimum feasible headway under the flexible-route strategy
q	Mean number of passengers served during a bus trip
q_{cap}	Capacity (passenger spaces) of each vehicle used for fixed-route service
q_{cap}'	Capacity (passenger spaces) of each vehicle used for flexible-route service
t_F	Time scheduled for a bus trip under the fixed-route strategy
t_F'	Fixed component of the time scheduled for a bus trip under the flexible-route strategy
t_1	Average service time per passenger allowed under the flexible-route strategy
t_{iv}	Average travel time spent inside the vehicle per passenger trip
t_{ov}	Average travel time spent outside the vehicle per passenger trip

MATHEMATICAL MODEL

Mathematical Formulation of Problem and Simplifying Assumptions

An objective function of minimizing the sum of operating costs and the expectation of the users' costs are assumed. So, if we define the operating costs to be C_o and the expectation of the users' costs to be C_u, then

$$Z = C_o + C_u \tag{1}$$

where Z is the total cost to be minimized. A similar objective function was used elsewhere (3-5). Because the magnitudes of the defined costs depend on the length of time considered, C_o and C_u are based on unit time.

An average ridership rate of Q passenger trips per unit time is assumed. Although Q is implicitly assumed to be dependent on the time of day, it is assumed to be unaffected by the strategy.

It is assumed that F buses are used for service within the unit time under consideration and that C_o can be approximately modeled as follows:

for fixed-route strategy,

$$C_o \simeq \gamma F \tag{2a}$$

and for flexible-route strategy,

$$C_o \simeq \gamma F + \Psi\lambda Q \tag{2b}$$

where

γ = average total cost of operating a vehicle per unit time,
Ψ = average cost (per passenger) of providing the communication medium for the flexible-route strategy, and
λ = proportion of passengers served who require a communication facility to register their demand.

Actually, the ridership rate is dependent on the time of day and, because of this, transit operators do not always provide the same quantity of service throughout the day. Also, the operators' costs consist partly of a component that directly varies with the quantity of service provided and partly of a fixed component that is independent of the quantity of service provided. Consequently, the values of γ and Ψ may not necessarily be the same throughout a whole day. The question of how to correctly apportion transit costs among different demand periods is not addressed in the present study. Both γ and Ψ are treated as constants. It is also possible that vehicle operating costs would be different for the two strategies. However, because a substantial portion of these costs is labor costs, it is the efficiency of labor use on the system rather than operating strategy that will significantly influence the value of γ. Thus, γ is taken to have approximately the same value for both strategies.

Users' costs consist of the fares and costs attributable to their total time commitment for the trip. The fares are considered to be internal to the bus transit system and are not included in the evaluation.

A user's time commitment to a trip consists of time actually spent on the vehicle and time spent outside of it. If we suppose that each unit of time spent inside the vehicle is worth γ_{iv} units of money while each unit of time spent outside is worth γ_{ov}, then

$$C_u = Q(\gamma_{iv}t_{iv} + \gamma_{ov}t_{ov}) \tag{3}$$

where $\gamma_{ov} \geq \gamma_{iv}$, t_{iv} is the average time spent inside the vehicle per passenger trip, and t_{ov} is the average time spent outside the vehicle per passenger trip.

By using a constant time value, it erroneously implies that all users attach the same value of travel time and that the marginal utility of travel time is constant, regardless of the time length of the trip. It should be pointed out, then, that because of the assumption, wealthy users, who are likely to attach higher values to their travel times, and long distance travelers, who are likely to have higher marginal time utility, are disadvantaged. In studies directly aimed at estimating travel demands, both factors would have to be allowed for.

Substituting Equations 2a and b and 3 into Equation 1 gives the following:

for fixed-route strategy,

$$Z = \gamma F + Q(\gamma_{iv}t_{iv} + \gamma_{ov}t_{ov}) \tag{4a}$$

and for flexible-route strategy,

$$Z = \gamma F + Q(\lambda\Psi + \gamma_{iv}t_{iv} + \gamma_{ov}t_{ov}) \tag{4b}$$

In comparing the two strategies, all design variables are first chosen so as to optimize Z within each strategy. After this, the optimal values of Z for the two strategies are compared. It is on the basis of the results that a general methodology for comparing the strategies is proposed.

