Mathematical Model of Inland Waterway Port Operations

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An inland waterway port model suitable for port planning and operations studies is developed. The model design is based on a critical review of the literature and of available data sources and on field observations of port operations at typical inland waterway terminals. The model breaks port operations down into seven elementary processes. Submodels of each process are developed by using queuing theory, probability, and other appropriate tools.

Inland waterway planners must respond to a variety of challenges--economic, environmental, operational, and managerial. Many analytic methods are available to assist planners in meeting these challenges. However, these methods are notably deficient when applied to inland waterway ports.

The major objective of this research was to develop a quantitative mathematical model to study the operating characteristics of inland waterway port facilities. In particular, the model is to be used to estimate port capacity and the cost and time associated with port operations at various levels of cargo throughput. These estimates are useful for freight system policy and planning studies. The model will also determine the impacts of operational changes on port capacity and delay.

Although models have existed for some time for tow movements through inland waterway channels and locks and for operations at ocean ports, there is currently very little understanding of inland waterway port operations among waterway planners and analysts. The research described in this paper should encourage the incorporation of inland port considerations into the inland waterway planning process. The first phase of this research was limited to model development and definition, but a succeeding computer programming and model testing phase is nearing completion.

PREVIOUS RESEARCH

A comprehensive literature review revealed a scarcity of inland waterway port models. Most past research studies have concentrated on the analysis and modeling of seaports. Abstracts of 26 selected literature sources are presented in the final report (1). Recent research, such as the Inland Navigation Systems Analysis study (2) and the Mid-America Ports Study (3), was especially helpful. Ongoing activities such as the National Waterways Study and current U.S. Department of Transportation efforts in the area of energy transportation were also reviewed. This literature search verified that the model to be developed would fill a definite gap in current analytic methodology.

Probably the most valuable data source reviewed in this study is the extensive inland terminal facility inventory completed by the U.S. Maritime Administration as part of the Mid-America Ports Study. This port data base covers 1200 terminals on the Mississippi River and its tributaries and includes both physical and operational data, such as

1. Location,
2. Fleeting area characteristics,
3. Pier dimensions and construction characteristics,
4. Type of cargo handled,
5. Availability of truck and rail access,
6. Cargo storage areas, and
7. Types and characteristics of handling equipment.

These data, which include port throughput capacities, can be used as model input data for validation testing.

INLAND PORT OPERATIONS

The following inland river ports and terminals were visited during this research:

Port	Terminal
Port of Metropolitan St. Louis	St. Louis Terminals Corporation
	Tri-City Regional Port (Granite City, Illinois)
	Bulk Services Corporation
	Apex Oil Company
	Art's Fleeting Service
	Granite City Terminals Corporation
	St. Louis Grain
	Peabody Coal Company
	American Commercial Terminal
Wood River, Illinois	American Oil Company
Memphis, Tennessee	Memphis-Shelby County Port Authority
	Ten-Tex Marine, Inc.
	Mid-South Terminals Corporation
	Island Terminal Company
Pine Bluff, Arkansas	Pine Bluff-Jefferson County Port Commission
	Arkansas River Terminal (Port of Pine Bluff)
	Cargo Carriers, Inc.
	Martin Terminals Company
Little Rock, Arkansas	Inland Rivers Terminal Company
Cincinnati, Ohio	Valley Terminal Company
	Tresler Oil Company
	River Transportation Company
	Columbia Marine Services
Louisville, Kentucky	Riverway Louisville Terminal Company
	Missouri Portland Cement Company
	River Road Terminals, Inc.
	American Commercial Terminal
	Chemtec Industries, Inc.

The visits to the Port of Metropolitan St. Louis and Cincinnati included a boat tour of the port facilities.

The major purpose of these visits was to observe layout, equipment, and operating practices of a variety of ports. Emphasis was placed on characterizing the range of normal operations, observing typical practices, and obtaining photographs and plan drawings of port facilities. Labor practices, market penetration, port economics, and the interactions between port operators, fleeting and harbor services, and towing companies were also discussed.

Throughout the field observations, cooperation by the terminal operators was excellent. The operators' candid discussions of their operating experiences represent their best estimates of what constitutes "typical" conditions.

