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Selecting Two-Regime Traffic-Flow Models

SAID M. EASA

A procedure for selecting two-regime macroscopic models for a given set of traffic-flow data is presented. The procedure is based principally on the theoretical characteristics among the various regions of macroscopic models, which includes the limiting case and the convexity and concavity properties. The input to the procedure is represented by the basic traffic-flow criteria (free-flow speed, optimum speed, jam density, and so on) as well as auxiliary criteria to account for the variability of the traffic-flow relations in the intermediate ranges of flow. With these criteria, which are established from the data, the procedure can directly output model parameters, through simplified graphical tools, for the non-congested- and congested-flow regimes. Application of the procedure by using actual data was made to illustrate its use and to discuss some issues related to establishing the traffic-flow criteria from the data. This application also illustrates the flexibility of the procedure and the ease with which the specified criteria can be adjusted to further improve the data fitting. The procedure presented in this paper significantly reduces the need for using computer facilities in estimating traffic-flow relations and as such should prove useful in many transportation applications.

Macroscopic traffic-flow models have been widely used in the field of transportation, including freeway operations, highway levels of service, environmental studies, and transportation planning. Generally, these models can be used to describe the traffic-flow relations in two ways: single-regime and two-regime representations. In the former, the entire range of operation is represented by a single model, whereas in the latter, two models are used--one for the non-congested-flow regime and the other for the congested-flow regime. The idea of the two-regime representation was first proposed by Edie (1). The general macroscopic models, their estimation approaches, and the scope of this paper are discussed first.

GENERAL MICROSCOPIC AND MACROSCOPIC MODELS

The general car-following (microscopic) equation developed by Gazis and others (2,3) is given as follows:

$$\ddot{X}_{n+1}(t+T) = \alpha\left\{\dot{X}^m_{n+1}(t+T)/[X_n(t) - X_{n+1}(t)]^\ell\right\}[\dot{X}_n(t) - \dot{X}_{n+1}(t)] \quad (1)$$

where

$\dot{X}_n$, $\dot{X}_{n+1}$ = speed of leading and following vehicles, respectively;
$\ddot{X}_{n+1}$ = acceleration (or deceleration) rate of following vehicle;
T = time lag of response to stimulus;
α = constant of proportionality (referred to throughout as a model parameter); and
ℓ, m = model parameters.

By integrating Equation 1, the general form of macroscopic models has been developed by Gazis and others (3). By using this general form, a matrix of macroscopic models has been established for different combinations of ℓ and m parameters by May and Keller (4). This matrix has undergone some adjustments by Ceder (5) and by Easa and May (6). The final version of the matrix is shown in Figure 1, along with illustrations of its use for the two-regime representation.

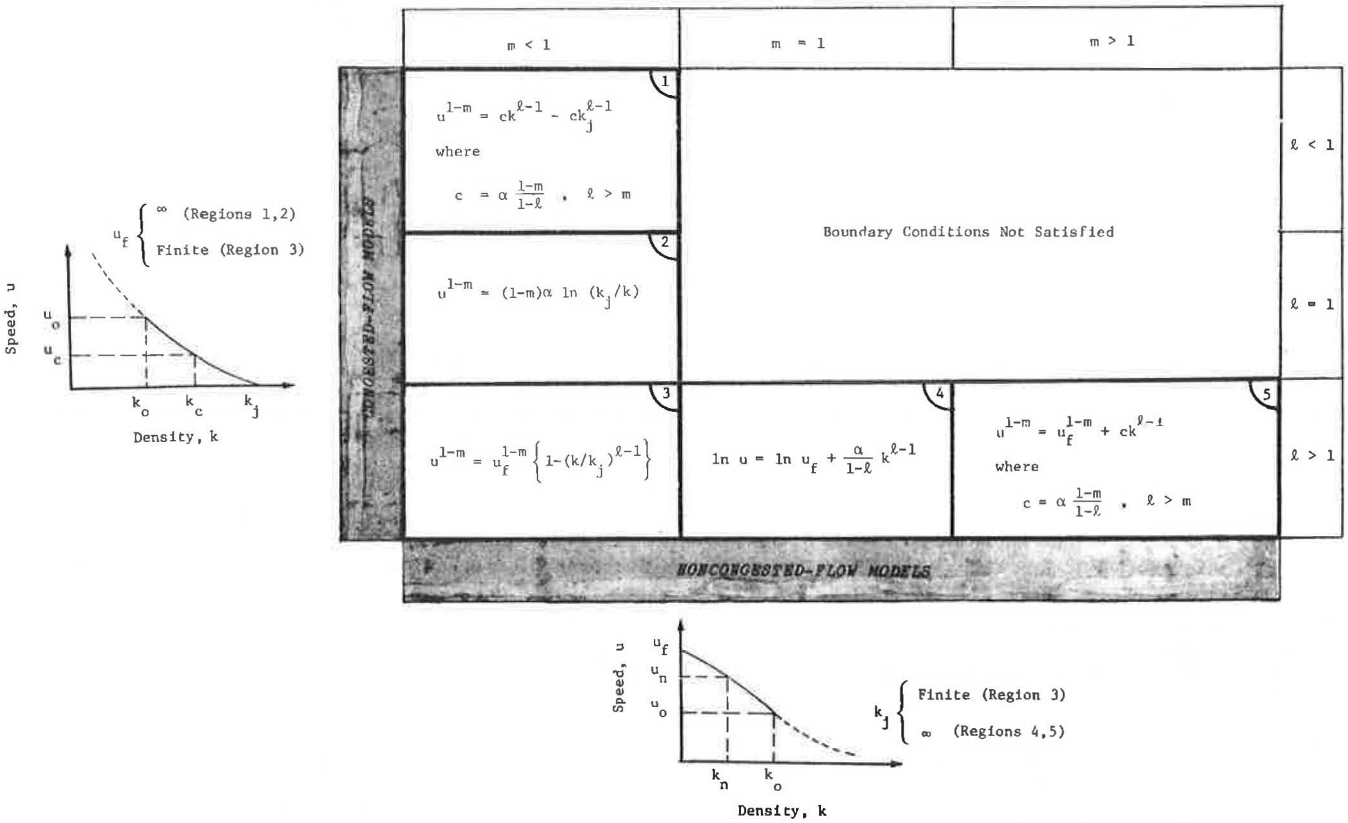
Figure 1 shows the speed-density relations and consists of five regions. In regions 1 and 2, models have no intercept with the speed axis, $u_f \to \infty$. In regions 4 and 5, models have no intercept with the density axis, $k_j \to \infty$. Models in region 3 have intercepts with both the speed and the density axes. Single-regime representation is usually accomplished by using models from region 3. The two-regime representation can be made, as illustrated in Figure 1, by using models from regions 1, 2, or 3 for the congested-flow regime and from regions 3, 4, or 5 for the non-congested-flow regime.

ESTIMATION APPROACHES

Estimation of macroscopic models is an essential task. For a given set of traffic-flow data, one often needs to estimate model parameters that best represent these data. In this regard, an approach employing computer techniques has been developed by May and Keller (4). This approach uses regression analysis to estimate model parameters for specified values of traffic-flow and statistical criteria. These criteria include free-flow speed u_f, optimum speed u_0, jam density k_j, optimum density k_0, maximum flow q_m, and a mean-deviation criterion.

In an attempt to significantly reduce the need for using computer systems, another theoretical-graphical approach has been recently proposed and applied to the estimation of single-regime models (7). This approach is based principally on the theoretical relations among the first five criteria mentioned above and model parameters ℓ, m, and α. A simplified graphical tool was used to represent those relations and could directly provide model parameters that satisfy specified traffic-flow criteria. This approach was applied later to the estimation of a special case of two-regime models by Easa and May (6). The procedure that has been developed for estimating the single-regime models corresponds to region 3 and that developed for the two-regime models corresponds to regions 2 and 4 and in a preliminary fashion to region 3. In both pro-

Figure 1. Macroscopic models and two-regime representation.



cedures, the traffic-flow criteria may be specified as single values or as ranges.

