Structural Design Method for Precast Reinforced-Concrete Pipe

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A comprehensive direct structural design method for buried concrete pipe is presented that has been included in a new section (1.15.4-Reinforced Concrete Pipe, Precast) of the bridge specifications of the American Association of State Highway and Transportation Officials. The method is based on the ultimate-strength and crack-control behavior of reinforced-concrete pipe and other structures observed in various tests of pipe, box sections, slabs, and beams under known loading conditions that encompass both concentrated and distributed test loads. The new design method includes criteria for ultimate flexural strength based on both tensile yield of reinforcement and compressive strength of concrete, ultimate shear (diagonal tension) strength, and ultimate flexural strength as limited by radial tension in pipe without radial ties. Also included is a crack-control criterion. Additional design equations are provided for radial ties when radial tension or shear strength is inadequate without such reinforcement. In order to adequately predict the ultimate shear and radial tension strengths of buried concrete pipe, it was necessary to develop new relations between significant variables that go beyond or extensively modify existing design methods. These are based on an extensive evaluation of new and existing tests of pipe, box sections, slabs, beams, and frames without web reinforcement that failed in shear by tests of curved slabs that failed in radial tension without simultaneous application of shear and by pipe industry design practices derived from accumulated test data. Design relations proposed for crack control also differ significantly from crack-control criteria available in existing standards. They also have been based on extensive tests of pipe, box sections, and slabs. The design method may also be applied to pipe for three-edge bearing strength and for buried box sections.

During the past 10 years, the American Concrete Pipe Association (ACPA) has sponsored several long-range research projects to develop improved methods for determining earth loads and pressure distributions on buried concrete pipe. As a part of this research effort, Heger and McGrath developed an accurate method for determining the ultimate strength and crack-control characteristics of reinforced-concrete pipe under any load distribution ($\underline{1}$). This work forms the basis of a direct design method that has recently been adopted by the Rigid Culvert Liaison Committee of the American Association of State Highway and Transportation Officials (AASHTO) for incorporation in a new Section 1.15.4 of the AASHTO bridge specifications ($\underline{2}$) covering the design of concrete pipe. The new Section 1.15.4 (3) is entitled Reinforced Concrete Pipe, Precast. It includes two alternative design methods for buried concrete pipe:

- 1. Indirect method: Based on pipe strength in three-edge bearing tests and bedding factors that convert these test strengths to design earth loads for embankment and trench installations with various classes of bedding, and
- 2. Direct method: The pipe is analyzed for moments, thrusts, and shears produced at governing sections by the design earth load and pressure distribution. The pipe wall thickness and reinforcement are designed for adequate strength and crack control under the combined effects of the design moments, thrusts, and shears. Appropriate load and capacity-reduction factors are applied when designing for strength.

The purpose of this paper is to present and explain the new direct design method in Section 1.15.4. A comprehensive presentation of the test programs, analyses of test results, and comparisons of various test parameters with predicted results by using the design equations given below are found elsewhere $(\underline{1})$.

The analyses of test results presented elsewhere

(1) show that existing equations (2,4) for shear (diagonal tension) strength and crack control do not correlate with test strengths and would give erroneous and impractical pipe designs. In view of this, the equations for shear strength and crack control presented in this paper were developed to obtain improved correlations between predicted and test strengths. Also, an equation for radial-tension strength was developed to predict this potential mode of failure in a curved member. Equations for flexural strength were developed by using the same basic theory given (2,4) for ultimate strength of reinforced-concrete flexural members. Correlations of predicted strengths with test strengths for the above four criteria are given elsewhere (1) and are not presented here because of space limitations. Also, separate technical papers explaining the development and correlation of equations for shear strength, radial tension strength, and crack control will be presented elsewhere.

LOADS

With either the direct or the indirect design methods in Section 1.15.4, the total earth load is determined by an analysis that accounts for soilstructure interaction. The total earth load $(W_{\rm E})$ is given by the following:

$$W_{E} = F_{e}WB_{c}H \tag{1}$$

where

W = unit weight of earth (psi),

B_C = outside horizontal projection of pipe (ft), H = height of earth cover over crown of pipe (ft), and

F_e = soil-structure interaction factor.

 ${
m F_e}$ is greater than 1 for installations such as embankments, where earth adjacent to the pipe settles relative to earth supported on the pipe, and may be less than 1 in installations such as vertical wall trenches because the trench sides resist consolidation of earth over the pipe.

