3. Traffic volumes had an effect on both the speed of the test vehicle and the time required to complete the maneuver. In general, the speed of the test vehicle decreased as the traffic volume increased and the time required to complete the maneuver increased as the volume increased.

4. The number of lanes affected maneuvering distances. As the number of lanes increased, the maneuvering distance also increased. Therefore the sign-placement distance should take into account the maximum number of lane changes for the particular freeway.

5. The number of lanes affected both the time required to complete the maneuver and the speed of the test vehicle for maneuvers performed in light, medium, and heavy HS conditions. The number of lanes did not affect either speed or time in light, medium, and heavy LS conditions. This was due to large differences in both time and speed between the heavy LS and heavy HS conditions.

6. The lane-maneuvering distances (mean distance) from this research are as follows: (a) 917 ft on a three-lane freeway during light traffic, (b) 1004 ft on a three-lane freeway during medium traffic, (c) 809 ft on a three-lane freeway in heavy LS traffic, (d) 1164 ft on a three-lane freeway in heavy HS traffic, (e) 1090 ft on a four-lane freeway during light traffic, (f) 1534 ft on a four-lane freeway during medium traffic, (g) 1178 ft on a four-lane freeway in heavy LS traffic, and (h) 1453 ft on a four-lane freeway in heavy HS traffic.

ACKNOWLEDGMENT

I wish to express my sincere gratitude to my friend and chairman of my graduate committee, R. Dale Huchingson, for his support and guidance throughout this research effort.

REFERENCES

1. Manual on Uniform Traffic Control Devices for Streets and Highways. U.S. Government Printing Office, 1978.
2. D.J. Mace, R.S. Hostetter, and E.L. Seguin. Information Requirements for Exiting at Interchanges. HRB-Singer, Inc., State College, University Park, PA, Publ. 89211-F, Sept. 1967.
3. M. Levin. Some Investigations of the Freeway Lane Changing Process. Texas A&M Univ., College Station, Ph.D dissertation, 1970.
4. J.W. Eberhard. Driver Information Requirements and Acceptance Criteria. HRB, Highway Research Record 265, 1969, pp. 19-30.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.

Exponential Filtering of Traffic Data

PAUL ROSS

Real-time traffic-control systems commonly filter their input data with exponential filters. The filtering constant β can be objectively determined by recasting the filter as a predictor and choosing the β that would have optimized the predictions for some observed time series of data—mostly taken to be 24 h of input here. Results, by using 70 consecutive days of traffic volume counts from Toronto plus some Los Angeles and District of Columbia data, indicate that the optimal β can be approximated by $\beta = (1 - a) \exp^{(-bT)} + a$, where T is the data-aggregation period and a and b are constants that depend on traffic-peaking characteristics. The filter constant β can be allowed to change during the day; the algorithms of this kind that were investigated all entail only a modest increase in mean-square prediction error. Algorithms that reoptimize β over the most recent N data points do not appear practical, but a simple recursive algorithm that gives good accuracy is presented.

Measurement noise in the usual sense constitutes no more than a few percent of traffic volume counts. However, traffic volumes from successive short periods often differ substantially from each other so that it is convenient to think of the traffic volume itself as being made up of some meaningful signal plus meaningless noise. Thus, detector-measured traffic data (usually volumes or occupancies) constitute a noisy time series. Time-series analysis is a well-established subject of study (1); the methods of Box-Jenkins analysis (2,3) and Kalman filtering (4-6) are quite sophisticated and powerful techniques.

With respect to traffic measurements, virtually all the time-series analysis effort has been directed toward short-term traffic prediction. Polhemus (7,8) codified the prediction problem at a high level (unfortunately without any immediately applicable results). The problem has been presented in a Box-Jenkins framework (9-11) and some specific prediction problems have been formulated in terms of point processes (12-15). Breiman (16) proposed an algorithm that has some intuitive appeal and showed how to choose its parameters to minimize the maximum (not mean) square prediction error.

Real, on-line computer-controlled traffic systems have to make predictions at many locations, and their computing power is usually taxed. Consequently, the prediction and filtering algorithms that have been used in practice have been economical of storage and simple in execution; they tend to be based on intuitive models of traffic.

The prediction algorithm used in the second-generation urban traffic control system (UTCS) program has been described by Ganslaw (17) and by Sperry (18); the predictor in the third-generation UTCS program has been described by McShane, Lieberman, and Goldblatt (19) and by Lieberman and others (20). More effort has gone into devising such algorithms than testing them; the only convincing evaluations of practical prediction algorithms were reported by Kreer (21,22). None of the algorithms tested was convincingly better than simple time-of-day historical averages.