SCOPE OF PAPER

The primary purpose of this paper is to establish the theoretical relations of the remaining regions (1 and 5) and to develop a procedure for selecting the particular region--and the model within that region--that satisfies specified traffic-flow criteria for both non-congested-flow and congested-flow regimes. Specifically, the selection procedure will determine one of the three regions 1, 2, or 3 for the congested-flow regime and one of the three regions 3, 4, or 5 for the non-congested-flow regime and will provide the respective model parameters for each regime. The selection procedure is based principally on the theoretical properties of models among the five regions of Figure 1. In addition, two auxiliary criteria (u_n, k_n) and (u_c, k_c) were used to account for the variability of the traffic-flow patterns in the intermediate ranges of operations.

Throughout this paper, the traffic-flow criteria u_f, u_0, k_j, and k_0 (or a combination of them) will be referred to as the basic criteria, as distinguished from the auxiliary criteria. Furthermore, this paper is primarily concerned with selecting the two-regime models where the traffic-flow criteria--basic and auxiliary--are specified as single values. Thus, the maximum-flow criterion q_m is not needed since it is implicitly determined by the two criteria k_0 and u_0 ($q_m = k_0 u_0$).

The following section describes the theoretical properties of the models and regions of the non-congested-flow regime. The next section presents these properties for the congested-flow regime. Then, based on these properties, a description of the procedure of region (and model) selection for each regime is given. After a section in which the procedure is applied to an actual set of traffic-flow data, there are some concluding remarks concerning practical aspects of the procedure.

NON-CONGESTED-FLOW REGIME

This section describes the theoretical properties of the non-congested-flow models of regions 3, 4, and 5. Those properties are presented in three parts: a summary of the previously reported results of the special case of region 4 and extended results of region 3, the theoretical relations of models in region 5, and between-region properties.

Previous Results

The speed-density equation of models in region 4 ($\ell > 1$, $m = 1$) is given as follows:

$$\ln u = \ln u_f + [\alpha/(1-\ell)]k^{\ell-1} \quad (2)$$

The relations between the traffic-flow criteria u_f, u_0, and k_0 and model parameters ℓ and α are given by (6) the following:

$$\alpha = 1/k_0^{\ell-1} \quad (3)$$

$$u_0/u_f = \exp\{-[1/(\ell-1)]\} \quad (4)$$

Equations 3 and 4 have been used to estimate model parameters based on specified traffic-flow criteria.

For models in region 3 ($\ell > 1$, $m < 1$), the speed-density equation is given as follows:

$$u^{1-m} = u_f^{1-m}[1-(k/k_j)^{\ell-1}] \quad (5)$$

Based on Equation 5, the relations among the traffic-flow criteria u_f, u_0, k_j, and k_0 and model parameters ℓ and m are given by the following:

$$(k_0/k_j)^{\ell-1} = (1-m)/(\ell-m) \quad (6)$$

$$(u_0/u_f)^{1-m} = (\ell-1)/(\ell-m) \quad (7)$$

Equations 6 and 7 have been used for single-regime models to estimate model parameters based on specified traffic-flow criteria (6,7).

In order to use region-3 models for the non-congested-flow regime, a particular modification should be made. In selecting a model for the non-congested-flow regime, the intercept with the density axis (k_j) is obviously not important. Therefore, it would seem reasonable to employ another (auxiliary) criterion within the non-congested-flow regime instead of the criterion k_j. This auxiliary criterion corresponds to a selected point from the data with speed and density denoted by u_n and k_n. The relation between this auxiliary criterion and the k_j criterion is given by the following:

$$k_j^{\ell-1} = k_n^{\ell-1}/[1-(u_n/u_f)^{1-m}] \quad (8)$$

Properties of Models in Region 5

Models in region 5 ($\ell > 1$, $m > 1$) are more general than the special-case models of region 4. For specific values of the basic criteria u_f, u_0, and k_0, region 5 provides a range of models, while region 4 provides only one model.

The speed-density relation of models in region 5, shown previously in Figure 1, is given by the following:

$$u^{1-m} = u_f^{1-m} + ck^{\ell-1} \qquad \ell > m \quad (9)$$

where

$$c = \alpha\,[(1-m)/(1-\ell)] \quad (10)$$

Equation 9 represents a model that has no intercept with the density axis, $k_j \to \infty$. The model contains three parameters--ℓ, m, and α. The relations between these parameters and the basic criteria can be established. From Equation 9, the relations between q and k and q and u can be obtained as follows:

$$q = k\,(u_f^{1-m} + ck^{\ell-1}) \quad (11)$$

$$q^{\ell-1} = (u^{\ell-1}/c)\,(u^{1-m} - u_f^{1-m}) \quad (12)$$

At maximum flow, $q'(k)_k = 0$ and $q'(u)_u = 0$. Therefore, by differentiating Equations 11 and 12 with respect to k and u, respectively, and equating the derivatives to zero, one obtains the following:

$$\alpha = [(\ell-1)/(\ell-m)]\,(u_f^{1-m}/k_0^{\ell-1}) \qquad \ell > m \quad (13)$$

$$(u_0/u_f)^{1-m} = (\ell-1)/(\ell-m) \qquad \ell > m \quad (14)$$

(Equation 14 is the same as Equation 7 of region 3.)

Equations 13 and 14 contain three parameters and in order to estimate these parameters, it is necessary to have a third condition. This condition may be represented by the auxiliary criteria (u_n,k_n). Thus, substituting these criteria into Equation 9, we have the following:

$$u_n^{1-m} = u_f^{1-m} + ck_n^{\ell-1} \quad (15)$$

Now, Equations 13, 14, and 15 can be solved for specified values of u_f, u_0, k_0, u_n, and k_n to determine model parameters ℓ, m, and α.

It is important to note that models in region 5 are considered valid for only $\ell > m$. For $\ell \leq m$, neither the q - k nor the q - u relations will have a maximum point; $q'(k)_k > 0$ for all k and $q'(u)_u < 0$ for all u. Although such relations may be used to represent the non-congested-flow regime, they clearly do not describe properly the behavior of the traffic flow near capacity. Consequently, models corresponding to $\ell \leq m$ are considered undesirable for the non-congested-flow regime (they are certainly not meaningful for the single-regime representation).

Between-Region Properties

The theoretical properties of models among regions 3, 4, and 5 will now be presented. A fundamental property is that models in region 4 are a limiting case of both models in region 3 and models in region 5. This property is proved below for only models in regions 3 and 4; the proof for regions 5 and 4 will be similar.

Substituting for k_j from Equation 6 into Equation 5 (region 3) gives the following:

$$u^{1-m} = u_f^{1-m}\left\{1-(k/k_0)^{\ell-1}\,[(1-m)/(\ell-m)]\right\}$$

To prove that the above model approaches the one corresponding to region 4, Equation 2, as $m \to 1$, let us first express k as a function of u:

$$k^{\ell-1} = k_0^{\ell-1}\,[(\ell-m)/(1-m)]\,[1-(u/u_f)^{1-m}] \quad (16)$$

As $m \to 1$, the limit of Equation 16 is equal to zero divided by zero. Therefore, one may apply L'Hospital's rule.

Let

$$f(m) = (\ell-m)\,k_0^{\ell-1}\,[1-(u/u_f)^{1-m}] \quad (17)$$

$$g(m) = 1-m \quad (18)$$

Then

$$\lim_{m\to 1}\,[f(m)/g(m)] = \lim_{m\to 1}\,[f'(m)/g'(m)] \qquad g'(m) \neq 0 \quad (19)$$

where the prime represents the first derivative with respect to m.

