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## Design of a Skew, Reinforced Concrete Box-Girder Bridge Model

RAYMOND E. DAVIS

A 1:2.82 scale model of a two-span, continuous, reinforced concrete box-girder bridge, which has supports skewed at 45°, was constructed and tested at the University of California, Berkeley. The cross section and significant dimensions were similar to those of two previously tested models, one straight on orthogonal supports and one curved on radial supports. The objective of the research was to compare behavior of the three models and to verify an analytically predicted diminution of longitudinal stringer moments that result from skewing supports. All three models were designed by the California Department of Transportation. Because traditional design criteria make no provision for skewed supports, the skew model was designed by means of a sophisticated finite-element computer code called CELL. Girder moments proved to be significantly less than those in the orthogonally supported model and had a 19 percent reduction in the main longitudinal reinforcing steel. Distribution of girder shears was changed significantly from that of the model on normal bearings. As a basis for implementation, this paper discusses some features of the skew model design process.

For many years, the California Department of Transportation (Caltrans) has been interested in anomalies that characterize the structural behavior of reinforced concrete box-girder bridges with skewed supports. Initially, interest was centered on effects of skew on girder shears. Excessive cracking of webs observed at obtuse corners suggested enhancement of girder reactions that had commensurate increases in diagonal tension.

Complexities in the analysis of skew boxes restricted early efforts toward mitigation of observed excessive web cracking to establishment of curves for augmentation of exterior and first interior girder shears at obtuse corners of such boxes. (Traditionally, skewed boxes in California have been designed as structures of the same spans on orthogonal supports and detailed with skewed supports.) Curves for shear augmentation were established with little scientific basis and furnished, at best, only estimates.

A request in 1959 by design management for a more definitive study of this problem initiated a protracted study of reinforced concrete cellular structures performed jointly by Caltrans' Structural Research Unit and the University of California, Berkeley. The research effort included tests of full-scale prototypes and small and large-scale models. Structures of increasing complexity were studied on a progressive basis, as follows: (a) simple span boxes without diaphragms on normal supports; (b) simple span boxes with rigid intermediate diaphragms, or continuous boxes without intermediate diaphragms on normal supports; (c) continuous boxes with intermediate diaphragms, which consider effects of bent and diaphragm flexibility; (d) curved boxes with radial supports; (e) nonprismatic boxes; (f) skewed boxes; (g) prestressed boxes; and (h) composite concrete and steel boxes. Analytic methods employed in the development of computer codes by the University of California relied heavily on the folded-plate theory and finite-strip, finite-segment, and finite-element methods.

A valuable computer code developed as part of the research effort employs a finite-element analysis to assess behavior of cellular structures of arbitrary plan geometry. This program, called CELL, was first used within Caltrans to analyze a heavily skewed, and curved, box-girder bridge to carry rail traffic and to assess the influence of intermediate diaphragms on that behavior. The program has been used in studies of boxes of varying skews and aspect ratios to establish functional relations between skew angle and shear augmentation factors. Estimated curves of such factors previously used by Caltrans were proved to be unconservative.

A serendipitous result of these studies was the

Figure 2. Cross section of structure for sample analysis.

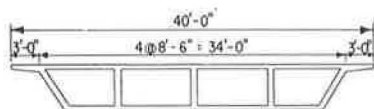


Figure 3. Sample nomographic analysis.

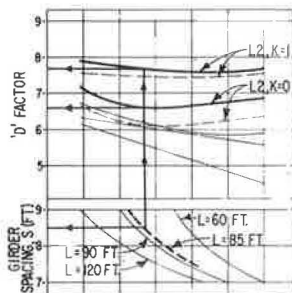
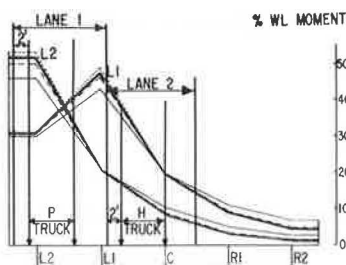


Figure 4. Sample influence-line analysis.