The determination of earth loads is not covered in detail here, since methods in current use are explained elsewhere (5). The following simplified relation is provided in Section 1.15.4 for determining F_e for an embankment or wide trench installation:

$$F_{e} = [1 + 0.2 (H/B_{c})]$$
 (2)

A maximum $F_{\rm e}$ of 1.5 is specified when side fills are not compacted, whereas a maximum $F_{\rm e}$ of 1.2 is specified for compacted side fills. When trench widths are less than the transition width, $F_{\rm e}$ is reduced as described in Section 1.15.4. Transition width is defined as the trench width for which the calculated trench $F_{\rm e}$ equals the calculated embankment $F_{\rm e}.$ Graphs and equations for determining transition width are given elsewhere $(\underline{3},\underline{5}).$

In addition to the earth load, a buried pipe is subject to its own weight $(W_{\rm p})$. Also, live loads applied on the surface may increase the earth pressure on the pipe. These effects may be approxi-

mately taken into account by distributing live load through the earth cover over the pipe in accordance with AASHTO rules (2). In this approach, the equivalent surface live load at the crown of the pipe per foot of pipe length (W $_{\rm L}$) is treated as additional total earth load to obtain a total equivalent external pressure load (W $_{\rm T}$) for use in designing the pipe:

$$W_{T} = W_{E} + W_{L} \tag{3}$$

where

 W_E = total weight of earth on unit length of buried structure (lbf/ft),

 W_L = total live load on unit length of buried structure (lbf/ft), and

 W_{T} = total live and earth load applied on pipe (lbf/ft).

DESIGN APPROACHES

The use of the traditional indirect design method avoids the need to estimate the earth pressure distribution and then to calculate moments, thrusts, and shears in the pipe because it provides empirically determined bedding factors that relate total earth load to the concentrated load and reactions applied in the three-edge test (5). This approach has the advantage of simplicity and a direct relationship to test strengths. However, it has obvious limitations since it cannot accurately reflect the many different conditions that may affect structural behavior of pipe in the ground.

The availability of more rigorous analytical soil-structure interaction theories based on finite-element computer methods and an improved understanding of the ultimate-strength and crack-control characteristics of reinforced-concrete members suggests that a more accurate procedure can be devised to achieve more economical designs for buried reinforced-concrete pipe. This approach has been under development by ACPA in several long-range research programs that have been sponsored and carefully monitored during the past 10 years.

At present, a practical computer program that

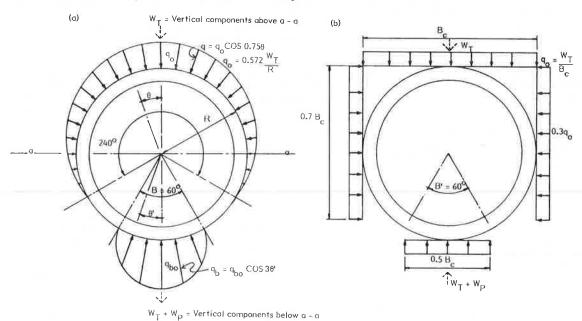
determines total earth load and pressure distribution in many typical embankment and trench installations has been developed and partly tested and evaluated by comparison of calculations and a limited number of test results. The program, Soil-Pipe Interaction Design and Analysis (SPIDA), also contains subroutines for determining the moments, shears, and thrusts caused by the applied earth pressure and for designing the required circumferential reinforcement when a trial wall thickness is specified for circular pipe. In addition, pipe strength is checked for adequate resistance to failure in shear (diagonal tension), radial tension, and compression, and pipe reinforcement is checked for adequate crack control. Design is automatically modified, if necessary, to meet all of these criteria.

The first part of ACPA's computerized design system that determines earth load and pressure distribution based on soil-structure interaction analysis is still being evaluated and tested. Sufficient results are not yet available to provide improved procedures for determining total earth load and pressure distribution. Thus, in applying the direct design method at present, loads and pressure distributions must still be determined by previously available approximate methods. Total earth load is estimated by using the Marston-Spangler theory (5)or the soil-structure interaction factor (Fe) described above. Earth pressure distribution is estimated by using a method suggested by Olander (6) or by uniformly distributed vertical and lateral pressures (7). Such distributions are shown in Figure 1 for the traditionally defined (5) Embankment Class C bedding and in Figure 2 for Embankment Class B bedding. In Figure 1(a), the applied earth pressure is modified slightly from the assumptions proposed by Olander by limiting the lateral pressure to that provided when the pressure bulb extends 30 degrees below the springline. Different pressure distributions are appropriate for the Class B and Class C beddings shown in the ACPA design manual (5) for trench installations.

ANALYSIS OF LOAD EFFECTS

Moments, thrusts, and shears in the pipe are deter-

Figure 1. Earth pressure assumptions for Embankment Class C bedding.



mined by elastic analysis of the pipe ring under the assumed earth pressure and pipe weight. The effect of cracking on pipe stiffness is taken into account in analyses performed with SPIDA but is usually neglected when pipe design is based on elastic analyses that use the estimated earth pressure distributions described above. This follows common structural engineering practice in design of other structures. The results of elastic analyses are given elsewhere $(\underline{1,6})$ for several "bulb"-type distributions like the one shown in Figure 2(a) and in the paper by Paris (7) for uniformly distributed pressure assumptions like the ones shown in Figures 1(b) and 2(b).

