

- to Calculate Optimal Cycle-Based Traffic Timing Patterns. Federal Highway Administration, 1975.
5. D.I. Robertson. TRANSYT: A Traffic Network Study Tool. Ministry of Transport, Transport and Road Research Laboratory, Crowthorne, Berkshire, England, TRRL Rept. LR 253, 1969.
  6. D.I. Robertson and P. Gower. User Guide to TRANSYT, Version 6. Ministry of Transport, Transport and Road Research Laboratory, Crowthorne, Berkshire, England, TRRL Supplementary Rept. 255, 1977.
  7. C.E. Wallace. Development of a Forward Link Opportunities Model for Optimization of Traffic Signal Progression on Arterial Highways. Univ. of Florida, Gainesville, Ph. D. dissertation, 1979.
  8. C.E. Wallace. Forward Link Opportunities: A New Concept for Optimizing Traffic Signal Progression. 11th Annual Pittsburgh Conference on Modeling and Simulation, Pittsburgh, May 1980.
  9. P.P. Jovanis, A.D. May, and A. Diekman. Further Analysis and Evaluation of Selected Impacts of Traffic Management Strategies on Surface Streets. Institute of Transportation Studies, Univ. of California, Berkeley, Rept. UCB-ITS-RR-77-9, 1977.

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## Macroscopic Traffic Delay Model of Bus Signal Preemption

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Productivity enhancement of public transportation is an essential goal, and bus signal preemption at intersections is one of the transportation system management strategies that strives for this goal. Improvements in bus speed and reductions in delay are the anticipated benefits accrued from such strategy. A macroscopic traffic delay model, which applies stochastic procedure, is presented to evaluate different bus preemption signal strategies at an isolated intersection. The model permits the user to evaluate a certain operational strategy provided for bus traffic on both main and cross streets. The signal controller modeled in this paper has a green extension and red truncation capabilities. A comparison between preemption on both main and cross street and preemption on main street only is provided to validate the model's logic. Sensitivity analyses were implemented and it was found that the delay savings due to signal preemption are sensitive to saturation flow rate and to bus passenger load. Potential applications and further enhancement are suggested.

Transportation and traffic engineers realize the importance of system productivity and its major role in minimizing passenger delays and maximizing passengers throughput. Several transportation systems management (TSM) strategies have been identified to achieve such a goal, and one of those is the provision of bus priority treatment at urban intersections by means of signal preemption strategies. The federal government currently can fund the capital costs of TSM projects and it is necessary to investigate the worthiness of bus preemption.

Bus preemption demonstration experiments were conducted in Los Angeles, Miami, and Melbourne, Australia (1-3). All studies concluded that bus signal preemption could reduce total passenger delay. Two bus signal preemption studies (4,5), one in Sacramento, California, and the other in Concord, California, reported similar results and showed that, with low bus frequencies, the added delays to automobile occupants are negligible.

Two computer simulation models were developed and tested for bus signal preemption (6,7). These models are of a microscopic nature in which the status of the vehicle with regard to its location, speed, and delay is updated every small time interval. The Urban Traffic Control System-Bus Priority System (known as UTCB-BPS) and the network simulation-bus priority system (NETSIM-BPS) computer programs are, perhaps, the only packages available that provide bus preemption at urban intersections (8,9).

The complexity and high cost involved in developing and validating such software packages lead to

the consideration of other macroscopic approaches. An analytical model of bus preemption, by using a deterministic vehicle arrival process, was developed for the purpose of evaluating pretimed signal priority treatments at isolated intersections (10). Delay values derived from this model are believed to be underestimated due to the deterministic nature. Another analytical model, which uses a stochastic approach, was developed to evaluate and assess priority treatment of buses at signalized intersections (11). The model provides green extension and red truncation signal strategies, however, it is limited to one direction signal capability (preemption on main street only).

Evaluation of signal strategy effects on traffic flow requires, in general, a detailed analysis of a vehicle's speed, location, acceleration and deceleration capabilities, and the status of the signal. Use of microscopic computer simulation packages for system evaluation and justification can be an accurate way. However, the time and cost involved in running the program constrains and sometimes prohibits the completion of an extensive analysis. A solution for this problem is to use macroscopic analytical models that can reasonably do the job with possibly 10 percent the price of the microscopic model. The analytical models cited in the literature lack the ability to evaluate the bus signal preemption option on both main and cross street approaches.

