Expected Costs

The next step in the decision analysis is to determine the expected costs of failure (ECF) of each alternative. Expected costs are the product of the cost of that event and the probability of its occurring. The costs of failure, i.e., $1 million for a delay and $3 million for an impact, are estimates based on likely delay times, injuries, damage to track and equipment, cleanup of rock, and slope stabilization. Indirect costs such as legal fees and insurance premiums should also be included in the costs of failure. The expected cost of failure for each alternative is shown on the decision tree (Figure 4).

The final step in the analysis is to determine the likely cost of implementing each decision, i.e., the stabilization costs. The costs are estimated from previous construction projects and are added to the expected cost of failure. As shown on the decision tree, the most effective means of stabilization is to unload the crest and install rock bolts, even though this is more expensive than bolting only. This shows the sensitivity to expected costs of the probability of failure.

Conclusions

In this paper, use of both limit-equilibrium and probability methods to design stabilization measures for a wedge failure in a rock slope is described. The advantages of using both methods are that limit-equilibrium analysis has been well proved in rock engineering practice, whereas probability analysis allows the designer to assess the effect of uncertainty in the input data on the design. The calculation of the probability of failure of the slope for different courses of action allows the relative merits of the alternatives to be evaluated. The use of decision analysis requires an assessment of the consequences of failure, which can involve the owner in the decision-making process.

References


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ment design methodology. Soil failure in this context is described as the accumulation of excessive residual deformations in soil, which may not necessarily be the result of a shear failure.

Strength and deformation characteristics of compacted and consolidated cohesive soils subjected to repetitive loading have been documented as functions of water content and soil suction value by many researchers (1-7). One of the earliest studies reported (8) indicated a steep rise in the resilient modulus of a till and a corresponding drop in the residual deformation with decreasing water content at wet-of-optimum and more gradual changes at dry-of-optimum. Richards and others (9), measuring the suction in a subgrade soil by psychrometers, looked for a possible correlation between the deformation characteristics of compacted laboratory samples and those of the subgrade soils as a function of soil suction. Richards and Gordon (10) observed a linear correlation between the soil suction and the resilient modulus on a logarithmic plot. Shackel (11) indicated a slight dependence of soil suction on dry density and observed a decrease in soil suction during repetitive loading. Fredlund, Bergan, and Sauer (12) proposed a set of stress-state variables—confining pressure, deviator stress, and soil suction—controlling the deformation characteristics of a soil under repetitive loading. Later, Fredlund, Bergan, and Wong (13) gave equations relating the resilient modulus to deviator and confining stress through a set of parameters that are functions of matrix suction. Edris and Lytton (14) reported negligible changes in the resilient modulus of three different soils (with group symbols of ML, CL, and CH according to the Unified Soil Classification System) at psychrometrically measured initial total suction values higher than those corresponding to moisture content at approximately 2 percent dry-of-optimum. Luh (15) reported a tendency in the dynamic shear modulus of kaolinite specimens tested in a resonant column apparatus to increase substantially up to 400 kPa, which was followed by a decrease at 800 kPa. The drop in the shear modulus at 800 kPa was attributed to the formation of microcracks at high suction levels.

In this study, the response of soil under unconfined repetitive loading has been investigated as a function of matrix and total suction values and number of cohesive soils over a range of plasticity characteristics. Consideration was given to the dependence of the resilient modulus and residual deformation of soils tested on the matrix component of the total suction values along with the modifying effects of the osmotic component, if any, on the soil behavior. The possible existence of a unique set of curves relating the resilient modulus to the matrix suction was investigated in particular.

EXPERIMENTAL PROGRAM

Soil Properties

Four series of compacted specimens were prepared for repetitive testing (Table 1) by using three different soils.

Sample Preparation and Moisture Equilibration

For series 1 and 2, three sets of samples were prepared by using a Harvard miniature compactor at the optimum moisture content and 2 percent dry- and wet-of-optimum (16). For series 3 and 4, two sets of samples were prepared at 2 percent dry- and wet-of-optimum moisture content by using static compaction and a compaction mold similar to that described by Shackel (17).

