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Estimation of Average Phase Durations for Full-Actuated Signals

FENG-BOR LIN

A model for estimating the average green durations that result from full-actuated signal control is described. The model is developed primarily on the basis of probabilistic interactions between traffic flows and the control. It is structured according to three flow and control conditions: (a) the control employs motion detectors, (b) right-turn-on-red is either prohibited or does not affect the signal timing, and (c) left turns are made only from exclusive left-turn lanes. The discussions are focused on the formulation of the model. Applications of the model are also illustrated. The model can be used either manually or with the aid of a simple computer program.

Traffic-related phenomena at a signalized intersection, such as lane capacity, delays, queue length, and passenger car equivalent of left-turn vehicles, are influenced by the cycle splits and cycle length of the signal control. Under a full-actuated control, the cycle splits and the corresponding cycle length vary from one cycle to another. Consequently, it becomes desirable to estimate the average cycle splits of a full-actuated control to facilitate rational planning, design, and operation of signalized intersections. However, reliable and convenient methods for estimating full-actuated cycle splits are currently not available. This weakness may become increasingly critical when more full-actuated controls are used for intersection control.

To alleviate this problem, this paper presents a model that can be used either manually or with the aid of a simple computer program to obtain estimates of average full-actuated cycle splits. The model is structured on the basis of the following conditions:

1. The control relies on motion detectors to obtain information on traffic flow,
2. Right-turn-on-red is either prohibited or does not affect the signal timing, and
3. Left turns are made only from exclusive left-turn lanes.

In the following, a model for estimating the average cycle splits of a two-phase control is illustrated. The model is then expanded for application to cases that involve multiphase controls.

CONTROL LOGIC AND FLOW CHARACTERISTICS

A typical full-actuated signal control that employs motion detectors has the following control parameters for each signal phase i:

I_i = initial portion of green duration,
U_i = unit extension,
$(G_{max})_i$ = maximum allowable green duration,
Y_i = yellow duration, and
S_i = detector setback.

The logic of the control is simple. If phase i receives the green light, the first stage of its green duration will include the initial portion. After the time of the initial portion is completed, the green will be extended for an interval equal to the unit extension. If a vehicle actuates a detector of this phase during this unit extension, the green will be extended from the moment of actuation for another unit extension. The green can be extended repeatedly in the same fashion until the maximum allowable green is reached. If no vehicles actuate the detectors during an extended interval of one unit extension, the green will be terminated at the end of that unit extension.

In a given time interval vehicles in a traffic lane can be assumed to arrive at the upstream side of an intersection at random. With this approximation, the arrival pattern in each lane can be represented by a Poisson distribution (1):

$$P_j(n/t) = (\lambda_j t)^n \exp(-\lambda_j t)/n! \tag{1}$$

where $P_j(n/t)$ is the probability of having n arrivals in the jth lane in time interval t, and λ_j is the average flow rate in the jth lane during the same time interval t.

The headway distribution associated with $P_j(n/t)$ can be approximated by a shifted negative exponential function (1):

$$F_j(h \geq t) = \exp[-(t-\tau)/(H_j-\tau)] \tag{2}$$

where

$F_j(h \geq t)$ = probability that a vehicle headway h in the jth lane is greater than or equal to t,
$H_j = 1/\lambda_j$ = average headway of vehicles in in the jth lane, and
τ = minimum headway of vehicles in a traffic lane, equal approximately to 1 s.

When more than one traffic lane is present, several vehicles can cross a given reference point simultaneously. Therefore, the minimum headway of the vehicles in the combined flow approaches zero. Under this condition, the headway distribution of the combined flow can be approximated by

$$\begin{aligned} Z(h \geq t) &= \exp(-\lambda_1 t)\exp(-\lambda_2 t)\ldots\exp(-\lambda_m t) \\ &= \exp[-(\lambda_1 + \lambda_2 + \ldots + \lambda_m)t] \end{aligned} \tag{3}$$

where $Z(h \geq t)$ is the probability that a headway h of a combined multilane flow is greater than or equal to t and m is the number of traffic lanes involved.

Equations 2 and 3 can be combined into a single probability distribution:

$$f(h \geq t) = \exp[-\lambda(t-\tau)] \tag{4}$$

where

$$\lambda = 1/(H_1-\tau) + 1/(H_2-\tau) + \ldots + 1/(H_m-\tau) \tag{5}$$

and

$$\tau = \begin{cases} 1 & \text{if } m = 1 \\ 0 & \text{if } m > 1 \end{cases} \tag{6}$$

The probability density function associated with $f(h \geq t)$ is

$$f(h = t) = \lambda\exp[-\lambda(t-\tau)] \tag{7}$$

BASIC MODEL FOR TWO-PHASE CONTROL

Under full-actuated control, vehicles in each phase can extend the green according to the control logic and the settings of the control parameters. Figure 1 shows a representative two-phase timing sequence that results from such a control. The time interval $[\beta_i(i = 1,2)]$ as shown in the figure represents the length of the green beyond the initial portion for phase i. This time interval can be divided into two components:

1. D_{ni}--This component is the additional green extended by n vehicles that form moving queues upstream of the detectors after the initial portion of time has elapsed.
2. E_{ni}--This is the additional green extended in the absence of the moving queues by vehicles that cross the detectors at headways of no more than one unit extension after the initial portion of time has elapsed or after the moving queues disappear.