### DELAY MODEL AT AN INTERSECTION

Several models have been developed for estimating queues and delays at signalized intersections. Winsten and coworkers were the first to use the binomial distribution in an analysis of delays at pretimed signals (12). The Poisson distribution that describes the arrival of vehicles at intersections has been used by Adams, Webster, and Wardrop (13-15). Newell used a model in which the arrival headways were assumed to have a shifted exponential distribution (16). Most models assume departures at equal time intervals, providing a queue exists and the first departure is at the start of the effective green time.

One of the better known models for delay is the one developed by Webster (14) by using data result-

ing from computer simulation of intersection operation. Because Webster's delay model has been tested at several locations in England and the United States and has been proven to be reliable, it was adapted for this paper. The average delay per vehicle as given by Webster is determined from the following formula:

$$d = [C(1-\lambda)^2/2(1-\lambda X)] + [X^2/2q(1-X)] - 0.65(C/q^2)^{1/3} X^{(2+5\lambda)} \quad (1)$$

where

- d = average delay per vehicle on the particular intersection approach;
- C = cycle time;
- $\lambda$  = proportion of the cycle that is effectively green for the phase under consideration (g/c);
- q = flow;
- S = saturation flow; and
- X = degree of saturation; this is the ratio of the actual flow to the maximum flow that can be passed through the intersection from this approach and is given by  $X = q/\lambda S$  (if d and c are in vehicles per second).

The third term of Equation 1 was found to range from 5 to 15 percent of the total mean delay, and Allsop suggested (17) that the average delay may be taken as

$$d = 9/10 \{ [C(1-\lambda)^2/2(1-\lambda X)] + [X^2/2q(1-X)] \} \quad (2)$$

Equation 2 was used to develop an analytical model described in this paper. The basic concept of the model was to investigate all possible bus detection events at an intersection and list the corresponding signal cycle lengths and splits. Cycle lengths, proportions of the cycle that were effectively green, degrees of saturation, and flow rates were substituted in Equation 2 to determine the total delay per approaching vehicle. Appropriate adjustments and assumption were made to calculate passenger car delays and bus delays. For each bus detection event, the probability of signal preemption was estimated by assuming a Poisson distribution of vehicle arrivals. The expected delay figures were then calculated and compared with the initial delay figures for no preemption.

#### MODEL ASSUMPTIONS

The following are the assumptions made to formulate the analytical model:

1. Pretimed signal controller with a two-phase plan and a cycle length determined from Webster's optimum cycle formula (14):

$$C_o = (1.5L + 5)/(1 - Y) \quad (3)$$

where

- $C_o$  = optimum cycle time (s),
- L = total lost time per cycle (5 s/phase), and
- Y = sum of the maximum ratios of flow to saturation flow;

2. Minimum red phase durations for main and cross streets are determined from Webster's minimum cycle formula:

$$C_m = L/(1 - Y) \quad (4)$$

3. Absolute minimum cycle length of 40 s and absolute maximum cycle length of 120 s;
4. Minimum green phase duration of 12 s;

5. Detectors set up around 250-ft upstream of the intersection with average time required for a bus to clear the intersection of 7 s; (The location of the detector does not allow for nearside bus stops.)

6. Green extension and red truncation strategies are provided;

7. Saturation flow rate of 1800 passenger car equivalent per hour per lane;

8. Bus weight of 2.25 passenger car equivalent; and

9. Average bus load of 35 passengers and a passenger car load of 1.4 passengers.

#### MODEL DEVELOPMENT

##### Probability Expressions

The general case of bus preemption (preemption on main and cross streets) was developed first. Some assumptions concerning preemption priorities were made:

1. Main street green extension,
2. Cross street green extension,
3. Main street red truncation, and
4. Cross street red truncation.

The minimum red phase constraints, estimated from Webster's minimum cycle length, combined with the green detection time period within which green extension is requested (last 6 s of green) created four possible operational scenarios, as shown in Figure 1. The four possible scenarios are as follows:

$$\begin{aligned} R_{1min} &> (G_2 - 6) \text{ and } R_{2min} > (G_1 - 6), \\ R_{1min} &> (G_2 - 6) \text{ and } R_{2min} < (G_1 - 6), \\ R_{1min} &< (G_2 - 6) \text{ and } R_{2min} > (G_1 - 6), \\ \text{and} \\ R_{1min} &< (G_2 - 6) \text{ and } R_{2min} < (G_1 - 6). \end{aligned}$$

In general, a total of 10 operational cases exist for any signal cycle:

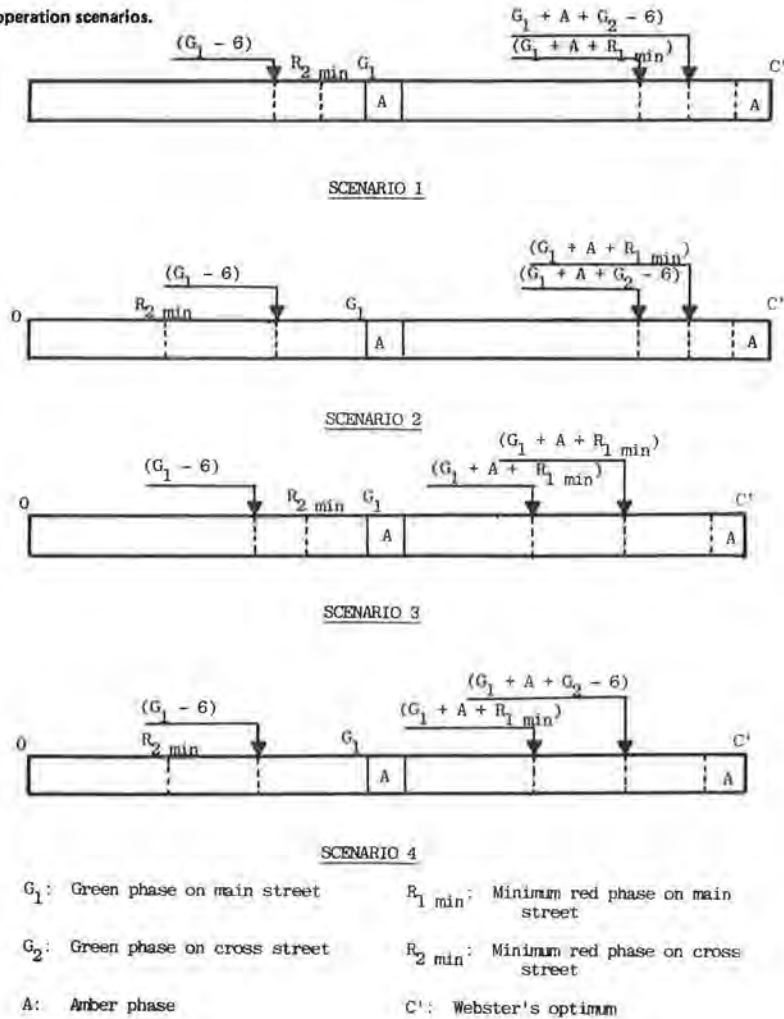
- Case 1: No buses in a cycle,
- Case 2: Buses arrive but there is no preemption,
- Case 3: Main street green extension,
- Case 4: Cross street green extension,
- Case 5: Main street red truncation with red phase =  $R_{1min}$ ,
- Case 6: Main street red truncation between ( $G_1 + R_{1min}$ ) and ( $G_1 + A + G_2 - 6$ ),
- Case 7: Main street red truncation after ( $G_1 + A + G_2 - 6$ ),
- Case 8: Cross street red truncation with red phase =  $R_{2min}$ ,
- Case 9: Cross street red truncation between  $R_{2min}$  and ( $G_1 - 6$ ), and
- Case 10: Cross street red truncation between ( $G_1 - 6$ ) and  $G_1$ .

Each of these cases has unique characteristics that may include cycle length, splits, saturation flow rates, and special bus delay terms. The 10 cases listed are the possible operational cases that can occur for scenario 4. Examination of Figure 1 would reveal that the number of cases for Scenarios 1, 2, and 3 is 8, 9, and 9 cases, respectively.

To develop the probability of each of these cases it was necessary to divide the cycle into six appropriate intervals:

- Interval 1: from ( $C' - A$ ) to  $C'$ , and from 0 to ( $R_{2min}$ );
- Interval 2: from ( $R_{2min}$ ) to ( $G_1 - 6$ );
- Interval 3: from ( $G_1 - 6$ ) to  $G_1$ ;
- Interval 4: from  $G_1$  to ( $G_1 + R_{1min}$ );

Figure 1. Four possible signal operation scenarios.



Interval 5: from  $(G_1 + R_{1 \min})$  to  $(G_1 + A + G_2 - 6)$ ; and  
 Interval 6: from  $(G_1 + A + G_2 - 6)$  to  $(C' - A)$ .

The probability of no arrivals during each of the six intervals was developed by assuming a Poisson arrival distribution. The probability expressions are presented in a matrix format (Figure 2) with the element  $S_{ij}$  as the probability of interval  $i$  from approach  $j$ . Main street approaches and cross street approaches are subscripted 1 and 2, respectively.