DIRECT DESIGN METHOD

Once the moments, thrusts, and shears produced by earth load, surface load, and pipe weight are determined throughout the pipe structure, the pipe is designed by using an appropriate load factor (ultimate strength) design procedure for determining the required combination of wall thickness, concrete strength, and reinforcement characteristics at governing design sections. The conventional design approach for any reinforced-concrete structure is to select a geometry of structure, trial wall thickness, concrete strength, and reinforcement type and to calculate the required area of reinforcement at governing design sections based on factored values of the moments, thrusts, and shears obtained in the analysis.

Ultimate Flexural Strength Based on Tensile Reinforcement Yield

Usually, reinforcement area is first selected based on ultimate flexural strength. The required reinforcement is as follows:

$$A_{s}f_{y} = g\phi d - N_{u} - \left\{ g[g(\phi d)^{2} - N_{u}(2\phi d - h) - 2M_{u}] \right\}^{\frac{1}{2}}$$
(4)

where

$$g = 0.85bf_C^*$$
;

Figure 2. Earth pressure assumptions for Embankment Class B bedding.

 W_T = Vertical components above a - a = q_o COS 0.750 = 0.572 WT Вс 240° 1200 $q_b = q_{bo} COS 1.50^{\circ}$

W_T + W_P = Vertical components below a - a

 A_S = tension reinforcement area on width b (in²);

b = width of section that resists M, N, V (in);

d = distance from compression face to centroid of tension reinforcement (in);

 $f_C' = design$ compressive strength of concrete (lbf/in2);

f_v = specified yield strength of reinforcement (lbf/in2);

h = overall thickness of member (wall thickness) (in);

 M_{ij} = ultimate moment acting on cross section of width b (in lbf);

 N_{ij} = ultimate load axial thrust acting on cross section of width b (lbf); and

 ϕ = capacity-reduction factor for variability in manufacture.

A form of the above design equation that is more familiar to many structural engineers is the following:

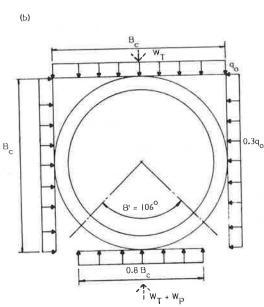
$$A_{s}f_{y} = \{M_{u} - N_{u}[(h - a)/2]\}/(\phi d - a/2)$$
(5)

where a = $(f_yA_s + N_u)/g$. The use of Equation 4 instead of Equation 5 avoids the trial calculations often needed to obtain the depth of the stress block (a) and thus is more appropriate for computerized solutions.

Load and Capacity-Reduction Factors

The ultimate bending moment $(\mathtt{M}_{\mathbf{u}})$ and compressive axial thrust $(\mathtt{N}_{\mathbf{u}})$ are obtained by multiplying the moment and thrust at governing sections as obtained in the above-described elastic analysis by an appropriate load factor. A load factor of 1.3 is provided in the AASHTO specifications ($\underline{2}$, Table 1.2.22). This table is referenced in the new section (3).

Special provisions for capacity-reduction factors are given in the new section (3). A capacityreduction factor o no greater than 1.0 is proposed for flexure and no greater than 0.90 for shear. The 1.0 capacity-reduction factor is justified because of the good quality control achieved in plant-



manufactured products such as pipe. Although philosophically $\boldsymbol{\phi}$ should always be less than 1.0 (since it is impossible to assure perfect construction of any structure), the current provision of φ = 1.0 for prestressed plank in the AASHTO bridge specifications (2) is cited as a precedent. I recommend that actual designs sometimes be based on \$\phi\$-values less than 1.0 to reflect the tolerances in reinforcement placement and the reinforcement strength actually expected in manufacture. However, pipe made with some reinforcements, such as welded wire fabric, has ultimate flexural test strengths essentially equal to the ultimate strength of the reinforcement. This is substantially higher than the strength indicated by the yield strength of the reinforcement. Where such pipe has full-strength splices or splices located in regions of lower expected stress and properly controlled tolerances for reinforcement placement, the use of 6 = 1.0 combined with a maximum reinforcement strength of f. or 65 000 psi, whichever is less, can be justified for determining reinforcement area based on ultimate flexural strength.

Minimum Reinforcement

In some practical cases involving shallow burial without surface loading, reinforcement areas obtained by using Equation 4 may be less than desirable minimum reinforcement areas for handling. Thus, the following relations for determining minimum reinforcement were recommended by the ACPA Technical Committee to provide adequate handling strength in reinforced-concrete pipe:

For inside face of pipe:

$$A_s = (S + h)^2 / 65 000 \tag{6}$$

For outside face of pipe:

$$A_s = 0.75(S + h)^2/65\ 000 \tag{7}$$

For elliptical reinforcement in circular pipe and for 33-in-diameter pipe and smaller with a single cage of reinforcement in the middle third of the pipe wall:

$$A_s = 2(S+h)^2/65\ 000 \tag{8}$$

where S is the horizontal span between the inside of the walls in inches. In no case shall the minimum reinforcement be less than 0.07 in²/linear ft. This is considered a lower limit of reinforcement areas that are practical for manufacturing.