The portion of the green represented by D_{ni} precedes that represented by E_{ni}. Since the green in any cycle cannot exceed $(G_{max})_i$, D_{ni} should not exceed $(G_{max})_i - I_i$ and E_{ni} can at most equal $(G_{max})_i - I_i - D_{ni}$. Collectively, these individual constraints can be replaced by $I_i + D_{ni} + E_{ni} \leq (G_{max})_i$ without affecting the estimated values of the average green durations.

The values of D_{ni} and E_{ni} vary from one cycle to another. Thus, estimates of their expected values are necessary in order to determine the average green duration for each phase. Let the average values of D_{ni} and E_{ni} be denoted, respectively, as D_i and E_i. Furthermore, assume for the time being that no opposed left turns are involved. Then, D_i and E_i can be estimated according to the procedures described below.

Value of E_i

After the initial portion of phase i has elapsed two things may happen. One is that there are no moving queues upstream of the detectors in the lanes associated with this phase. The other is that moving queues may exist in some or all of the lanes. In the former case, the green can be extended if vehicles of phase i actuate at least one detector within U_i seconds of each other after the initial portion of time has elapsed. In the latter case, vehicles can still extend the green in the same manner after the moving queues disappear.

The probability that the green will be extended exactly k times is $[f(h \leq U_i)]^k f(h > U_i)$. Let the average length of each extension be denoted as J. Then, the additional green that results from the k extensions is $kJ + U_i$. This $kJ + U_i$ represents the value of E_{ni} in a given cycle for phase i. As indicated previously, this extended portion of the green cannot exceed $(G_{max})_i - I_i - D_{ni}$. For estimating the average green of phase i, however, this individual constraint may be neglected as long as the collective constraint $I_i + D_{ni} + E_{ni} \leq (G_{max})_i$ is satisfied. If we neglect the individual constraint, the expected value of E_{ni}, (i.e., E_i) can be conveniently estimated from

$$E_i = \sum_{k=0}^{\infty} (kJ + U_i)[f(h \leq U_i)]^k f(h > U_i)$$
$$= U_i + J\{\exp[\lambda(U_i - \tau)] - 1\} \quad (8)$$

where

$$J = \left[\int_\tau^{U_i} t\, f(h = t)dt\right] / f(h < U_i)$$
$$= \{-[U_i + (1/\lambda)] + [\tau + (1/\lambda)] \exp[\lambda(U_i - \tau)]\} / \{\exp[\lambda(U_i - \tau)] - 1\} \quad (9)$$

Therefore,

$$E_i = -(1/\lambda) + [\tau + (1/\lambda)] \exp[\lambda(U_i - \tau)] \quad (10)$$

The sum of I_i and E_i cannot be allowed to exceed $(G_{max})_i$. Therefore, the value of E_i obtained from Equation 10 should be limited to a maximum of $(G_{max})_i - I_i$. If E_i exceeds $(G_{max})_i - I_i$, the average green will reach $(G_{max})_i$ and the estimation of D_i becomes unnecessary.

Value of D_i

After a green duration is terminated for phase i, arriving vehicles will begin to accumulate and form a stationary queue in each of the lanes associated with this phase. Once the light is turned to green again, the vehicles in the queues will start moving downstream while additional arriving vehicles may join the queues. If the number of the vehicles in a stationary queue is large and the flow rate of the same lane is high, a moving queue may exist upstream of the detector in that lane by the time the initial portion of time has elapsed. When this happens and a reasonably long unit extension is implemented, vehicles in the moving queue may cross the detection area at headways of no more than one unit extension. Under this condition, the green will be extended continuously until the moving queue disappears from every lane or until the maximum allowable green is reached.

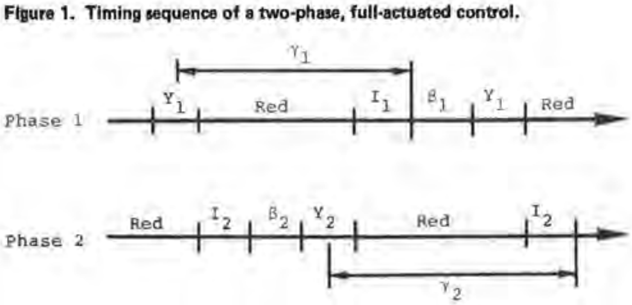
The growth and decay of a moving queue in a given cycle depends on the number of arrivals in time period γ_i (refer to Figure 1) and also on the flow rate of the traffic lane. γ_i represents a time interval in a cycle approximately from the beginning of the yellow duration of phase i until the end of the initial portion. For a two-phase control, the values of γ_1 and γ_2 are as follows:

$$\gamma_1 = 0.5Y_1 + I_2 + \beta_2 + Y_2 + I_1 \quad (11)$$

$$\gamma_2 = 0.5Y_1 + I_1 + \beta_1 + Y_1 + I_2 \quad (12)$$

In these two equations, it is assumed that 50 percent of the yellow duration in each phase will be used as the green duration by the vehicles in that phase. Therefore, vehicles that arrive in the first half of the yellow duration will enter the intersection instead of waiting for the next green.

Figure 1. Timing sequence of a two-phase, full-actuated control.



