

Fatigue Reliability Analysis of Highway Bridges

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A method of calculating the expected fatigue failure probability of a structural detail, given the distribution of resistance and load, is presented. The resistance data, in terms of cycles to failure, come from previous laboratory tests. The load data come either from stress-range histograms recorded on bridges or from loadometer surveys. The proposed method replaces each histogram by an equivalent stress range and converts the latter into a distribution in terms of number of cycles. The problem is thus cast into the standard format for reliability analysis and allows one to calculate failure probabilities. Application of the method to designs in accordance with American Association of State Highway and Transportation Officials specifications showed that fatigue failure probabilities for redundant load-path (RLP) structures are inconsistent and vary greatly from $P_F = 9.2 \times 10^{-2}$ for category B to $P_F = 9.2 \times 10^{-10}$ for category E' and for nonredundant load-path (NRLP) structures from $P_F = 5.1 \times 10^{-2}$ for category A to $P_F = 2.1 \times 10^{-22}$ for category E. It is proposed that the specifications be revised to include (a) allowable stress ranges for RLP and NRLP structures with uniform failure probabilities; (b) explicit formulation of the specifications in terms of the actual number of single fatigue trucks, each causing an equivalent stress range; and (c) continuous definition of allowable stress range versus truck traffic volume. An example illustrates the design of a bridge not covered by the specifications to a specified failure probability.

Design methods based on statistical reliability concepts have recently been developed for many areas of static design of members and connections. Code-writing bodies are now incorporating them into their specifications to ensure consistent reliability throughout the structure. Still lacking is a reliability method for fatigue that can then be used as a basis for a load and resistance factor approach to fatigue design. The present study addresses this need.

This paper briefly reviews the equivalent stress range and the reliability concepts needed herein. Thereafter, the load and resistance curves are constructed and transformed in a manner suitable for writing the governing equations. The failure probability of designs to American Association of State Highway and Transportation Officials (AASHTO) fatigue specifications are assessed, and an illustrative design example is presented for a special bridge not covered by the AASHTO specifications (1). The previous work (2) is extended to cover designs for over 2 million cycles and nonredundant load-path (NRLP) structures.

EQUIVALENT STRESS RANGE

Recent studies have employed, with good success, the concept of an equivalent stress range to correlate data from variable amplitude cyclic-load tests with data from constant-amplitude tests. The concept states that, for an equal number of cycles, the equivalent (constant-amplitude) stress range will cause the same fatigue damage as the sequence of variable-amplitude stress ranges it replaces. For convenience of applying the concept later in the paper, the equivalent stress range (f_{re}) is expressed in the following form (2):

$$f_{re} = [\sum \gamma_i (\phi_i \alpha f_{rd})^m]^{1/m} \quad (1)$$

where

- γ_i = frequency of occurrence of *i*th stress range;
- ϕ_i = ratio of an individual load to the design load, or ratio of corresponding stress ranges;
- α = ratio of measured to computed stress range for the design load;

- f_{rd} = computed stress range that corresponds to the design load; and
- m = slope of S-N curve

Figure 1 illustrates the meaning of the parameters in Equation 1 for a typical stress-range histogram (note for Figure 1 that if detail is designed to the allowable stress range, then $f_{rd} = F_{sr}$).

Because f_{rd} and α are constant for a given stress-range histogram, taking them out of the summation in Equation 1 gives

$$f_{re} = (\sum \gamma_i \phi_i^m)^{1/m} \alpha f_{rd} \quad (2)$$

or

$$f_{re} = \rho \alpha f_{rd} \quad (3)$$

where ρ is defined in this paper as

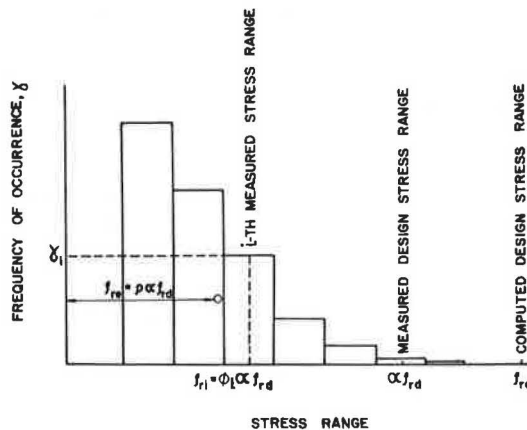
$$\rho = (\sum \gamma_i \phi_i^m)^{1/m} \quad (4)$$

The equivalent stress-range concept is needed in calculations of the fatigue failure probability of structures subjected to variable amplitude stress cycling.