7.25 ft, and slope factor  $(K) = (2.81/4.69) = 0.60$ .

By using the (typical) nomograph shown in Figure 3, and noting carefully the correction factors indicated by asterisks, a designer can obtain D-factors, where values of  $S/D$  (or, for exterior girders,  $W_e/D$ ) represent distribution factors.

The typical influence-line analysis is illustrated (in part) in Figure 4. Interpolations are required for span and slope factors and eight separate diagrams must be considered (for three girders each). Resulting moment percentages are obtained for P-series and H-series trucks separately.

Influence-line analysis (which, incidentally, is a misnomer) may be easily applied without a computer, and it is compatible with arbitrary loading conditions. It is probably the only simple approach to distribution of such loads. The method becomes cumbersome in a production environment, so the computer code has been written in FORTRAN IV language. Caltrans' program (LANELL) is immediately accessible in the time-share option system via a cathode-ray tube (CRT) remote terminal. The designer enters seven parameters from the keyboard and obtains as readout suitably interpolated values for the number of wheel lines of P-series and H-series trucks to be distributed to each girder and to the whole structure. Factors have also been established for curvature correction.

For the rare design that cannot be treated by the first three methods, the Berkeley programs may be used, e.g., for spine beams, MUPDI3 and SIMPLA2; for heavily skewed structures or boxes with arbitrary plan geometry, CELL; for short radius structures, CURDI; and for composite steel boxes, FINPLA.

#### ACKNOWLEDGMENT

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Opinions, findings, and conclusions expressed in this paper are ours and do not necessarily reflect official views or policies of Caltrans or FHWA. This report does not constitute a standard, specification, or regulation.

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demonstration that significant diminutions of longitudinal bending moments might be realized in skewed structures. A potential for significant economies was manifested by curves drawn for simple span structures, which suggested the possibility of reducing dead load resisting moments by nearly one-half in structures skewed  $45^\circ$  and by nearly 70 percent for skews of  $60^\circ$ .

The reduction of resisting moments is usually explained as the result of a tendency to span the normal distance between supports. This explanation is overly simplistic. Diminution of longitudinal resisting moments in the girders is realized at the expense of increasing torsional moments that act on the structure. As orthogonal supports are changed to skewed, formerly symmetrical reactions become asymmetrical as those at obtuse corners are increased and those at acute corners are decreased in magnitude. Resultants of support reactions move away from the centroidal axis of the structure, and torsional forces are introduced. Closed cellular sections possess high torsional rigidity, and increases in torsional moments are less significant than diminutions of longitudinal moments. It was desirable to evaluate qualitatively the influence of torsional forces in skewed structures.

Tests conducted by the University of California of small-scale aluminum models that had varying skews and aspect ratios verified the accuracy of CELL. However, Caltrans' Structures Design management was understandably reluctant to adopt the indicated large reductions in reinforcement quantities because they lacked verification with a larger-scale reinforced concrete model, the behavior of which might also be directly compared with that of the (orthogonal) straight and curved boxes tested previously.

#### SCOPE OF PAPER

Responsibility for construction, testing, and analysis of behavior of the skew model was assumed by the University of California, Berkeley. Responsibility for design of the model and implementation of results was assumed by Caltrans' Structural Research Unit. This paper describes the techniques used in the model design. [This paper is a condensation of the work by Davis (1), in which the design of the model has been described in detail.]

#### MODEL DESCRIPTION

The model comprised reinforced concrete elements that had a linear scale reduction of 1:2.82. A No. 4 reinforcing bar in the model, which was built to this scale exactly, simulates main No. 11 reinforcing bars in the prototype. A 6-mm (0.236-in) bar in the model (smallest available with deformations) approximately simulates a No. 5 or a No. 6 bar in the prototype with minor variations in spacing.