The separate but related filtering problem has been largely ignored. Only Houpt and others (23) describe the filtering of traffic data in more than a passing manner; the technique used is the powerful but cumbersome "extended Kalman filter."

In real traffic problems it is almost universal to use exponential filtering. Briefly, the use of exponential filtering assumes that the data are generated by

$$a = a^* + \text{noise} \quad (1)$$

where a is the observed traffic count and a* is an underlying true traffic volume, which varies only slowly from measurement to measurement. The noise is assumed to have a mean value of zero. The problem is to make a good estimate $\hat{a}$ of a*.

If it is true that a* varies slowly with time, then one intuitively wants to discount the older observations, and the simplest way to do so is by exponential weighting:

$$\hat{a}(t) = (1 - \beta) \sum_{i=1}^{t} \beta^{t-i} a(i) \quad (2)$$

$$= (1 - \beta)\, a(t) + \beta \hat{a}(t - 1) \quad (3)$$

where β is the filter constant.

This use of exponential filtering has many advantages. First, the calculation is recursive--as can be seen from Equation 3. The recursive character means that data-storage requirements are minimal; only the previously calculated estimate $\hat{a}(t - 1)$ and the value of the filter constant itself need be stored. Second, the calculation could hardly be simpler or quicker. And finally, this simple filter is optimal for the common MA(1) class of Box-Jenkins processes. For the purpose of this paper, the use of exponential filtering as defined above will not be questioned.

FILTERING CONSTANT β INDEPENDENT OF TIME OF DAY

Figure 1 shows a typical day of raw traffic counts and the resulting values after the data have been filtered with β = 0.2, 0.4, 0.6, and 0.8. [The principal data used in this paper were obtained in Toronto in the fall of 1973. That data set, the most extensive that I have located, consists of 5-min counts for each of eight detectors 24 h/day for 77 consecutive days. For a description of this data set and some of its properties, see the report by Pignataro and others (24) and by McShane and Crowley (25). Figure 1--and all other cases where a single day is used in this paper--is from detector 1 on September 24, 1973, the first Tuesday of the study.]

Which value of the smoothing constant should be used appears to be a matter of personal taste--how well filtered one prefers the data. One finds such statements as "the constant (β) is typically 0.7 to insure that about five previous speed measurements make a significant contribution to the correct average" (26) and "a value of (β) in the range of 0.4-0.5 is generally quite satisfactory in reducing the (difference between the data and their filtered values) to low-variance white noise" (20).

However, a close look at the fundamentals of the problem indicates that one can establish an objective criterion for the value of β. The filtered data point $\hat{a}(t)$ from Equation 2 or 3 is the best available estimate of the signal a* and hence is the best prediction of the next datum a(t + 1). After data have been collected for an entire day, one can compare the predicted value $\hat{a}(t)$ with the actual value a(t + 1) and determine which value of β would have given the best estimate; such is the approach that will be used in this paper. The criterion of best will mean minimum mean-square error in the predictions made. Figure 2 shows how the mean-square prediction error depends on the value of β for the traffic data used in Figure 1. The best predictions occur when β = 0.424.

Location of this minimum point is most conveniently done by differentiating the mean-square error curve and locating the β that makes this derivative zero. The derivative of the total square error with respect to β can be written as follows:

$$dE(t)/d\beta = [dE(t - 1)/d\beta] + [a(t) - \hat{a}(t - 1)]\,(d/d\beta)\,\hat{a}(t - 1) \quad (4)$$

where

Figure 1. Five-minute traffic-volume counts filtered with various values of filtering constant β.