BASIC RELIABILITY CONCEPTS

The reliability concepts employed in this paper are well documented in the literature (3). Structural reliability can be defined as the probability that a structural component will not fail within its design life. In other words, it is the probability that a member's resistance to load is higher than the applied load. In deterministic design, one assumes a high value of load and a low value of resistance and specifies that the distance between the two shall not be less than a preselected safety factor. In probabilistic design, one recognizes that neither the resistance (R) nor the load (Q) are single valued; both have a mean and a distribution. The objective is then to compute the probability of failure, i.e., the likelihood of the undesirable cases where a high value of load will exceed a low value of resistance. Conversely, for purposes of

Figure 1. Illustration of terms in Equations 1 and 3.



writing fatigue design specifications, one wishes to determine the distance between the mean resistance and the mean load ($\tau = \bar{R} - \bar{Q}$), so that the failure probability (P_f) does not exceed a specified value. This is done with the equation for the safety index (β):

$$\beta = (\tau/s_r) = (\bar{R} - \bar{Q})/\sqrt{s_R^2 + s_Q^2} \quad (5)$$

where the difference (τ) between mean load and mean resistance is equal to a number (β) of standard deviations of that difference (s_r). If load and resistance are normally distributed, so is their difference, and the value of β that corresponds to a specified failure probability can be read from tables for the standard normal variable.

Note that increasing the safety index (β) will decrease the failure probability. This can be achieved either by moving the mean load (\bar{Q}) farther away from the mean resistance (\bar{R}) or by reducing the standard deviation. The second option is usually not available in most designs.

When applying Equation 5 to fatigue design, the resistance is given by the number of cycles to failure and the load is given by the applied stress-range history. This leads to two difficulties. One is the need to find a mean and standard deviation of many stress-range histograms, each of which describes in itself a distribution of stress ranges. Second, the resistance data, which consist of the number of cycles to failure, are distributed along a horizontal line in a S-N plot, but the stress-range data are distributed along a vertical line. The following solution to the two difficulties is proposed:

1. Replace each histogram by one equivalent constant-amplitude stress range,
2. Calculate the mean and standard deviation of

Figure 2. Construction of equivalent stress-range distribution.

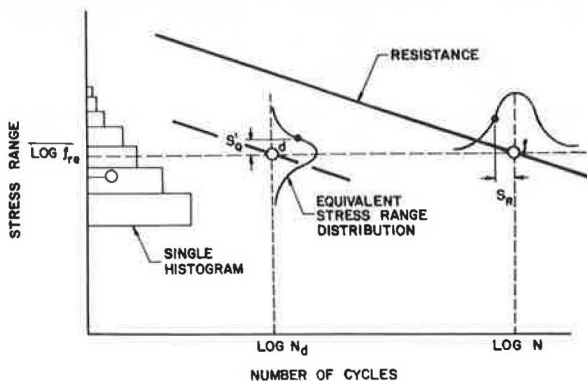


Table 1. Resistance curve parameters.

Category	Type of Detail Tested	No. of Data Points		Regression Coefficients		Standard Deviation (s_R)
		Included	Excluded	Intercept ^a (b)	Slope (m)	
A	Rolled beam	28	16	11.121	3.178	0.221
B	Welded beam	55	1	10.870	3.372	0.147
C*	Stiffeners	135	5	10.085	3.097	0.158
C	2-in attachments	14	0	10.0384	3.25	0.0628
D	4-in attachments	44	8	9.603	3.071	0.108
E	Cover plate end	193	0	9.2916	3.095	0.1006
E'	Cover plate end, $t > 0.8$ in	18		9.1664	3.2	0.1943

Note: 1 in = 25 mm.

^aValues of f_r substituted in units of ksi.

all equivalent stress ranges, and

3. Convert the resulting distribution of equivalent stress ranges into one given in terms of number of cycles.

The problem is then reduced to the form to which Equation 5 applies.

RESISTANCE CURVE

The AASHTO fatigue specifications (1) are based on constant-amplitude fatigue test data for steel beams (4,5). The statistical analysis of the S-N data has shown that the mean regression line with the best fit was of the log-log linear form:

$$\log N = b - m \log f_r \quad (6)$$

with the intercept $\{b[\text{at } f_r = 1 \text{ ksi (7 MPa)}]\}$, and the slope (m) as the regression coefficients. The log-log plot of Equation 6 gives a straight S-N line, which is labeled resistance in Figure 2. The data points were found to be log-normally distributed about the mean regression line with about equal standard deviation at all stress-range levels. This held true for all details. Thus, one may assume that for any point (f) on the mean regression line, the fatigue life of replicate specimens tested at the same stress range would be log-normally distributed about that point with mean $\bar{R} = \log N$ and standard deviation, i.e.,

$$s_R = s_{\log N} \quad (7)$$

The mean and the standard deviation of the number of cycles to failure define the resistance.

The regression coefficients (b and m) and the standard deviation (s_R) for the six categories A through E' are summarized in Table 1 (4-7). They provide the resistance curve parameters needed in Equation 5. The data listed under categories C and E' require some explanation. In the AASHTO specifications, category C covers both transverse stiffeners and 2-in (50-mm) attachments. The mean regression line of the former falls higher than that of the latter, thereby making it appear that the 2-in attachment data would govern. In reality, one must also consider the standard deviation, which is about 2.5 times larger for stiffeners than for 2-in attachments. This creates a peculiar situation. If one moves to the left of each mean by up to 2.56 standard deviations, the 2-in attachment governs. Beyond that, however, the transverse stiffener data become critical. Accordingly, this study employs that data for category C that governs at the value of β being considered. In contrast, the AASHTO fatigue specifications are solely based on the 2-in attachment data, although safety indices for category C reach a value of 7.12 for NRLP structures (6).