Constructed on the test floor of Raymond E. Davis Hall at the University of California, the model was 3.66 m (12 ft) wide (from edge to edge of deck), 25.6 m (84 ft) long (measured between acute corners), and approximately simulated a 10.4-m (34-ft) wide continuous bridge with two 31-m (101.5-ft) spans. The structure was identical in cross section to the curved and straight models tested previously. Transverse reinforcement in the top and bottom slabs was the same as in previous models in order to maintain similitude.

The cross section and transverse reinforcement patterns were originally established in the design of the Harrison Street Undercrossing, a full-sized prototype tested by the California Division of Highways in 1960 (2,3) and a 1:3.78 scale model tested

contemporaneously by the University of California (4,5) in the initial phases of the box-girder research program.

#### MODEL DESIGN

The methods used to design the skew model appear tedious but were characterized by much more thoroughness than would be expected for a full-sized structure. Caltrans' Structural Research Unit employs its own modified version of CELL, which permits storage of the decomposed stiffness matrix for future use in analyses of various loading conditions. An optional mesh-plotting routine is included, since errors in the geometry of large meshes are easily made.

The mesh employed is depicted in plan to a small scale in Figure 1 and, in part, to a larger scale in Figure 2. The same geometrical mesh is used for the top and bottom slabs (this mesh-generating scheme in the current version of CELL mandates vertical webs); however, material properties of elements in the two slabs may differ. A current research project will remove some of the deficiencies in CELL, thereby permitting sloping webs and adding a prestressing facility and automated girder moment integration. This last feature will eliminate much of the effort expended in the design of this model.

The mesh was made rectangular to satisfy a requirement of the CELL postprocessor (CELLPOP) (6) that all cross sections have the same number of girders (e.g., if girder moments within longitudinal limits of end supports are desired). All elements beyond supports are made null elements (i.e., with zero thickness) in the materials properties section of the input data. A second study made with a skewed mesh without null end elements yielded similar results.

Careful choice of numerical designations of nodes and elements allows maximum use of program mesh-generation features. Although punched-card input is conceivable, the repetitive nature of the data greatly decreases key data entry if this work is done on a cathode-ray tube (CRT) terminal with standard utility routines that allow rapid proliferation of data blocks (e.g., in Caltrans' IBM System, the INCLUDE routine). In all, 688 slab elements were described on 50 card images, and 362 vertical elements, including all transverse diaphragm elements followed by longitudinal web elements, were described on 22 card images. Materials properties for upper and lower slabs were specified separately.

Figure 1. Finite-element mesh for design of model with CELL program.

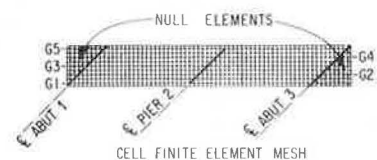
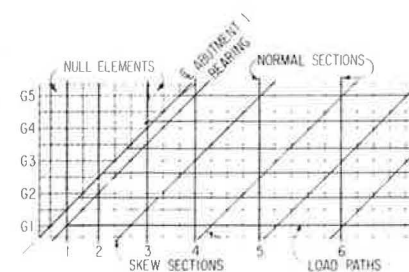


Figure 2. Partial finite-element mesh showing skew reference lines for vehicle location, orthogonal reference lines for moment, and shear calculations and wheel paths.



Specifications of nodal coordinates required separate card images for each of 737 nodes; however, because of the rectangular nature of the mesh, ordinate values are repetitive in blocks of 11, and the proliferating routine was used to advantage. A single block with 11 different ordinates was established. The block successively proliferated to 10, 100, and 800, followed by deletion of 63 card images. Numerical nodal designations (different for each card image) and the abscissae (all the same for each card image in an 11-card group) may be readily entered.

Materials properties were described on eight card images and included separate elastic moduli in the x- and y-directions, shear moduli, mean values of Poisson's ratio in two directions, and element thicknesses. One null element with zero thickness was included to represent nonexistent elements in the bottom slab that correspond geometrically to cantilever upper slab elements and nonexistent elements outside abutment bearings.