All the samples were placed on saturated ceramic plates in pressure extractors and permitted to absorb as much water as possible under atmospheric pressure. In series 3 and 4, sodium and calcium chloride were added to water saturating the extractor plate to minimize the water transfer between the plate and the samples through osmotic processes. Subsequently, all samples were moisture-equilibrated in extractors along a desorption cycle by increasing the applied air pressure in steps to achieve matrix suction values of 50, 100, 200, 400, 800, and 1500 kPa, and a number of them were tested under repetitive loading after equilibration at each step (16). At equilibrium, the moisture remaining in the soil specimen is in equilibrium with the applied air pressure, which has a value corresponding to the matrix suction component of the total suction. Total suction, which can be directly measured by using a psychrometer, is the sum of the matrix and osmotic components. Some of the samples in series 3 and 4 were further dried over a sulfuric acid solution in a desiccator to moisture contents lower than those corresponding to 1500-kPa suction prior to repetitive loading in order to examine the soil behavior at suction values beyond 1500 kPa (at an estimated matrix suction of 9000 kPa). A moisture sorption cycle (400 and 100 kPa) was also followed in series 3 and 4. In addition, three samples that were previously tested were retested subsequent to oven drying, which resulted in very large suction values.

Repetitive Loading

For series 1 and 2, the samples were tested uniaxially on an MTS Systems Corporation repetitive-loading system at a loading frequency of 0.5 cycle/s; a haversine load curve that lasted 0.3 s was used (16). For series 3 and 4, a pneumatic repetitive-loading system applying uniaxial loading was used at a frequency of 0.5 cycle/s with a continuous haversine load curve. During the repetitive testing of series 3 and 4 samples, top and bottom loading caps made of Plexiglas and equipped with open psychrometers were used (Figure 1) to monitor the total soil suction before, during, and after the repetitive-loading tests. Load-deformation curves on an X-Y recorder were taken several times during the test and more fre-
quently within the first 1000 cycles. Resilient and residual deformations were monitored by a 6.25-mm travel linear variable differential transformer (LVDT) and a dial gage reading $2.5 \times 10^{-3}$ mm per division, respectively. Axial load was measured by a load cell as illustrated in Figure 1. Minimum and maximum deviator stresses during the tests were kept fairly constant around 20 kPa and 100 kPa, respectively, to eliminate the deviator stress as a variable. Moisture loss in samples during repetitive loading was prevented by surrounding the membrane-covered samples by a cylinder containing wetted Styrofoam packing material, which was found to be very effective. Maximum number of load repetitions in series 1 and 2 were 10 000 and 5000, respectively. In series 3 and 4, tests were terminated after approximately 40 000 repetitions. An unconfined-compression test was run on each sample after repetitive testing.

EVALUATION OF EXPERIMENTAL DATA

Resilient modulus ($M_r$, cyclic deviator stress divided by resilient strain), axial residual strain ($\varepsilon_{pl}$), strain-energy absorption capacity, and postrepetitive testing unconfined strength ($q_u$) have been examined in this section as a function of number of load repetitions (N) and pre-test-induced matrix suction ($\theta_m$). Justification for the choice of matrix suction instead of total suction ($\theta_t$) will be given later in this section.

Load-Deformation Curves

During the repetitive-loading tests, basically three types of load-deformation curves were obtained as illustrated in Figure 2, where the repetitive stress level is indicated as a percentage of the unconfined strength. Curve 1 represents a tendency for recovery in the soil after a relatively large deformation has been imposed on it, which results in a rapid buildup of residual deformations. Repetitive-load level is relatively close to the unconfined strength, and the frequency of loading can play a significant role in controlling the residual deformations. With increasing suction, linear and concave upward loading curves result (curves 2 and 3). The lateral inertia of the sample is not expected to play a significant role in curve 3, since the loading cannot be classified as impact type. However, the initiation of a desaturation after 800 kPa as measured on series 3 and 4 specimens is believed to leave some relatively large pores free of water and easier to deform, but elastically, because of the increasing integrity of the soil structure under increasing suction. Also, the formation of micro-cracks with decreasing water in grundite, as observed by Luh (15) at macroscale in kaolinite samples after 800-kPa suction, and the closure of such cracks at small compressive stresses compared with the unconfined compressive strength can result

Figure 1. Arrangement of psychrometers and load cell during repetitive-loading test.