The probability expression for each case that corresponds to each scenario was then derived as a function of the X-matrix components. A summary of the probability terms is shown in Figure 3. The sum of all the probabilities of each scenario is equal to unity.

Cycle Lengths

The cycle lengths that correspond to the 10 operational cases are given in the list below.

Case 1,

$$C = C' \tag{5}$$

Case 2,

$$C = C' \tag{6}$$

Case 3,

$$C = \sum_{n=1}^3 (p_1 q_1^{n-1}) Y + C' \tag{7}$$

Case 4,

$$C = \sum_{n=1}^3 (p_2 q_2^{n-1}) Z + C' \tag{8}$$

Case 5,

$$C = G_1 + A + R_{1 \min} \tag{9}$$

Case 6,

$$C = G_1 + A + R_{1 \min} + (G_2 - R_{1 \min} - 6)/2 \tag{10}$$

Case 7,

$$C = G_1 + A + G_2 - 3 \tag{11}$$

Case 8,

$$C = G_2 + A + R_{2 \min} \tag{12}$$

Case 9,

$$C = G_2 + A + R_{2 \min} + (G_1 - R_{2 \min} - 6)/2 \tag{13}$$

Figure 2. Probability of no bus arrival matrix.

$$\begin{aligned}
 X_{11} &= \exp\left[-\frac{B_1 R_2 \min}{3600}\right] & X_{12} &= \exp\left[-\frac{B_2 R_2 \min}{3600}\right] \\
 X_{21} &= \exp\left[-\frac{B_1(G_1 - R_2 \min + A - G)}{3600}\right] & X_{22} &= \exp\left[-\frac{B_2(G_1 - R_2 \min + A - G)}{3600}\right] \\
 X_{31} &= \exp\left[-\frac{G B_1}{3600}\right] & X_{32} &= \exp\left[-\frac{G B_2}{3600}\right] \\
 X_{41} &= \exp\left[-\frac{B_1(B_1 \min)}{3600}\right] & X_{42} &= \exp\left[-\frac{B_2(B_1 \min)}{3600}\right] \\
 X_{51} &= \exp\left[-\frac{B_1(C' - G_1 - A - R_1 \min - G)}{3600}\right] & X_{52} &= \exp\left[-\frac{B_2(C' - G_1 - A - R_1 \min - G)}{3600}\right] \\
 X_{61} &= \exp\left[-\frac{G B_1}{3600}\right] & X_{62} &= \exp\left[-\frac{G B_2}{3600}\right]
 \end{aligned}$$

where:  $B_1$  and  $B_2$  are bus flow rates for main street and cross street, respectively.

\* Napierian exponent

Figure 3. General probability expressions.

Case Number	Scenario Number			
	1	2	3	4
$P_1 =$		$\exp\left[-\frac{(B_1 + B_2) C'}{3600}\right]$		
$P_2 =$		$\exp\left[-\frac{G[(B_1 + B_2) + B_2(G_1 + A) + B_1(C' - G_1 + A)]}{3600}\right]$		$\exp\left[-\frac{(B_1 + B_2) C'}{3600}\right]$
$P_3 =$	$(1 - X_{31})$	$(1 - X_{31})X_{12}X_{22}$	$(1 - X_{31})$	$(1 - X_{31})X_{12}X_{22}$
$P_4 =$	$(1 - X_{62})X_{31}$	$(1 - X_{62})X_{31}X_{12}X_{22}X_{32}$	$(1 - X_{62})X_{31}X_{41}X_{51}$	$(1 - X_{62})X_{31}X_{41}X_{51}X_{12}X_{22}X_{32}$
$P_5 =$	$(1 - X_{41})X_{31}X_{62}$	$(1 - X_{41})X_{31}X_{62}X_{12}X_{22}X_{32}$	$(1 - X_{41})X_{31}$	$(1 - X_{41})X_{31}X_{12}X_{22}X_{32}$
$P_6 =$	$(1 - X_{51}X_{61})X_{31}X_{41}X_{62}$	$(1 - X_{51}X_{61})X_{31}X_{41}X_{62}X_{12}X_{22}X_{32}$	$(1 - X_{51})X_{31}X_{41}$	$(1 - X_{51})X_{31}X_{41}X_{12}X_{22}X_{32}$
$P_7 =$	-	-	$(1 - X_{61})X_{31}X_{41}X_{51}X_{62}$	$(1 - X_{61})X_{31}X_{41}X_{51}X_{62}X_{12}X_{22}X_{32}$
$P_8 =$	$(1 - X_{12})X_{62}X_{31}X_{41}X_{51}X_{61}$	$(1 - X_{12})$	$(1 - X_{12})X_{62}X_{31}X_{41}X_{51}X_{61}$	$(1 - X_{12})$
$P_9 =$	$(1 - X_{22}X_{32})X_{62}X_{12}X_{31}X_{41}X_{51}X_{61}$	$(1 - X_{22})X_{12}$	$(1 - X_{22}X_{32})X_{62}X_{12}X_{31}X_{41}X_{51}X_{61}$	$(1 - X_{22})X_{12}$
$P_{10} =$	-	$(1 - X_{32})X_{12}X_{22}X_{31}$	-	$(1 - X_{32})X_{12}X_{22}X_{31}$