Eleven boundary-condition cards specified zero displacements in the z-direction at supports, in the x- and y-directions (also) at the center pier to prohibit rigid body translation, and in the y-direction (also) at the two central abutment supports (to prevent rotation of the whole structure about the z-axis). An additional entry on all of these cards located reactions at bottom slab nodes.

A designer who has developed skill in using a CRT and utility routines can establish data-input files without key data personnel. A partial listing of input for geometric and physical properties is shown in Figure 3.

#### CELL INPUT FOR MODEL DESIGN: LOADS

##### General

Current American Association of State Highway and Transportation Officials (AASHTO) specifications for bridge design call for 3.66-m (12-ft) traffic lanes and no fractional lanes. Tread of the AASHTO design vehicle is 1.83 m (6 ft), and minimum distance from a wheel line to edge of a lane is 0.6 m (2 ft). Six vehicle paths were chosen as the most probable critical paths for the five girders. Four paths would suffice but for asymmetry produced by having intermediate diaphragm elements in only one span. Six vehicle paths produce 12 separate wheel paths; because those in proximity to one another are separated by only 0.3 m (1 ft), six compromised wheel paths shown in Figures 2 and 4 were established by moving truck paths a maximum distance of 0.15 m (0.5 ft) transversely. Thirteen lines paralleling the skew were chosen to establish live load positions at intersections with parallel wheel paths, and 19 orthogonal cross sections were established for moment calculations (see Figure 2).

Exterior load paths are coincident with exterior girder webs, and intersections with skew lines fall on nodal points. Intersections of skew lines and inner wheel paths lie within elements, and concentrated loads may be distributed to element nodes by the tributary-area method.

Typical input for a single live load condition is shown in Figure 5. The requirement of a list of 219 nodes at which stresses are to be calculated for 78 live, and 1 dead, load cases studied again suggests tedium in the input; however, the repetitive nature of input again permits rapid proliferation of a single file, which is followed by a separate entry of nodal points and magnitudes of applied loads.

##### Determination of Longitudinal Girder Moments

Finite-element output from CELL was translated into

Figure 3. Partial list of input to CELL program.