Figure 2. Relative magnitude of repetitive stress level and form of repetitive-load axial-deformation curves at different suction values.

Figure 3. Rate of increase in resilient modulus during repetitive loading at different matrix suction values.
in a concave upward stress-strain curve. However, these cracks, if they actually exist, have to be very small, since thin sections prepared by using compacted grundite specimens freeze-dried under vacuum after equilibrium under 1500-kPa extractor pressure did not reveal any visible crack pattern when viewed under a petrographic microscope.

### Resilient Modulus Versus Number of Load Repetitions

The resilient modulus in series 1 and 2 increased monotonically from the beginning of the repetitive-loading test (16). However, in the majority of the samples in series 3 and 4, a decrease in \( M_0 \) was evident during the early part of the repetitive loading; \( M_0 \) reached a minimum value mostly within the first 100 repetitions and was accompanied by an increase thereafter. This is attributed to a small, time-dependent increase in the rigidity of the soil structure during the moisture-extraction process, which extended over a period of 4-12 months. After several applications of the repetitive load, the memory effects were removed. The considerable scatter of the mean of increase in \( M_0 \) (Figure 3) as determined beyond 1000 repetitions by the slope of the curves of \( M_0 \) versus log \( N \) shows that apparently identical compacted soil samples may display significantly different resilient moduli at large numbers of load repetitions even though the resilient response may be comparable at low repetitions during a repetitive-load test.

### Resilient Modulus Versus Matrix Suction

Samples of series 1 and 2 were tested at different matrix suction values by repetitive loading without any total-suction measurements. In series 3 and 4, the measured total-suction curve showed a decreasing osmotic suction component within the range of test moisture contents when superimposed on the curve of moisture content versus matrix suction and eventually approached it at higher induced matrix suction values. The osmotic suction measured on the pore-water extracts obtained during pressure equilibrium, on the other hand, remained fairly constant with increasing extractor pressure. In general, the contribution of the soluble salts in pore water to the changes in soil behavior did not seem to be clearly reflected by the magnitude of the measured osmotic suction. This is due to the apparently more complicated effects of the ionic distribution around the particles than can be deduced by osmotic suction measurements alone. If we consider the obvious changes taking place in soil behavior after moisture equilibration under increasing extractor pressure, the matrix suction was adopted as the meaningful soil-moisture stress parameter for the purpose of this study.

The differences in the resilient modulus of dry- and wet-of-optimum specimens at 5000 repetitions in all four series were relatively small below 100 kPa (Figures 4 and 5). Afterwards, an increasing range of \( M_0 \) in series 1 and 2 was observed, more significantly in series 2, whereas it remained comparatively small in series 3 and 4 throughout the suction range used (Figure 5). The same range of compaction moisture content \((W_{\text{opt}} \pm 2\text{ percent})\) was used in all series. The difference in resilient modulus between dry- and wet-of-optimum specimens and the average value of the resilient modulus increase with decreasing plasticity (or optimum moisture content of the soil) is shown in Figure 6.

The maximum resilient modulus in all series was found to lie at a matrix suction value of approximately 800 kPa; there is a drop at 1500 kPa. The indication is that \( M_0 \) increases again at matrix suction values beyond 1500 kPa as shown in the table below, which gives a set of samples tested after air drying that gave significantly higher resilient moduli:

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( M_0 ) (kPa•10^-')</th>
<th>( w ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series 1, wet of optimum</td>
<td>236.9</td>
<td>2.8</td>
</tr>
<tr>
<td>Series 3, wet of optimum</td>
<td>197.6</td>
<td>2.8</td>
</tr>
<tr>
<td>Series 4, dry of optimum</td>
<td>152.2</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Therefore, all the evidence suggests the form of \( M_0 \) versus log \( W_{\text{opt}} \) relationship as given qualitatively in Figure 7.

