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# Method to Establish Pay Schedules for Rigid Pavement

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**An equation is derived to compute the appropriate pay factor for any quality level of rigid pavement. The measure of quality used in this development is the estimated load-bearing capacity of the pavement although the results may be applied to specifications based on other quality measures. The appropriate pay adjustment is considered to be the present worth of any expense or savings expected to occur in the future as the result of a departure from the specified level of quality and may be positive or negative. Sensitivity tests demonstrate that the method is reliable provided the input variables are determined with reasonable accuracy. By using input values typical of a relatively urbanized area, this procedure indicates that a minimum pay factor of about 60 percent is appropriate for the poorest-quality work and a maximum pay factor of about 115 percent is justified for work of truly superior quality. Additional factors are cited that, although unquantified, would tend to lower the minimum pay factor and raise the maximum pay factor. Finally, pay schedules are developed, the operating-characteristic curves of which closely approximate the theoretically derived relationship.**

Statistical end-result specifications are now in widespread use and one of the reasons for their popularity among specification writers is that they provide a practical way to deal with work that is only slightly deficient. A construction item that falls just short of the specified quality level does not warrant rejection but neither does it deserve 100 percent payment. Accordingly, statistical specifications usually employ some form of adjusted pay schedule to award payment in proportion to the level of quality actually achieved.

Throughout the nearly 20 years that specifications of this type have been evolving, several methods (1-3) have been proposed to establish the level of payment appropriate for different levels of quality. In those cases for which there is little or no information relating quality measures to performance, this is an especially difficult task and the methods have necessarily been quite arbitrary. However, there are a few cases for which the quality-performance relationship is well established and these, at least, provide the opportunity to develop a rational and logical procedure for determining appropriate pay factors.

One type of construction for which there are ample data relating performance to various quality characteristics is rigid (portland cement concrete) pavement. The design guide (4) of the American Association of State Highway and Transportation Officials (AASHTO) has just been updated and now provides an equation that gives the expected number of equivalent 18-kip load applications that a rigid pavement can sustain as a function of several common quality characteristics. The details of the manner in which this equation can be used are presented in a separate paper by Weed in this Record. For the purposes of this paper, it is simply desired to establish that the technology required to design a pavement can also be used to assess the quality of a pavement, the as-built characteristics of which differ from the intended design values.

## BASIS FOR PAY ADJUSTMENTS

Ordinarily, a pavement is designed to sustain a specified number of load applications before major repair (overlaying with bituminous concrete) is required. If, due to construction deficiencies, the pavement is not capable of withstanding the design loading, it will fail prematurely. The necessity of repairing this pavement at an earlier date results in an additional expense that, since it usually occurs long after any contractual obligations have expired, must be borne by the highway agency. It is the purpose of the adjusted pay schedule to withhold sufficient payment at the time of construction to cover the extra cost anticipated in the future as the result of deficient-quality work.

Based on the procedure used to arrive at the original design parameters of the pavement, the as-built parameters can be used to estimate the fraction of design loadings the pavement will actually be able to sustain. For practical purposes, it is reasonable to assume that the yearly traffic volume is constant so that this fraction can be multiplied by the design life to obtain the expected life. Then, based on current construction costs and projected interest and inflation rates, it is possible to compute both future and present-worth values for credits and debits resulting from the rescheduling of the several generations of overlays that are required after the useful service life of the original pavement has been exhausted. The appropriate pay adjustment is the present worth of the sum of these credits and debits and, depending on the estimated life of the original pavement, this adjustment may be either positive or negative. As a result, the corresponding pay factors obtained by this method are not limited to a maximum of 100 percent.

In essence, pay schedules derived by this method comprise both a liquidated damages clause and a bonus provision. For those agencies not desiring to apply a bonus provision, the pay factors can be limited to a maximum of 100 percent. Alternatively, a crediting concept (5) can be used that allows pay factors greater than 100 percent to offset other pay factors lower than 100 percent while the overall average pay factor is still limited to a maximum of 100 percent.

## BASIC FORMULAS

Certain basic engineering economics formulas (6) will be found to be useful in the development of the pay-factor equation. The compound-interest formula can be modified slightly to compute the projected future cost of an item as follows:

$$C_n = C_o(1 + R_{INF}/100)^n \quad (1)$$

where

$C_n$ = future cost after n years,
$C_o$ = present cost, and
$R_{INF}$ = inflation rate (percent per year).

The present worth of this future cost is given by

$$W_o = C_n/(1 + R_{INT}/100)^n \quad (2)$$

where

$W_o$ = present worth,
$C_n$ = future cost after n years, and
$R_{INT}$ = interest rate (percent per year).

Then, by defining the ratio

$$R = (1 + R_{INF}/100)/(1 + R_{INT}/100) \quad (3)$$

and substituting Equation 1 into Equation 2, the expression for the present worth of a future cost can be simplified to

$$W_o = C_oR^n \quad (4)$$

which provides an effective means to estimate the present economic impact of the decision to make (or cancel) a future expenditure.

Finally, since the interest rate is often stated as compounded on some periodic basis of less than a year, it will be useful to have an expression to convert it to an equivalent annual interest rate of the form used in Equations 2 and 3. This can be accomplished by the following equation:

$$R_{INT} = 100[(1 + R_{COMP}/100m)^m - 1] \quad (5)$$

where

$R_{INT}$ = equivalent annual interest rate (percent per year),
$R_{COMP}$ = compound interest rate (percent), and
m = annual frequency of compounding.