DESIGN OF SKW BOX GIRDER MODEL USING PROGRAM CELL										Start Card
Static Load										1
Input Control Card										2
688	737	162	8	11	139	5	339	0		3
1	2	3	4	5	6	7	8	9	10	4
2	3	14	15	4	6					5
4	4	15	16	5	6	6	11			6
7	5	16	17	6	6	6	11			7
11	6	17	18	7	6	6	11			8
16	7	18	19	8	6	6	11			9
22	8	19	20	9	6	6	11			10
29	9	20	21	10	6	6	11			11
37	10	21	22	11	6	6	11			12
46	1	13	2	1	6	6	11			13
56	1	12	13	1	5	6	12			14
57	13	24	25	13	3					15
65	109	120	121	109	5	4	12			16
66	12	23	24	13	5	6				17
93	24	35	36	25	3	4	11			18
120	36	47	48	37	3	4	11			19
147	48	59	60	49	3	4	11			20
174	60	71	72	61	3	4	11			21
201	72	83	84	73	3	4	11			22
228	84	95	96	85	3	4	11			23
255	96	107	108	97	3	4	11			24
282	108	119	120	109	3	4	11			25
309	120	131	132	121	5	6	21			26
336	399	320	321	310	5	6	11			27
337	321	333	332	321	5	6				28
345	417	423	429	418	5	6	12			29
354	321	332	333	321	5	6	23			30
356	320	331	332	321	5	6				31
381	332	343	344	333	3	4	11			32
408	354	355	356	355	3	4	11			33
435	356	367	368	357	3	4	11			34
462	368	379	380	369	3	4	11			35
489	390	391	392	381	3	4	11			36
516	392	403	404	393	3	4	11			37
543	404	415	416	405	3	4	11			38
570	416	427	428	417	3	4	11			39
597	429	439	440	429	5	6	11			40
624	451	462	463	442	5	6	11			41
652	469	481	480	469	3	4				42
633	725	737	725	725	5	6	12			43
635	617	628	629	617	6	6				44
644	628	637	636	629	6	4	12			45
653	649	651	652	641	6	6	11			46
661	652	663	664	653	6	6	11			47
668	664	675	676	665	6	6	11			48
674	676	687	688	677	6	6				49
679	685	697	700	629	6	6				50
683	700	711	712	701	6	6	11			51
685	711	723	724	719	6	6	11			52
685	725	735	736	725	6	6	11			53
1	13	25	1							54
8	97	109	1	12						55
9	178	179	7							56
10	177	180	7							57
11	202	203	7							58
12	203	204	7							59
13	224	227	7							60
14	227	223	7							61
15	250	251	7							62
16	251	252	7							63
17	321	333	2	12						64
24	405	417	2	12						65
25	629	611	1	12						66
32	13	75	1							67
33	2	13	6							68
35	13	25	7							69
89	618	629	7	11						70
90	629	640	6							71
98	717	728	6	11						72
99	4	15	6							73
101	26	37	7	11						74
102	37	68	7							75
157	662	653	7	11						76
158	653	664	6							77
164	719	730	6	11						78
165	6	17	5							79
169	59	61	6	11						80
170	61	72	7							81
205	664	677	7	11						82
226	677	680	6							83
230	721	722	6	11						84
231	8	19	6							85
237	74	65	6	11						86
223	85	96	7							87
253	692	761	7	11						88
296	721	712	6							89
725	723	736	6	11						90
297	10	21	6							91
305	53	109	6	11						92
305	109	123	7							93
361	716	725	7	11						94
722	725	736	6							95
2	0	0	0							96
3	0	0	2.5							97
4	0	0	5.125							98
5	0	0	9.75							99
6	0	0	13.375							100
7	0	0	17							101
8	0	0	20.625							102
9	0	0	24.25							103
10	0	0	27.875							104
11	0	0	31.5							105
12	2.5	2.5	35							106
13	5	5	38.5							107
14	7.5	7.5	41.25							108
15	10	10	44							109
16	12.5	12.5	46.875							110
17	15	15	49.5							111
18	17.5	17.5	52.125							112
19	20	20	54.75							113
724	231.5		27.875							819
725	231.5		31.5							820
726	231.5		34							821
727	236		0							822
728	236		2.5							823
729	236		6.125							824
730	236		9.75							825
731	236		13.375							826
732	236		17							827
733	236		20.625							828
734	236		24.25							829
735	236		27.875							830
736	236		31.5							831
737	432000	432000	183950	0.18	0	2.5				833
2	432000	432000	183950	0.18	0	5.5				834
3	432000	432000	183950	0.18	0	0.531				835
4	432000	432000	183950	0.18	0	0.458				836
5	432000	432000	183950	0.18	0	0.72				837
6	432000	432000	183950	0.18	0	0				838
7	432000	432000	183950	0.18	0	0.667				839
8	432000	432000	183950	0.18	0	0.467				840
13	T	T	T							841
37	T	T	T							842
61	T	T	T							843
65	T	T	T							844
109	T	T	T							845
369	T	T	T							846
629	T	T	T							847
553	T	T	T							848
677	T	T	T							849
701	T	T	T							850
725	T	T	T							851
Displacement Boundary										852
Condition Array										853
										854
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										898
										899
										900



LIVE LOAD CRITICAL PATHS FOR:

G1, G2

G3

G4, G5

2.5' 6' 6' 5' 6' 6' 2.5'

\*COMPROMISED

G1 G2 G3 G4 G5

[illegible]

Reactions at supports may be taken from the generalized force vector in the CELL output for dead

Certain positions of the live load produced small negative moments in girders over abutment supports near obtuse corners of the structure due to the fact

Figure 6. Dead load shears from CELL program.