Table 1 also has an entry for category E', although no regression analysis was reported (7), presumably because of a lack of data at various stress-range levels in the finite life region of the S-N plot. For the purpose of this study, the finite life region for category E' details was defined by the 18 data points at the 8-ksi (55-MPa) stress range for which the mean life was 1 890 000 cycles, and it had a standard deviation of log of life of $s_R = 0.1943$ (6). Assuming a slope ($m = 3.2$) equal to the mean of the six other slopes in Table 1, the intercept is then given from Equation 6 as

$$b = \log(1.89 \times 10^6) + 3.2 \log(8.0) = 9.1664 \quad (8)$$

LOAD CURVE

The load-curve data can come either from field measurements of stress-range histograms or from loadometer surveys. In the former case, the strain ranges caused by the applications of a live load are obtained from a strain gage mounted at a suitable point on the bridge. In the latter, the trucks are weighed. In both cases, the results are usually reported as a histogram of stress range or truck weight versus frequency of occurrence.

The proposed construction of the load curve is illustrated below for a class of comparable structures, all of which are subjected to the same design load. For example, the fatigue design of short-span highway bridges is governed by the number of single truck crossings. Each application of the design load induces one stress-range cycle. It is assumed that one has available the stress-range histograms recorded on several bridges.

The construction of the load curve begins with a single stress-range histogram, such as the one plotted along the ordinate in Figure 2. The bar width in a histogram is usually constant; it varies in Figure 2 because the S-N plot scales are logarithmic. The equivalent stress range (f_{re}) of the single histogram is then calculated. It replaces the histogram in subsequent calculations and provides one point for the desired load curve. Plotting the distribution of all equivalent stress ranges on a vertical line through the design point (d) gives the load curve. To define its distribution, one needs the mean and the standard deviation. Assuming that the ratio of measured to computed stress range (α) is constant implies that αf_{rd} is also constant. Therefore, the computation of the log mean of all equivalent stress ranges is reduced to evaluating the log-mean of ρ for all histograms (see Equation 3):

$$\overline{\log \rho} = (1/h) \sum \log(f_{re}/\alpha f_{rd}) \quad (9)$$

where h is the number of histograms. The standard deviation of the load is then given by the standard deviation of the log ρ values:

$$s_Q' = s_{\log \rho} \quad (10)$$

The prime added to s_Q' and to any other symbol indicates a quantity measured along a vertical line. The line through the design point (d) drawn parallel to the resistance and the standard deviation of the equivalent stress ranges define the load. The load curve could be derived in analogous fashion from the results of loadometer surveys if one assumes that loads are proportional to stresses.

Estimation of the load curve is less certain when the structure is one of a kind and few data are available. In that case, one must construct an expected load histogram over the design life, compute the equivalent stress range for that histogram,

and estimate the standard deviation.

TRANSFORMATION OF LOAD AND RESISTANCE

Equation 5 applies only if the load and resistance curves are plotted side by side with the same base line. The load is distributed along a vertical line through the design point (d) shown in Figure 2, whereas the resistance is distributed along a horizontal line through the failure point (f). One of the two curves must, therefore, be transformed.

Albrecht (2) presented the transformation of the load curve, whereas this paper explains instead the transformation of the resistance curve in terms of self-evident geometrical relations. Figure 3 shows the solid resistance line and two dashed lines shifted above and below the mean resistance by a specified number of standard deviations, so that the survival probability (assuming a single-valued load) would be, say, 5 percent and 95 percent, respectively. The distribution of the resistance is drawn along a horizontal line through point f and also along a vertical line through point g. In both cases, the points with the same survival probability must lie on the same line parallel to the mean resistance. Because the slope is 1:m, it follows for reasons of geometry that

$$s_R' = s_R/m \quad (11)$$

where s_R and s_R' are the standard deviations of the resistance when its distribution is drawn about a horizontal and a vertical line, respectively. For the same geometrical reasons, the distance between the mean resistance and the mean load, measured along the vertical line g-d, is given by

$$\bar{R}' - \bar{Q}' = (1/m)(\bar{R} - \bar{Q}) \quad (12)$$

Rewriting Equation 5 for the distance g-d and substituting Equations 11 and 12 gives