Case 10,

$$C = G_2 + A + G_1 - 3 \tag{14}$$

where

$$\begin{aligned}
 C' &= \text{Webster's optimum,} \\
 P_1 &= \exp(-7 B_1/3600), \\
 q_1 &= 1 - p_1, \\
 Y &= (n-1) \left\{ \frac{3600}{B_1} - 7 \exp(-7 B_1/3600) - \frac{3600}{B_1} \exp(-7 B_1/3600) \right\} + 7, \\
 p_2 &= \exp(-7 B_2/3600), \\
 q_2 &= 1 - p_2, \text{ and} \\
 Z &= (n-1) \left\{ \frac{3600}{B_2} - 7 \exp(-7 B_2/3600) - \frac{3600}{B_2} \exp(-7 B_2/3600) \right\} + 7.
 \end{aligned}$$

The cycle length expression of cases 3 and 4 (green extension logic) was attained by adding the expected

green extension to Webster's optimum cycle length. We assumed that no more than three buses can request green extension in any cycle. The expected green extension period was calculated by summing the product of the probability of exactly  $n$  arrivals to produce one headway greater than 7 s times the expected extension of  $n$  arrivals. The probability term is explained by a geometric distribution:

$$p_i(n) = p_1 q_1^{(n-1)} \tag{15}$$

where  $q_1$  equals  $1 - p$  and  $p_1$  equals  $\exp[-7 B_1/3600]$  = probability of headway > 7.

As for the expected extension of  $n$  arrivals, the following derivation describes it:

$$\begin{aligned}
 \text{Expected headway}/(\text{all headway} < 7) &= \int_0^7 Y \lambda \exp(-\lambda Y) dY \\
 &= 1/\lambda - 7 \exp(-7\lambda) - 1/\lambda \exp(-7\lambda) \tag{16}
 \end{aligned}$$

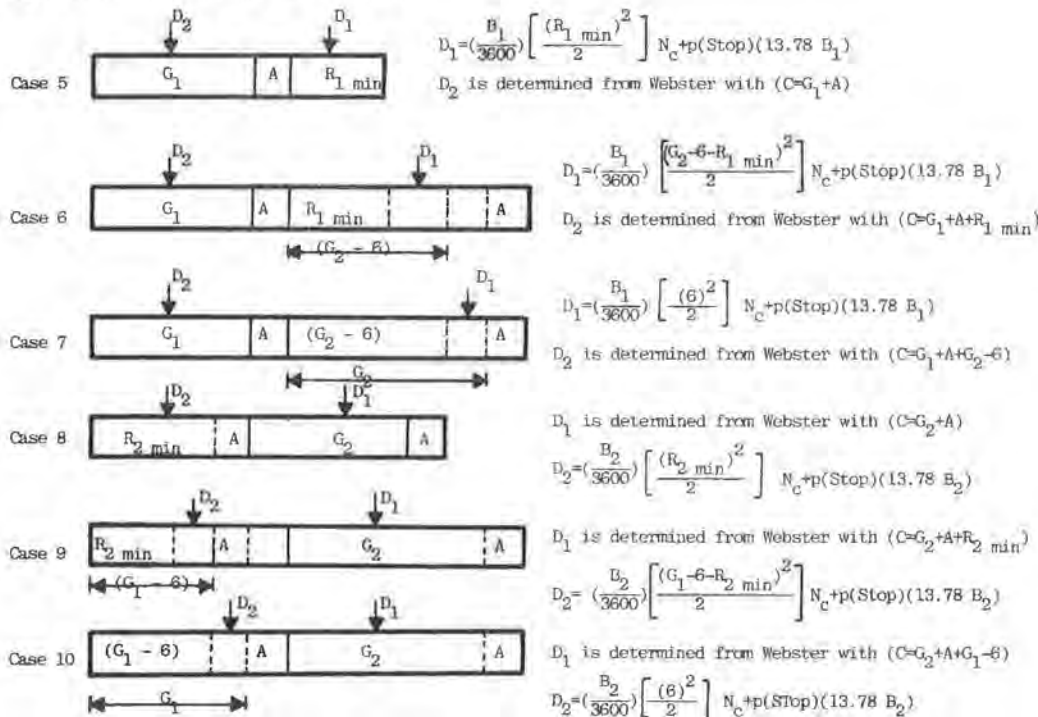


Table 1. Bus delay terms used on model.