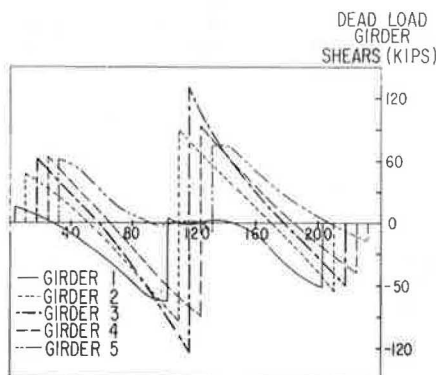
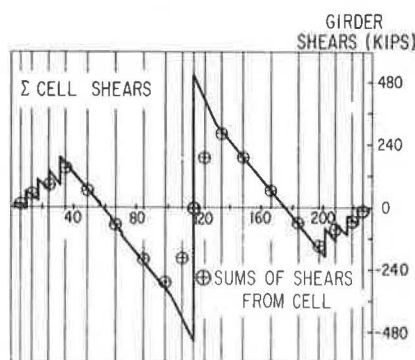


Figure 7. Comparison of CELL and calculated shears.



that end diaphragms provide components of longitudinal extension to these girders.

#### Total Moments

Digitized values of live load moments were tabulated at each cross section, modified by an impact factor, and added to dead load moments to create envelopes of total moments. The design for resisting moment subsequently differs little from standard design procedures.

#### Determination of Girder Shears

##### Dead Load Shears

Girder shears are obtainable directly from the program via a second postprocessing program called CELLSHR, which chooses proper values of  $N_{xy}$  at the tops and bottoms of girder webs from CELL output matrices and multiplies the averages of these two values by web depths. Signs must be reversed to agree with beam convention.

The dead load shears plotted in Figure 6 are very different from those that might be expected for a structure with orthogonal bearings. The shear at girder G1, which is at the acute corner at abutment 1, exhibits a very low value. Shears for center, first interior, and exterior girders at the obtuse corner exhibit nearly the same magnitudes. Shears for girders that frame into the bent on the acute angle side are very small. Maximum shears at the bent occur in the center girder.

Statics checks were made for dead load shears determined theoretically and by the program (see Figure 7). Agreement was not as good as in the case of bending moment. Calculated total shears from CELL fall very close to the curve of theoretical

total shear out in the span. At the pier, deviations of CELL shears are evident and result from the fact that orthogonal sections intersect the bent cap where there is no convenient way to determine shears. Lesser deviations are evident at the abutments, but shear predictions break down at supported nodes.

Statics checks were greatly improved when cross sections were taken parallel to supports. The total load of the superstructure in span 1 was calculated with precision, and the total reaction at abutment 1 was deducted to provide a total bent shear of 200 t (440 kips); the sum of CELL reactions at the bent was 180 t (397 kips).

##### Live Load Shears

Calculation of live load shears required an influence-line approach, since the CELL program was run for 78 separate locations of the unit load on the deck. The plotting of influence lines was automated, again with curves for two adjacent wheel paths on each plot, and the maximum shear values were obtained by trial-and-error digitization with the transparent overlay. Envelopes of live load shears for each girder were added to dead load shears, and stirrup spacings were determined in the usual manner.

#### Bent Cap Design

Considerable effort was expended in writing additional postprocessing programs to determine bending moments in the bent cap. CELL does not directly output membrane stresses in the direction of skew, and such data must be formulated from deck and bottom slab stresses by rotations of axes. The bent cap designed on the basis of these stresses proved to be very lightly reinforced because forces in outlying slabs were not taken into account; indeed, the effective width of deck slabs is moot. CELL does not output bent cap shears. For these reasons, the bent cap was finally designed for bending moment and shear in the usual manner by using bent cap reactions obtained from the program.