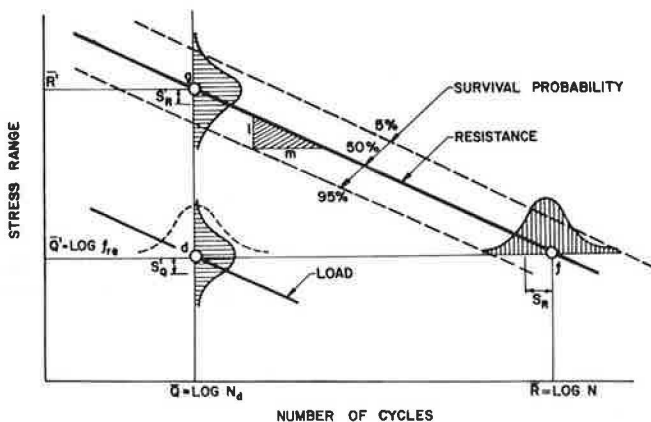
$$\beta = (\bar{R}' - \bar{Q}') / \sqrt{(s_R')^2 + (s_Q')^2} = (1/m)(\bar{R} - \bar{Q}) / \sqrt{(s_R/m)^2 + (s_Q')^2} \quad (13)$$

or, after simplifying

$$\beta = (\bar{R} - \bar{Q}) / \sqrt{(s_R)^2 + (ms_Q')^2} \quad (14)$$

Equation 13 applies to distributions of load and resistance along a vertical line through points d and g, respectively. It requires a 90-degree rotation of the resistance distribution, which decreases the standard deviation by a factor (m) in accordance with Equation 11. The load remains unchanged.

Figure 3. Transformation of load and resistance.



Equation 14 applies to distributions along a horizontal line through points d and f. In this case, the load distribution is rotated by 90 degrees, thus increasing its standard deviation to $s_Q = ms_Q'$, which is compatible with the geometrical condition stated in Equation 11. The resistance remains unchanged. Both equations give the same result for β .

Finally, substituting the identities $\bar{R} \equiv \log N$ and $\bar{Q} \equiv \log N_d$ into Equation 14 and using the abbreviation for the standard deviation of the difference between the resistance and the load:

$$s_r = \sqrt{(s_R)^2 + (ms_Q')^2} \quad (15)$$

Using the above equation yields the safety index for fatigue design:

$$\beta = (1/s_r)(\log N - \log N_d) \quad (16)$$

The terms s_R and s_Q' are given by Equations 7 and 10, respectively. The failure probability that corresponds to any numerical value of β can be read from tables for the normal distribution.

Equation 16 addresses the problem of computing the failure probability for a given design. The solution to the reverse problem, that of computing the design life (N_d) for a desired failure probability, follows from the same equation:

$$\log N_d = \log N - \beta s_r \quad (17)$$

On substitution of N from Equation 6 with $f_r = F_{re}$, one gets

$$\log N_d = (b - \beta s_r) - m \log F_{re} \quad (18)$$

or, taking the antilog:

$$N_d = 10^{(b - \beta s_r)/F_{re}^m} \quad (19)$$

The mean safety factor (F.S.) on life is, from Equation 17:

$$(F.S.)_N = N/N_d = 10^{\beta s_r} \quad (20)$$

and that on stress range:

$$(F.S.)_r = (F.S.)_N^{1/m} = 10^{\beta s_r/m} \quad (21)$$

For a fixed value of β , the safety factors vary with type of detail because the standard deviation of the resistance (s_R) varies.

In summary, Equations 16 and 19 yield the safety index and the allowable number of cycles, respectively, for a given design or a desired failure probability.

ASSUMPTIONS

Summarized below are the principal assumptions made in the development of the load and resistance. Many are based on data from previous studies and are referenced accordingly; others rely on engineering judgment when little or no data exist to support them.

Assumptions pertaining to the resistance curve are as follows:

1. The log-log linear S-N curve for constant-amplitude fatigue test data is extended below the constant-amplitude fatigue limit (F_L) downwards to a point where the equivalent stress range meets the variable-amplitude fatigue limit (8) at

$$F_{Le} = \rho F_L \quad (22)$$

2. Calculations that pertain to a specific type of detail employ the exponent m , which corresponds to the slope of the S-N curve for that detail. Calculations of equivalent stress range are insensitive to small changes in m . Therefore, a rounded value $m = 3$ is used, thereby lending to Equation 1 the meaning of a root-mean-cube (RMC) stress range (9).

3. Load interaction effects in high-low stress-range sequences are neglected because, in most civil engineering structures, the random nature of loading does not provide enough low-load cycles following a high-load excursion to retard crack growth (10).

4. The fabrication quality of the test specimens, from which the resistance data of Table 1 originate (4,5,7), is representative of all structures in service.

5. Except for thick cover plates, which are now covered by the newly adopted category E' (11), any effect of specimen size and plate thickness on the fatigue life is neglected.

6. Loss of life due to corrosion fatigue (all steels) and weathering (A588 steel) is neglected, although it can be large for high-fatigue strength details (12).