In general, the two main design variables are the dispatch headways and the dimensions and configuration of the service zones (for flexible-route strategy) or spacing between the routes (for fixed-route strategy). In this paper, the service area is assumed to be given and to be small enough to require only a single route under the fixed-route strategy or a single service zone under the flexible-route strategy. However, some publications (5,6) include the topic of route spacing under the fixed-route strategy while the problem of optimally partitioning an area into service zones is investigated in Ward (1).

If we represent the headway by h and the scheduled vehicle trip time as T, then:

$$F = T/h \tag{5}$$

Normally, T (the scheduled trip time) is comprised of a slack time and the estimated expectation of the actual driving time. The driving time is denoted as T_{rd}. Although T should be a constant for a given ridership rate, T_{rd} is random.

The following mathematical relations are assumed for the travel times contained in Equations 4a and b and 5:

for fixed-route strategy,

$$t_{ov} = 2\sigma_1 \left\{[h^2 + \mathrm{var}(T_{rd})]/h\right\} + t_{acc} \tag{6a}$$

and for flexible-route strategy,

$$t_{ov} = 2\sigma_1 \left\{[h^2 + \mathrm{var}(T_{rd})]/h\right\} \tag{6b}$$

for fixed-route strategy,

$$T = t_F \tag{6c}$$

and for flexible-route strategy,

$$T = t'_F + t_1 q \tag{6d}$$

for fixed-route strategy,

$$t_{iv} = \sigma_2 T \tag{6e}$$

and for flexible-route strategy,

$$t_{iv} = \sigma'_2 T \tag{6f}$$

for fixed-route strategy,

$$\mathrm{var}(T_{rd}) = 0 \tag{6g}$$

and for flexible-route strategy,

$$\mathrm{var}(T_{rd}) = \sigma_3 q \tag{6h}$$

where the notations are as follows:

1. σ_1, σ_2, σ_2', and σ_3 are constants such that $0 < \sigma_1 \leq 0.5$, $0 < \sigma_2 \leq 1$, $0 \leq \sigma_2' < 1$, and $\sigma_3 > 0$;
2. t_F, t'_F, t_1, and t_{acc} are also constants for a given service area; and
3. q is the average number of passengers served during a vehicle trip and, by definition

$$q = Qh \tag{6i}$$

Equations 6a and b indicate that t_{ov} consists of t_{acc}, the access time between the bus route and the passenger's trip-end point and the waiting times. Although t_{acc} is a function of the width of the service zone, the waiting time consists of (a) the waiting time at the point of boarding and (b) the schedule delay, which represents the extra time that a user commits to the trip because his or her preferred arrival time at the destination point differs from the vehicle's schedule. For example, in a situation where all the passengers' trip-inception times coincide with the bus arrival times at the boarding points, and the bus arrival times at the respective destination points are exactly the same as the passengers' preferred arrival times, then $\sigma_1 = 0$. However, in the extreme situation where both the passengers' trip-inception times and preferred arrival times at their destinations, independent of the bus schedule, are distributed uniformly over time, then $\sigma_1 = 0.5$. This latter value of σ_1 is assumed, for illustration purposes, in the subsequent discussion. The actual numerical value of σ_1 (provided $\sigma_1 > 0$) is, however, of little significance to the present work.

Equations 6c and d and 6g and h were found to be approximately true from a computer simulation of bus operation during intraurban service undertaken by Adebisi (7). In the simulations, the scheduled trip time was taken to be the sum of the total driving time expectation and twice its standard deviation. The number of passengers served was taken to be randomly distributed over time and space but with a fixed average value per unit time. Only the variability in the bus trip times due to randomness in the bus load was allowed for. One would expect the results, particularly for Equations 6g and h, to be different if the variability in the trip times due to interaction with other vehicles that use the roadway is considered.