To allow a moving queue to extend the green, the first queueing vehicle immediately upstream of the detector should be able to move into or across the detection area before the first unit extension of time has elapsed. This feature of the control can generally be ensured if the following condition is satisfied:

$$w(N + 1) < I_i$$

where w is the average time required for each queueing vehicle to start moving after the light has turned from red to green (approximately 1.5 s), and N is the maximum number of queueing vehicles that may be stored between the stop line and the detector.

The above condition implies that the first vehicle immediately upstream of the detector should have started moving toward the detector before the initial portion of time has elapsed. This will allow the vehicle a time interval equal at least to one unit extension to move into the detection area at a sufficiently high speed (2-3 mph) to actuate the detector. In this study, this condition is considered satisfied.

To extend the green continuously, the vehicles in the moving queue should cross the detection area at headways of no more than one unit extension. Studies of queue discharge headways at intersections have revealed that a moving queue formed by signal interference usually reaches a steady average headway of about 2.2 s (2). Therefore, if the unit extension is set at a value of 3.5 s, as recommended by the Southern Section of the Institute of Traffic Engineers (3), every vehicle in the moving queue should have little difficulty extending the green.

Assume that the unit extension is greater than the headways at which vehicles in a moving queue cross the detection area. Furthermore, let n represent the number of vehicles in a lane that have arrived during γ_i. If the nth vehicle is in the queue, the time (B_n) required for this vehicle to reach the detector after the initial portion of time has elapsed can be approximated by

$$B_n = nw + [2(nL - S_i)/A]^{0.5} - I_i \quad (13)$$

where L is the average longitudinal space occupied by a vehicle (approximately 25 ft) and A is the average acceleration rate of a vehicle from a standing position (approximately 6 ft/s²).

If $B_n \leq 0$, no moving queue exists upstream of the detector in a lane by the time the initial portion of time has elapsed. If B_n is greater than zero, the n arrivals will create a moving queue to extend the green after the initial portion of time has elapsed. By using representative values of w = 1.5 s, L = 25 ft, A = 6 ft/s², S_i = 120 ft, and I_i = 12.5 s, the smallest n needed to form a moving queue is 7 vehicles. In the following discussion the smallest n needed to form a moving queue will be denoted as n_{min}.

During time interval B_n additional vehicles may arrive and join the queue and, thus, continue to extend the green. Let μ be the rate at which the queueing vehicles move across the detector and λ_j, as defined previously, be the flow rate of the jth lane associated with phase i. If $\mu > \lambda_j$, the average time required for the moving queue to disappear from the upstream side of the detector in the jth lane is

$$\Delta_{nj} = \mu B_n / (\mu - \lambda_j) \quad (14)$$

The value of Δ_{nj} as given in this equation equals the time interval from the moment the initial portion of time has elapsed to the moment the moving queue disappears from the upstream side of the detector. To ensure that the maximum allowable green is not exceeded, a limiting value $(B_n)_{max}$ should be imposed on B_n. Given the value of E_i as obtained from Equation 10, this constraint can be stated as $I_i + \Delta_{nj} + E_i \leq (G_{max})_i$ or

$$B_n < (B_n)_{max} = [(\mu - \lambda_j)/\mu]\,[(G_{max})_i - I_i - E_i] \tag{15}$$

If phase i has m lanes, the longest Δ_{nj} in a cycle for this phase will determine the green extended by the moving queues in that cycle. In other words, D_{ni} for a given cycle is the maximum of Δ_{n1}, Δ_{n2}, ..., Δ_{nm}. Since the longest Δ_{nj} may not always be produced by the same lane flow in every cycle, the contribution of the moving queue in each lane to the extension of the green has to be estimated.

The probability that there will be no moving queue in the jth lane in a cycle is $P_j(n < n_{min})$. This probability is a function of the flow rate λ_j of the jth lane and the time interval γ_i. It can be determined from Equation 1 as the sum of $P_j(n = 0/\gamma_i)$, $P_n(n = 1/\gamma_i)$, ..., $P_j(n = n_{min} - 1/\gamma_i)$. The values of $P_j(n < n_{min})$ for $n_{min} = 7$ and various combinations of λ_j and γ_i are given in Figure 2.

When m lanes are associated with phase i, the probability that there will be no moving queue in any of the lanes in a cycle is the product of $P_1(n < n_{min})$, $P_2(n < n_{min})$, ..., $P_m(n < n_{min})$. Accordingly, the probability that there will be at least one moving queue in a cycle for phase i is

$$p_i = 1 - P_1(n < n_{min}) \cdot P_2(n < n_{min}) \ldots P_m(n < n_{min}) \tag{16}$$

Given that there is at least one moving queue in a cycle, the probability that the flow in the jth lane will result in the longest Δ_{nj} can be approximated by

$$g_j = \exp(0.0075\lambda_j) / \sum_{k=1}^{m} \exp(0.0075\lambda_k) \tag{17}$$

where λ_j is measured in vehicles per hour.

This equation was developed through a simple probabilistic simulation of the number of arrivals in each lane during time period γ_i and the resulting Δ_{nj} for various combinations of lane flows. Analytical formulation of g_j is possible but the computation requirements associated with it are prohibitively tedious.