7. The resistance data are log-normally distributed (4,5).

Assumptions pertaining to the load curve are as follows:

1. The maximum stress range in a recorded histogram is caused by the design load.

2. The measured-to-computed stress-range ratio (α) has a single value with no distribution.

3. Available loadometer surveys and stress-range histograms describe typical load variability for highway bridges.

4. The load data are log-normally distributed (13).

It should be emphasized that the safety index and failure probability computed from Equation 16 apply to one detail. Because all structures have more than one detail, the probability that the first detail will fail is about equal to the sum of the failure probabilities of all details. Finally, failure of the first detail does not necessarily induce collapse. This depends on the redundancy of the load path.

The user must evaluate the assumptions listed above and the remarks on failure probabilities when applying the proposed method to a specific problem.

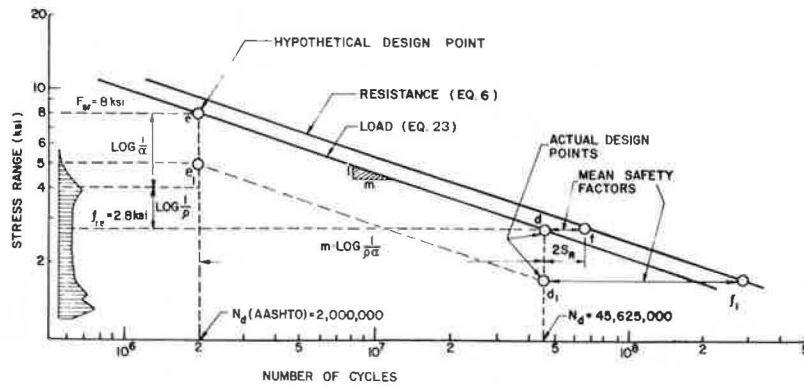
APPLICATION TO AASHTO SPECIFICATIONS

This section illustrates the first type of application, namely, to compute with Equation 16 the failure probability of a detail that was designed to a specified design line.

Background

The current fatigue specifications for highway bridges, railway bridges, buildings, and weldments state in identical tables the allowable stress range (F_{SR}) as a function of type of detail and number of loading cycles, N_d (1, Table 1.7.2A1). The listed pairs of stress range versus number of loading cycles are coordinates of points on the allowable S-N lines for each type of detail. For redundant load-path (RLP) structures, these lines were set at two standard deviations ($2s_R$) to the left of the resistance (14). They are loosely called in the literature the 95 percent confidence limit for 95 percent survival, although a design to those allowable S-N lines will not give a failure prob-

Figure 4. Illustration of AASHTO fatigue specification requirements for category E and ADTT = 2500.



ability of $P_F = 5$ percent (95 percent survival) for reasons that will become apparent in the following. It should also be noted that the current specification makes no allowance for load variability. Substituting $s_0' = 0$ and $\beta = 2$ into Equation 17 and computing $\log N$ for $f_r = F_{SR}$ from Equation 6 gives, indeed, the AASHTO fatigue design lines for RLP structures:

$$\log N_d = \log N - 2s_R = (b - 2s_R) - m \log F_{SR} \quad (23)$$

The tabulated values of F_{SR} and N_d are approximate coordinates of points on those lines. Equations 6 and 23 for category E details are plotted in Figure 4.

RLP Structures

Equation 16 is applied to loading case 1 for which the average daily truck traffic (ADTT) equals 2500 (or more). It cannot be applied to case 2 and case 3 without assuming a frequency of loading, because AASHTO does not specify the value of ADTT for those cases.

Consider, for example, a category E cover-plate end detail on an RLP structure designed to point e on the allowable S-N line shown in Figure 4. Its coordinates are $F_{SR} = 8$ ksi (55 MPa) and N_d (AASHTO) = 2 000 000 cycles. To locate the actual design point (d), one must find the equivalent stress range (f_{re}) and the actual number of loading cycles (N_d).

The equivalent stress range was extracted from the information reported in Fisher (14). In that report, a linear relation was assumed between gross vehicle weight and stress range. Accordingly, the coefficients ρ and α that relate the equivalent stress range (f_{re}) to the design stress range (F_{SR}) can be obtained from gross vehicle weight data. The gross vehicle weight distribution from the 1970 Federal Highway Administration (FHWA) nationwide loadometer survey yielded a summation of $\gamma_i \phi_i^3 = 0.35$ (14). This gives, in the manner of Equation 4, $\rho = 0.705$. The ratio of the actual stress range due to the passage of a design vehicle and the design stress range is $\alpha = 0.5$. It is obtained from Equation 6 in Fisher (14) for a 50-year design life. Note that F_{SR} is based on a distribution factor for wheel loads to girders on a bridge designed for two or more traffic lanes ($s/5.5$), where s is the girder spacing. The combination of $s/5.5$ with $\alpha = 0.5$ gives $s/11$, a plausible value for the distribution factor for bridges designed for one traffic lane. The equivalent stress range is then