The derivatives in Equation 19 can be obtained from Equations 17 and 18 as follows:

$$f'(m) = k_0^{\ell-1}\,[-(\ell-m)\,u^{1-m}\ln(u/u_f) - (u/u_f)^{1-m} + 1]$$

$$g'(m) = -1$$

Substituting into Equation 19 gives

$$\lim_{m\to 1}\,[f(m)/g(m)] = (\ell-1)\,k_0^{\ell-1}\ln(u/u_f)$$

Thus, as $m \to 1$, Equation 16 becomes

$$k^{\ell-1} = (\ell-1)\,k_0^{\ell-1}\ln(u/u_f)$$

or

$$\ln u = \ln u_f + [1/(\ell-1)]\,(k/k_0)^{\ell-1}$$

which is the same as Equation 2 of region 4; note that $\alpha = 1/k_0^{\ell-1}$ from Equation 3.

It follows from this limiting-case property that the characteristics of models in regions 3 and 5 should, as $m \to 1$, approach those of region 4. For example, it can be easily shown that by taking the limit of α and the limit of u_0/u_f (Equations 13 and 14) of region 5, as $m \to 1$, the corresponding formulas of region 4 are obtained (Equations 3 and 4). The continuity of the u_0/u_f ratio in regions 3, 4, and 5 is illustrated in Figure 2.

Another important aspect that will be useful in model selection is the continuity of the location of the inflection point and the associated convexity and concavity characteristics of the speed-density curves. For $\ell \leq 2$, models in the three regions are entirely concave, as illustrated in Figure 2. That can be expressed mathematically as follows:

$$u''(k)_k > 0 \quad 0 \leqslant k \leqslant k_j \tag{20}$$

for the ($\ell \leq 2$, $m \geq 0$) space except at $\ell = 2$, $m = 0$, where $u''(k)_k = 0$ (the double prime represents the second derivative with respect to k).

For $\ell > 2$, models in the three regions are mixed; they consist of two portions (convex and concave). The concave portion diminishes at $m = 0$; that is, models become entirely convex. When the inflection point coincides with k_0, the non-congested-flow regime becomes totally convex. If the density corresponding to the inflection point is denoted by k_t, the above characteristics can be expressed mathematically as follows:

$$u''(k)_k < 0 \quad 0 \leqslant k \leqslant k_j$$

for $\ell > 2$, $m = 0$, and

$$u''(k)_k < 0 \quad 0 \leqslant k < k_t$$
$$u''(k)_k > 0 \quad k_t < k \leqslant k_j \tag{21}$$

for $\ell > 2$, $m > 0$, and $0 < k_t < k_j$. The location of models where $k_t = k_0$ is given by $m = \ell - 2$. This linear function separates the models where the non-congested-flow regime is totally convex from those where it is mixed, as shown in Figure 2. Also shown in Figure 2 is the location of models with an inflection point lying at half the optimum density, $k_t = 0.5k_0$.

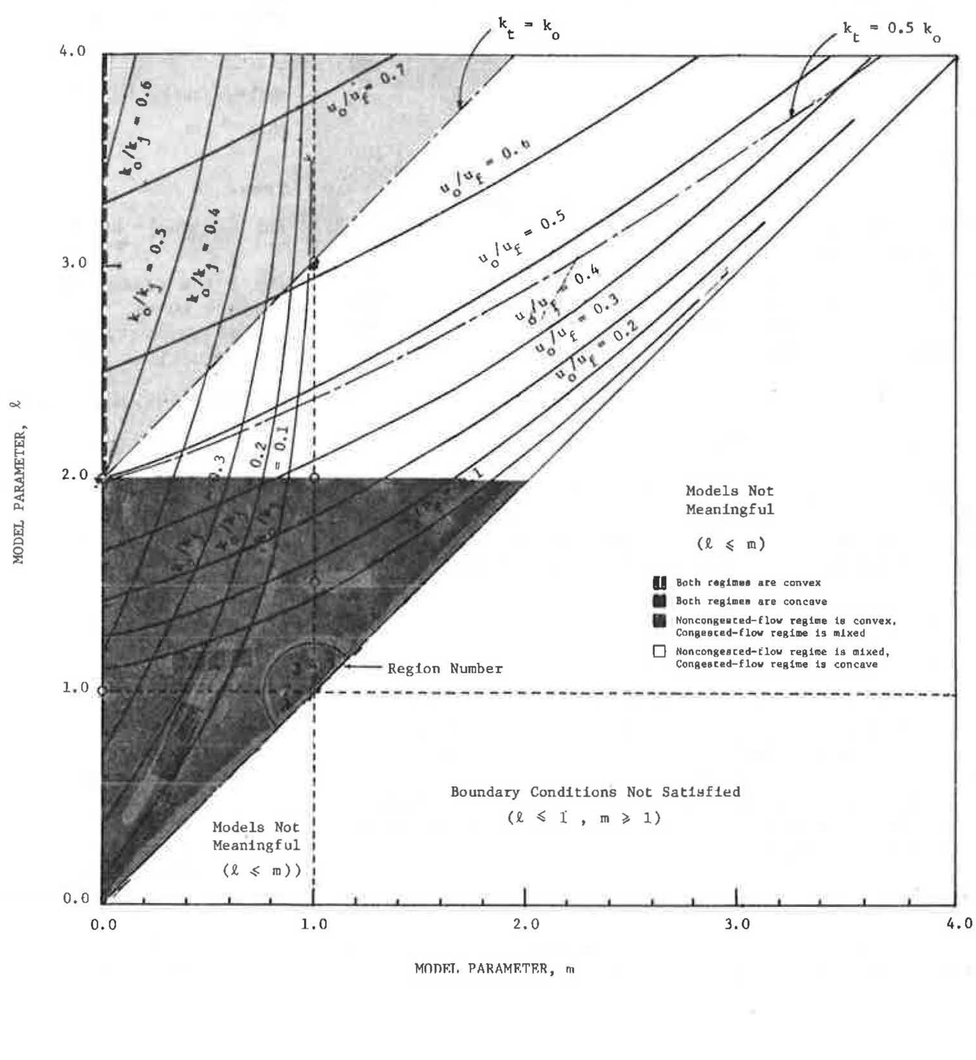
As noted in Figure 2, all convex, concave, and mixed models converge to a linear function at $\ell = 2$, $m = 0$ [Greenshields' model (8)]. Also, in region 4, the two models by Drew (9), $\ell = 1.5$, and by Underwood (10), $\ell = 2$, are totally concave. The model by Drake, Schoefer, and May, $\ell = 3$, is convex in the non-congested-flow regime and concave in the congested-flow regime; $k_t = k_0$.

A final property of the non-congested-flow models is that, for specific values of the basic criteria u_f, u_0, and k_0, the following relation can be shown for the entire range of the traffic density, except at $k = 0$ and $k = k_0$ where the values of u coincide:

$$u_{(3)} < u_{(4)} < u_{(5)} \tag{22}$$

The subscripts in Equation 22 refer to the region number, and the values of u correspond to Equations 5, 2, and 9, respectively. Both $u_{(3)}$ and $u_{(5)}$ approach $u_{(4)}$ as $m \to 1$, as proved earlier. This property is illustrated in Figure 3, which shows three models from regions 3, 4, and 5 drawn for the specific criteria u_f = 50 mph, u_0 = 25 mph, and

Figure 2. Continuity of model characteristics, convexity, and concavity.