Given that the jth lane has a moving queue and that this moving queue results in the longest Δ_{nj} in a cycle for a phase, the expected length of the green extended by the moving queue in this lane can be approximated by

$$\left[\sum_{n=n_{min}}^{\infty} P_j(n/\gamma_i) \cdot \Delta_{nj}\right] / [1 - P_j(n < n_{min})]$$
$$= [\mu/(\mu - \lambda_j)] \sum_{n=n_{min}}^{\infty} [B_n P_j(n/\gamma_i)] / [1 - P_j(n < n_{min})]$$
$$= [\mu/(\mu - \lambda_j)] \cdot \alpha_j \tag{18}$$

The values of α_j for B_n that satisfies Equation 15 are given in Figure 3 for various combinations of λ_j, γ_i, and $(B_n)_{max}$.

Taking into account the contribution of each lane flow to the extension of the green, the value of D_i can be determined as

$$D_i = \sum_{j=1}^{m} [\mu/(\mu - \lambda_j)]\, \alpha_j p_i g_j \tag{19a}$$

This equation is applicable if $\mu > \lambda_j$. If $\mu \leq \lambda_j$, then

$$D_i = (G_{max})_i - E_i - I_i \tag{19b}$$

With the aid of Figures 2 and 3, the estimation of D_i can be simplified. Table 1 gives an example computation of D_i for a phase that involves three lanes.

The value of D_i can also be estimated from a simple Monte Carlo simulation (4). Table 2 compares the values of D_i estimated from Equation 19a and those generated from such a simulation. The differences between the values obtained from the two methods are negligibly small.

Figure 2. Value of $P_j(n<n_{min})$ for n_{min} = 7.

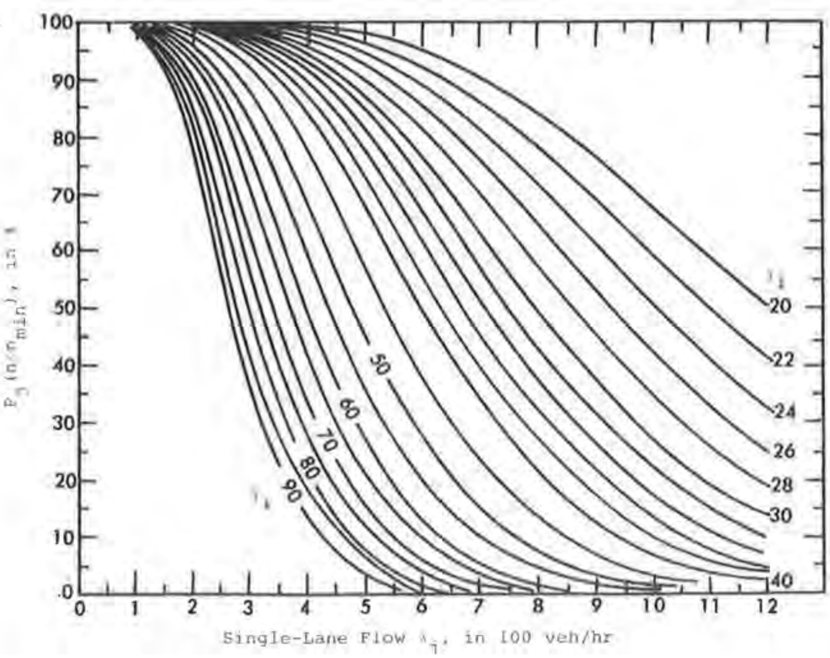


Figure 3. Value of α_j as a function of $\lambda_j\gamma_i$ and $(B_n)_{max}$.

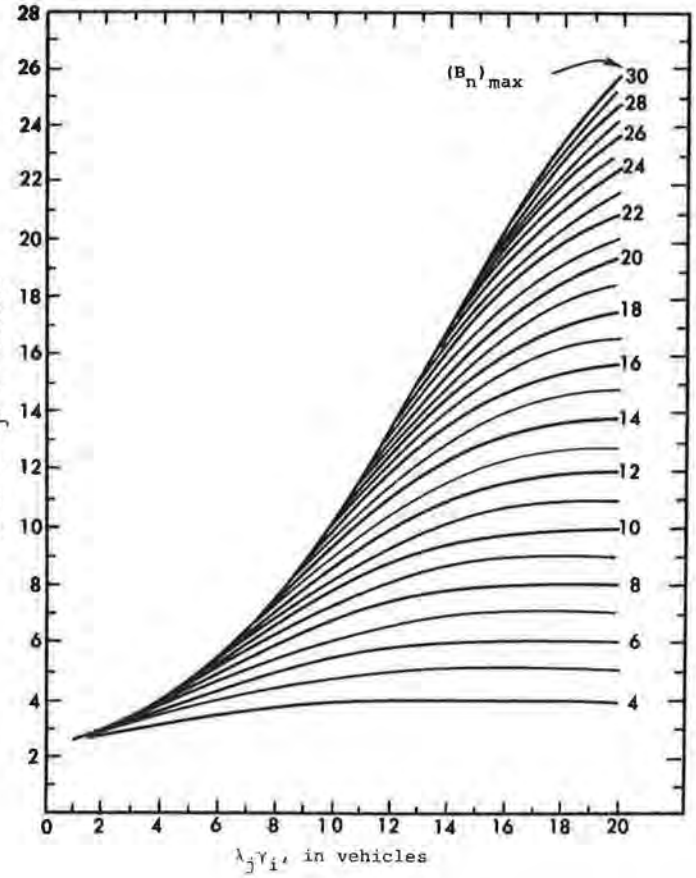


Table 1. Example computation of D_i.