Some typical materials-handling rates at inland marine terminals for various commodities and typical crew sizes are given below (none of the terminals visited reported unloading activity for coal):

Commodity	Handling Rate (tons/h) Loading	Unloading	No. in Crew
Dry bulk	200-450	100-325	3-5
Coal	700-1400	--	5-6
Liquid bulk	275-400	250-325	2
Iron and steel	100-150	50-150	5-7
General cargo	100-200	40-175	5-7

These represent effective transfer rates rather than the instantaneous capabilities of the crews and equipment. These rates are noticeably lower than those typical of tidewater and Great Lakes ports, primarily because of lower mechanization and smaller crew size. Loading is faster than unloading due to the gravity assist.

MODEL CONCEPT

Figure 1 shows the basic elements of an inland waterway port. The port model is defined to include all cargo transfer facilities lying between designated river miles (A, B, C, and D in Figure 1). Some ports may contain only one terminal whereas more complex ports will be composed of multiple terminals. Each terminal serves one type of commodity and is usually either a loading or unloading terminal, although some terminals, such as those that handle general cargo, both load and unload barges. A terminal, then, may have one or more docks, aprons, storage areas, and waterside and landside cargo transfer facilities. Thus, the port the modeler desires to study can be represented by the number and types of terminals, their associated docks, and any fleeting areas that lie between designated river miles. The flexibility of designating specific river-mile boundaries will allow the model to be compatible with existing data collection and analysis systems.

The model is intended to apply to river ports only. Where an inland waterway port interfaces with Great Lakes or ocean shipping, the other waterborne mode will be treated similarly to a land mode. Although the model emphasizes tows rather than self-propelled vessels, it has sufficient flexibility to accommodate self-propelled vessels when necessary.

Figure 2 shows the general traffic patterns within an inland waterway port complex. The diagram indicates that tows stopping at the port will travel either to a fleeting area or directly to the appropriate terminal whereas other tows may pass directly through without interfacing with any port activity.

Figure 3 defines the terminal elements. Cargo is interchanged at the terminal either with a waterside plant or with another transportation mode. The transfer between the other mode and the inland waterway may occur directly or may be buffered through the terminal storage areas. The port activities are defined to include the transfer of cargo to or from the user's storage facilities and to or from the interfacing cargo vehicle (railcar, truck, ocean barge, etc.). Subsequent material movements within the plant and dispatching of loaded or unloaded cargo vehicles are not included because it is assumed that these activities can occur without interfering with terminal operations. On the other hand, delays that occur while equipment is being marshalled to facilitate the transfer of cargo across the system boundary will be considered within the model. Most terminals serve either a waterside user (who usually owns the terminal) or other transportation modes, but not both.

MODEL STRUCTURE AND LOGIC

As mentioned earlier, the port is defined as a set of geographically contiguous terminals. Each terminal may have several docks operating in parallel, where docks are unique to commodity groups (i.e., all handle liquid bulk, dry bulk, or general cargo). Each dock may also have several berths that operate in parallel.

This model structure permits each terminal to be modeled in considerable detail, or all terminals of a particular type can be aggregated and treated as a single terminal, depending on the desires of the analyst. When individual terminals are grouped into a single terminal (e.g., a liquids terminal), the single terminals can be modeled by considering them

Figure 1. Inland waterway port.

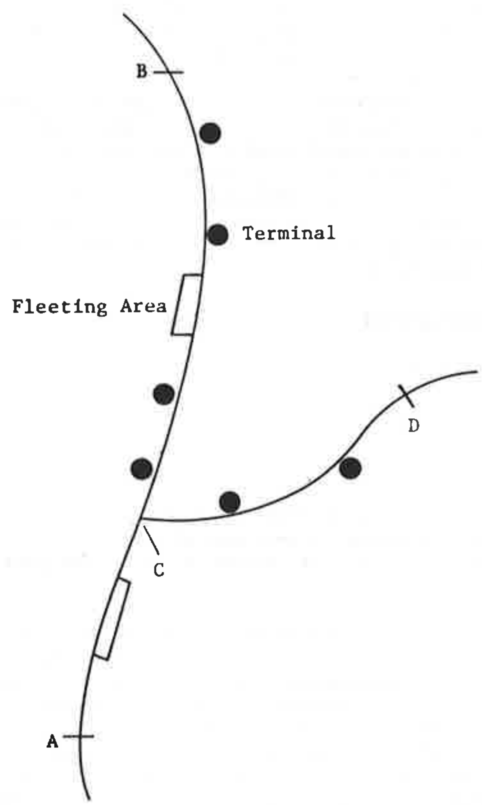
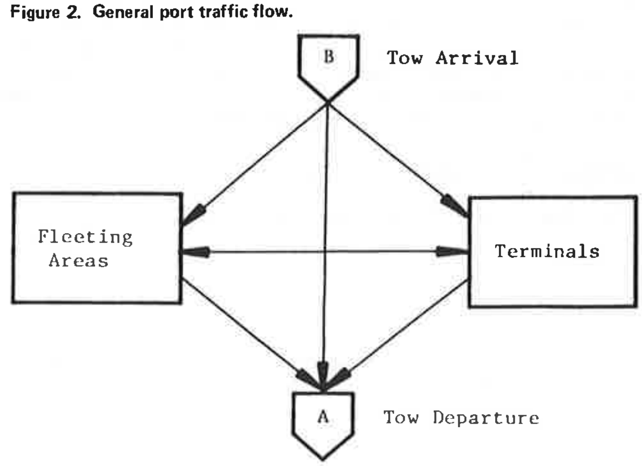