#### Girder Deflections and Camber

Vertical girder displacements under dead load are given directly in the program output. Engineers who have been involved in the construction of heavily skewed structures are aware of the difficulties involved in establishing camber diagrams that resemble actual deflection patterns in finished structures because of the propensity for warping. Direct output of reasonable configurations of dead load displacements is a great aid in the design of skewed structures.

#### Postprocessor Program Modifications

After completion of the skew model design, the two postprocessing programs--CELLPOP (for calculation of individual girder moments) and CELLSHR (for calculation of girder shears)--were combined into one program, called CELL moments and shears (CELLMOSH), which had automatic tabulated output and plot tapes for automated plotting of moment and shear influence lines.

#### CONCLUSIONS

Volume of longitudinal No. 4 reinforcement in the skew model proved to be about 81 percent of that in the model on orthogonal supports. Previous calculations suggest even greater savings for simple spans,

and it is believed that they would also be greater for wider continuous structures of equal length. Savings are realized at the expense of significant increases in design effort. The alternative to expenditure of this effort for heavily skewed boxes may be very unrepresentative designs. Automation of program input, output interpretation, and development of expertise on the part of designers would be essential to realization of appreciable savings.

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The opinions, findings, and conclusions expressed in this paper are mine and do not necessarily reflect the official views or policies of Caltrans or FHWA. This paper does not constitute a standard, specification, or regulation.

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## Response of 45° Skew, Reinforced Concrete Box-Girder Bridge Model to AASHTO Trucks and Overload Construction Vehicles

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A detailed study of the structural response of a 45° skew, two-span, four-cell, reinforced concrete box-girder bridge model under different types of vehicle loading is presented. The model, which was a 1:2.82 scale replica of a typical California highway prototype bridge, was 72 ft (21 m) long by 12 ft (3.7 m) wide and was supported by 45° skew end abutments and a 45° skew center bent supported by a single column. The vehicle loadings used consisted of scale models of standard American Association of State Highway and Transportation Officials HS 20-44 trucks and overload construction vehicles (class 2). In addition, influence lines for reactions and deflections were obtained by positioning a forklift truck at selected points on the bridge deck. The experimental response of the bridge model in the form of reactions, deflections, moments, and steel and concrete strains is compared with the theoretical response values obtained from a finite-element computer program CELL. The influence of skewness on the major design quantities is also assessed.

Multicell reinforced concrete box-girder bridges are widely used in the California highway system. The growing number of complex intersections, the lack of space in crowded urban areas, and the demand for road layouts without abrupt changes in direction frequently necessitate the use of bridges with skew, curved, or arbitrary plan geometry. Most design calculations for live load distribution in straight, skew, and curved box-girder bridges are based on the same empirical formula in which the effects of skewness or curvature are generally ignored.

The 1977 American Association of State Highway and Transportation Officials (AASHTO) specifications (1) specify a design method wherein a box-girder bridge is divided up into a number of interior

girders plus two exterior girders. Each of these girders is designed as a separate member by applying to it a certain fraction of a single longitudinal line of wheel loads from a standard AASHTO HS 20-44 truck. The fraction is  $N_{WL} = S/7$ , in which  $S$  is the web spacing.

California uses a design procedure in which the whole bridge width is considered as a single unit and the distribution factor for the whole width unit is given by  $N_{WL} \text{ (total)} = \text{deck width in feet}/7$ . The total moment at any section is assumed to be uniformly distributed across the width of the bridge.

In current practice for a skew bridge, design live load moments are determined for either of the above empirical wheel loadings by analyzing a straight bridge that has the same span but without any skew. Empirical rules, approximations, and engineering judgment are then used to account for skewness in determining longitudinal reinforcement cutoff points and some increase in web reinforcement for shear in the obtuse corners of the bridge.

In fact, the presence of skew generally reduces the total midspan moments in box-girder bridges because of the distribution of the reactions along the end abutments. The reduced moment for a simple span, 45° skew, four-cell box-girder bridge is shown in Figure 1 for a uniformly distributed surface load calculated with the finite-element computer program CELL (2) and compared with the generally used solu-