High resilient modulus is rather expected at very high suction values and increasing rigidity of the soil skeleton. The hump occurring at 800 kPa represents a change more likely in the role of soil moisture than in the mechanical behavior of the soil. Increasing the dry unit weight with suction should also be considered as a contributing factor, although it does not play a significant role in \( M_0 \) after 800 kPa. Buildup of residual deformation is very small above 400 kPa, which is indicative of the more-elastic character of soil deformations under repetitive loading (see figure 12).

A desaturation initiated at 800 kPa (Figure 8) is thought to have resulted from emptying of relatively large pores as mentioned earlier. This confines the soil moisture to extremely small pores, particle surfaces, and interparticle contacts under very high tension and possibly results in microcrack formation. This distribution of moisture is believed to give the soil structure a more elastically deformable character while the development of residual strains is resisted. The eventual development of particle-to-particle bonds beyond a certain suction level, then, results in a stiff soil structure that displays high resilient modulus as observed in oven-dried tested samples at very high suction values.

The advantage of relating the resilient modulus of soil suction instead of moisture content is evident in Figure 9 due to the fact that the matrix suction fixes the position of the maximum resilient modulus at 800 kPa for all soils tested, whereas no such trend is evident when moisture content is used as the main moisture variable. Furthermore, the use of moisture content results in considerable scatter of the data.

Because of the small differences in moisture content along the desorption and sorption branches of the moisture-retention curve, the samples tested at 400 and 100 kPa suction on the sorption curve did not provide significant insight into the effects of moisture hysterisis on the resilient behavior of the samples in series 3 and 4 and were not included in Figure 5. Series 1 and 2 samples were equilibrated only along a desorption curve.

### Effects of Dry Unit Weight and Deviator Stress Level on Resilient Modulus

The samples compacted on the same side of the optimum moisture content and tested at matrix suction values up to 800 kPa indicate approximately a linear variation between \( M_0 \) and \( Y_d \) (Figure 10). However, this situation is believed to be due as much to generally higher soil suction in dry of optimum at a given unit weight as to the differences in the soil structure. Also, any possible relation between \( M_0 \) and \( Y_d \) breaks down after 800 kPa due to the change in soil behavior.

After the termination of the repetitive loading, axial deformation-load curves were taken at increas-
Figure 4. Resilient modulus versus matrix suction in series 1 and 2 samples.

Figure 5. Resilient modulus versus matrix suction in series 3 and 4 samples.

Figure 6. Range of resilient modulus values as function of plasticity index of soil.

Figure 7. Qualitative relationship of resilient modulus versus matrix suction including extremely high matrix suction range.

Figure 8. Degree of saturation in series 3 and 4 samples as function of matrix suction.

The ratio of the area inside the hysteretic load-deformation curve ($A_1$) to the area of the triangle ($A_2$) in Figure 11 is defined as the relative energy absorption capacity of the soil. It decreases rapidly with increasing suction up to 400 kPa in series 3 and 4; the minimum is in the vicinity of 800-1500 kPa. Desiccator-equilibrated samples had an absorption capacity somewhat higher than the minimum for both soils, which signified a delayed elastic response of the soil within a suction zone beyond 1500 kPa and attainment of nearly perfect elastic response afterwards.

Estimation of this ratio at different stress levels and suction values for a soil profile underlying a structure subject to vibratory loading may help in assessing the degree the soil against vibration effects. A similar, suction-dependent relationship expressed in terms of shear stresses will be useful in computing the soil
response to earthquake excitation whether there is an already established steady state or a transient suction profile.