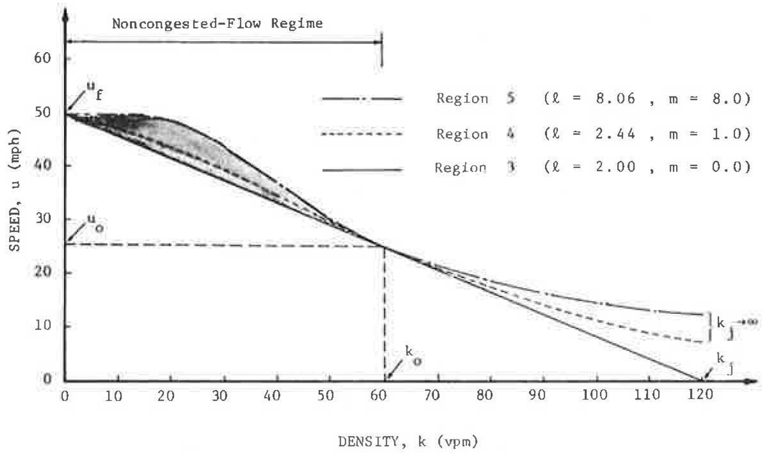


Figure 3. Continuity of non-congested-flow models.

k_0 = 60 vehicles per mile (vpm). There is only one model from region 4 (dashed curve) that satisfies the above criteria. The range below the dashed curve corresponds to models from region 3, while the range above it corresponds to models from region 5. For these specified criteria, the model from region 3 represents a lower bound of the models from that region since it corresponds to m = 0. On the other hand, there exist other models from region 5 that lie above the one shown in Figure 3.

Selecting a particular model from the range of models shown in Figure 3 can be accomplished by the auxiliary criteria. Such a model will satisfy the five criteria of that regime. In addition, an even better fit to the data between these criteria can be obtained by the knowledge of the convexity and concavity of the models. These aspects represent the basic elements of the procedure of model selection to be described later.

CONGESTED-FLOW REGIME

The congested-flow regime corresponds, as stated previously, to the models of regions 1, 2, and 3. The purpose of this section is to describe the theoretical properties of models and regions of this regime. The description is given in three parts: a summary of the previously reported results of region 2 and extended results of region 3, the theoretical relations of models in region 1, and between-region properties.

Previous Results

The speed-density relation of models in region 2 ($\ell = 1$, $m < 1$), shown previously in Figure 1, is given by the following:

$$u^{1-m} = \alpha (1-m) \ln (k_j/k) \tag{23}$$

The relations between the traffic-flow criteria k_j, u_0, and k_0 and model parameters m and α are given by (6) the following:

$$\alpha = u_0^{1-m} \tag{24}$$

$$k_0/k_j = \exp\left\{-[1/(1-m)]\right\} \tag{25}$$

For models in region 3, the speed-density relation and the relations between the basic traffic-flow criteria and the model parameters are as given previously in Equations 5, 6, and 7. In order to use these models for the congested-flow regime, a similar modification to the one employed previously should be made. Specifically, for the congested-flow regime, the intercept with the speed axis, u_f, is not of major importance and is replaced here by an auxiliary criterion (u_c, k_c), which lies somewhere within the congested-flow regime. The relation between this auxiliary criterion and the u_f criterion is given by the following:

$$u_f^{1-m} = u_c^{1-m}/[1-(k_c/k_j)^{\ell-1}] \tag{26}$$

Properties of Models in Region 1

Models in region 1 ($\ell < 1$, $m < 1$) are more general than the special-case models of region 2. For specific values of the basic criteria k_j, u_0, k_0, region 1 provides a range of models, while region 2 provides only one model.

The speed-density relation of models in region 1, shown previously in Figure 1, is given by

$$u^{1-m} = c(k^{\ell-1} - k_j^{\ell-1}) \qquad \ell > m \tag{27}$$

where $c = \alpha[(1-m)/(1-\ell)]$.

Equation 27 represents a model that has no intercept with the speed axis, $u_f \to \infty$, and contains three parameters. The relations between these parameters and the basic criteria can be established. From Equation 27 one can obtain q as a function of k and as a function of u as follows:

$$q = k\,[c(k^{\ell-1} - k_j^{\ell-1})]^{1/1-m} \tag{28}$$

$$q^{\ell-1} = u^{\ell-1}\,[(u^{1-m}/c) + k_j^{\ell-1}] \tag{29}$$

Following a similar procedure to that described for region 5, α and k_0/k_j can be obtained as follows:

$$\alpha = [(\ell-m)/(1-m)]\,(u_0^{1-m}/k_j^{\ell-1}) \qquad \ell > m \tag{30}$$

$$(k_0/k_j)^{\ell-1} = (1-m)/(\ell-m) \qquad \ell > m \tag{31}$$

(Equation 31 is the same as Equation 6 of region 3.)

The auxiliary criteria (u_c,k_c) can now be used to provide a third relation so that the three parameters may be estimated. Substituting for these criteria into Equation 27, one obtains the following:

$$u_c^{1-m} = c(k_c^{\ell-1} - k_j^{\ell-1}) \tag{32}$$

Equations 30, 31, and 32 can be solved to determine model parameters ℓ, m, and α for specified values of k_j, u_0, k_0, u_c, and k_c.

It is noted that models in region 1 are considered valid for only $\ell > m$. For $\ell \leq m$, the q - k and q - u relations will not have a maximum point; $q'(k)_k < 0$ for all k and $q'(u)_u > 0$ for all u. For example, for $\ell = m = 0$, Equation 28 will be a straight line having an intercept α with the flow axis, and Equation 29 will be an increasing function; q approaches α as u approaches infinity. The characteristics of these models near capacity are considered here undesirable for representing the congested-flow regime. Moreover, these models are certainly not meaningful for the single-regime representation.

Between-Region Properties

The congested-flow models of regions 1, 2, and 3 exhibit similar properties to those of the non-congested-flow models described in the previous section. Specifically, as $\ell \to 1$, models in region 2 are a limiting case of both models in region 1 and models in region 3. As a result, there is also a continuity of model characteristics and for specific values of k_j, k_0, and u_0, models in the three regions exhibit a particular pattern.

The limiting-case property of only models in regions 1 and 2 (both regions have no intercept with the speed axis, $u_f \to \infty$) will now be proved. This will perhaps complement the presentation in the previous section where the limiting-case property of regions 3 and 4 (one region with finite k_j and the other with $k_j \to \infty$) was proved.

Substituting for c into Equation 27 (region 1) gives the following:

$$u^{1-m} = \alpha [(1-m)/(1-\ell)] \,(k^{\ell-1} - k_j^{\ell-1}) \qquad (33)$$

Since the limit of the product is the product of the limits, the limit of Equation 33 is equal to the limit of α multiplied by the limit of the remainder, as $\ell \to 1$. The limit of α, Equation 30, is equal to u_0^{1-m} and the limit of the remaining part can be obtained by applying L'Hospital's rule, given previously in Equation 19.

Let

$$f(\ell) = (1-m)\,(k^{\ell-1} - k_j^{\ell-1}) \qquad (34)$$

$$g(\ell) = 1 - \ell \qquad (35)$$

Applying Equation 19 gives the following:

$$\lim_{\ell \to 1} [f'(\ell)/g'(\ell)] = (1-m)\ln(k_j/k) \qquad (36)$$

Thus, as $\ell \to 1$, Equation 30 becomes the following:

$$u^{1-m} = u_0^{1-m}\,(1-m)\ln(k_j/k)$$

which is the same as Equation 23 of region 2; note that $\alpha = u_0^{1-m}$ from Equation 24.

It follows from this limiting-case property that the characteristics of models in regions 1 and 3, α and (k_0/k_j), should approach those of region 2 as $\ell \to 1$. The continuity of the (k_0/k_j) ratio is illustrated in Figure 2. It is noted that region 3 exhibits a wide range of this ratio, while regions 1 and 2 show a narrow range with a maximum value of $(k_0/k_j) = 0.37$, which corresponds to Greenberg's model (12).