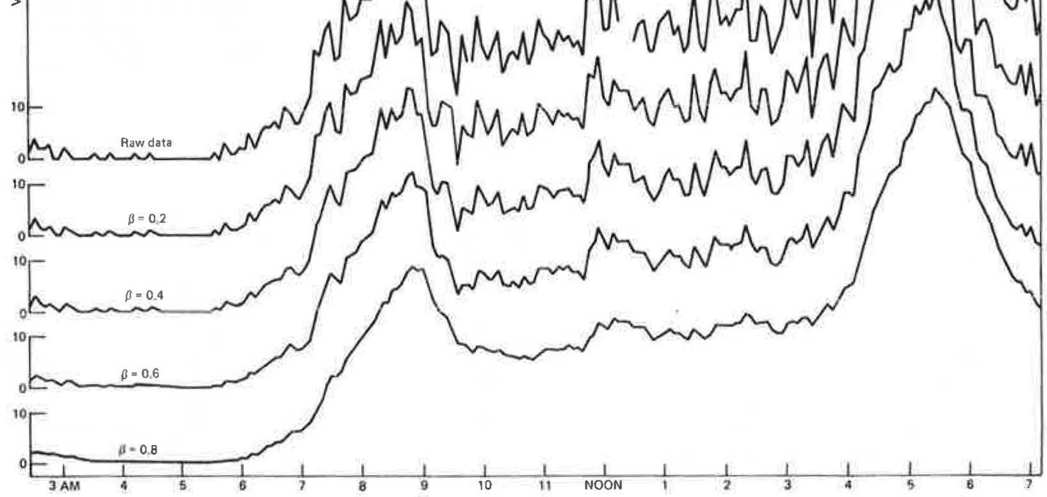
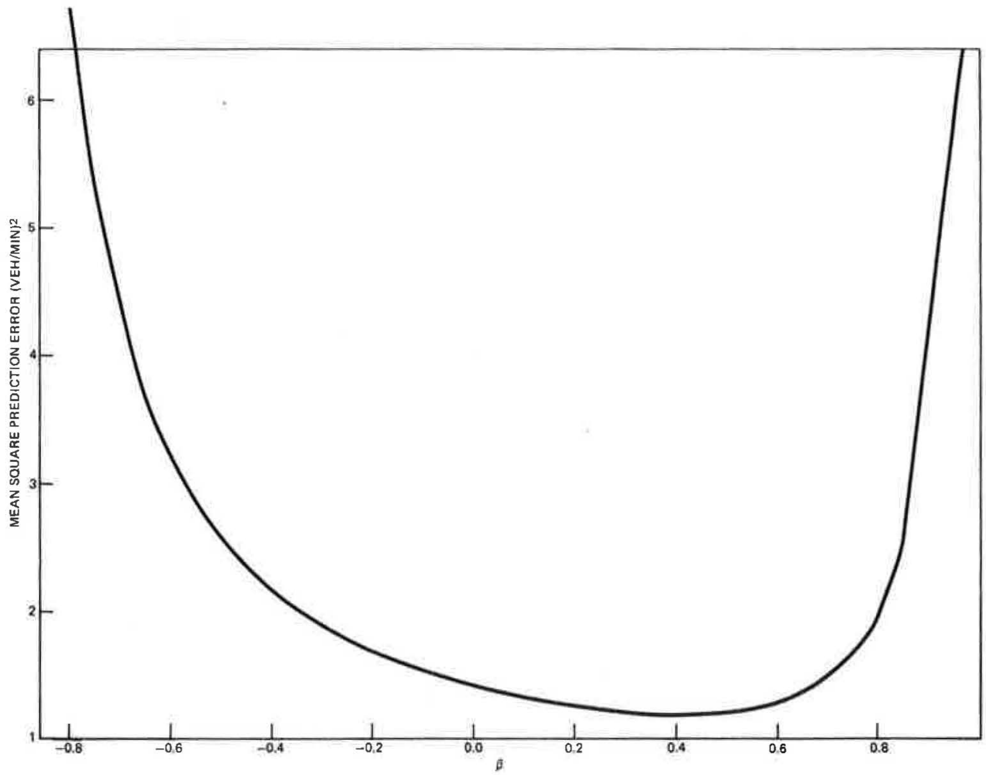


Figure 2. Mean-square prediction error as function of filtering constant β.



$$\hat{a}(t-1) = (1-\beta)\, a(t-1) + \beta\, \hat{a}(t-2)$$

$$(d/d\beta)\, \hat{a}(t-1) = -a(t-1) + \hat{a}(t-2) + \beta\, (d/d\beta)\, \hat{a}(t-2)$$

The optimal β is a root of $dE(t)/d\beta = 0$ from Equation 4. In practice there appears to be only one real root. [Since $dE(t)/d\beta$ is of odd degree in β, it must have at least one real root.] The complex roots complicate many root-finding methods; a bisection algorithm was used in this study. Equation 4 is recursive and therefore easy to program for electronic computers.