Figure 2. General port traffic flow.



to be individual docks (e.g., chemical docks, petroleum docks, etc.), each of which may have several berths.

Based on previous research and on the field observation of port operations, a port model consisting of a series of seven submodels was developed. Each submodel represents one type of activity, requires a significant amount of time to complete, and represents one segment of the total port operation.

The seven activities for which submodels were developed include

1. Tow travel,
2. Harbor tow travel,
3. Barge pickup and delivery,
4. Fleet dispatching,
5. Barge loading and unloading,
6. Dock access, and
7. Tow dispatching.

Mathematical relations for each of these seven submodels were developed and range in complexity from simple equations to more complex algorithms and queuing theory equations.

The following sections detail the mathematical relations used to represent the seven activities. In each case, the submodel produces an estimate of the average time required to complete the activity. Figures 4 and 5 show the sequence of inbound and outbound activities, respectively, and the numbers in the boxes correspond to the activity numbers and submodels defined below.

Activity 1: Tow Travel

The process for tow travel is

$$t_{1d} = X_d/v_t \quad (1)$$

where

t_{1d} = travel time to or from terminal d,
X_d = travel distance to terminal d, and
v_t = average tow travel speed within the port area.

Tow travel speed is actually a function of many variables, including towboat horsepower; tow draft, length, and width; and channel depth and width. In addition, tow speed depends on travel direction (upstream or downstream) and current velocity. However, at the scale of analysis of the port model, tow travel times are on the order of 1 h or less whereas several other port processes require many hours or even days. Hence, there is no need to incorporate a sophisticated model of tow speed. Average travel speed provides sufficient accuracy.

Several mathematical models of tow speed are available (2,4). The analyst may wish to use these models to estimate average tow speeds. Such results should be tempered by any speed restrictions that may apply to the harbor area being modeled.

Activity 2: Harbor Tow Travel

The process for harbor tow travel is

$$t_{2d} = X_d/v_h \quad (2)$$

where

t_{2d} = travel time of harbor tows to terminal d and
v_h = average harbor tow travel speed.

Harbor tows are normally small tows that consist of one to four barges pushed by a harbor boat of

Figure 3. Elements of a terminal facility.

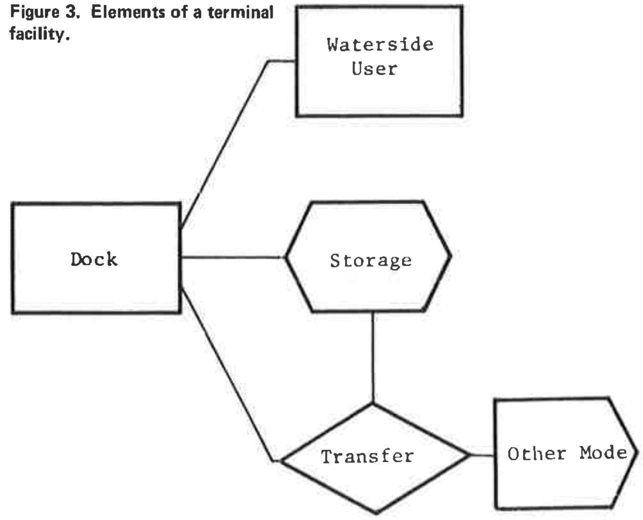


Figure 4. Inbound activities.