$$f_{re} = 0.5 \times 0.705 \times 8 = 2.8 \text{ ksi (19 MPa)} \quad (24)$$

For purposes of illustration, a stress-range histogram proportional to the gross vehicle weight histogram from the nationwide loadometer survey, as used in Fisher (14), is shown along the ordinate of Figure 4 [see also Fisher (14), Figure 25]]. For loading case 1 and ADTT = 2500, AASHTO specifies that a category E detail be designed to the hypothetical point e in Figure 4 and for N_d (AASHTO) = 2 000 000 cycles of the allowable stress range $F_{SR} = 8$ ksi (55 MPa). The $\alpha F_{SR} = 4$ ksi (28 MPa) stress range corresponds to one 72 000-lbf (320-kN) truck on a bridge designed for one traffic lane. The $f_{re} = \alpha \rho F_{SR} = 2.8$ ksi (19 MPa) stress range corresponds to one ($\rho \cdot 72\ 000$) = 50 760-lbf (226-kN) fatigue truck on a bridge designed for one traffic lane (15, Table 5). The actual number of cycles for a 50-year design life is $N_d = 2500$ trucks/day \times 365 days \times 50 years = 45 625 000 cycles. The actual design point (d) has therefore the coordinates $f_{re} = 2.8$ ksi and $N_d = 45\ 625\ 000$ cycles. That the actual design point (d) also lies on the load curve at $2s_R$ to the left of the resistance follows from the identities:

$$(F_{SR}/f_{re})^m = [1/(0.5 \times 0.705)]^3 = 22.83 \quad (25)$$

$$N_d/[N_d(\text{AASHTO})] = 45\ 625\ 000/2\ 000\ 000 = 22.81 \quad (26)$$

Equations 25 and 26 reflect the geometrical relation that the slope times the rise must equal the flat of the log-log linear design S-N line.

Evidently, the intentional mismatch between the AASHTO number or cycles of F_{SR} stress range and the actual number of cycles of f_{re} stress range means that the AASHTO specifications apply, in reality, to one traffic lane loaded by a single fatigue truck. The fatigue design to the hypothetical point e is mathematically identical to a design to the actual point d. This conclusion is illustrated in Figure 4 for a category E detail, but it holds equally true for all other categories.

The horizontal distance between the failure point (f) and the actual point (d) is therefore $2s_R$. Substituting this value into Equation 16 gives the safety index for main longitudinal load-carrying members in RLP structures designed for ADTT = 2 500, i.e.,

$$\beta = 2s_R/s_T \quad (27)$$

Note again that neglecting load variability implies that $s_0' = 0$ and leads to $\beta = 2$, as in Equation 23. AASHTO requires that "members shall also be investigated for over 2 million stress cycles produced by placing a single truck on the bridge distributed to the girders as designated in Article 1.3.1(B) for one traffic lane loading" (1). (The distribution factor for one traffic lane loading is

S/7.) This requirement lowers the hypothetical design point in Figure 4 from e to e₁ and the actual design point from d to d₁. It increases the numerator of Equation 27 from the distance d-f to d₁-f₁. The safety index for more than 2 million cycles is then given by

$$\beta = (1/s_r) \{ 2s_R + m \cdot \log [F_{sr}(2 \times 10^6)/F_{sr}(\text{over } 2 \times 10^6)] \} \quad (28)$$

in which the allowable stress ranges for 2 million cycles and over 2 million cycles are read from AASHTO Table 1.7.2A1 (1) for RLP structures. There is really no need to check both loading conditions for every design, since the one that governs can be determined a priori. The condition for 2 million cycles of loading always governs for categories A, B, and C* because $[F_{sr}(2 \times 10^6)/F_{sr}(\text{over } 2 \times 10^6)] < [(S/5.5)/(S/7)]$. The condition for over 2 million cycles always governs for categories C, D, E, and E' because $[F_{sr}(2 \times 10^6)/F_{sr}(\text{over } 2 \times 10^6)] > [(S/5.5)/(S/7)]$. Hence, Equation 27 always applies to categories A, B, and C*, and Equation 28 applies to categories C, D, E, and E'. Both are in reality for single truck loading; the former with a distribution factor $\alpha(S/5.5) = S/11$, as shown previously, and the latter with $\alpha(S/7) = S/14$. The double-check requirement is superfluous and leads to inconsistent failure probabilities.

The numerical evaluation of Equations 27 and 28 was carried out for all categories. The values of s_R and m needed to calculate s_T with Equation 15 are listed in Table 1. Lacking variability information for fatigue truck weights, the standard deviation of the load was set equal to the standard deviation of the equivalent stress ranges that were obtained from 104 histograms recorded on 29 bridges in eight states [s_Q' = 0.0492 (13)]. The allowable stress ranges for main longitudinal load-carrying members in RLP structures were taken from AASHTO Table 1.7.2A1 (1). The calculated safety indices and failure probabilities are shown in the left part of Table 2. The results reveal extreme variations in failure probability, which range from a high of P_F = 9.2x10⁻² for category B to a low of P_F = 9.2x10⁻⁸ for category E'.