Figure 3 shows the optimal smoothing constants β for detector 1 during the first seven days of Toronto data. β is shown as a function of the period of aggregation. Intuitively, it seems that β ought to be large (nearly unity) for very small aggregation periods (because short periods of data are very noisy) and nearly zero for long periods; this seems approximately true for the Saturday and Sunday data. However, the weekday data definitely imply that the optimal β is negative for long aggregation periods. This reflects a violation of the assumption that the underlying signal varies only slowly from period to period; on weekdays the traffic volumes vary substantially (compared with the noise) when the aggregation period is 30 min or more.

The appearance of negative values for β does not prohibit the use of exponential smoothing even though the original assumption about the character of the process is violated for large aggregation periods; the actual prediction error (Figure 4) remains small even for large aggregation periods. The observed increase in prediction error with increased aggregation period is due to two separate effects--the model failure discussed above and the fact that there are fewer data periods when the periods are long. The increase in prediction error with the Saturday and Sunday data is almost wholly due to this latter effect.

A summary of all the available data is shown in Figure 5. Optimal smoothing constants were calculated for every detector for every day (except that the last seven days, which had a great deal of data missing, were deleted from the data set). The curve shown for each detector is the average of 50 daily curves (weekdays) or 20 daily curves (weekends). Also shown are two curves based on District of Columbia data that recorded each vehicle detection for 90 min (with two short breaks to remount the tapes). These arterial data appear to be of the same character as the Toronto weekday data.

The third set of curves in Figure 5 represents 10 detectors on a Los Angeles freeway. These 10 detectors are randomly selected from the four lanes in the northbound direction on the San Diego Freeway; the detector array spans 1 mile. These data contain individual vehicle actuations uninterrupted for 150 min.

All three sets of curves fit a family that can be written as follows:

$$\beta = (1-a)\exp(-bT) + a \qquad (5)$$

where a and b are constants that depend on the peaking characteristics of the traffic and T is the data-aggregation period in minutes. The values of a and b shown below give a good visual fit to the curves in Figure 5:

Curve	a	b (min^{-1})
Freeway	0.00	0.86
Arterial (Sat.-Sun.)	-0.05	0.13
Arterial (Mon.-Fri.)	-0.042	0.10

FILTERING CONSTANT β ALLOWED TO VARY DURING DAY

The foregoing discussion has assumed that a single, fixed value of the filtering constant must be used throughout the day. Intuition says that there may be substantial benefits in allowing β to vary. A

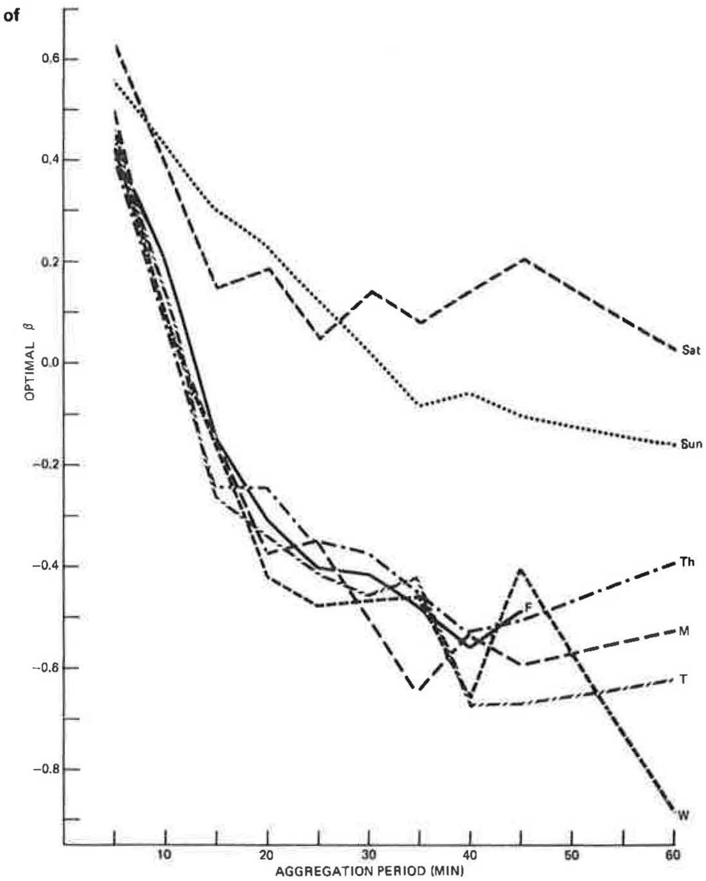


Figure 3. Optimal filtering constant β as function of aggregation period on seven consecutive days.