The congested-flow models of regions 1, 2, and 3 may be convex, concave, or mixed. These characteristics were described previously for region 3. Models in regions 1 and 2 are entirely concave as illustrated in Figure 2. Thus,

$$u''(k)_k > 0 \qquad 0 \leq k \leq k_j \qquad (37)$$

for $\ell \leq 1$ and $m \geq 0$.

Finally, for specific values for the basic criteria k_j, u_0, and k_0, it can be shown that the following relation holds:

$$u_{(3)} < u_{(2)} < u_{(1)} \qquad (38)$$

for the entire range of the traffic density, except at $k = k_0$ and $k = k_j$, where the values of u coincide. Both $u_{(3)}$ and $u_{(1)}$ approach $u_{(2)}$ as $\ell \to 1$, as proved previously.

An illustration of this property is shown in Figure 4 for the specific criteria $k_j = 200$ vpm, $u_0 = 25$ mph, and $k_0 = 60$ vpm. The dashed curve represents the (only) model from region 2 that satisfies these criteria. The model from region 1 represents an upper bound of models from that region. On the other hand, the model from region 3 is a specific selection and other models below the solid curve do exist. Thus, the range above the dashed curve corresponds to models from region 1 and the range below it corresponds to models from region 3. Selecting a model from the range of models shown in Figure 4 can be accomplished by means of the auxiliary criteria. The selection procedure for the non-congested-flow and congested-flow regimes is described in the following section.

MODEL SELECTION

The continuity property of the non-congested-flow and congested-flow models described in the previous sections represents the basis for the selection procedure presented in this section. The procedure is described first for the non-congested-flow regime and then for the congested-flow regime.

Non-Congested-Flow Regime

As mentioned previously, the non-congested-flow regime can be represented by a model from regions 3, 4, or 5. For specified traffic-flow criteria u_f, u_0, k_0, u_n, and k_n, it is necessary to determine first the region that contains the model satisfying these criteria and then to determine that model. It is important to note that since models in the three regions exhibit the continuity property, these models do not intersect, and therefore there is only one specific model that satisfies the above five criteria. The relations between the traffic-flow criteria and model parameters for regions 3, 4, and 5 will now be obtained.

For region 5, the formula that considers the auxiliary criterion (Equation 15) can be written as follows:

$$u_n^{1-m} = u_f^{1-m} + c\,(\beta_n k_0)^{\ell-1} \qquad (39)$$

where β_n is a standardized variable given by the following:

$$\beta_n = k_n/k_0 \qquad (40)$$

Substituting for c from Equation 10 (and for α from Equation 13) and dividing both sides of Equation 39 by u_f^{1-m} gives the following:

$$(u_n/u_f)^{1-m} = 1 - [(1-m)/(\ell-m)]\,\beta_n^{\ell-1} \qquad (41)$$

From Equation 14, we have

$$\ell = [m\,(u_0/u_f)^{1-m} - 1]/[(u_0/u_f)^{1-m} - 1] \qquad (42)$$

Equations 41 and 42 relate model parameters ℓ and m to the traffic-flow criteria u_f, u_0, u_n,

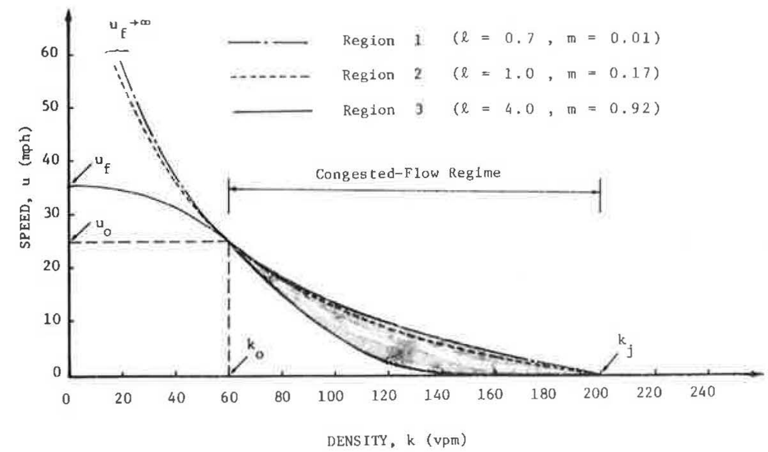


Figure 4. Continuity of congested-flow models.

and β_n (which incorporates k_n and k_0). For region 3, identical formulas to the above can be obtained from Equations 6, 7, and 8.

For region 4, the relations between ℓ and the traffic-flow criteria can be obtained, based on Equations 2, 3, and 4, as follows:

$$\ln(u_n/u_f) = [1/(1-\ell)]\,\beta_n^{\ell-1} \qquad (43)$$

$$\ell = 1 - [1/\ln(u_0/u_f)] \qquad (44)$$

The above formulas are shown in Figure 5 for $\beta_n = 0.5$ and for various ranges of the other traffic-flow criteria (the value $\beta_n = 0.5$ indicates that the auxiliary criterion u_n, k_n lies at a point where k_n is half the optimum density k_0). The thick solid curve in Figure 5 corresponds to region 4. This curve corresponds to Equations 43 and 44 (or equivalently to the limits of Equations 41 and 42 as $m \to 1$). The area above that curve corresponds to region 5 and the one below it corresponds to region 3. Models below the curve m = 0 (region 3) correspond to negative values of m and are considered undesirable; a negative m has the effect of shifting the speed term in Equation 1 from the numerator to the denominator. Figure 5 also shows the curvature properties of models.

Now, the selection procedure is as follows:

1. Determine from the data the criteria u_f, u_0, k_0, u_n, and k_n where k_n is to be chosen as 0.5 k_0.
2. Follow the sample arrows shown in Figure 5 and determine the intersection point. Read the corresponding values of ℓ and m. If the intersection point lies above the thick solid curve, the model satisfying the specified criteria lies in region 5. Estimate the value of α from Equation 13 and obtain the traffic-flow relations corresponding to this model from Equations 9 through 12.
3. If the intersection point lies below the thick solid curve, the model we are after is located in region 3. Obtain the traffic-flow relations corresponding to the model from Equation 5 (the q-u and q-k relations can be easily obtained from this equation).
4. If the intersection point lies on the thick solid curve, the model lies in region 4. Estimate the value of α from Equation 3 and obtain the traffic-flow relations corresponding to this model from Equation 2.

In order to help the user determine the values of ℓ and m accurately, the numerical values on which the nomograph of Figure 5 is based are provided in Table 1. Table 1 will be particularly useful for the upper portion of the nomograph where ℓ becomes very close to m and the difference cannot be easily detected from Figure 5.

It is important to note that, based on the exhibited pattern of the non-congested-flow regime data, some adjustments of the traffic-flow criteria may be deemed necessary in order to select the model that is compatible with such a pattern. For example, if the data exhibit a convex shape, the traffic-flow criteria may be properly adjusted so that the intersection point in Figure 5 lies in the area delineated in the upper portion of this figure. This adjustment will ensure an even better fit to the data between the traffic-flow criteria.

Congested-Flow Regime

The objective here is to select one of the three regions--1, 2, or 3--and the model within the selected region that satisfies the specified criteria k_j, u_0, k_0, u_c, and k_c. Because of the continuity property of the congested-flow models, there is only one specific model that satisfies these criteria. Let us first estimate model parameters for regions 1 and 2 and then outline the procedure of model selection.