Equations 6c and d imply that it is only with flexible-route service that the passengers' service time constitutes a significant portion of the scheduled trip time and also confirms the assumptions (3,8) that passengers' service time in a scheduled, fixed-route operation has a negligible effect on the scheduled trip time. Results of the simulations indicate that, while t_F is mostly a function of the length of the fixed route and vehicle speed characteristics, it generally increases with the length of the bus route. Similarly, t_F', t_1, and σ_1 generally increase with the area of the service zone.

The parameters σ_2 and σ_2' in Equations 6e and f represent the fraction of a bus trip time that constitutes the average duration of a passenger trip time. Consequently, their numerical values depend on the passengers' origin-destination (O-D) pattern. No specific values are assumed for σ_2 and σ_2' in the subsequent analysis.

Objective Function

Combining Equations 4a and b, 5, and 6a-i gives the following:

for fixed-route strategy,

$$Z = (\gamma t_F/h) + Q\,[\gamma_{iv}\sigma_2 t_F + \gamma_{ov}(h + t_{acc})] \tag{7a}$$

and for flexible-route strategy,

$$Z = (\gamma t_F'/h) + Q\,[\Psi\lambda\gamma + \gamma_{iv}\sigma_2'(t_F' + Qht_1) + \gamma_{ov}(h + \sigma_3 Q)] \tag{7b}$$

In order to minimize Z, we must choose an appropriate value of h, which is the only design variable in the present model. Differentiating Z with respect to h gives h_0* (the optimum headway for the fixed-route strategy) as follows:

$$h_0^* = (\gamma t_F/\gamma_{ov}Q)^{1/2} \tag{8a}$$

Thus, we have from Equations 6g and h and 8a that the mean vehicle load, when optimal headways are used, is as follows:

$$q = (\gamma t_F Q/\gamma_{ov})^{1/2} \tag{8b}$$

The implication from Equation 8b is that, because vehicles have finite capacities, optimal headways may not always be feasible when Q assumes large values. It is therefore necessary to distinguish between optimal feasible headway and the theoretically optimal headway. If we let q_{cap} represent the vehicle capacity and h_0' represent the optimal feasible headway under fixed-route operation, then:

$$h_0' = \min\,[h_0^*, (q_{cap}/Q)] \tag{8c}$$

Because of the randomness in the vehicle load, one may need to allow a safety factor in selecting the value of q_{cap} and not simply use actual capacity. However, if all passengers board the vehicle at the dispatch point and a sufficient supply of vehicles is always available, one does not need a safety factor.

Similarly, if we let h_1' and h_1* represent the optimal feasible headway and the theoretically optimal headway, respectively, for the flexible-route strategy, we have the following:

$$h_1^* = \left\{ t_F'/[(\gamma_{ov}Q) + (\gamma_{iv}\sigma_2' t_1 Q^2)] \right\}^{1/2} \tag{9a}$$

while

$$h_1' = \min\,[h_1^*, (q_{cap}'/Q)] \tag{9b}$$

where q_{cap}' is the capacity of each vehicle used for flexible-route service. Based on the present practice whereby transit operators use small buses for flexible-route service and large buses for fixed-route service, it is reasonable to suppose that $q_{cap}' \leq q_{cap}$. It should be observed that any other headways beside h_0' and h_1' would only lead to suboptimal states for the strategies. Comparisons based on such suboptimal headways would obviously give biased results. Let us denote as z_0* and z_1* the minimum values of Z divided by Q for the fixed-route strategy and the flexible-route strategy, respectively. Thus, z_0* and z_1* represent the average total minimum disutility per passenger. Because Q is assumed to be independent of the strategy, it should not make any difference whether we use Z or z_0* and z_1* in our comparison. However, because it considerably simplifies subsequent analysis, the latter option is adopted. Thus,

$$z_0^* = (\gamma t_F/Qh_0') + \gamma_{iv}\sigma_2 t_F + \gamma_{ov}(h_0' + t_{acc}) \tag{10a}$$

and

$$z_1^* = (t_F'/Qh_1') + \gamma\Psi + \gamma_{iv}\sigma_2'(t_F' + Qh_1't_1) + \gamma_{ov}(h_1' + \sigma_3 Q) \tag{10b}$$