Case Number	Main Street	Cross Street
1. No buses in a cycle	Webster; S is adjusted for no bus arrivals	Webster; S is adjusted for no bus arrivals
2. Buses arrive, no preemption	Webster	Webster
3. Main street green extension	Webster	Webster
4. Cross street green extension	Webster	Webster
5. Main street red truncation ( $R = R_{1 \min}$ )	Compound delay model 1 <sup>a</sup>	Webster
6. Main street red truncation ( $G_1 + R_{1 \min} < R < G_1 + A + G_2 - 6$ )	Compound delay model 2 <sup>a</sup>	Webster
7. Main street red truncation ( $G_1 + A + G_2 - 6 < R < C$ )	Compound delay model 3 <sup>a</sup>	Webster
8. Cross street red truncation ( $R = R_{2 \min}$ )	Webster	Compound delay model 4 <sup>a</sup>
9. Cross street red truncation ( $R_{2 \min} < R < G_1 - 6$ )	Webster	Compound delay model 5 <sup>a</sup>
10. Cross street red truncation ( $G_1 - 6 < R < G_1$ )	Webster	Compound delay model 6 <sup>a</sup>

<sup>a</sup>Shown in detail in Figure 4.

Figure 4. Red truncation compound delay models.



Where:  $D_1$  and  $D_2$  are delay figures of buses arriving during the time periods indicated by arrows.

Total bus delay =  $D_1 + D_2$

$N_c$  = Number of Cycles per hour.

$\lambda = B_1/3600$  (17)

from Equations 16 and 17 the expected extension of n arrivals is defined as:

$Y = (n - 1)[(3600/B_1) - 7 \exp(-7B_1/3600) - (3600/B_1) \exp(-7B_1/3600)] + 7$  (18)

Delay Estimation

Webster's delay model, shown in Equation 2, provides the average delay per approaching vehicle at a pretimed signalized intersection. The model does not differentiate between passenger cars and buses, therefore, some assumptions and adjustments had to be made to count for the difference. The average delay per vehicle was divided into stop time delay and delay due to speed change cycles. The derivation of both delay components is as follows:

Total delay = Total passenger car stopped time delay

+ passenger car time lost due to acceleration and deceleration + total bus stopped delay + bus time lost due to acceleration and deceleration.

The time lost in the queue was neglected in this derivation, and the speed profile adopted for estimating time lost due to acceleration and deceleration was as follows:

Vehicle Type	Speed		
	Initial (ft/s)	Final (ft/s)	Rate of Change (ft/s <sup>2</sup> )
Passenger car	0	20	+8
	20	V	+4
	V	0.90V	-1
	0.90V	0	-7
Bus	0	V	+2
	V	0	-4

Table 2. Saturation headway and bus passenger load sensitivity results.

Item	Total Passenger Delay Savings (passenger-s)	
	Main Street	Cross Street
Saturation headway		
1800 vehicles/h	19 769	-2442 <sup>a</sup>
1980 vehicles/h	18 747	1651
2160 vehicles/h	17 353	2083
Passenger bus load		
20	11 572	-2755 <sup>a</sup>
35	19 769	-2442 <sup>a</sup>
50	27 965	-2127 <sup>a</sup>

<sup>a</sup>Delay was increased.

where  $V$  is the target speed in feet per second. This unimpeded speed profile was borrowed from the UTCS-BPS computer program (9).

It was assumed that the average target speed of heavily traveled urban streets is 25 mph. The time lost per passenger car speed change maneuver based on the same distance was found to be 5.56 s, and the corresponding value was 13.78 s/bus.