Similar to the procedure adopted for region 5 of the non-congested-flow regime, the formulas relating model parameters to the traffic-flow criteria in region 1 can be obtained, based on Equations 30, 31, and 32, as follows:

$$(u_c/u_0)^{1-m} = [(\ell - m)/(1-\ell)]\,[(\beta_c k_0/k_j)^{\ell-1} - 1] \qquad (45)$$

$$m = [1 - \ell(k_0/k_j)^{\ell-1}]/[1 - (k_0/k_j)^{\ell-1}] \qquad (46)$$

where β_c is a standardized variable given by the following:

$$\beta_c = k_c/k_0 \qquad (47)$$

For region 3, identical formulas to the above can be obtained from Equations 6, 7, and 26.

For region 2, the corresponding relations, based on Equations 23, 24, and 25, are given:

$$(u_c/u_0)^{1-m} = (1-m)\ln(k_j/k_0\beta_c) \qquad (48)$$

$$m = 1 + [1/\ln(k_0/k_j)] \qquad (49)$$

The above formulas are shown in Figure 6 for $\beta_c = 1.5$ and 2.0. The thick solid curve corresponds to region 2. The area above that curve corresponds to region 1 and the one below it corresponds to region 3. The curvature properties of models are also shown in Figure 6.

Now, the selection procedure is as follows:

1. Determine from the data the criteria k_j, u_0, k_0, and k_c, where k_c is to be chosen as either $1.5k_0$ or $2k_0$.
2. Follow the sample arrows shown in Figure 6 and determine the intersection point. Read the corresponding values of ℓ and m. If the intersection point lies above the thick solid curve for the corresponding β_c, the model lies in region 1. Estimate the value of α from Equation 30 and obtain the traffic-flow relations corresponding to this model from Equations 27 through 29.
3. If the intersection point lies below the thick solid curve, the model lies in region 3 and the corresponding traffic-flow criteria can be obtained from Equation 5.
4. If the intersection point lies on the thick solid curve, the model lies in region 2. Estimate the value of α from Equation 24 and obtain the corresponding traffic-flow relations from Equation 23.

Figure 5. Nomograph for non-congested-flow regime (regions 3, 4, and 5).

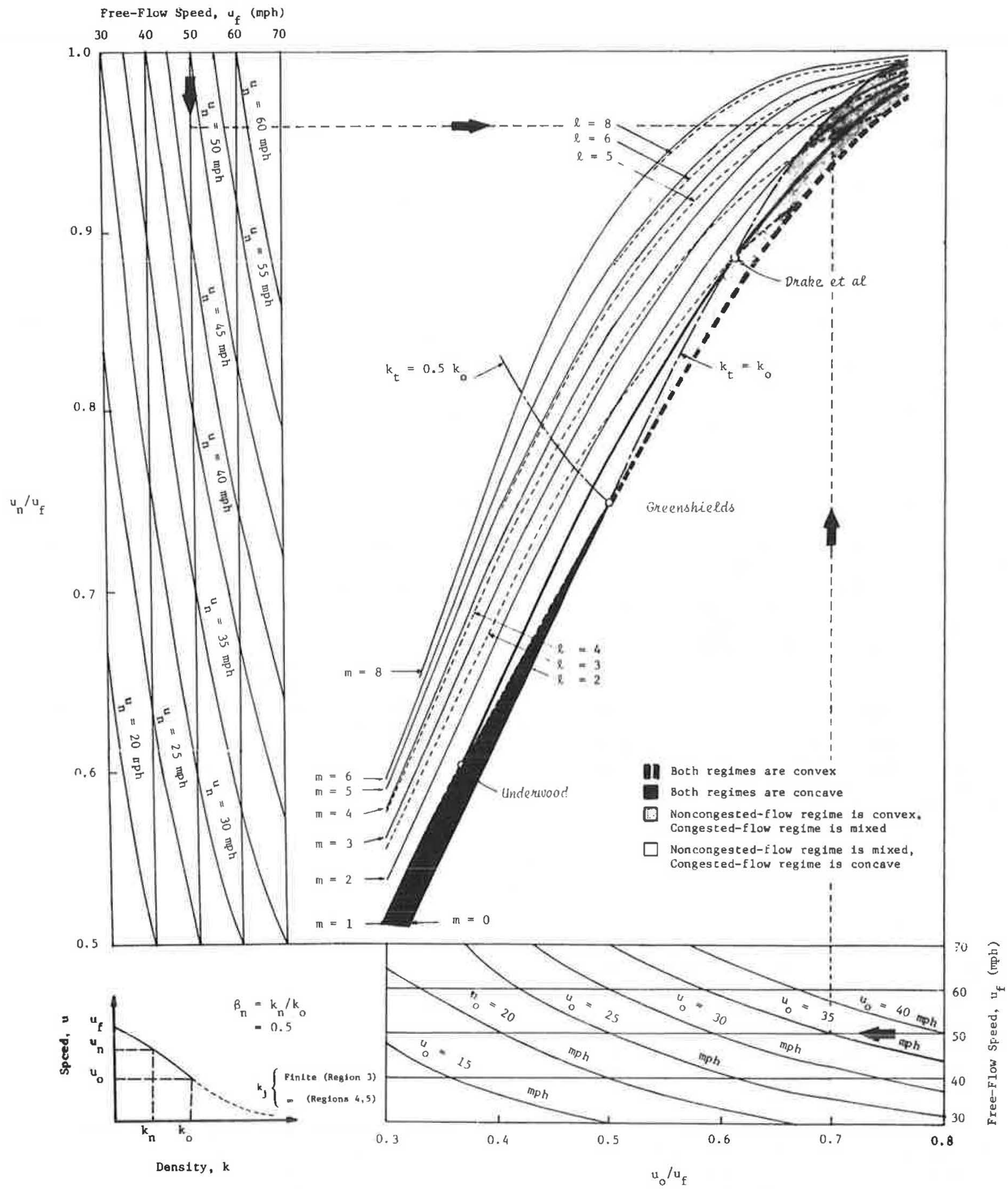


Table 1. Model parameters for non-congested-flow regime (regions 3, 4, and 5).

(u_0/u_f)	ℓ	m	(u_n/u_f)	(u_0/u_f)	ℓ	m	(u_n/u_f)
0.3	1.429	0.000	0.480	0.6	2.500	0.000	0.859
	1.494	0.200	0.485		2.585	0.200	0.862
	1.566	0.400	0.491		2.673	0.400	0.866
	1.647	0.600	0.497		2.764	0.600	0.870
	1.735	0.800	0.502		2.859	0.800	0.873
	1.831	1.000	0.508		2.958	1.000	0.877
	2.429	2.000	0.536		3.500	2.000	0.895
	3.198	3.000	0.559		4.125	3.000	0.911
	4.083	4.000	0.575		4.827	4.000	0.927
	5.033	5.000	0.586		5.596	5.000	0.941
	6.012	6.000	0.592		6.422	6.000	0.953
	7.004	7.000	0.596		7.294	7.000	0.962
	8.002	8.000	0.598		8.202	8.000	0.970
	9.001	9.000	0.599		9.137	9.000	0.977
0.4	1.667	0.000	0.662	0.7	3.333	0.000	0.940
	1.740	0.200	0.628		3.423	0.200	0.942
	1.819	0.400	0.633		3.514	0.400	0.944
	1.904	0.600	0.639		3.608	0.600	0.946
	1.994	0.800	0.645		3.705	0.800	0.948
	2.091	1.000	0.650		3.804	1.000	0.950
	2.667	2.000	0.679		4.333	2.000	0.959
	3.381	3.000	0.706		4.922	3.000	0.967
	4.205	4.000	0.729		5.566	4.000	0.974
	5.105	5.000	0.747		6.264	5.000	0.980
	6.052	6.000	0.761		7.010	6.000	0.985
	7.025	7.000	0.772		7.800	7.000	0.989
	8.011	8.000	0.779		8.628	8.000	0.992
	9.005	9.000	0.785		9.489	9.000	0.994
0.5	2.000	0.000	0.750	0.75	4.00	0.000	0.969
	2.079	0.200	0.755		4.091	0.200	0.970
	2.163	0.300	0.760		4.185	0.400	0.971
	2.252	0.600	0.765		4.280	0.600	0.972
	2.345	0.800	0.770		4.377	0.800	0.973
	2.443	1.000	0.775		4.476	1.000	0.974
	3.000	2.000	0.800		5.000	2.000	0.980
	3.667	3.000	0.824		5.571	3.000	0.984
	4.429	4.000	0.846		6.189	4.000	0.988
	5.267	5.000	0.866		6.851	5.000	0.991
	6.161	6.000	0.883		7.556	6.000	0.993
	7.095	7.000	0.897		8.299	7.000	0.995
	8.055	8.000	0.909		9.078	8.000	0.997
	9.031	9.000	0.918		9.890	9.000	0.998

It shoulu be noted that if the intersection point lies above the curve m = 0 (models not desirable), the traffic-flow criteria should be adjusted so that the intersection point lies on or below that curve.