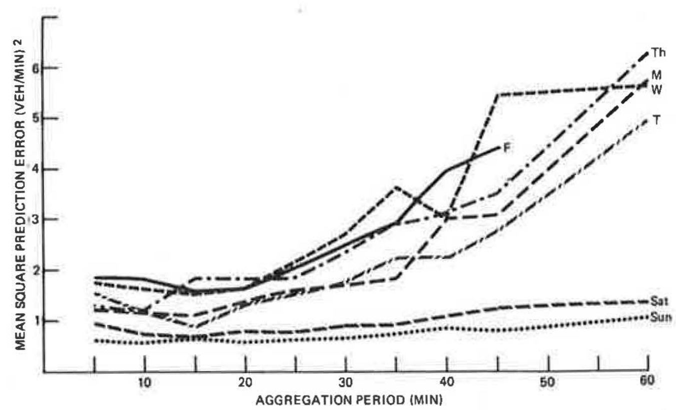


Figure 4. Mean-square prediction error as function of aggregation period by using same data as those in Figure 3.

properly chosen algorithm for the variation will, in effect, allow β to "tune" itself, obviating the need to guess in advance what the correct value of the smoothing constant will be. A variable β can conceivably even produce smaller prediction errors than using a single fixed value all day.

The obvious way to calculate such a variable smoothing constant is to recalculate the optimal β every period or two by using the most recent several data points. Figure 6 shows how some smoothing constants vary during the day. (Shortly after midnight, when fewer than the desired number of data points were available, the optimal calculation was limited to the available data only.)

Figure 7 shows how the mean-square prediction error depends on the number of data points used. As one would expect, the accuracy appears to be approaching a limit as the reoptimization includes more data. The limiting value of the mean-square error is about 2 percent worse than using the 24-h optimal smoothing constant throughout the day. Since the variable smoothing constant is based on the data available up to the time of the calculation only, it is not surprising that its limit is worse than that of the fixed 24-h optimal constant, which, in effect, is chosen with foreknowledge of the data yet to come.

One can evaluate the practicality of direct reoptimization by using the latest N points. Examination of Figure 6 indicates that the 5-min smoothing constant is likely to vary appreciably in 10 min, so that it will be necessary to recalculate β at least that often. How many of the latest data points must be used to produce acceptable accuracy

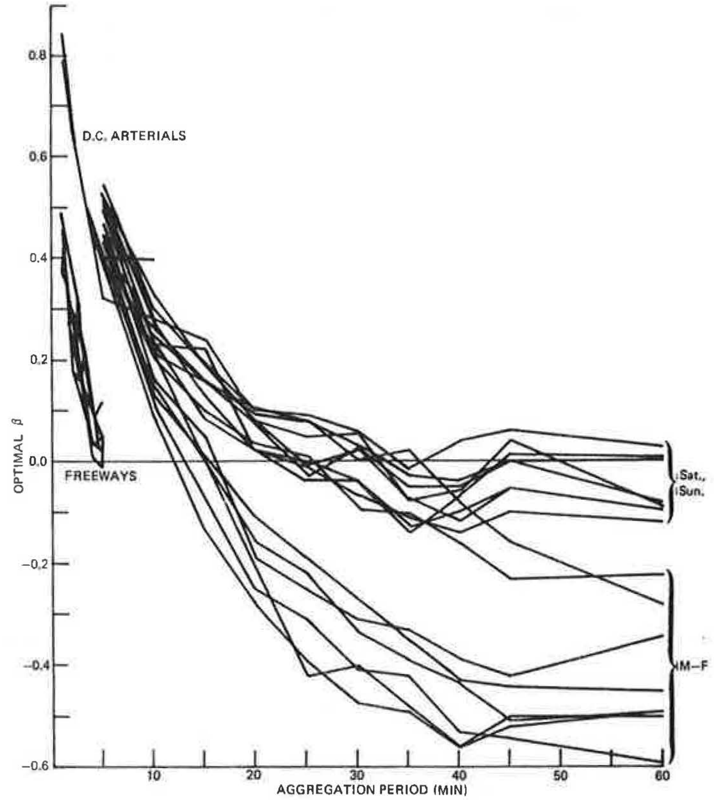


Figure 5. Average optimal filtering constants β for 8 arterial detectors in Toronto, 2 arterial detectors in Washington, and 10 freeway detectors in Los Angeles.