The probability of stopping more than once, as defined by Webster, was applied for the delay estimation. The breakdown of the delay components is

$$\text{Total delay per approach} = X + p(\text{stop})[5.56V_1 + 13.78B_1] \quad (19)$$

where

$$\begin{aligned} X &= \text{total stopped delay,} \\ V_1 &= \text{hourly passenger car flow,} \\ B_1 &= \text{hourly bus flow, and} \\ p(\text{Stop}) &= \text{probability of stopping} = (1 - \lambda) / \\ &\quad (1 - Y) \text{ and } \lambda \text{ and } Y \text{ were defined} \\ &\quad \text{earlier in Equation 7.} \end{aligned}$$

$$\text{Total delay per passenger car per approach} = [X(V_1)/(V_1 + B_1)] + p(\text{stop})(5.56V_1) \quad (20)$$

The delay term defined in Equation 20 applies only for passenger cars and buses that operate under normal cycle length and phase splits with no preemption. The delay terms of buses for all possible signal cases are listed in Table 1 and detailed in Figure 4. As for passenger cars, Webster's model was assumed to apply to their delay estimation, and their benefit from signal preemption is reflected in the  $(G/c)$  term. The probability expressions and the delay equations were then coded into a computer program. The program calculates internally the total delay of passenger cars and buses under both preemption and nonpreemption strategies and provides the total delay saving (or losses) due to preemption.

#### Model Testing and Sensitivity Analyses

The model was applied to the following hypothetical setting:

Main street passenger car volume = 500 cars/h,  
Cross street passenger car volume = 500 cars/h,  
Main street buses = 40 buses/h,  
Cross street buses = 10 buses/h, and  
Saturation flow rate = 1800 vehicles/h.

The hypothetical setting resulted in operational strategies of scenario 1. The total delay per hour of passenger cars and buses for no preemption and preemption and the total passenger delay gains are given in the table below.

Results	Main Street		Cross Street	
	Passenger Cars	Buses	Passenger Cars	Buses
Vehicle delay, no preemption (s)	8887	991	9 583	263
Vehicle delay, preemption (s)	8425	445	11 855	242

Savings (losses) attributed to signal preemption were 19 769 passenger-s for main street traffic but a delay of 2442 passenger-s was found for cross-street traffic. Total intersection savings were therefore 17 327 passenger-s. The results proved to be consistent and as expected in the sense that bus signal preemption helped both main street and cross street buses, with more benefits to main street. The main street bus delay saving amounted to 122 percent due to the preemption and the corresponding cross street saving was 9 percent. The bus delay saving on cross streets did not offset the passenger car delay loss, hence a total passenger delay loss was observed (2442 passenger-s).

A set of sensitivity analysis were implemented on the saturation headway and bus passenger load and they are given in Table 2. The increase in saturation headway caused less savings for main street and higher savings to cross street. As the results show, the model proved to be sensitive to cross street passenger delay savings between saturation headways of 1800 and 1980 vehicles/h.

An increase in the passenger bus load of 15 passengers caused a delay savings increase of 8197 passenger-s of main street passenger delay. This shows the significance of improving the bus passenger load. On the contrary, the reduction in delay losses on cross streets were insensitive to the increase in the passenger bus load.

The last step in the analysis was to test the model for bus preemption for main street only. This was attained by using a zero bus flow on the cross street. The total passenger saving for the basic hypothetical setting was found to be 21 549 passenger-s (4222 passenger-s higher than the preemption logic on both streets). This result is as expected because the zero bus flow on the cross street resulted in higher saturation flow rate, which provided higher total intersection delay savings, as proved earlier in Table 2.

#### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This paper presented an analytical model for evaluating and testing a possible bus preemption strategy at an isolated intersection. Four possible signal operational scenarios were identified and their corresponding probability terms were fully documented. Webster's delay model was adopted to estimate the average delay per vehicle per approach. The model was applied to a set of hypothetical demand rates to further validate the logic.

Sensitivity analyses were implemented, and it was concluded that the model was sensitive to an increase in the saturation flow rate from 1800 to 1980 vehicles/h and that the delay saving was insensitive beyond that point. In addition, the model results were found to be sensitive to bus passenger load.

The analytical model presented in this paper can be incorporated in the traffic network study tool (TRANSYT) computer package to develop signal splits and offsets in urban networks with provision for bus signal preemption on a main transit artery, an option that TRANSYT can not handle. The model can also be extended and enhanced to evaluate bus preemption strategies for a multiphase signal operation.