APPLICATION

The selection procedure described in the previous section was applied to traffic-flow data that have been collected on the Eisenhower Freeway at Harlem. The speed-density and flow-density data are shown in Figure 7 (top and bottom, respectively). The data exhibit some discontinuity, which can be seen more clearly from the flow-density graph. An excellent discussion of the possible reasons for the existence of discontinuity in some traffic-flow data is given by Gazis, Herman, and Rothery (3).

The traffic-flow criteria can now be established for both non-congested-flow and congested-flow regimes. For the non-congested-flow regime, the criteria include u_f, u_0, k_0, u_n, and k_n. Those for the congested-flow regime include k_j, u_0, k_0, u_c, and k_c. Because of the discontinuity, however, the optimum density u_0 need not be the same for both regimes. These criteria can be established from the data in such a way that the model corresponding to the respective criteria for each regime provides a good visual fit to the data of the particular regime. The specified criteria are given in the table below and are indicated by circles in Figure 7 (NA = not applicable):

	Regime	
Criterion	Non-Congested Flow	Congested Flow
u_f (mph)	50	NA
u_n (mph)	48	-
k_n (vpm)	25	-
u_0 (mph)	31	35
k_0 (vpm)	50	50
k_j (vpm)	-	200
u_c (mph)	-	12
k_c (vpm)	-	100

It is useful at this point to comment on establishing the traffic-flow criteria. As noted in the table above and Figure 7, the optimum speed is 35 mph for the non-congested-flow regime and 31 mph for the congested-flow regime. These values were determined by establishing first the maximum flow values in Figure 7 (bottom) corresponding to k_0 = 50 vpm, which was arbitrarily chosen. Then these values were used to determine the optimum speed. The auxiliary criteria were chosen at the mid-range of the non-congested-flow regime (β_n = 0.5) and at a point twice the optimum density for the congested-flow regime (β_c = 2.0). Establishing the remaining criteria u_f and k_j will generally require a knowledge of the characteristics of the highway facility. In particular, data points near the jam density are not usually obtainable and one would rely on previous data on the respective freeway or other similar freeways. As a general guideline, the jam density ranges from 180 to 250 vpm, which correspond to low-design and high-design facilities, respectively. Since the Eisenhower Freeway is an old-design facility of the 1950s, the jam density was arbitrarily chosen as 200 vpm. Now we will determine the non-congested-flow and congested-flow models that satisfy the specified traffic-flow criteria by following the procedure described in the previous section.

For the non-congested-flow regime, we first use Figure 5 and draw lines corresponding to the specified criteria shown in the table above. Since the intersection point lies above the thick solid curve, the model we are seeking lies in region 5. The values of model parameters can be read from the nomograph as ℓ = 4.3 and m = 2.0. Since the model lies in the upper-delineated portion of Figure 5, the result will be that the speed-density relation is convex in the entire non-congested-flow regime. Because the speed-density data also appear to exhibit a convex shape, the model will provide a good fit to the data between the traffic-flow criteria. In cases where the convexity and concavity characteristics are not the same for the data and the determined model, one may adjust the traffic-flow criteria to achieve this similarity. This adjustment can be made with the aid of the curvature properties shown in Figure 5.

With the above values of ℓ and m and the specified criteria, the value of α can be calculated from Equation 13. Thus, the three parameters in the traffic-flow relations of the non-congested-flow regime are known. The speed-density relation (Equation 9), for example, becomes the following:

$$u = 1/(0.02 + 2.15 \times 10^{-8} k^{3.3}) \qquad 0 \leq k \leq 50 \qquad (50)$$

The speed-density relation of Equation 50 is shown in Figure 7 (top) and is convex, as expected. The flow-density relation is also shown in Figure 7 (bottom). As seen, the selected model provides an excellent fit to the data, which is largely achieved by the proper selection of the traffic-flow criteria.

For the congested-flow regime, the intersection point corresponding to the specified criteria was determined as shown in Figure 6. As seen, this point lies above the thick curve corresponding to $\beta_c = 2.0$. Therefore, the model we are seeking is located in region 1 and the speed-density relation will be concave in the entire congested-flow regime. The values of model parameters can be read from Figure 6 as $\ell = 0.5$ and $m = 0$.

With the above values of ℓ and m and the traffic-flow criteria, the value of α can be calculated from Equation 30. The three parameters for the congested-flow regime are now known and so are the traffic-flow relations. The speed-density relation (Equation 27), for example, becomes the following:

$$u = (438.4/k^{0.5}) - 31.0 \qquad 50 \leqslant k \leqslant 200 \tag{51}$$

The speed-density relation of Equation 51 is shown in Figure 7 (top). The flow-density relation is also shown in Figure 7 (bottom). Again, the selected model provides excellent fit to the data.

CONCLUDING REMARKS

This paper has presented a theoretical procedure for

Figure 6. Nomograph for congested-flow regime (regions 1, 2, and 3).