NRLP Structures

Failure probabilities of main longitudinal load-carrying members in NRLP structures can be calculated in a similar manner. Again, the condition for 2 million cycles of loading always governs for categories A, B, and C*, for which the safety index is as follows:

$$\beta = (1/s_r) \{ 2s_R + m \cdot \log [F_{sr}(2 \times 10^6; \text{RLP})/F_{sr}(2 \times 10^6; \text{NRLP})] \} \quad (29)$$

The condition for over 2 million cycles of loading

always governs for categories C, D, and E that have a safety index of

$$\beta = (1/s_r) \{ 2s_R + m \cdot \log [F_{sr}(2 \times 10^6; \text{RLP}) / F_{sr}(\text{over } 2 \times 10^6; \text{NRLP})] \} \quad (30)$$

The results, shown to the right in Table 2, reveal once more extreme variations in failure probability, from a high of P_F = 5.1x10⁻² for category A to a low of P_F = 2.1x10⁻²² for category E. Note also that the failure probabilities for category A, B, and C* details on NRLP structures exceed the failure probabilities for category C, D, E, and E' details on RLP structures. The original intent of adding a table of F_{sr} values for NRLP structures had been to lower failure probabilities to less than those for RLP structures.

DESIGN EXAMPLE

This section illustrates with one example the second type of application, namely, the fatigue design of a detail for a desired failure probability. This type of example is not covered by the AASHTO specifications. Additional examples are presented in Albrecht (2).

Problem

A one-lane bridge consists of two plate girders and a concrete deck. It is located on a private access road from an ore concentrating plant to the mine. The trucks cross the bridge empty on the way to the mine and full on the way back to the plant. The net vehicle weight (NVW) and gross vehicle weight (GVW) are NVW = 40 kips (178 kN) and GVW = 140 kips (625 kN). Because all other vehicles weigh much less than the empty ore truck, their contribution to fatigue damage is neglected. About 150 daily round trips will keep the plant working at full capacity. At that rate, all ore will be mined in 12 years. Because the bridge has low clearance, crosses a shallow river, and serves no public roads, a low safety index of β = 3 (P_F = 1.35x10⁻³) is assumed. Compute the allowable stress range for the category B flange-to-web weld.

Solution

The load- and resistance-curve data are determined and substituted into Equation 18. Its solution yields the allowable stress range for the specified number of loading cycles (see Figure 5).

1. The load-curve data for vehicle weights and frequencies are as follows:

$$\phi_{NVW} = 40/140 = 0.286; \gamma_{NVW} = 0.5.$$

Table 2. Fatigue failure probabilities for main longitudinal load-carrying members designed for ADTT = 2500.

Category	Standard Deviation of R - Q ^d (s _r)	RLP Structures				NRLP Structures			
		2 Million Cycles		Over 2 Million Cycles		2 Million Cycles		Over 2 Million Cycles	
		β ^b	P _F	β ^c	P _F	β ^d	P _F	β ^e	P _F
A	0.2707	1.633	5.1 x 10 ⁻²	-	-	1.633	5.1 x 10 ⁻²	-	-
B	0.2217	1.326	9.2 x 10 ⁻²	-	-	2.104	1.8 x 10 ⁻²	-	-
C*	0.2195	1.440	7.5 x 10 ⁻²	-	-	1.930	2.7 x 10 ⁻²	-	-
C	0.1718	-	-	2.887	1.9 x 10 ⁻³	-	-	3.752	8.8 x 10 ⁻⁵
D	0.1857	-	-	3.725	9.5 x 10 ⁻⁵	-	-	6.141	4.1 x 10 ⁻¹⁰
E	0.1825	-	-	4.564	2.5 x 10 ⁻⁶	-	-	9.669	2.1 x 10 ⁻²²
E'	0.2501	-	-	6.012	9.2 x 10 ⁻¹⁰	-	-	-	-

^aEquation 15.

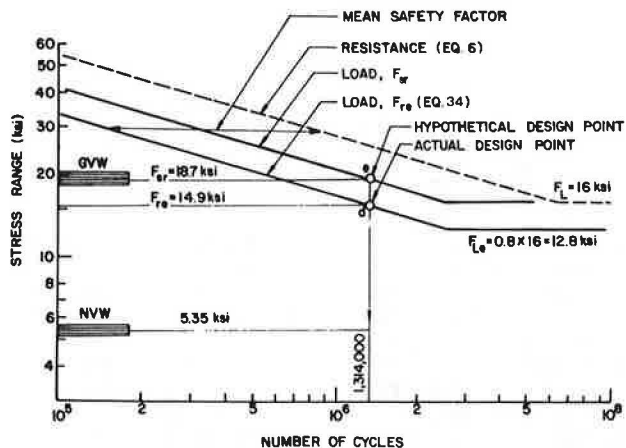
^bEquation 27.

^cEquation 28.

^dEquation 29.

^eEquation 30.