Parameter	Lane, j 1	2	3	Source
$P_j(n < n_{min})$	0.076	0.975	0.675	Figure 2
ρ_i	0.950	0.950	0.950	Equation 16
$\exp(0.0075\lambda_j)$	403	4	20	
ε_j	0.944	0.009	0.047	Equation 17
$(\mu - \lambda_j)/\mu$	0.50	0.88	0.75	
$(B_n)_{max}$	15	26.4	23.5	Equation 15
$\lambda_j\gamma_i$	11.1	2.8	5.6	
α_j	10.8	3.4	5.3	Figure 3
D_i	20.1	20.1	20.1	Equation 19a

Note: This table is based on the following set of data:
$(G_{max})_i - I_i - E_i = 30$ s,
$n_{max} = 7$,
$\gamma_i = 50$ s,
$\lambda_1 = 800$ vehicles/h,
$\lambda_2 = 200$ vehicles/h,
$\lambda_3 = 400$ vehicles/h, and
$\mu = 1600$ vehicles/h.

A Model

Given I_i, E_i, and D_i, the average green (G_i) for phase i can be determined as

$$G_i = I_i + D_i + E_i \tag{20}$$

In this equation D_i is a function of γ_i, which, in turn, depends on β_i (refer to Figure 1 and Equations 11 and 12). The length of β_i varies from one cycle to another. As an approximation, β_i can be used to represent the average length of the extended portion of the green for phase i. With this approximation, the sum of I_i

Table 2. Comparison of simulated and estimated D_i (μ = 1600 vehicle/h).

Case	Flow Rate Lane, Lane j (vehicles/h) 1	2	3	4	Simulated	Estimated
1	800	200	400	0	21.1	20.1
2	600	200	400	0	10.2	9.4
3	600	800	300	100	21.6	19.2
4	500	500	300	300	8.1	7.4
5	500	300	200	100	5.2	5.8
6	1000	800	400	200	29.1	26.6
7	1000	800	400	200	9.2	9.2
8	600	800	300	100	3.2	3.8
9	1000	400	0	200	7.8	7.5
10	600	300	200	200	0.7	1.1
11	800	500	200	100	3.2	3.5

Note: For cases 1 through 6, $\gamma_i = 50$ s and $(G_{max})_i - I_i - E_i = 30$ s; for cases 7 through 11, $\gamma_i = 25$ s and $(G_{max})_i - I_i - E_i = 20$ s.

and β_i equals the average green of phase i and Equations 11 and 12 can be rewritten as

$$\gamma_1 = 0.5Y_1 + Y_2 + I_1 + G_2 \tag{21}$$

$$\gamma_2 = 0.5Y_2 + Y_1 + I_2 + G_1 \tag{22}$$

Thus, γ_1 depends on G_2 and γ_2 varies with G_1. Since G_1 and G_2 are unknown, an iteration process has to be used to determine their values. The iteration process may include the following steps:

Step 1: Let $G_2 = I_2 + U_2$;
Step 2: Determine γ_1 from Equation 21 and use Equations 10, 19, and 20 to obtain an estimate of G_1; denote the estimate as G_1;

Step 3: Use $\tilde{G}_1$ in Equation 22 to determine γ_2 and use Equations 10, 19, and 20 to obtain an estimate of G_2; denote the estimate as $\tilde{G}_2$; and

Step 4: Compare the assumed G_2 and the estimated $\tilde{G}_2$; if the difference is sufficiently small (e.g., ≤ 1 s), stop the iteration. Otherwise, use the estimated $\tilde{G}_2$ as a new trial value and go back to the second step.

This process generally requires less than two iterations to obtain estimates of G_1 and G_2 that are within 1 s of the final solutions.

Comparisons with Simulation Results

Prior to the current study, a microscopic simulation model has been developed for analyzing the performance of pretimed and traffic-actuated signal controls. The model contains a traffic flow processor and a signal processor. The flow processor generates random arrivals in each lane at an arbitrarily specified distance from the intersection. The arriving vehicles in each lane are then processed downstream according to a set of car-following, gap acceptance, and car-signal interaction behaviors. The location, speed, and acceleration of each vehicle are updated once every second. The processing of vehicles is dependent only on signal indications. The signal indications for each 1-s interval are determined by the signal processor according to the logic of a control.

Table 3 gives the average green durations (G_1 and G_2) obtained from the simulation model and from Equations 10, 19, and 20 for 12 combinations of flows. It shows that Equations 10, 19, and 20 result in estimates that are practically the same as those generated from the complex simulation model. Similar comparisons based on other combinations of $(G_{max})_1$, $(G_{max})_2$, and lane flows reveal that the differences between the simulated and the theoretical values of G_1 and G_2 are generally less than 2 s. In these comparisons, the simulation model employs (a) a driver reaction time of 1.5 s in a stationary queue that corresponds to $w = 1.5$ s in Equation 13 and (b) a minimum spacing of 25 ft between two successive vehicles that corresponds to $L = 25$ ft in the same equation. Furthermore, the value of μ in Equation 19a is assumed to be 1600 vehicles/h (i.e., about 1 vehicle in every 2.2 s) and that of A in Equation 13 equals 6 ft/s². The settings of the control parameters are $S_i = 120$ ft, $I_i = 12.5$ s, $U_i = 3.5$ s, and $Y_i = 3.5$ s. It can be shown that a 15-20 percent variation in the value of any w, L, A, and μ from the assumed values has negligible effects on the estimated values of G_1 and G_2.