Complications in choosing between the strategies are examined by exploring the sensitiveness of z_0* and z_1* to changes in Q. Thus, we differentiate z_0* and z_1* with respect to Q and obtain the following:

if theoretically optimal headways are used, i.e., if $h_0' = h_0*$,

$$\partial z_0^*/\partial Q = -(\gamma_{ov} t_F/Q^3)^{1/2} \tag{11a}$$

and if capacity-constrained headways are used, i.e., if $h_0' = q_{cap}/Q$,

$$\partial z_0^*/\partial Q = -(\gamma_{ov} q_{cap}/Q^2) \tag{11b}$$

and if $h_1' = q_{cap}'/Q$,

$$\partial z_1^*/\partial Q = \gamma_{ov}\sigma_3 - (\gamma_{ov}\, q_{cap}'/Q^2) \tag{11c}$$

and if $h_1' = h_1*$,

$$\partial z_1^*/\partial Q = \gamma_{ov}\sigma_3 - \left\{ \gamma_{ov}\, t_F'/Q^3\, [1 + (\gamma_{iv}\sigma_2/\gamma_{ov}\sigma_2') \cdot t_1 Q] \right\}^{1/2} \tag{11d}$$

Equations 11a and b indicate that z_0* always decreases with Q and thus confirms the general belief that the fixed-route strategy is characterized by economies of scale. Equations 11c and d, on the other hand, indicate that z_1* might actually increase with Q such as when

$$\gamma_{ov}\sigma_3 > \left\{ \gamma_{ov} t_F'/Q^3\, [1 + (\gamma_{iv}\sigma_2/\gamma_{ov}\sigma_2') \cdot t_1 Q] \right\}^{1/2}$$

or when

$$\gamma_{ov}\sigma_3 > (\gamma_{ov} q_{cap}'/Q^2).$$

The conditions that lead to z_1* increasing with Q are more likely to be met when Q assumes large values than when it is small. This finding also affirms the reasonableness of the general aversion to recommend a flexible-route strategy for high levels of ridership demands.

COMPARISON OF STRATEGIES

At this stage, useful inferences on the relative performance of the two strategies under consideration can be drawn. The situation when headways under the two strategies are constrained by vehicle capacity (thus being more straightforward) is discussed first. Numerically, $\sigma_3 > 0$, and $q_{cap}' \leqslant q_{cap}$. It is implied from Equations 11a-d that, when the vehicles used for service under both strategies are always fully loaded, z_1* never decreases with Q at a faster rate than z_0*.

Also, it will be recalled from Equations 6c and d that t_F represents the scheduled trip time under the fixed-route strategy while t_F' represents just the fixed portion of the scheduled trip time under the flexible-route strategy. It is true that when the same demands are served, flexible-route service is likely to take a longer time to complete than fixed-route service, but the service times (i.e., buses' access times to demand points and consequent loading times) that t_1q represents make up the bulk of the scheduled times. It will therefore be appreciated that t_F' will almost never exceed t_F. Given that $t_F > t_F'$, it follows from Equations 11a-d that, when theoretically optimal headways are used for both strategies, z_1* never decreases with Q at a faster rate than z_0*.

It is possible that, for some values of Q, theoretically optimal headways are appropriate with one strategy, but for the other strategy the headways must be based on vehicle capacity. Therefore, the above does not constitute sufficient proof that z_1* never decreases with Q at a faster rate than z_0* in all situations, but it is sufficient to prove lemma 1, which is stated as follows: When it is optimal for both strategies that theoretically optimum headways always be used or that the headways always be based on vehicle capacity and if we find that for a specific value of Q that $z_0^* < z_1^*$, then $z_0^* < z_1^*$ for all higher values of Q. Similarly, if we find that for a specific value of Q that $z_1^* < z_0^*$, then $z_1^* < z_0^*$ for all smaller values of Q.