## REFERENCES

1. H.K. Evans and G.W. Skiles. Improving Public Transit Through Bus Preemption of Traffic Signals. *Traffic Quarterly*, Vol. 24, No. 4, Oct. 1970.
2. J.A. Wattleworth, K.G. Courage, and C.E. Wallace. Evaluation of Bus-Priority Strategies on Northwest Seventh Avenue in Miami (Abridgment). *TRB, Transportation Research Record* 626, 1977, pp. 32-35.
3. A.J. Richardson and K.W. Ogden. Evaluation of Active Bus-Priority Signals. *TRB, Transportation Research Record* 718, 1979, pp. 5-12.
4. W.J. Elias. The Greenback Experiment--Signal Preemption for Express Buses: A Demonstration Project. California Department of Transportation, Sacramento, Rept. DMT-014, 1976.
5. C.D. Kinzel and others. City of Concord, Evaluation of Bus Priority Signal System. TJKM Transport Consultants, Walnut Creek, CA, 1978.
6. K. Wood. Bus-Actuated Signal Control at Isolated Intersections--A Simulation Model. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, Supplementary Rept. 373, 1978.
7. R.A. Vincent, B.R. Cooper, and K. Wood. Bus-Actuated Signal Control at Isolated Intersections--Simulation Studies of Bus Priorities. Transport and Road Research Laboratory, Crowthorne, Berkshire, England, Laboratory Rept. 813, 1978.
8. G. Radelat. Bus Priority Systems Supplement to the Subroutine Documentation of the MITRE's Version of the Original UTCA-1 Model. Traffic Systems Division, Office of Research, Federal Highway Administration, 1973.
9. J.F. Torres and B. Mikhalkin; JFT Associates. Integration of BPS Strategies with NETSIM. FHWA, Oct. 1980.
10. I.B. VanBilderbeek. Priority Treatment of Buses at Traffic Signals. Proc., Technology Assessment Review, Organization for Economics and Development, Directorate for Scientific Affairs, Paris, France, 1979.
11. J. Jacobson and Y. Sheffi. Analytical Model of Traffic Delays Under Signal Preemption: Theory and Application. Paper presented at 59th Annual Meeting, TRB, 1980.
12. M. Beckmann, C.B. McGuire, and C.B. Winsten. Studies in the Economics of Transportation, Yale Univ. Press, New Haven, CT, 1956.
13. W.P. Adams. Road Traffic Considered as Random Series. *Journal of the Institute of Civil Engineers*, Vol. 4, 1936, pp. 121-130.
14. F.V. Webster. Traffic Signal Settings. Road Research Tech. Paper No. 39, Road Research Laboratory, London, 1958.
15. J.G. Wardrop. Some Theoretical Aspects of Road Traffic Research. Proc., Institute of Civil Engineers, Vol. 1, 1952, pp. 325-362.
16. G.F. Newell. Statistical Analysis of the Flow of Highway Traffic Through a Signalized Intersection. *Quarterly Applied Mathematics*, No. 13, 1956, pp. 353-369.
17. R.E. Allsop. Delay at a Fixed Time Traffic Signal. I: Theoretical Analysis, *Transportation Science*, Vol. 6, No. 3, pp. 260-285, 1972.

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## Estimation of Average Phase Durations for Full-Actuated Signals

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A model for estimating the average green durations that result from full-actuated signal control is described. The model is developed primarily on the basis of probabilistic interactions between traffic flows and the control. It is structured according to three flow and control conditions: (a) the control employs motion detectors, (b) right-turn-on-red is either prohibited or does not affect the signal timing, and (c) left turns are made only from exclusive left-turn lanes. The discussions are focused on the formulation of the model. Applications of the model are also illustrated. The model can be used either manually or with the aid of a simple computer program.

Traffic-related phenomena at a signalized intersection, such as lane capacity, delays, queue length, and passenger car equivalent of left-turn vehicles, are influenced by the cycle splits and cycle length of the signal control. Under a full-actuated control, the cycle splits and the corresponding cycle length vary from one cycle to another. Consequently, it becomes desirable to estimate the average cycle splits of a full-actuated control to facilitate rational planning, design, and operation of signalized intersections. However, reliable and convenient methods for estimating full-actuated cycle splits are currently not available. This weakness may become increasingly critical when more

full-actuated controls are used for intersection control.

To alleviate this problem, this paper presents a model that can be used either manually or with the aid of a simple computer program to obtain estimates of average full-actuated cycle splits. The model is structured on the basis of the following conditions:

1. The control relies on motion detectors to obtain information on traffic flow,
2. Right-turn-on-red is either prohibited or does not affect the signal timing, and
3. Left turns are made only from exclusive left-turn lanes.

In the following, a model for estimating the average cycle splits of a two-phase control is illustrated. The model is then expanded for application to cases that involve multiphase controls.

### CONTROL LOGIC AND FLOW CHARACTERISTICS

A typical full-actuated signal control that employs motion detectors has the following control param-