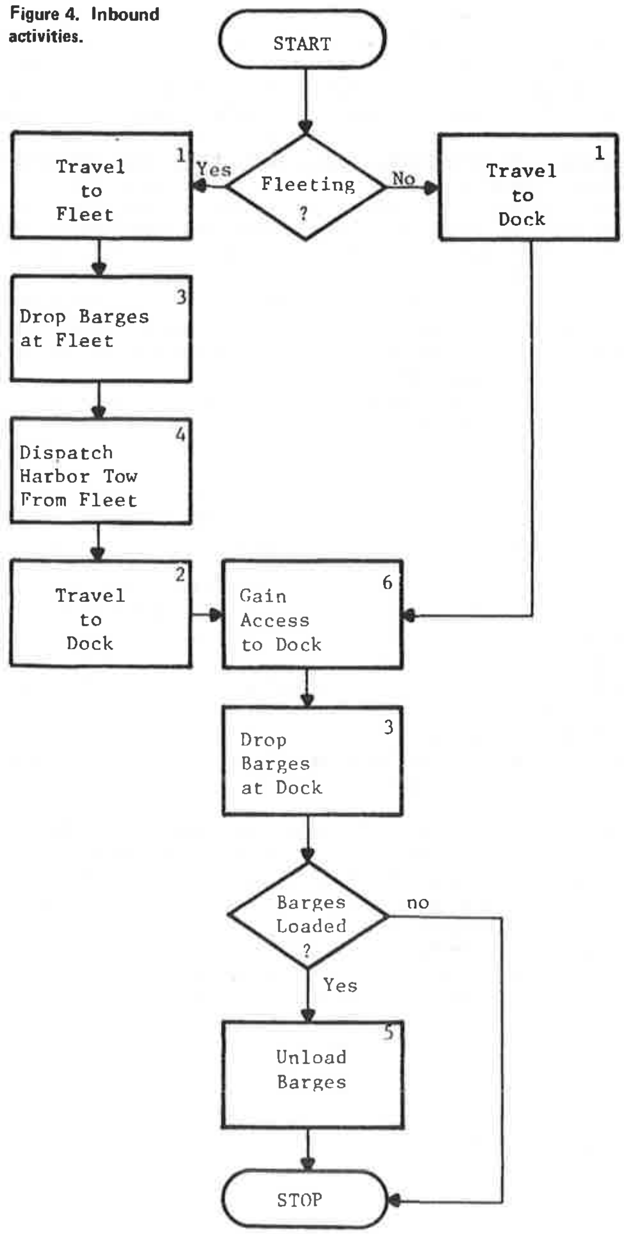
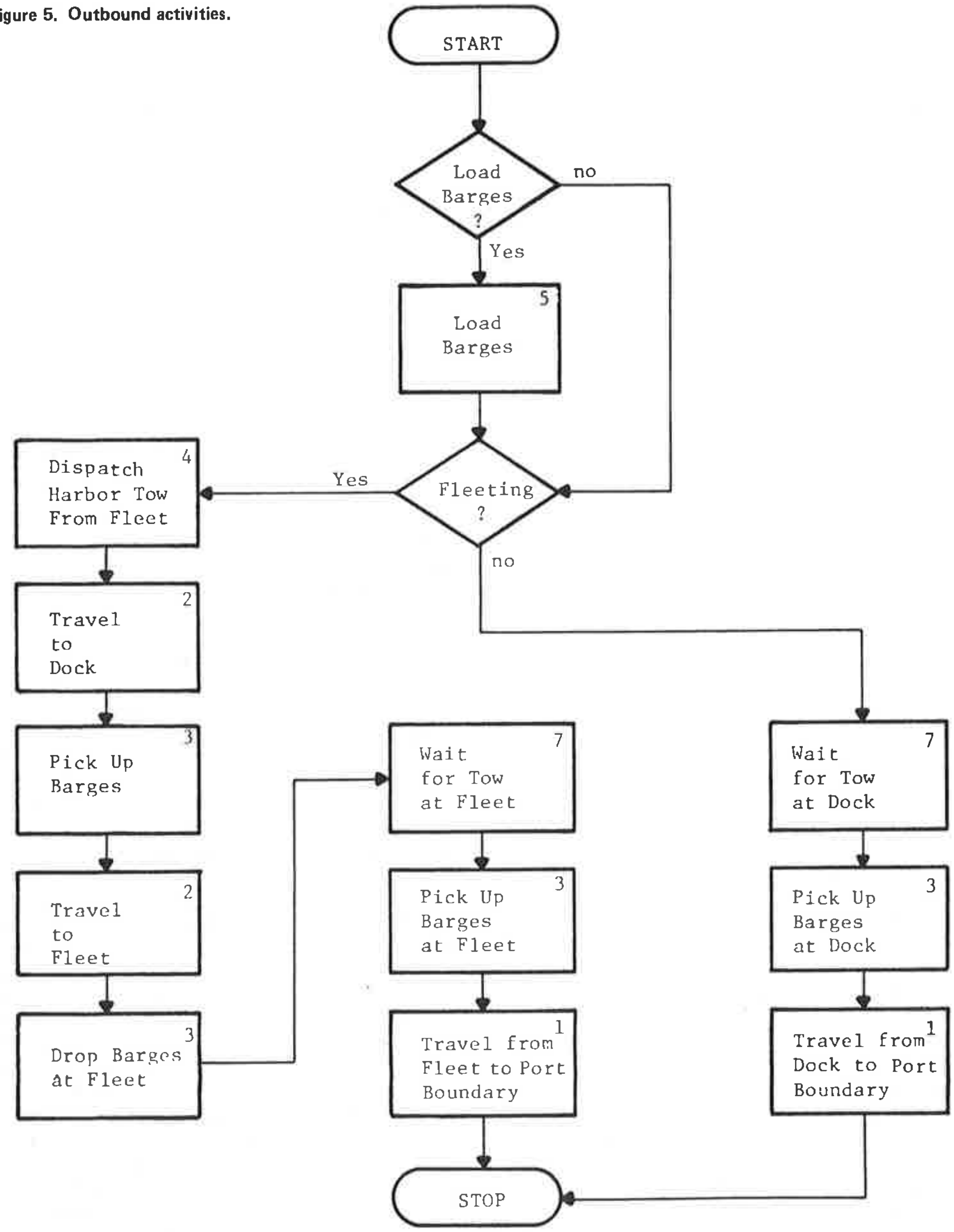