Residual Deformation Versus Matrix Suction

Total residual strain decreased sharply in series 1 and 2 up to 100-kPa suction; it remained approximately constant afterwards (Figure 12). The same trend was observed in series 3 and 4, and the residual strain evened out after 200-kPa suction (Figure 13). There was not a clearly defined indication as to which side of the optimum compaction yielded higher residual deformation.

The increase in residual strain with the number
of load repetitions \((\Delta \epsilon_p / d \log N)\) on a logarithmic plot against the soil suction is approximately a straight line, although some scatter at suction values less than 400 kPa is evident (Figure 14). In other words, the rate of increase of residual strain is a transcendental function of suction. This suggests that the following relationship may be helpful in evaluating the buildup of residual strain with an increasing number of load repetitions:

\[
\log (\Delta \epsilon_p / d \log N) = a + b \log \psi
\]

If we integrate Equation 1, the residual strain is obtained as follows:

\[
\epsilon_p = ah_{\psi \psi_i}^{N_f} (\psi^{-1} / N_i) d \log \psi + C
\]

where

- \(a\) and \(b\) = empirical coefficients,
- \(\psi_i\) and \(\psi_f\) = initial and final matrix suction values,
- \(N_i\) and \(N_f\) = initial and final numbers of load repetitions, and
- \(C\) = value of \(\epsilon_p\) at \(\psi_i\) and \(N_i\).

However, considering the complexity of the field conditions, in the absence of supporting data, Equation 2 should be regarded only as a first approximation to the actual residual settlement.

**Postrepetitive Loading Unconfined Compressive Strength**

Unconfined compressive strength increases with increasing matrix suction in series 1 and 2; dry-of-optimum samples generally yield higher strength (Figure 15). Variation of the unconfined compressive strength in series 3 and 4 with matrix suction is given in Figure 16. The wet-of-optimum specimens yield higher strength throughout the moisture-content range; the difference between series 3 and series 4 specimens increases with increasing matrix suction. However, as is evident from the comparison of Figures 16 and 17, unconfined compressive strength correlates better with moisture content than with matrix suction. Since the data presented in Figures 15 and 16 do not indicate the type of decrease observed in the resilient modulus beyond a suction value of 800 kPa, no interdependence between the unconfined compressive strength and the resilient modulus of the soil exists after 800 kPa. Therefore, the static unconfined compression test results should not be taken as indicative of the resilient modulus of the soil at high suction and at relatively low repetitive-stress levels.

**SUMMARY AND CONCLUSIONS**

The changes in the deformation characteristics of a number of cohesive soils as a function of the initial matrix suction have been investigated for repetitive unconfined loading. The following conclusions are drawn from an evaluation of the data:
1. The rate of change in the resilient modulus with the number of load repetitions in samples tested at the same initial matrix suction can be significantly different even for soils with similar grain size, plasticity, and compaction characteristics.

2. The resilient modulus displayed a maximum at a matrix suction of about 800 kPa followed by a drop and a subsequent increase for the soils tested at the repetitive deviator-stress level used. However, it is possible that both the magnitude of the maximum modulus and the corresponding critical matrix suction change at a different stress level.

3. The similarity of the curves of resilient modulus versus matrix suction obtained on different soils supports the use of matrix suction as the basic soil-moisture parameter rather than the moisture content for indexing the deformation behavior of cohesive soils.

4. The scatter in the curves of resilient modulus versus matrix suction can be minimized by normalizing the resilient modulus with respect to a factor related to the plasticity index of the soil. Another parameter, which involves the difference between the dry- and wet-of-optimum compaction moisture contents compared with the optimum compaction moisture content, can possibly be used for this purpose.

5. The effects of dry unit weight on the resilient behavior of soil with increasing matrix suction can meaningfully be investigated for compaction only on one side of the optimum moisture content, either dry or wet, because of the fabric effects.

6. Unconfined compressive strength does not provide a good indicator of the resilient behavior of cohesive soils tested at matrix suction values above 800 kPa, since it continues to increase while the resilient modulus decreases. Strength appears to be correlated with moisture content better than soil suction, whereas the opposite is true for the resilient modulus.

REFERENCES


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