By applying lemma 1, it is shown in the next section that, in most cases, where only one strategy is required the appropriate strategy is uniquely determined by considering the extreme ridership rates only.

Proofs of Relevant Theorems

Let us suppose that the maximum and minimum ridership rates likely to be served on a typical day within the service area are Q_{max} and Q_{min}. Let us also represent the values of Q when the theoretically optimal headways are exactly equal to the headways based on vehicle capacity as $Q_{c,0}$ and $Q_{c,1}$ for the fixed- and flexible-route strategies, respectively. For convenience, we denote the value of z_0* as z_0 when the headways are based on the vehicles' capacities but as z_0** when theoretically optimal headways are used. The corresponding values for z_1* are z_1 and z_1**. Thus,

when $Q \leqslant Q_{c,0}$,

$$z_0^* = z_0^{**} \tag{12a}$$

and when $Q > Q_{c,0}$,

$$z_0^* = z_0 \tag{12b}$$

and when $Q \leqslant Q_{c,1}$,

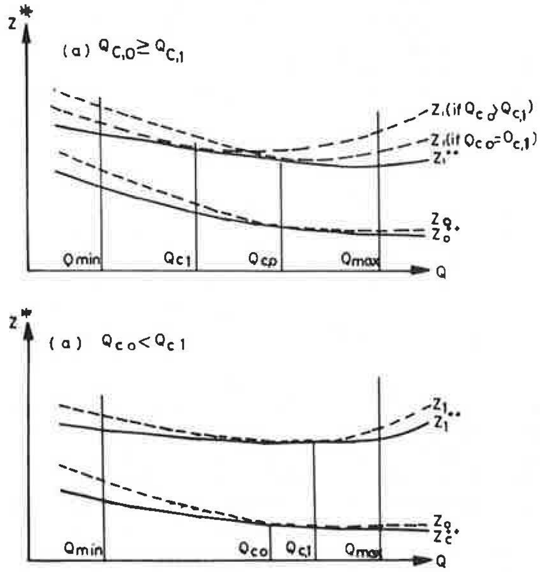
$$z_1^* = z_1^{**} \tag{12c}$$

and when $Q > Q_{c,1}$,

$$z_1^* = z_1 \tag{12d}$$

Let us first consider the case where the fixed-route strategy performs better than the flexible-route strategy at $Q = Q_{min}$; i.e., $z_0^* < z_1^*$ at $Q = Q_{min}$. The following can then be deduced:

Figure 1. Conceptualization of some situations when fixed-route strategy is more appropriate at Q_{min}.



1. If $Q_{c,0} > Q_{max}$ and $Q_{c,1} > Q_{max}$, then theoretically optimal headways should be used under both strategies for all relevant values of Q; i.e., $Q \in [Q_{min}, Q_{max}]$. It follows from Equations 12a-d that $z_0^* = z_0^{**}$ and $z_1^* = z_1^{**}$ for $Q \in [Q_{min}, Q_{max}]$. Since $z_0^{**} < z_1^{**}$ at $Q = Q_{min}$, it follows from lemma 1 that $z_0^{**} < z_1^{**}$ for $Q \in [Q_{min}, Q_{max}]$ and the fixed-route strategy is more appropriate than the flexible-route strategy for all relevant ranges of demand.

2. If $Q_{c,0} < Q_{min}$ and $Q_{c,1} < Q_{min}$, by implication headways should be based on vehicle capacity under both strategies for all $Q \in [Q_{min}, Q_{max}]$. Therefore, it follows from Equations 12a-d that for all $Q \in [Q_{min}, Q_{max}]$, $z_0^* = z_0$ and $z_1^* = z_1$. Because $z_0 < z_1$ at $Q = Q_{min}$, it follows from lemma 1 that $z_0 < z_1$ for all $Q \in [Q_{min}, Q_{max}]$ and the fixed-route strategy is more appropriate within the relevant ranges of demand.