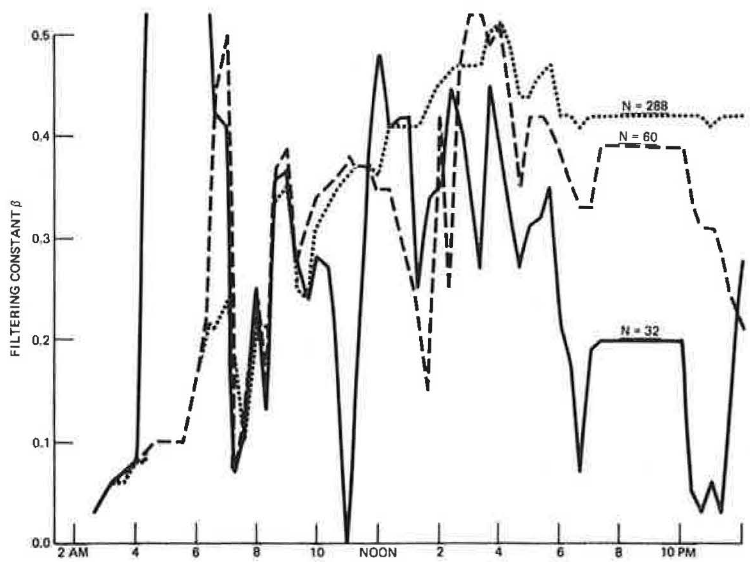


Figure 6. Variation in optimal β based on most recent 32, 60, and 288 observations as function of time of day.

is a somewhat subjective question; based on Figure 7, I would use at least N = 80 points. The use of 80 data points implies finding a root of a 159th-degree polynomial. If this must be done every 10 min for every detector in the system, it is clear that no exact reoptimization based on the latest N data points can be workable.

After abandoning exact reoptimization of β, one wonders if an approximate formulation can be found. Recursive algorithms are particularly attractive since they require only modest storage of data and preceding results and are generally simple in execution. Such an algorithm has been devised as follows.

At time t, the most recent prediction error is

$$a(t) - \bar{a}(t-1) = \beta_t[a(t-1) - \bar{a}(t-2)] - a(t-1) + a(t) \tag{6}$$

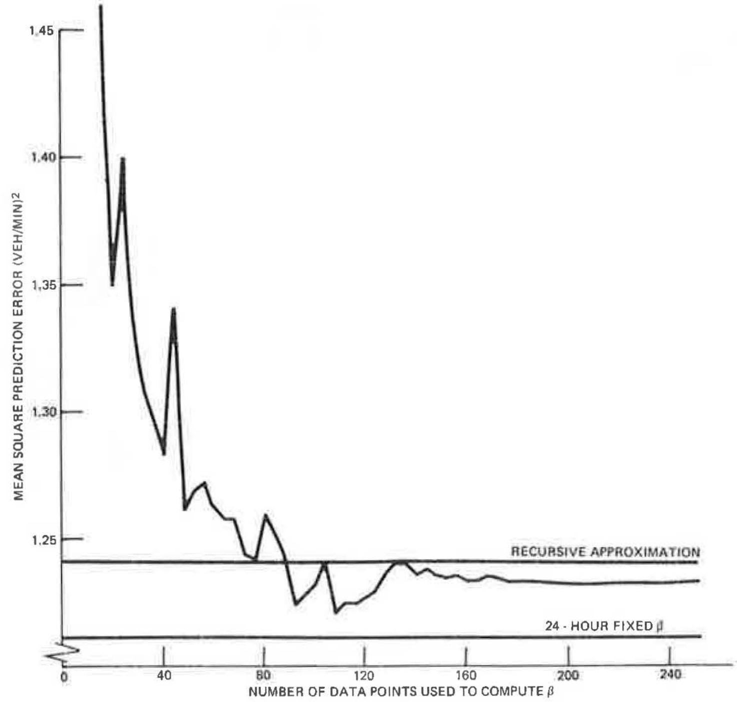
If the value of β_t had been as follows,

$$\beta_t^* = [a(t-1) - a(t)]/[a(t-1) - \bar{a}(t-2)] \tag{7}$$

the prediction would have been perfect, i.e., zero prediction error.

It is desired that the new filtering constant β_{t+1} be a linear combination of β_t^* and β_t. There seems to be no logically imperative choice for the weights associated with β_t^* and β_t; I have used, somewhat arbitrarily, the square prediction error in the (t − 1)st term and

Figure 7. Mean-square prediction error by using optimal filtering constant based on N floating data points.



the total preceding square prediction error as the weights for β_t* and β_t, respectively. This combination seems to work well; the mean-square prediction error is less than 0.5 percent worse than using all available data (i.e., up to 288 values) in a rigorous optimization when tested with the 5-min aggregation test case.