Figure 5. Design example.



$$\phi_{GVW} = 140/140 = 1.0; \gamma_{GVW} = 0.5.$$

Then substitute into Equation 4; i.e.,

$$\rho = [0.5(0.286)^3 + 0.5(1.0)^3]^{1/3} = 0.80 \quad (31)$$

For a one-lane bridge assume that $\alpha = 1.0$. A 15 percent coefficient of variation ($C = 0.15$) is estimated for the equivalent truck weight, so that

$$s'_C = \sqrt{0.4343 \log_{10}(1 + C^2)} = 0.0648 \quad (32)$$

2. Resistance-curve data for category B, which are taken from Table 1, are as follows: $b = 10.870$, $m = 3.372$, and $s_R = 0.147$.

3. The design equations (Equations 15 and 19) are as follows:

$$s_r = \sqrt{(0.147)^2 + (3.372 \times 0.0648)^2} = 0.2634 \quad (33)$$

$$N_d = 10^{(10.870 - 3 \times 0.2634)/F_{re}^{3.372}} = (12.0 \times 10^9)/F_{re}^{3.372} \quad (34)$$

4. The number of loading cycles is as follows:

$$N_d = (2 \times 150 \text{ trips})(365 \text{ days})(12 \text{ years}) = 1\,314\,000 \text{ cycles} \quad (35)$$

5. The allowable stress range for a design based on equivalent truck weight is

$$F_{re} = [(12.0 \times 10^9)/(1.314 \times 10^6)]^{1/3.372} = 14.9 \text{ ksi (103 MPa)} \quad (36)$$

and for a design based on GVW is

$$F_{sr} = F_{re}/\rho\alpha = 14.9/(0.80 \times 1.0) = 18.7 \text{ ksi (129 MPa)} \quad (37)$$

6. Check the fatigue limit; i.e.,

$$f_r(\text{GVW}) = 18.7 \text{ ksi} > F_L = 16 \text{ ksi (110 MPa)} \quad (38)$$

Therefore, fatigue must be checked.

7. The safety factor on life, which is taken from Equation 20, is as follows:

$$(F.S.)_{N_d} = 10^{(3 \times 0.2634)} = 6.2 \quad (39)$$

The results of the previous calculations are shown in Figure 5. The histogram is plotted along the ordinate. The upper and lower load lines are for designs to F_{sr} and F_{re} , respectively. Both give analogous results, since the two lines are shifted by the ratio $\alpha\rho = F_{re}/F_{sr}$. The safety factor on life is the horizontal distance between the resistance curve and the F_{re} load curve.

CONCLUSIONS AND RECOMMENDATIONS

A method of calculating fatigue failure probabilities based on the S-N approach was presented in a form suitable for examining designs to current fatigue specifications and for performing designs to any desired level of risk. The method is general. It was illustrated here in detail for highway bridges. The main findings were as follows:

1. The design for the AASHTO number of cycles of maximum stress range (F_{sr}) calculated for multiple HS-20 trucks with a distribution factor for two or more traffic lane loadings ($S/5.5$) is mathematically identical to the design for the actual number of single fatigue trucks with a distribution factor of $S/11$.

2. The dual requirement to check all case 1 designs for 2 million cycles with $S/5.5$ and for over 2 million cycles with $S/7$ is superfluous because the former always governs for categories A, B, and C*, and the latter always governs for categories C, D, E, and E'. It is also inconsistent because both are, in reality, for single fatigue truck loading. But the former is for a higher stress range with an actual distribution factor $\alpha(S/5.5) = S/11$, while the latter is for the fatigue limit with an actual distribution factor $\alpha(S/7) = S/14$.

3. The failure probabilities lack uniformity. For RLP structures, they vary from a high of $P_F = 9.2 \times 10^{-2}$ for category B to a low of $P_F = 9.2 \times 10^{-10}$ for category E'. For NRLP structures they vary from a high of $P_F = 5.1 \times 10^{-2}$ for category A to a low of $P_F = 2.1 \times 10^{-22}$ for category E.

4. The failure probabilities for category A, B, and C* details on NRLP structures are higher than those for category C, D, E, and E' details on RLP structures. This violates the intent of the requirements for NRLP structures.

There are dangers of either giving too much credence to the accuracy of calculated failure probabilities or of dismissing the results out of hand because of unavoidable uncertainties and a lack of data. The most important benefit is the ability to compare the values relative to each other.

The pressing need for specifications that are based on uniform failure probabilities mandates that AASHTO choose two values, one for RLP and one for NRLP structures. Thereafter, allowable stress ranges can be derived with methods such as the one outlined here. This need alone calls for a revision of the current specifications. In addition, AASHTO should consider the following points, which have also been made by other investigators in the past:

1. Define the allowable stress range as a continuous function of truck traffic volume instead of the step function approach by loading case in Table 1.7.2B (1).

2. Explicitly formulate the fatigue specifications in terms of the actual number of single fatigue trucks, each of which cause an equivalent stress range, instead of a hypothetical number of HS-20 design trucks that cause the maximum stress range.

3. Examine what impact the distribution factors would have on the fatigue specifications if they were expressed in terms of number of girders and lanes instead of girder spacing.

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