Other Design Criteria

The above trial pipe design, based on requirements for ultimate flexural strength, is checked to determine whether other ultimate-strength or service-load criteria may require design modifications. Other ultimate-strength criteria include radial tension strength, flexural strength as limited by concrete compression strength instead of tensile yield of reinforcement, and shear (diagonal tension) strength. The primary service-load criterion is crack control. This is governed by the arrangement and type of reinforcement as well as by the reinforcement area, as described later.

Radial Tension

Bending moments that produce tension on the inside of a pipe also produce radial tension that is maximum in the concrete wall between the reinforcement

and the neutral axis of the pipe ring. This radial tension may be envisioned as the distributed internal force that prevents the curved tension reinforcement from straightening. The nominal radial stress is as follows:

$$t_{ru} = (M_u - 0.45N_u d)/\phi b d I_s$$
 (9)

where $\mathbf{r}_{\mathbf{S}}$ is the radius to the inside reinforcement in inches.

Based on a limited number of special curved-slab tests and experience in three-edge bearing testing of concrete pipe $(\underline{1},\underline{8})$, the nominal radial tension strength of concrete pipe, as determined by Equation 9, should be limited to the following (8):

$$t_{rc} = 1.2F_{pr}(f_c')^{1/4}$$
 (10)

The term F_{pr} is a factor used to reflect the variations that local materials and manufacturing processes can have on the tensile strength of concrete. Experience within the precast concrete pipe industry has shown that such variations are significant. F_{pr} may be determined from three-edge bearing test data:

$$F_{pr} = [(DL_{ut} + 9W_p/S)/1230r_s d(f_c')^{\frac{1}{2}}] S(S+h)$$
 (11)

where $\mathrm{DL_{ut}}$ is a statistically valid test strength obtained by using ASTM C655 and test pipe with inner reinforcement areas equal to or greater than $\mathrm{A_S}$ from Equation 12 below and $\mathrm{W_p}$ is the weight of a unit length of pipe.

Once determined, $F_{\rm pr}$ may be applied to other pipe built by the same process and with the same materials. If Equation 11 yields values of $F_{\rm pr}$ less than 1.0, a value of 1.0 may still be used if a review of test results shows that the failure mode was diagonal tension and not radial tension. $F_{\rm pr}=1.0$ gives predicted three-edge bearing strengths of about 0.9 times the highest strength classification (Class V in ASTM C76), which further justifies $F_{\rm pr}=1.0$ for use in design where specific tests for $F_{\rm pr}$ are not available. The radial tension strength given by Equation 10

The radial tension strength given by Equation 10 will exceed the radial tension stress given by Equation 9 if the maximum strength of reinforcement provided to resist $M_{\rm u}$ that produces tension on the inside of a pipe (Equations 4 or 5) is no greater than the following:

$$\max A_s f_y = 1.33 \, \text{br}_s (f_c')^{1/3} F_{pr}$$
 (12)

For b = 12 in,

$$\max A_s f_y = 16 r_s (f_c')^{\frac{1}{2}} F_{pr}$$
(13)

Also, max $f_{\text{C}}' = 7000$ psi, since neither test data nor sufficient experience is available for pipe with concrete strengths above this value.

When ${\bf A_sf}_{\bf y}$ obtained in Equation 4 (or Equation 5) exceeds ${\bf maxA_sf}_{\bf y}$ given by Equation 12, either radial ties must be used to preclude radial tension failure, as described later, or pipe wall thickness may be increased to reduce the ultimate tension force in the reinforcement for a given ${\bf M_u}$ required in the design.

Ultimate Flexural Strength Based on Concrete Compressive Strength

The ultimate strength of most conventional reinforced-concrete pipe is seldom limited by the ability of the pipe to resist concrete compressive failure prior to tensile yield of the reinforcement. However, in special designs for deep burial, flex-

ural compressive capacity may have to be investigated. Flexural compressive strength will not govern design of a pipe if the maximum reinforcement strength required by Equation 4 (or Equation 5) does not exceed the following:

$$\max A_s f_y = [(5.5 \times 10^4 \text{g}/\phi \text{d})/(87\ 000 + f_y)] - 0.75 N_u$$
 (14)

 $g' = b f_c' \{ 0.85 - 0.05 [(f_c' - 4000)/1000] \}$

 $g'_{max} = g = 0.85 bf'_{c}$

 $g'_{min} = 0.65 \, b \, f'_{c}$

The above equations reflect the provisions contained in the general reinforced-concrete design Section 1.5.32 (2) for compressive strength of reinforcedconcrete flexural members.