This algorithm reduces to three fairly elegant recursion equations:

$$Z_t = a(t-1) - \hat{a}(t-2)$$

$$E_t = E_{t-1} + Z_t^2$$

$$\beta_{t+1} = \left\{ E_{t-1}\beta_t + Z_t\,[a(t-1) - a(t)] \right\} / E_t \tag{8}$$

In the above equations, Z_t can be interpreted as the (t - 1)st prediction error and E_t is the cumulative square prediction error through time t.

SUMMARY

Exponential filtering of traffic volume counts is simple, quick, and reasonably accurate. A constant value for the filter constant may be used with good accuracy; if so, the appropriate value of the filter constant depends on the aggregation period and the daily traffic variation as shown in Equation 5 and the tabulation below it.

The necessity of predicting the best value for the smoothing constant and updating it as conditions change can be obviated by using on-line updating of the filtering constant. Updating by exact reoptimization--even over short histories--is not feasible, but Equation 8 presents a simple recursive method for calculating the filter constant that gives good results.

REFERENCES

1. M.G. Kendall. Time-Series. Charles Griffin and Co., London, 1973.
2. C.R. Nelson. Applied Time Series Analysis for Managerial Forecasting. Holden-Day, San Francisco, 1973.
3. G.E.P. Box and G.M. Jenkins. Time Series Analysis, Forecasting and Control. Holden-Day, San Francisco, 1977.
4. R.E. Kalman. A New Approach to Linear Filtering and Prediction Problems. Transactions of ASME: Journal of Basic Engineering, March 1960, pp. 35-45.
5. A.E. Bryson, Jr., and Y.-C. Ho. Applied Optimal Control. Blaisdell Publishing Co., Waltham, MA, 1969.
6. A.P. Sage and J.L. Melsa. Estimation Theory with Applications to Communications and Control. McGraw Hill, New York, 1971.
7. N.W. Polhemus. Statistical Analysis of Traffic Systems: A Time Series Approach. Department of Civil Engineering, Princeton Univ., Princeton, NJ, Ph.D. dissertation, 1976.
8. N.W. Polhemus. Time Series Analysis of Local Fluctuations in Traffic Parameters. Transportation Research, Vol. 10, 1976, pp. 311-317.
9. Data Sciences, Inc. Mathematical Models for Traffic Volume Prediction. TRW Systems, Houston, TX, Nov. 17, 1972.
10. Integrated Systems, Inc. A Traffic Volume Predictor for Networks. FHWA, April 20, 1972.
11. M. Levin and Y.-D. Tsao. On Forecasting Freeway Occupancies and Volumes. TRB, Transportation Research Record 773, 1980, pp. 47-49.
12. R.L. Lopez and P.K. Houpt. Estimation of Traffic Variables Using Point Processing Techniques. Research and Special Programs Administration, Office of Systems Engineering, U.S. Department of Transportation, Rept. DOT-TSC-RSPA-78-9, 1978.
13. J.S. Baras, W.S. Levine, A.J. Dorsey, and T.L. Lin. Advanced Filtering and Prediction Software for Urban Traffic Control Systems. Transportation Studies Center, Univ. of Maryland, College Park, 1977.
14. J.S. Baras and W.S. Levine. Estimation of Traffic Flow Parameters in Urban Traffic Networks. Proc., 1977 IEEE Annual Conference on Decision and Control, San Francisco, CA, pp. 428-433.