TRANSFORMATION OF LEFT-TURN FLOWS

When left-turn flows are not opposed they can be treated essentially the same as straight-through flows. When they are opposed, however, left-turn vehicles may not freely actuate the detectors to extend the green duration. Consequently, a left-turn vehicle is less effective than a straight-through vehicle in extending a green duration.

The complexity of the interactions between opposed left turns and the signal control makes it difficult to examine analytically the influence of the turning movements on the average green durations. For this reason, the microscopic simulation model described previously was used as a tool to identify a mechanism for transforming an opposed left-turn flow into an equivalent straight-through flow. The simulation model has proved to be capable of providing a reasonable representation of opposed left-turn movements (5).

Based on an examination of more than 50 combinations of flow patterns that involve opposed left turns of up to 900 vehicles/h, the simulated average greens reveal that an opposed left-turn flow can be transformed into a straight-through equivalent flow according to the following constraint:

$$(Q_s)_{max} = 900 - Q_o \tag{23}$$

where $(Q_s)_{max}$ is the maximum straight-through flow equivalent of an opposed left-turn flow in vehicles per hour, and Q_o is the opposing flow rate in vehicles per hour.

Equation 23 implies that, if an opposed left-turn flow exceeds its $(Q_s)_{max}$, then it can be transformed into a straight-through flow with a flow rate equal to $(Q_s)_{max}$. Otherwise, it can be treated directly as a straight-through flow. For example, a left-turn flow of 400 vehicles/h opposed by a flow of 600 vehicles/h has a $(Q_s)_{max}$ of 300 vehicles/h. Therefore, the left-turn flow of 400 vehicles/h is equivalent to a straight-through flow of 300 vehicles/h. If the left-turn flow is 200 vehicles/h instead, then this left-turn flow can be treated directly as a straight-through flow.

Figure 4 shows a comparison of simulated average green durations for a number of traffic patterns that involve opposed left turns and the corresponding durations estimated from Equations 10, 19, and 20 based on transformed left-turn flows. The differences are generally less than 4 s. The settings of the control parameters used in the comparison are $U_i = 3.5$ s, $I_i = 12.5$ s, $S_i = 120$ ft, $Y_i = 3.5$ s, and $(G_{max})_i$ ranges from 35 to 50 s.

IMPACT OF FLOW VARIATION ON CYCLE SPLITS

The model described previously can be applied to any time period within which the flow rate in each lane does not vary significantly. If the flow rate varies significantly with respect to time, an adjustment of the average green as estimated from Equations 10, 19, and 20 may be necessary.

One way to account for the impact of the flow variation on the cycle splits is to divide the time period into several intervals, each of which has a more or less constant flow rate. With this approximation, the model can be applied to each time interval and the resulting average green for each interval can then be used to obtain an estimate of the average green for the entire period. Figure 5 shows three approximated hourly flow patterns with varying degrees of flow variation. Pattern A has a constant average 5-min flow rate. Pattern B has a moderate variation in its average 5-min flow rate. The flow variation represented by pattern C is substantially higher than that of pattern B.

The extent of the variation in the flow rate can be conveniently defined in terms of the peak-hour factor described in the Highway Capacity Manual (6). The peak-hour factors associated with patterns A, B, and C are, respectively, 1.0, 0.85, and 0.70. Based on pattern B and pattern C, Equations 10, 19, and 20 were applied to each 5-min interval to obtain estimates of average green durations for various combinations of hourly lane volumes. The results are given in Table 4 along with estimates generated directly from the microscopic simulation model. This table shows that repeated application of Equations 10, 19, and 20 can adequately account for the impact of the flow variation on the average green of a signal phase.

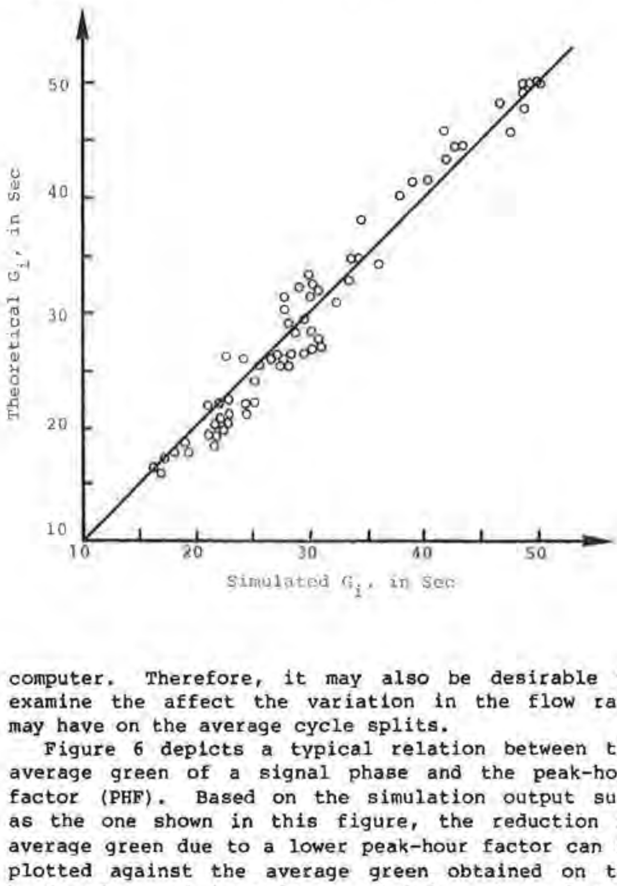
The repeated application of Equations 10, 19, and 20, however, is tedious unless it is aided with a

Table 3. Simulated values of G_1 and G_2 versus values obtained from $(G_{max})_1 = (G_{max})_2 = 35$ s.