If ${\rm A_Sf_y}$ obtained in Equation 4 exceeds ${\rm maxA_Sf_y}$ given by Equation 14, compression reinforcement and radial ties to support the compression reinforcement against buckling are required. This rarely occuring special case requires a design investigation based on provisions given by AASHTO (2)for design of flexural members with compression reinforcement. In cases where the axial compressive thrust (N_D) predominates over the bending moment (M1), the pipe may have to be designed like a column subject to combined bending and axial load.

Shear (Diagonal Tension)

The shear design criteria given in Section 1.15.4 are based on an extensive new study of the shear strength of pipe, box sections, and slabs (1,9). The new shear tests of slabs as well as the extensive review of previous shear tests of pipe, box sections, slabs, and beams under both concentrated loads and uniformly distributed loads $(\underline{1})$ show that the general provisions for shear strength (2,4) give excessively high strengths (unconservative) for certain flexural members under concentrated loads (particularly pipe in three-edge bearing) and excessively low strengths (too conservative) for flexural members with distributed loads (such as buried pipe and box sections).

New shear strength relations are given $(\underline{1},\underline{9})$ that provide an accurate evaluation of shear strength for both of these load conditions. These new provisions are particularly applicable to pipe, box sections, slabs, and other flexural members without web reinforcement and with reinforcement ratios below about 0.015. Shear strength equations in existing design standards (2,4) give erroneous results for such members. Another new method (10) gives results that are more accurate than existing equations (2,4) for members with concentrated loads but much less accurate and too conservative for members with distributed loads.

The direct design method for determining shear strength of buried pipe given in Section 1.15.4 first locates the critical section for shear and then compares the shear strength of that section with the factored shear force at the same section as follows:

1. The critical section for shear strength is that in which the ratio $M_u/V_u\phi d$ is 3.0 (V_u is the ultimate shear force acting on a cross sec tion of width b in pounds). For buried pipe with distributed bedding, this section is not the section of maximum shear stress resultant (V_umax). This section is located by calculating the M_u/V_u ϕd ratio at several trial locations as determined from

the shear and bending-moment diagrams for the earth pressure distribution used in the design analysis described previously. For most types of bedding, the critical section is usually located in a region between 10 and 20 degrees from the invert.

2. The calculated shear strength at the above critical section is the minimum shear strength of the pipe. This also applies to structures with straight flexural members such as box sections and slabs under high uniformly distributed loads. This minimum strength is termed the basic shear strength (V_b) and is given as follows:

$$\phi V_b = \phi b d(f_c')^{1/4} (1.1 + 63 p) (F_d/F_c F_N) F_{pv}$$
(15)

where

 V_{b} = basic shear strength of sections where

 $\begin{array}{ll} M_U/V_U\phi d \geq 3.0\,, \\ p = A_S/\phi \, bd \quad \mbox{(it is conservative to ne-} \end{array}$ glect the use of ϕ in this equation and in Equations 16 and 17 below),

 $p_{\text{max}} = 0.02,$

f_C max = 7000 psi, and F_C = proces

 r_{pv} = process and material factor for radial tension strength that differs from theoretical strength.

The constants F_d , F_C , and F_N are modifying factors for crack depth, curvature, and axial thrust, respectively. Shear strength is reduced by flexural cracking. Wall thickness affects crack depth, and thinner walls, which have a smaller ratio of crack depth to crack spacing, can support a higher nominal shear stress than thicker walls. Curvature results in an increase in circumferential shear stress over the stress given by the conventional equation for nominal shear stress in prismatic members (v = V/bd) due to the additional relationship of change in thrust to change in bending moment in a curved member. Compressive axial thrust increases shear strength and tensile axial thrust reduces it relative to a flexural member without thrust. The following relations for the above modifying factors were determined semiempirically from derived relations and evaluations of conventional and special tests of pipe, slabs, and box sections, as described elsewhere (1):

$$F_d = 0.8 + (1.6/\phi d)$$
 $F_d \max = 1.25$ (16)

$$F_c = 1 \pm (\phi d/2r) \tag{17}$$

where r is the radius to the centerline of the pipe wall in inches and the plus indicates tension on the inside of the pipe and the minus, tension on the outside of the pipe.

For compressive thrust $(N_{ij} is +)$,

$$F_N = 1.0 - 0.12(N_u/V_u)$$
 $F_N \min = 0.75$ (18)

For tensile thrust $(N_{ij} \text{ is -})$,

$$F_N = 1.0 - 0.24(N_u/V_u)$$
 up to $(N_u/V_u) = 1.0$ (19)

The term $\mathbf{F}_{\mathbf{DV}}$ is a factor used to reflect the variations that local materials and manufacturing processes can have on the tensile strength of concrete. Experience within the precast concrete pipe industry has shown that such variations are significant. F_{pv} may be determined with Equation 20 below when a manufacturer has a sufficient amount of test data on pipe that fails in diagonal tension to determine a statistically valid test strength (DLut) by using the criteria given in ASTM C655.

$$F_{pv} = F_c(DL_{ut} + 11W_p/S)S/293F_d(1.1 + 63p)d(f_c')^{1/2}$$
(20)

Table 1. Methods of obtaining B₁ and C₁.