15. J.S. Baras, W.S. Levine, A.J. Dorsey, and T.L. Lin. Advanced Filtering and Prediction Software for Urban Traffic Control Systems. U.S. Department of Transportation, Feb. 1981.
16. L. Breiman. Predicting Input Flows. Systems Development Corp., Santa Monica, CA, Tech. Memorandum TM-4638/016/01, Sept. 1971.
17. M.J. Ganslaw. A 5-Minute Volume and Speed Predictor for the UTCS/BPS Second Generation Software. TRW Systems, Houston, TX, Sept. 1975.
18. Sperry Systems Management. Development of Traffic Logic for Optimizing Traffic Flow in an Intercity Corridor. FHWA, April 1, 1976.
19. W.R. McShane, E.B. Lieberman, and R. Goldblatt. Developing a Predictor for Highly Responsive System-Based Traffic Signal Control. TRB, Transportation Research Record 596, 1976, pp. 1-2.
20. E. Lieberman, W.R. McShane, R. Goldblatt, and D. Wicks. Variable Cycle Signal Timing Program: Volume 4--Prediction Algorithms, Software and Hardware Requirements, and Logical Flow Diagrams. KLD Associates, Inc., Huntington, NY, June 1974.
21. J.B. Kreer. Factors Affecting the Relative Performance of Traffic Responsive and Time-of-Day Traffic Signal Control. Transportation Research, Vol. 10, 1976, pp. 75-81.
22. J.B. Kreer. A Comparison of Predictor Algorithms for Computerized Traffic Control Systems. Traffic Engineering, Vol. 45, No. 4, April 1975, pp. 51-56.
23. P.K. Houpt, M. Athans, D.G. Orlahac, and W.J. Mitchell. Traffic Surveillance Data Processing in Urban Freeway Corridors Using Kalman Filter Techniques. Research and Special Projects Administration, U.S. Department of Transportation, Nov. 1978.
24. L.J. Pignataro, W.R. McShane, K.W. Crowley, B. Lee, and T.W. Casey. Traffic Control in Oversaturated Street Networks. NCHRP, Rept. 194, 1978.
25. W.R. McShane and K.W. Crowley. Regularity of Some Detector-Observed Arterial Traffic Volume Characteristics. TRB, Transportation Research Record 596, 1976, pp. 33-37.
26. D. Ghosh and C.H. Knapp. Estimation of Traffic Variables Using a Linear Model of Traffic Flow. Transportation Research, Vol. 12, No. 6, Dec. 1978, pp. 395-402.

Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.

Operational Effects of Two-Way Left-Turn Lanes on Two-Way Two-Lane Streets

PATRICK T. McCOY, JOHN L. BALLARD, AND YAHYA H. WIJAYA

The two-way left-turn lane (TWLTL) has been installed on two-way streets under a wide variety of conditions as a solution to the safety and operational problems caused by the conflict between midblock left turns and through traffic. Although the safety effectiveness of the TWLTL has been the subject of many studies, very few studies have been made of its operational effectiveness. Consequently, its effects on the efficiency of traffic flow have not been precisely measured. The objective of this study was to quantify the effects of a TWLTL on the efficiency of traffic flow on a two-way two-lane street. By using computer simulation models specifically developed and validated for the purpose of this study, traffic operations were simulated over a range of traffic volumes and driveway densities. From the outputs of these simulation runs, the reductions in stops and delays that result from a TWLTL were computed. Isograms of the stop and delay reductions were prepared to facilitate the use of the results of this study to evaluate the potential cost effectiveness of TWLTL installations.

The two-way left-turn lane (TWLTL) is recognized as a possible solution to the safety and operational problems on two-way streets that are caused by the conflict between midblock left turns and through traffic. The primary function of the TWLTL is to eliminate this conflict by removing the deceleration and storage of vehicles making these turns from the through lanes, thereby enabling through traffic to move past them without delay. However, the extent to which a TWLTL can improve the efficiency of traffic operations depends on the traffic volumes and density of driveways involved. Although the principle of the complex relationship between these factors and the operational effectiveness of the TWLTL is intuitively apparent, it has yet to be quantitatively expressed. Consequently, traffic engineers have not been able to precisely predict the amount of improvement in the efficiency of traffic operations that would result from the installation of a TWLTL.

An extensive review of the literature and nationwide survey of experience with the TWLTL were conducted by Nemeth (1) in developing guidelines for its application. This effort revealed that the TWLTL has been installed under a wide variety of conditions. In most cases, it was considered to have noticeably improved the quality of traffic flow. Numerous before-and-after accident evaluations were found that provided measures of the safety effectiveness of the TWLTL. But similar studies of its effect on the efficiency of the traffic were rare, and measures of the operational effectiveness of the TWLTL were not found.

Likewise, in developing guidelines for the control of access on arterial streets, Glennon and others (2) found that empirical data pertinent to the determination of the operational effectiveness of the TWLTL were lacking. This deficiency precluded the precise estimate of the delay-reduction potential of the TWLTL. And this in turn limited the specificity with which the conditions that warrant installation of a TWLTL could be defined.

In response to the need of traffic engineers to be able to more precisely predict the operational effectiveness of a TWLTL and more clearly define those circumstances that justify its installation, a