Figure 5. Outbound activities.



500-1500 hp. They may have travel speeds substantially different from those of line-haul tows and so are treated separately in the port model. However, the analyst may safely use $v_h = v_t$ for typical fleeting activities without introducing significant error.

Activity 3: Barge Pickup and Drop-Off

The process for barge pickup and drop-off is

$$t_{3i} = \sum_n p_{ni} [a_1 + a_2 (n - 1)] \quad i = d, f \tag{3}$$

where

- t_{3d} = average time required for a line-haul or harbor tow to pick up or drop off barges at terminal d,
- t_{3f} = average time required for a line-haul or harbor tow to pick up or drop off barges at fleeting area f,
- p_{nd} = probability that n barges are picked up or dropped off at one time ($\sum_n p_{nd} = 1.0$),
- a_1 = time to pick up or drop off the first barge handled, and
- a_2 = time to pick up or drop off each additional barge.

Barges are picked up and dropped off at both fleeting areas and terminals, by both line-haul tows and fleeting-service tows. This time element includes only the actual barge handling time, exclusive of any dispatching or congestion delays. Port operators indicate that this operation requires 10-20 min/barge, 15 min being a representative average. There is some time savings when more than one barge is handled during one stop, so the incremental time (a_2) is on the order of 10 min. These typical values seem to hold across all types of terminals handling a variety of products.

Activity 4: Fleet Dispatching

The process for fleet dispatching is

$$t_{4dg} = 1/(A \cdot B_{g-1} \cdot B_g) \quad g = 1, 2, \ldots G \tag{4}$$

where

t_{4dq} = average time elapsed between a request for fleeting service and the departure of a harbor tow for terminal d in priority group q,

$$A = h![(h/t_f - y_f)/r^h] \sum_{j=0}^{h=1} (r^j/j!) + (h/t_f),$$

$$B_g = 1 - \left(\sum_{i=1}^{g} y_{fi}\right)/(h/t_f),$$

$B_0 = 1$,

G = number of terminal priority groups ($G \geq 1$),

h = number of harbor boats in service at any one time ($h \geq 1$),

t_f = average harbor tow service time $= [\sum_d (Q_{fd}/b_{hd})(t_{3d} + 2t_{2d})/\sum_d (Q_{fd}/b_{hd})] + t_{3f}$,

y_{fq} = average rate of requests for service by terminal priority group g = $(2Q_g/b_{hg})/T_f$,

r = average harbor boat use = $t_f y_f < h$, $y_f = \sum_g y_{fg}$,

Q_{fd} = total barges fleeting for terminal d,

Q_g = total barges fleeted for priority group g = $\sum_{d\epsilon g} Q_{fd}$,

T_f = total time fleeting service is in operation,

b_{hd} = average number of barges per harbor tow trip to terminal d = $\sum_n np_{nd}$, and

b_{hg} = average number of barges per harbor tow trip to a terminal in priority group g = $\sum_{d\epsilon g} Q_{fd}b_{hd}/Q_g$.