Type Reinforcement	B_1	C_1
Smooth wire or plain bars Welded smooth wire fabric, 8-in maximum spacing of longitudinals	$(0.5 t_b^2 s_{\ell}/\pi)^{1/3}$ 1.0	1.0
Welded deformed wire fabric, deformed wire, deformed bars, or any reinforcement with stirrups anchored thereto	$(0.5 t_b^2 s_{\ell}/n)^{1/3}$	1.9

Notes: Use n=1 when the inner and the outer cages are each a single layer. Use n=2 when the inner and the outer cages are each made from multiple layers. For type-2 reinforcement that has $(t_0^2 \log)/n > 3.0$, also check A_{SCT} by using coefficients B_1 and C_1 for type-3 reinforcement and use larger value for A_{SCT} .

Once determined, F_{pv} may be applied to other pipe built by the same process and with the same materials. $F_{pv}=1.0$ gives predicted three-edge bearing test strengths in reasonably good agreement with pipe-industry experience, as reflected in the pipe designs for Class 4 strengths given in ASTM C76. Thus, it is appropriate to use $F_{pv}=1.0$ for pipe manufactured by most combinations of process and local materials. Available three-edge bearing test data show minimum values of F_{pv} of about 0.9 for poor-quality materials and/or processes, as well as possible increases up to about 1.1 or more with some combinations of high-quality materials and manufacturing process.

Prior to making the above-described check for ultimate shear strength, the reinforcement area should be calculated based on both ultimate flexural strength (Equation 4) and the crack-control criteria described below. The larger of these required reinforcement areas should be used for the reinforcement ratio in Equation 15.

If the shear strength given by Equation 15 is less than the shear force (V_U) at the critical section for shear (where $\rm M_U/V_U\phi d=3.0)$, increased shear strength may be obtained by increasing $f_C^{'}$ (but $f_C^{'}$ may not be taken greater than 7000 psi), by increasing $\rm A_S^{'}$ (but $\rm A_S^{'}$ may not be taken greater than 0.02bd), or by the use of radial ties (stirrups) as described later.

Shear strength at sections where $M_u/V_u\phi d < 3.0$ may be determined by using the following more general expression for shear strength:

$$\phi V_c = 4\phi V_b (M_u/V_u \phi d + 1) \qquad M_u/V_u \phi d \le 3.0$$
 (21)

$$\max \phi V_c = 4.5\phi \operatorname{bd}(f_c')^{1/2}/F_N \tag{22}$$

Design investigations have shown that the overall shear strength of buried pipe and box sections is governed by the section where $\rm M_u/V_u\phi d=3.0$ in a region where $\rm M_u$ produces tension on the inside of the pipe.

Crack Control

The proper service-load performance of reinforced-concrete pipe requires that the reinforcement area, spacing, and type be adequate to limit flexural cracking to acceptable widths. Reinforcements with a deformed surface or with welded cross wires at proper longitudinal spacing exhibit superior crack-control capability compared with smooth wire or bar reinforcements, primarily because they produce a greater number of more closely spaced cracks of smaller width than those that occur at the same stress with smooth reinforcements. The following semiempirical relationship, based in part on derived relations between variables and in part on analysis of pipe, box section, and slab flexural behavior in tests (1), provides a design procedure for limiting

crack width in buried concrete pipe:

$$F_{cr} = (B_1/30\ 000\ dA_s) \left[\left(\left\{ M_s + N_s \left[d - (h/2) \right] \right\} / ij \right) - C_1 bh^2 (f_c')^{1/2} \right]$$
 (23)

where

 $j\approx 0.74 + 0.1e/d \qquad j_{max}\approx 0.9$

$$e = (M/N) + d - (h/2)$$
 $i = 1/[1 - (jd/e)]$

and where

B₁ = crack-control coefficient for effect of cover and spacing of reinforcement;

C₁ = crack-control coefficient for type of reinforcement;

e = thrust eccentricity, as given by Equation 23
 (in);

i = coefficient for effect of axial force at service load stress (f_s);

j = coefficient for moment arm at service load stress;

 M_S = service load moment acting on cross section of width b (in•lbf); and

 ${\rm N_S}$ = service load axial thrust acting on cross section of width b (lbf).

(The approximations for j and i are only valid when e > 1.15d.)

 $\rm B_1$ and $\rm C_1$ are obtained from Table 1 where n is number of layers of reinforcement in a cage (type 1 or 2); $\rm s_1$ is the spacing (longitudinal) of circumferential wires or bars, in inches; and $\rm t_b$ is the clear cover distance from tension face of concrete to tension reinforcement, in inches.