3. If $Q_{min} < Q_{c,0} < Q_{max}$ or $Q_{min} < Q_{c,1} < Q_{max}$, then both optimum headways and headways based on vehicle capacity should be used for at least one of the strategies. Figure 1 gives a conceptual representation of the possible situations where $Q_{c,0} \geqslant Q_{c,1}$ and $Q_{c,0} < Q_{c,1}$.

When $Q_{c,0} \geqslant Q_{c,1}$, then the flexible-route strategy requires that optimal feasible headways be based on vehicle capacity at the same or lower ridership rates than the fixed-route strategy requires. But from lemma 1 and Equations 12a-d we know that if $z_0^* < z_1^*$ at Q_{min}, then $z_0^{**} < z_1^{**}$ and $z_0 < z_1$ for $Q \geqslant Q_{min}$. Furthermore, because $z_1^{**} \leqslant z_1$ for all values of Q, it follows that $z_0^{**} < z_1^{**} \leqslant z_1$ for all $Q \geqslant Q_{min}$. Following from Equations 12a-d, we know that for $Q \in [Q_{min}, Q_{c,1}]$, $z_0^* = z_0^{**}$ and $z_1^* = z_1^{**}$. Thus, $z_0^* < z_1^*$. For $Q \in (Q_{c,1}, Q_{max}]$, $z_1^* = z_1$; but $z_0^* = z_0^{**}$ for $Q \in (Q_{c,1}, Q_{c,0}]$ and $z_0^* = z_0$ for

$Q \in (Q_{c,0}, Q_{max}]$. Thus, $z_0^* < z_1^*$ for $Q \in (Q_{c,1}, Q_{max}]$, too. Hence, $z_0^* < z_1^*$ for all relevant values of Q when $Q_{c,0} \geq Q_{c,1}$. Therefore, the fixed-route strategy should be used throughout. When $Q_{c,0} < Q_{c,1}$, then the fixed-route strategy requires that optimal feasible headways be based on vehicle capacity at lower ridership rates than with the flexible-route strategy. As in the former case, we can deduce that for $Q \geq Q_{min}$, $z_0^{**} < z_1^{**}$ and $z_0 < z_1$.

In addition, $z_0^{**} \leq z_0$ and $z_1^{**} \leq z_1$. However, while it is true that $z_0^{**} \leq z_1^{**} \leq z_1$ for all $Q \geq Q_{min}$, it is not necessarily true that $z_0 < z_1^{**}$ for all $Q \geq Q_{min}$. However, for $Q \in [Q_{min}, Q_{c,0}]$, $z_0^* = z_0^{**}$ and $z_1^* = z_1^{**}$. Thus, $z_0^* < z_1^*$. For $Q \in (Q_{c,0}, Q_{max}]$, $z_0^* = z_0$, but $z_1^* = z_1^{**}$ for $Q \in (Q_{c,0}, Q_{c,1}]$ and $z_1^* = z_1$ for $Q \in (Q_{c,1}, Q_{max}]$. Therefore, the fixed-route strategy is more appropriate for $Q \in [Q_{min}, Q_{c,0}]$ and $Q \in (Q_{c,1}, Q_{max}]$ but, since z_0 is not necessarily less than z_1^{**}, a more detailed investigation is required to determine if, in fact, the fixed-route strategy is appropriate within the interval $(Q_{c,0}, Q_{c,1}]$.

The above deductions are sufficient to prove theorem 1, which is stated below.

Theorem 1

If the fixed-route strategy is found to perform better than the flexible-route strategy at $Q = Q_{min}$, then the fixed-route strategy is more appropriate for all relevant values of Q whenever (a) theoretically optimal headways are always feasible under both strategies or (b) optimal feasible headways are always based on vehicle capacity for both strategies or (c) both theoretically optimal headways and headways based on vehicle capacity are used for at least one of the strategies but $Q_{c,0} \geq Q_{c,1}$.