Flow Rate (vehicles/h)				Avg Green (s)			
Phase 1		Phase 2		Simulation Model		Equations[a]	
Lane 1	Lane 2	Lane 1	Lane 2	G_1	G_2	G_1	G_2
600	200	300	100	22.1	17.6	20.7	17.0
600	200	600	200	23.0	23.0	22.3	22.3
600	200	750	250	25.1	29.2	24.3	28.0
600	200	900	300	27.0	34.1	25.8	32.4
600	200	1200	400	27.4	35.0	26.7	34.8
600	200	1500	500	27.4	35.0	26.7	35.0
450	150	300	100	19.0	17.4	18.1	16.9
450	150	600	200	18.9	21.2	18.6	21.1
450	150	750	250	19.8	27.3	19.3	25.8
450	150	900	300	20.4	31.1	20.1	30.5
450	150	1200	400	20.8	35.0	21.0	34.5
450	150	1500	500	20.8	35.0	20.1	35.0

[a]Values were obtained from Equations 10, 19, and 20.

Figure 4. Simulated and theoretical average greens that involve opposed left turns.

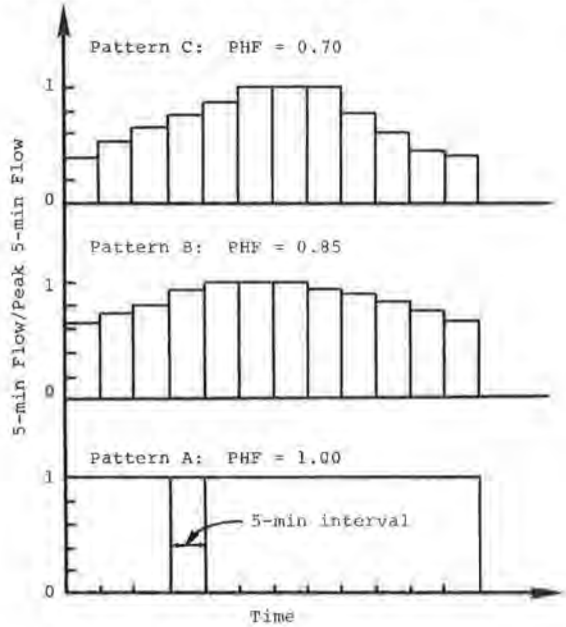


computer. Therefore, it may also be desirable to examine the affect the variation in the flow rate may have on the average cycle splits.

Figure 6 depicts a typical relation between the average green of a signal phase and the peak-hour factor (PHF). Based on the simulation output such as the one shown in this figure, the reduction in average green due to a lower peak-hour factor can be plotted against the average green obtained on the basis of a peak hour factor of 1.0. Figure 7 presents such reductions at four levels of maximum allowable green: 35, 40, 45, and 50 s. This figure indicates that, for all practical purposes, the impact of the flow variation can be neglected if the maximum allowable green for a phase is less than 35 s. For a higher setting of the maximum allowable green, an adjustment may be readily obtained from the figure.

For example, if the lane flows of a given signal phase have a peak-hour factor of 0.7 and the average green duration as estimated from Equations 10, 19, and 20 is 42 s, then Figure 7 indicates that the

Figure 5. Flow patterns with different degrees of flow variation.



estimated green should be reduced by 7 s if the maximum allowable green is 50 s and by 6 s if the maximum allowable green is 45 s.

Note that Figure 7 gives at best an approximate picture of the impact of the flow variation on the average green duration. A simple chart that can be used with an equal degree of accuracy for estimating the impact of the flow variation under varying traffic and signal control conditions is difficult to develop. If a greater degree of accuracy is desired, the repeated application of Equations 10, 19, and 20 may become necessary. I have a simple computer program that implements these equations for general applications.

MODEL FOR MULTIPHASE CONTROL

If the signal phases of a multiphase control are arranged in sequence without overlapping, Equations 10, 19, and 20 are still applicable. The only parameter that requires modifications is γ_i. Under a multiphase control, the value of γ_i can be generalized into the following form:

$$\gamma_i = 0.5Y_i + I_i + \sum_k (Y_k + G_k) \tag{24}$$

where Y_k, G_k are the yellow duration and the green duration of a competing phase k ($k \neq i$), respectively.