The term $F_{\rm Cr}$ is a crack-control index factor. When $F_{\rm cr}=1.0$, the reinforcement area $(A_{\rm S})$ will produce an average maximum crack width = 0.01 in. If the value calculated for $F_{\rm Cr}$ is too high, the designer may improve crack control by using a type of reinforcement with higher bond, a closer spacing of circumferential bars or wires (but not less than about 2 in), multiple layers of reinforcement, or a larger reinforcement area $(A_{\rm S})$ than the minimum area required for ultimate flexural strength. Note that the maximum $A_{\rm S}$ limits given by Equations 12, 13, and 14 do not apply when $A_{\rm S}$ is increased for crack control.

If the designer wishes to tighten crack control, $F_{C\Gamma}$ may be reduced somewhat but should probably not be taken less than about 0.7; for less stringent crack control, $F_{C\Gamma}$ may be increased somewhat but probably not more than 1.5. This suggested range in $F_{C\Gamma}$ reflects the fact that the data used to develop empirical constants in the above equations were from 0.01-in crack-strength tests of pipe, box sections, and slabs (1). If the designer wishes to account for variability in crack formation and control to minimize the occurrence of crack widths exceeding 0.01 in, $F_{C\Gamma}$ = 0.9 may be used.

In tests, the use of radial ties (stirrups) improves the crack control provided by smooth reinforcements. Thus, the highest crack-control coefficients recommended for deformed reinforcements, B_1 and C_1 , may also be used for pipe with any reinforcement type plus radial ties.

DESIGN OF RADIAL REINFORCEMENT

Occasionally, pipe subject to very heavy loads requires circumferential tensile reinforcement strengths that exceed the limits given previously for the radial tension (Equation 12), concrete compression (Equation 14), or shear (Equations 15 and 21) strengths of pipe without radial ties. In such cases, the circumferential tensile reinforcement

required for ultimate flexure (or crack control) may be provided together with radial ties. Since pipe walls generally are thin, ties are usually designed to be spaced at their maximum effective (allowable) circumferential spacing. Because these ties resist the combined effects of shear and radial tension, which makes the inclination of a potential diagonal crack flatter than the 45 degree angle assumed with prismatic members, their maximum allowable spacing is increased over the nominal 0.5d maximum stirrup spacing permitted for prismatic members by AASHTO (2). Thus, in typical pipe, the maximum allowable circumferential spacing is taken as follows:

$$\max s = 0.75 \phi d \tag{24}$$

Longitudinal spacing of ties must coincide with longitudinal spacing of inside circumferential tensile reinforcement. When radial ties are needed to resist radial tension, each line of inside circumferential reinforcement must be restrained by radial ties anchored around the circumferential reinforcement and into the compression zone on the opposite side of the pipe wall. Anchorage strength must at least equal the effective ultimate tensile strength $(f_V A_V)$ used to design the tie. In most practical cases, f_v is probably limited by the anchorage strength rather than the yield strength (f_{vy}) of the tie material. In this case, the anchorage strength of any specific type of tie should be proved by tests.

Ties may be designed for adequate radial tension strength and combined shear and radial tension strength as follows (8):

$$A_{vr} f_v = 1.1s(M_u - 0.45N_u \phi d)/r_s \phi d$$
 (25)

Shear and radial tension ties,

$$A_{v}f_{v} = (1.1s/\phi d)(V_{u}F_{c} - \phi V_{c}) + A_{vr}f_{v}$$
(26)

Vc is given by Equation 21, except that

$$\max \phi V_c = 2.0 \phi b \, d(f_c')^{\frac{1}{2}} \tag{27}$$

See the report by Heger and McGrath (1) for the derivation of the above equations for radial ties. The maximum contribution of concrete shear strength after diagonal cracking, $V_{\rm C}$ in Equation 26, is taken to be the same as concrete strength used in the design of web reinforcement for prismatic reinforced-concrete flexural members, as given by Equation 27 (2).

Equations 25 and 26 have been evaluated based on a very limited number of tests of pipe with ties in three-edge bearing and curved beams with ties, supported and loaded to simulate the invert region of a pipe in three-edge bearing. See the report by Heger and McGrath $(\underline{1})$ for references and further discussion. Additional confirmation and experience would be desirable to validate the use of these equations for general design of highly loaded pipes. The user is especially cautioned to use a conservative value of f_{V} based on tie-anchorage strength unless limited by tie-material strength.

CONCLUSIONS

The design method presented in this paper was developed to represent the structural behavior of concrete pipe as accurately as possible and still be practical. The conventional ultimate flexural strength theory for under-reinforced sections was found to provide a practical basis for the design of reinforcement in most applications. The maximum allowable yield strength is increased to 65 000 psi for welded wire fabric reinforcement, and less allowance for capacity reduction due to construction variations often is acceptable since the ultimate tensile strength of cold-drawn wire reinforcement is reached in test pipe that fails in flexure.