In addition, if the fixed-route strategy is found to be more appropriate than the flexible-route strategy for $Q = Q_{min}$, and both optimal headways and headways based on the vehicles' capacities are used for the fixed-route strategy with $Q_{c,0} > Q_{c,1}$, then the fixed-route strategy is appropriate for $Q \in [Q_{min}, Q_{c,0}]$ and $Q \in (Q_{c,1}, Q_{max}]$.

By using a similar procedure and logic similar to the above, we can also prove theorem 2.

Theorem 2

If the flexible-route strategy is found to perform better than the fixed-route strategy for $Q = Q_{max}$, then the flexible-route strategy is more appropriate for $Q \in [Q_{min}, Q_{max}]$ whenever (a) optimal headways are always used under both strategies or (b) optimal feasible headways are always based on vehicle capacity for both strategies or (c) both optimal headways and headways based on the buses' capacities are used for at least one of the strategies but $Q_{c,1} \geq Q_{c,0}$.

In addition, if the flexible-route strategy is found to be more appropriate for $Q = Q_{max}$ and both optimal headways and headways based on on the buses' capacities are used for flexible-route strategy with $Q_{c,1} < Q_{c,0}$, then the flexible-route strategy is more appropriate for $Q \in [Q_{min}, Q_{c,1}]$ and $Q \in (Q_{c,0}, Q_{max}]$.

In real-life situations, the values of $Q_{c,1}$ and $Q_{c,0}$ would be very large and, if not larger than Q_{max}, should be close to it. Otherwise, the obvious implication from the present work is that improvements are possible simply by having larger vehicles. Thus, the range of demand $(Q_{c,1}, Q_{c,0}]$ or $(Q_{c,0}, Q_{c,1}]$ where the proven theorems indicate inconclusive results should not be of much practical importance. A general methodology for comparing the strategies is proposed in the next section.

General Methodology for Comparing Strategies

Theorems 1 and 2 indicate that, if a bus transit operator wants to choose one of the fixed-route and flexible-route strategies, he or she needs to compare them at $Q = Q_{max}$ and $Q = Q_{min}$. Theorem 1 indicates that if the fixed-route strategy is found to be superior to the flexible-route strategy at $Q = Q_{min}$, then it will, for most practical situations, be superior for all relevant ranges of demand. It is therefore desirable to operate exclusively with the fixed-route strategy in such a situation. Similarly, if the flexible-route strategy is found to be more appropriate than the fixed-route strategy at $Q = Q_{max}$, then it is desirable to operate exclusively with the flexible-route strategy. However, when the fixed-route strategy is found to be the better strategy at $Q = Q_{max}$ but the flexible-route strategy is more suitable at $Q = Q_{min}$, then neither strategy should be used exclusively. In such a case, the transit operator could determine the range of demand for which one strategy is superior to the other and then draw up a schedule for switching strategies as the demand rate changes from one region to the other. Such an arrangement, however, poses some operational problems. In a situation whereby the fixed-route strategy is exclusively adopted, large vehicles are likely to be used, but where the flexible-route strategy is exclusively adopted, mostly small buses will be used because they are easier to maneuver. Therefore, some compromise may have to be made on fleet composition when operating strategies are routinely switched. This compromise may increase overall operating costs. When allowance is made for the extra costs, it is possible that one finds it better to operate exclusively with one strategy. However, making a rational choice precludes the adoption of an all-or-nothing principle in choosing between the fixed-route and flexible-route strategies.

CONCLUSION

It has been shown that choosing between a fixed-route strategy and a flexible-route strategy is more complicated than earlier studies indicated. Evidently, a comparison method based solely on the determination of the critical ridership rate is inadequate, since it indirectly assumes that demand is invariant with time. Such a comparison method can only identify the dominant strategy that should be used more widely than the other strategy, since it does not conclusively show that the chosen strategy performs better for all relevant ranges of demand. Also, by comparing the strategies on the basis of the costs required to achieve a specified level of service, one runs the risk of comparing them at nonoptimum states. The indication from the present work is that, when the strategies are compared at states other than the optimum, the results are likely to be biased.