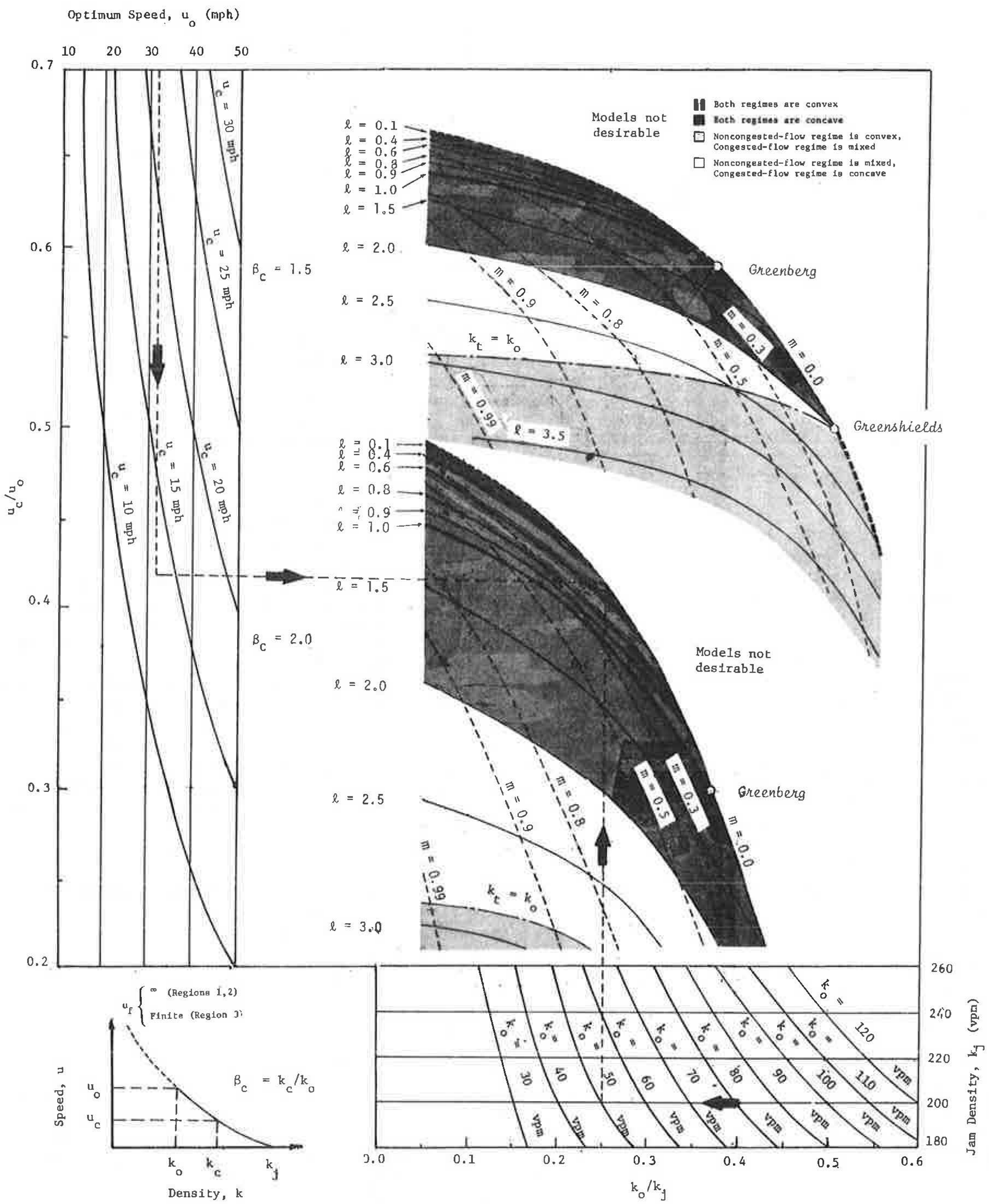
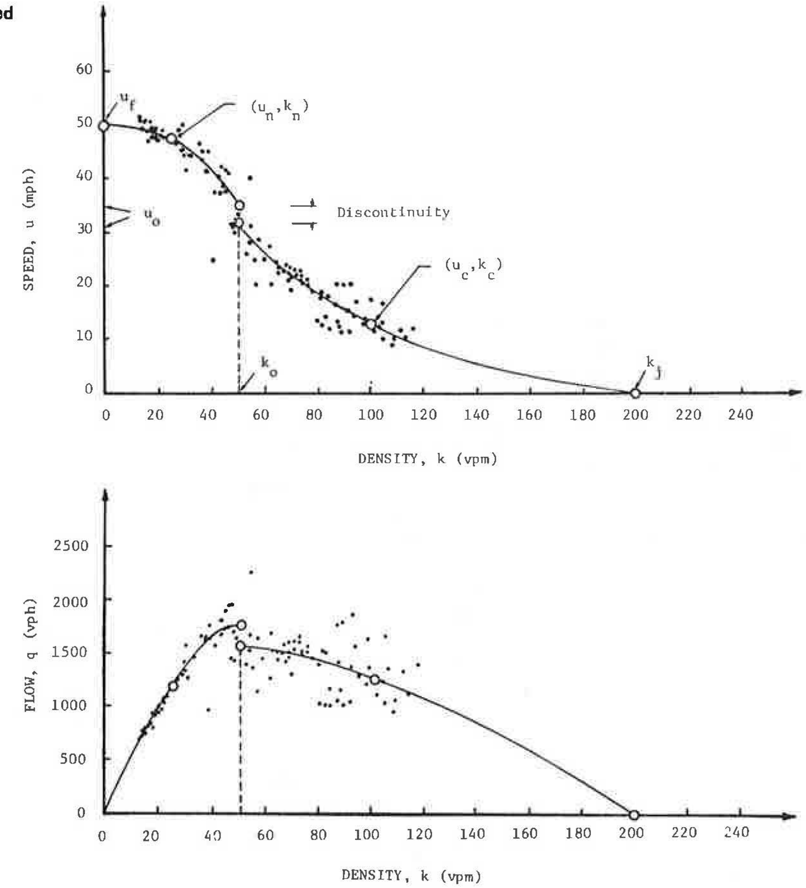


Figure 7. Speed-density and flow-density data and selected models.



selecting two-regime traffic-flow models and has demonstrated its use by applying it to actual data. The procedure is based principally on the between-region properties, including the continuity of models and the convexity and concavity characteristics. These properties have allowed a systematic procedure for selecting the region and the model within the region that satisfies the specified traffic-flow criteria and further maintains a good fit to the data in the intermediate ranges of flow. A few observations concerning the selection procedure and some of its practical applications are worthy of note:

1. The two-regime representation will generally provide a better fit to the traffic-flow data than the single-regime representation, especially if there is a wide range of flow disturbance (discontinuity) near capacity. This is mainly due to the fact that two-regime models account for the variability of the data in the intermediate ranges of operations, by means of the auxiliary criteria, while single-regime models consider only the basic criteria. This superiority of the two-regime over the single-regime representation has also been demonstrated by using the regression-analysis procedure (13). It is therefore desirable that the two-regime representation be employed in practice whenever possible. In this regard, it is important to point out that the two-regime representation can be used whether there is discontinuity or not in the traffic-flow data.

2. As illustrated in the application described in this paper, the proposed procedure provides a level of data fit that appears to be reasonable for most practical applications. In situations in which a particularly high degree of accuracy is deemed necessary, the regression-analysis procedure (4) may be used. In this case, however, the procedure presented can be first used to determine the specific region of Figure 1 (or perhaps a portion of it) that will most likely contain the best-fit model. The regression-analysis procedure can then be used to search for that model within the identified region rather than within all three regions of each regime. This will obviously reduce the computer time and the data-analysis effort required for the application of this computer-based procedure.

3. The procedure presented requires that the traffic-flow criteria be specified as single values. With a modest effort, however, the procedure can be extended to deal with ranges of the traffic-flow criteria, in which case the procedure would provide a feasible region (rather than specific values) of model parameters. Previous work on single-regime models and the special case of two-regime models (6,7) consider, in addition to single values, ranges of the traffic-flow criteria. Similar logic to the one used in these references may be used here to determine the feasible region of model parameters.

4. The general family of models located in region 5 exhibits certain features that would be of practical use. For large values of m, the speed-flow relation of this region is almost flat for a portion of the low-density end of the curve and then decreases rapidly as density increases. These characteristics have been observed on highways with a rigidly enforced speed limit that is lower than the average highway speed (14). The flattening of the curve occurs since normal average speed cannot be attained due to the speed limit; at some point, as

density increases, the speed limit would no longer govern. Thus, models in region 5 are particularly useful for representing such situations.

5. Another practical phenomenon that can be accounted for by the non-congested-flow models presented is that traffic-flow relations on highways with the same average highway speed exhibit different shapes in the range between the free flow and capacity as a result of a different number of lanes (14). For such highways, the traffic-flow criteria u_f, u_0, and k_0 are generally the same; only the shape of the traffic-flow relations in the intermediate operation differs. Specifically, the Highway Capacity Manual shows that the speed-flow curve of an eight-lane highway is higher than that of a six-lane highway, which is higher than that of a four-lane highway. The proposed procedure can capture this variability in the traffic-flow relations by modifying the auxiliary criteria while retaining the basic criteria fixed.

6. The procedure of selecting two-regime models presented in this paper and the one reported previously for estimating single-regime models (6,7) significantly reduce the need for using computer facilities in estimating traffic-flow relations. As such, the procedure may be useful in such transportation areas as highway capacity and level of service, freeway operations, transportation planning, and environmental studies.

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