The fleeting service is modeled as a nonpreemptive priority queuing system (5). There are G priority groups (group 1 has the highest priority and group G the lowest), and the requests for service from the highest-priority group in the queue are answered on a first-come, first-served basis. Assignment of priority groups is entirely left to the discretion of the model user. One assignment rule that seems to correspond to actual practice in many cases is to rank the terminals on fleeting service use and to form groups so that $Q_1 > Q_2 > \ldots > Q_G$.

In this queuing model, the system has h service channels, where h is the number of harbor boats operated by the fleeter. Each harbor boat is assumed to have the same average service time (t_f), which is the average time (weighted by number of trips) needed to pick up or drop off barges at the fleeting area, to travel to and from the terminal, and to drop off or pick up barges at the terminal. Furthermore, the service time is assumed to be an exponentially distributed random variable. Although the assumption of equal service time is not generally true, in most cases the service-time differentials are not very large. The distribution assumption is also questionable, but not critical.

The rate of service requests is different for each priority group and is calculated as twice the number of barges fleeted divided by the average tow size and distributed over the total operating time. A Poisson input process is assumed.

Activity 5: Load and Unload Barges

The processes for loading and unloading barges are

$$T_{5dki} = q_k/u_{dki} \quad i = 1, u \tag{5}$$

$$t_{5dki} = T_{5dki} + (T_{5dki}/H_{dk})_I (24 - H_{dk}) + P_w(168 - 24D_{dk}) \quad i = 1, u \tag{6}$$

where

t_{5dkl} = time to load one barge with commodity k at terminal d,

t_{5dku} = time to unload one barge of commodity k at terminal d,

q_k = average barge load for commodity k,

u_{dkl} = average effective barge loading rate for commodity k at terminal d,

u_{dku} = average effective barge unloading rate for commodity k at terminal d,

H_{dk} = average hours per day that berths serving commodity k are in service at terminal d,

$(x)_I$ = largest integer contained in x,

$P_w = \{1 - \exp[-L(168 - 24D_{dk})]\}[(7 - D_{dk})/7]$,

D_{dk} = average days per week that berths serving commodity k are in service at terminal d,

$L = (\sum_d \sum_i N_{dki})/(W_{dk}D_{dk}H_{dk})$,

N_{dkl} = total number of barges of commodity k loaded at terminal d,

N_{dku} = total number of barges of commodity k unloaded at terminal d, and

W_{dk} = weeks during the analysis period that berths serving commodity k are in service at terminal d.

In Equation 6, the first term is the number of hours required for the commodity transfer process. The second term captures delays incurred during daily nonoperating periods, if any, for barges that require more than one day to load or unload. The last term accounts for weekend periods of inactivity. In this term, P_w is the probability that a barge would complete service during the weekend hours. The expression for P_w assumes that barge arrivals for service during operational hours follow a Poisson process and barge service times are exponentially distributed. This implies that the completion process is also Poisson. The exponential factor is simply the probability that one or more barges will finish the loading or unloading processing during any period of $(168 - 24D_{dk})$ consecutive hours, and the remaining factor is the probability that this period falls on a weekend. The distribution assumptions made here are required by the submodel for activity 6; hence, they are used here primarily to maintain consistency rather than for any compelling theoretical or empirical reasons. The loading and unloading rates used here should be the controlling rate for the berths servicing commodity k, including the effects of the system used to move materials between the apron and the storage or intermodal transfer facilities of the terminal.

Equation 6 produces estimates of the total time required to process one barge completely. The average transfer time for the cargo involved will be approximately half the barge holding time. It is possible to compute the exact cargo delay time, but the approximation is sufficiently accurate.