The method proposed in this study has tried to remove the limitations described above that are inherent in the current methods of comparing the strategies. However, this new method is also not without its own limitations. For one, the fact that the analysis was focused on a very small service area limits its application. Another limitation is that no allowance was made for the interplay of demand and level of service. Further research on the subject should aim at removing the identified weaknesses in the model.

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Publication of this paper sponsored by Committee on Bus Transit Systems.

Concurrent-Flow High-Occupancy Vehicle Treatment on Freeways—Success Story in Houston

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On March 30, 1981, a 3.3-mile concurrent-flow lane began operation within the median shoulder on North Freeway (Interstate-45). The concurrent-flow lane operates inbound only from 6:00 to 8:30 a.m. and is available to authorized vehicles, which include registered and approved buses and eight-passenger vanpools. The concurrent-flow lane is an extension of contraflow preferential treatment provided further downstream; it provides a travel-time savings of about 4 min. This project is one of seven nationwide that is currently operating, is the only project to be implemented within an existing paved emergency shoulder, and is the first operation to restrict use to authorized vehicles that display an appropriate permit. A general report on the unique characteristics and results of Houston's concurrent-flow operation is presented. Comparative evaluations are presented that measure the success of this project with other concurrent-flow applications on freeways. In the first three months, an average of 257 vehicles (78 percent vanpools and 21 percent buses) traveled the lane inbound during each daily 2.5-h peak period, which facilitated the movement of 3752 commuters. The North Freeway concurrent-flow project was jointly implemented by the Texas State Department of Highways and Public Transportation and the Metropolitan Transit Authority of Harris County. Both agencies funded construction of the project with local monies and jointly managed daily operation. The success of the concurrent-flow project, as illustrated in this paper, has resulted in increased person trips on a severely congested freeway facility and has provided a travel-time incentive to vanpool and bus transit users until such time that a more permanent transitway facility can be constructed.

In 1979 the Metropolitan Transit Authority (MTA) of Harris County, Houston, and the Texas State Department of Highways and Public Transportation (TSDHPT) opened a 9.6-mile contraflow lane on Interstate-45N (North Freeway). The $2.1 million project, funded under a Service and Methods Demonstration program grant (Sections 5 and 6) of the Urban Mass Transportation Act of 1964 (as amended), was very successful in attracting riders into vanpools and buses. These were the only authorized vehicles that could benefit from the project, and rather rigid authorization procedures were adopted to help ensure safe operation. The contraflow lane bypassed about 6 miles of severe traffic congestion and saved users about 30-min of travel time daily. Use increased 350 percent from the 1st through 82nd week of operation to 10 900 daily trips (1). However, during the contraflow planning and implementation period from 1975 through 1979, severe traffic congestion was growing and began extending several miles upstream of the northern terminus to contraflow. An extension of the contraflow concept to alleviate this problem was complicated by several factors. Unacceptable traffic conditions upstream did not permit borrowing a lane for contraflow. Also, facility design would not accommodate a safe project termination farther north. Other alternatives were studied for bypassing congestion outside the contraflow limits.

BACKGROUND

The concurrent-flow concept was first proposed as an extension to the contraflow lane in early 1980 to alleviate congestion in the morning period. The concept could be readily implemented within an existing paved median shoulder along a 3.3-mile segment, as shown in Figure 1. The segment was unique in that the termination of the concurrent-flow lane could be transitioned directly to the contraflow lane. This segment encompassed most of the regularly recurring traffic congestion. Median drainage inlets and superelevations prohibited easy conversion of the inbound median any further. In the afternoon peak period, traffic conditions at the time did not warrant implementation of a similar treatment on the outbound shoulder.

TSDHPT subsequently designed the necessary signing and striping modifications to convert the median shoulder for bus and vanpool use. A connection ramp was designed at the downstream terminus to facilitate direct access from the concurrent-flow shoulder to the entry of contraflow. An exception was granted from Interstate standards by the Federal