The estimation of G_i ($i = 1,2,\ldots$) still has to rely on an iteration procedure that may include the following steps:

Step 1: Transform opposed left-turn flows into equivalent straight-through flows according to the condition set forth in Equation 23;

Step 2: Let $G_i = I_i + U_i$ for $i = 2,3,4,\ldots$;

Step 3: Use Equation 24 to determine γ_1 (i.e., $\gamma_1 = 0.5Y_1 + I_1 + Y_2 = G_2 + Y_3 + \ldots$);

Step 4: Use Equations 10, 19, and 20 to obtain an estimate of G_1; denote this estimate as $\bar{G}_1$;

Step 5: Use $\bar{G}_1$ in Equation 22 to determine γ_2 (i.e., $\gamma_2 = 0.5Y_2 + I_2 + Y_1 + \bar{G}_1 + Y_3 + \bar{G}_3 + \ldots$) and then obtain an estimate of G_2 from Equations 10, 19, and 20; denote the estimate as $\bar{G}_2$; use $\bar{G}_1$ and $\bar{G}_2$ to estimate G_3; continue this task until an estimate

Table 4. Average green durations that result from nonuniform average flow rates for $(G_{max})_1 = (G_{max})_2 = 45$ s.

Hourly Flow Rate (vehicles/h)				Avg Green Duration(s), Peak-Hour Factor = 0.70				Avg Green Duration(s), Peak-Hour Factor = 0.85			
Phase 1		Phase 2		Equations[a]		Simulation Model		Equations[a]		Simulation Model	
Lane 1	Lane 2	Lane 1	Lane 2	G_1	G_2	G_1	G_2	G_1	G_2	G_1	G_2
200	200	200	200	17.1	17.1	17.6	17.5	16.8	16.8	17.4	17.5
200	200	600	600	17.2	25.8	18.0	25.8	16.9	24.5	17.8	25.7
200	200	800	800	17.5	32.1	18.5	32.0	17.1	32.8	17.9	35.1
200	200	1000	1000	17.6	36.7	17.7	37.9	17.3	39.6	17.7	41.2
600	600	200	200	25.8	17.2	25.7	17.7	24.5	16.9	25.8	17.4
600	600	600	600	30.5	30.5	28.2	28.4	30.2	30.2	31.7	32.4
600	600	800	800	31.9	35.2	31.4	38.9	33.0	38.4	34.2	40.2
600	600	1000	1000	32.7	38.4	32.3	39.6	34.4	42.6	35.4	42.8

[a]Values were obtained from Equations 10, 19, and 20.

Figure 6. Variation of average green duration related to peak-hour factor.

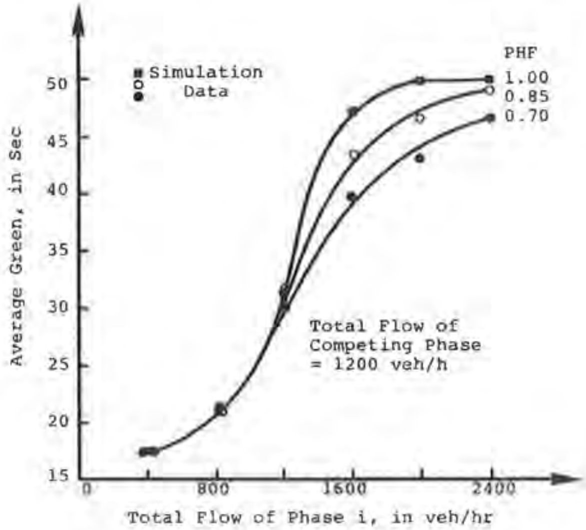
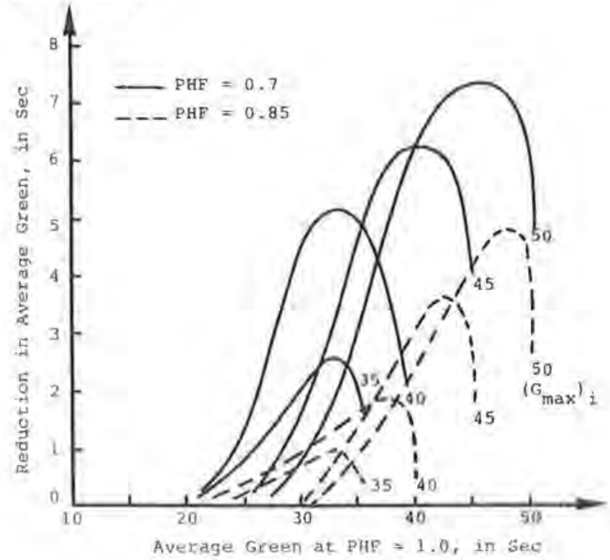


Figure 7. Reductions in average green durations at peak-hour factors smaller than 1.0.



of the average green is obtained for the last phase; and

Step 6: Compare the assumed G_2, G_3, ... with the estimated $\bar{G}_2$, $\bar{G}_3$, ...; if any pair of G_i and $\bar{G}_i$ (i = 2,3,...) has a significant difference (e.g., >1 s), use $\bar{G}_2$, $\bar{G}_3$, ... as the new trial values for G_2, G_3, ... and go back to step 3.

This procedure generally requires only two iterations to obtain the needed estimates.

As in the case of the two-phase control, the impact of the flow variation on the average green can be taken into account in two ways. One is to use Figure 7 to estimate the needed adjustments in the values of the average greens. The other requires repeated application of the iteration procedure to successive time intervals. The computer program mentioned previously can also be used for this type of application.

CONCLUSIONS

The model as represented by Equation 10, 19, and 20 provides traffic engineers with a convenient tool to estimate full-actuated cycle splits. With the aid of charts to simplify computations, the model can be readily and manually used. It can also be implemented in the form of a simple computer program that requires limited computing facilities. The model can be incorporated into current methodologies that are being used for the planning, design, and operation of signalized intersections.

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