This submodel can also be used to compute the towboat delay for unit tows. In this case, the towboat and barges will be held for as long as it takes to complete a number of consecutive loading or unloading operations. To account for this, Equation 5 is modified as follows:

$$T'_{5dki} = (n_{dkt}/n_{dk})\, T_{5dki}$$

where n_{dkt} is the number of barges in a unit tow of commodity k at terminal d and n_{dk} is the number of (simultaneously operating) berths serving commodity k at terminal d.

Activity 6: Access Dock

The process for access-dock activity is

$$X_{6dk} = \left\{ [(Y_{dk}/M_{dk})^{n_{dk}} R_{dk}] / [Y_{dk} n_{dk}! (1 - R_{dk})^2] \right\} C_0 \tag{7}$$

where

t_{6dk} = average time between arrival of a barge of commodity k destined for terminal d and the time that the terminal is ready to receive the barge,
Y_{dk} = average rate of arrival of barges of commodity k at terminal d = $(n_{dkl} + N_{dku})/(W_{dk}D_{dk}H_{dk})$,
M_{dk} = average barge service rate per berth serving commodity k at terminal d = $1/[2t_{3d} + (N_{dkl}T_{5dkl} + N_{dku}T_{5dku})/(N_{dkl} + N_{dku})]$,
n_{dk} = number of berths available to serve commodity k at terminal d,
R_{dk} = average use of berths serving commodity k at terminal d = $Y_{dk}/(n_{dk}M_{dk}) < 1$, and

$$C_0 = 1/\left\{ \sum_{j=0}^{n_{dk}-1} [(Y_{dk}/M_{dk})^j/j!] + [(Y_{dk}/M_{dk})^{n_{dk}}/n_{dk}!\,(1 - R_{dk})] \right\}$$

Then

$$t_{6dk} = X_{6dk} + \left\{1 - \exp[-Y'_{dk}(24 - H_{dk})]\right\} [(24 - H_{dk})^2/48] + \left\{1 - \exp[-Y'_{dk}(168 - 24D_{dk})]\right\} [(168 - 24D_{dk})^2/336] \tag{8}$$

where $Y'_{dk} = Y_{dk}D_{dk}H_{dk}/168$.

The terminal is modeled as a queuing system with identical parallel servers, where each berth servicing a commodity is one server. Service time is exponentially distributed with mean $1/M_{dk}$ and includes the average time the berth is occupied by a barge (drop off, load-unload, pick up). Arrivals at the dock are Poisson at mean rate Y_{dk}. Equation 7 is the expression for average waiting time due to berth occupancy. Equation 8 adds to this waiting time in port due to arrivals during nonoperating periods (nights and weekends). Again, Poisson arrivals and exponential service are assumed. In this case, however, the arrival rate is adjusted to reflect arrivals occurring during 168 h/week. The general form of both the second and third terms of Equation 8 is delay = probability of arrival during T hours x probability that T is a nonoperating period x average delay during period T.

Submodel 6 assumes that each barge is a separate traffic unit. This is incorrect for unit tows. The correct result can be obtained by considering such tows to be single "barges", with capacity equal to the tow capacity. This adjustment does not affect delays due to berth use (Equation 7) but causes a reduction in the probability of arrival during nonoperational periods. A commensurate adjustment to the number of berths must also be made. In most cases, unit tows serve exclusive terminals, so there is little problem with applying the model in this fashion. As a first approximation, it is acceptable to ignore the unit tow problem, with the recognition that delay will be somewhat overestimated. A more detailed model would consider unit tows as bulk arrivals at an n-server queuing system.

Activity 7: Tow Dispatching

The process for tow dispatching is

$$t_{7k} = T/N_k \tag{9}$$

where

t_{7k} = average time that barges with commodity k wait for a tow at a dock or fleeting area,
T = total time in the analysis period, and
N_k = number of tows serving commodity k that call at the port during time T.

Equation 9 depends on the assumptions that barges become ready for pickup randomly following a Poisson process and tow arrivals are also Poisson. Hence, the time from a barge arrival to a tow arrival is a random variable exponentially distributed with parameter N_k/T. If unit tows are involved, Equation 9 does not apply, since the towboat waits for its barges. In this case, $t_{7k} = 0$.

RESEARCH IN PROGRESS

Example calculations made with the model are presented in the project final report (1). Based on those examples, the model appears to give reasonable results. More thorough examples and computations are being performed now that the model is programmed for the computer, since manual calculations can become quite tedious. The final report of the second research phase will include program documentation and the results of model testing.

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