Accurate representation of strength in shear requires extensive modification of existing shearstrength methods. The procedure suggested here, although somewhat more complex than methods in existing standards (2,4), predicts the lower shear strength of pipe under concentrated load (three-edge bearing test) as well as the much higher shear strength of pipe under distributed load (buried pipe) (9). The latter result occurs because the critical section for shear is found at a location where the $M_u/V_{u}\phi d$ ratio is 3.0. This is the location of both maximum shear and maximum moment in the three-edge bearing test. However, it is not the location of maximum shear for load distributions representative of buried concrete pipe.

Specific consideration of radial tension as a separate ultimate strength limit was suggested for the first time by Heger and McGrath (1,8) and is incorporated in the design method described here. It is shown that if the required tensile strength of reinforcement provided to resist flexural tension on the inside of a pipe at invert and crown does not exceed a specific radial tension limit, radial tension strength will not limit the flexural strength

Although usually only needed for special designs with concentrated bedding and/or very high fills, radial ties may be provided to increase the flexural strength of pipe beyond limits defined by radial tension, shear, or compressive strength. Design equations are provided to determine spacing and area of such reinforcement ties. Anchorage requirements for ties are also defined.

Extensive modifications in existing equations for crack control given elsewhere (2,4) are also necessary to predict accurately the crack-control behavior of buried pipe under service loads. New crack-control equations were developed applicable to the type and arrangement of reinforcements typically used in precast concrete pipe, and these indicate whether the reinforcement area required for ultimate strength needs to be increased for proper crack control. Equations are formulated to permit the design engineer to vary the basic crack-control criterion-average maximum crack width above or below an index value of, 0.01 in--that has been widely used in three-edge bearing pipe tests.

Once the magnitude and distribution of earth pressure caused by earth and surface loads have been established with sufficient accuracy, the pipe may be analyzed by conventional methods of elastic stress analysis to obtain moments, thrust, and shears that act at all sections around the pipe. The design procedures presented in this paper may then be used to calculate the required wall thickness, concrete strength, and reinforcement area and strength or to evaluate accurately the expected minimum strength of an existing design. Furthermore, existing reinforced-concrete design methods, as available elsewhere (2,4), do not provide suitable procedures for the design of pipe structures. No provisions are included in these standards for determining radial tension strength, and the procedures for shear and crack control do not reflect the actual performance of buried pipe. Various arbitrary limits and design provisions are not appropriate for design of buried pipe.

The design methods presented here may also be used to design pipe for three-edge bearing strength and to design prismatic structures such as buried box sections, slabs, or one-way spanning footings without web reinforcement.

FURTHER RESEARCH

I am currently directing the next phase of ACPA's long-range research program at Simpson Gumpertz & Heger Inc. (SGH), the development of a direct soilstructure interaction analysis for earth loads, earth pressure distributions, and moments, thrusts, and shears in a buried concrete pipe. This involves development of a finite-element representation of the soil and the pipe and a computerized analysis of the system as it is loaded incrementally by the soil and surface loads. Ernest Selig is consultant to SGH on the soil model and its properties. As mentioned previously, the computer program that results from this effort will be known as SPIDA and will provide a direct design for a buried pipe with specified earth cover, bedding, and pipe conditions.

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Abridgment

Behavior of Aluminum Structural Plate Culvert

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A corrugated aluminum culvert 17 ft 10 in high with a 28-ft 6-in span was instrumented to obtain measurements of strain and displacement during backfilling and under static live load. Values of circumferential bending moment and thrust at 16 locations spaced around the structure's circumference at midspan are reported for each 2 ft of backfill from the springline to 2 ft over the crown. Despite bending moments 70 percent of the fully plastic value and stresses exceeding the nominal yield point of the aluminum, it is concluded that the structural behavior is satisfactory. Discrepancies between measured values and design predictions are discussed.

Corrugated metal culverts can be economical replacements for short-span bridges and have been used for spans as long as 51 ft $(\underline{1})$. Traditionally, culvert design has been largely empirical, but with the increasing demand for large-span structures the need for a rational analytical procedure has grown. The purpose of the research described here was to obtain strain and displacement measurements on a typical structure to provide data for comparison with ana-

lytical predictions. The work is described completely elsewhere $(\underline{2})$. The structure is a 28.5-ft span pipe arch with a rise of 11 ft 9 in and a total height of 17 ft 10 in. The invert length is 140 ft. The structure was manufactured by Kaiser Aluminum and Chemical Sales, Inc., which contributed to this research.

The structure carries Van Campen Creek under State Route 275 in the town of Friendship, New York. With a filled invert, the culvert provides a clear opening of 346 ft². It is constructed of 0.175-in aluminum (5052-H141 alloy) structural plate with corrugations of 9-in pitch and 2.5-in depth. Bulb angle stiffening ribs (6061-T6 alloy) were bolted to the crown on 2-ft 3-in centers. Seven plates were assembled with 0.75-in diameter galvanized steel bolts on 9.75-in centers to form a complete circumference of the structure as shown in Figure 1. Circumferential seams are staggered.