## NUMERICAL EXAMPLE

Before the general expression for the appropriate pay factor is derived, it will be instructive to work out a numerical example by using data representative of a moderately urbanized area. A typical in-place bid price for concrete pavement is approximately $\$30/yd^2$. Because the pay factors to be developed will be based on the economic effect of rescheduling the successive overlays that will eventually be installed, the overlay costs must account for all operations normally included in a resurfacing contract. A review of construction costs for several projects suggests that $\$8/yd^2$ and $\$7/yd^2$ are typical costs for the first and subsequent overlays, respectively. It will be assumed that the design life of the pavement is 20 years, its expected life based on as-built measurements is 16 years, and the expected life of all overlays is 10 years. The annual interest and inflation rates are assumed to be 15 percent and 10 percent, respectively.

Based on this information, it is required to determine the economic impact on the highway agency of the expected premature failure of the original pavement. This will include not only the effect of installing the first overlay four years sooner than planned but in addition the effects of installing all subsequent overlays an equal amount of time ahead of schedule. The credits and debits resulting from the rescheduling of the overlays are computed by means of Equation 4 by using R = 1.10/1.15 = 0.9565. The computations for the first three overlay generations are tabulated as follows:

| Year | Overlay Status | Present Worth | Credit or Debit |
|---|---|---|---|
| n = 16 | Scheduled | $\$8 \times R^{16} = \$3.93$ | Debit |
| n = 20 | Cancelled | $\$8 \times R^{20} = \$3.29$ | Credit |

(The net effect of rescheduling the first overlay is a debit of \$3.93 - \$3.29 = \$0.64.)

| Year | Overlay Status | Present Worth | Credit or Debit |
|---|---|---|---|
| n = 26 | Scheduled | $\$7 \times R^{26} = \$2.20$ | Debit |
| n = 30 | Cancelled | $\$7 \times R^{30} = \$1.84$ | Credit |

(The net effect of rescheduling the second overlay is a debit of \$2.20 - \$1.84 = \$0.36.)

| Year | Overlay Status | Present Worth | Credit or Debit |
|---|---|---|---|
| n = 36 | Scheduled | $\$7 \times R^{36} = \$1.41$ | Debit |
| n = 40 | Cancelled | $\$7 \times R^{40} = \$1.18$ | Credit |

(The net effect of rescheduling the third overlay is a debit of \$1.41 - \$1.18 = \$0.23.)

The total effect of rescheduling the successive generations of overlays is the sum of the individual effects. Unless it is known that the pavement is planned to be phased out of existence at some specific time in the future, it is appropriate to continue this computational procedure until the terms become so small that they contribute nothing further to the total. The continuation of this procedure produced the following results:

| Overlay | Debit (\$) |
|---|---|
| 1 | 0.640 |
| 2 | 0.359 |
| 3 | 0.230 |
| 4 | 0.147 |
| 5 | 0.095 |
| 6 | 0.061 |
| 7 | 0.039 |
| 8 | 0.025 |
| 9 | 0.016 |
| 10 | 0.010 |
| 11 | 0.007 |
| 12 | 0.004 |
| 13 | 0.003 |
| 14 | 0.002 |
| 15 | 0.001 |
| 16 | 0.001 |
| Total | 1.640 |

The total unit debit resulting from the early scheduling of the successive generations of overlays is $\$1.64/yd^2$. Since the unit cost of this pavement is $\$30/yd^2$, the appropriate pay factor for this particular example can be expressed in decimal form as follows:

$$F = (30 - 1.64)/30 = 0.945 \quad (6)$$

This will be used as a check value for the general expression that is to be derived.

It is interesting to note at this point that a pavement, the expected life of which is 80 percent of its design life, is deemed worthy of 94.5 percent of the contract price. More will be said about this later.

DERIVATION OF PAY-FACTOR EQUATION

Like the numerical example, the pay-factor equation will be derived as a function of the following variables:

$C_P$ = present unit cost of original pavement (\$/yd²),
$C_{01}$ = present unit cost of first overlay (\$/yd²),
$C_{02}$ = present unit cost of subsequent overlays (\$/yd²),
$L_{PD}$ = design life of original pavement (years),
$L_{PE}$ = expected life of original pavement (years),
$L_{OE}$ = expected life of overlays (years),
$R_{INT}$ = annual interest rate (percent),
$R_{INF}$ = annual inflation rate (percent), and
$R$ = $(1 + R_{INF}/100)/(1 + R_{INT}/100)$.

The variable $C_P$ represents the bid price of the original pavement and is included as a reference on which the pay factors will be based. The variables $C_{01}$ and $C_{02}$ represent the total costs of the resurfacing projects that must be moved forward or backward in time depending on the expected life of the pavement. Two overlay costs are included because the first resurfacing often includes items of work not required for subsequent overlays.

By inspecting the computations for the numerical example, for which R = 1.10/1.15 = 0.9565, it can be seen that the portion of the pay adjustment resulting from the early scheduling of the first overlay is the following:

$$A_1 = C_{01}(R^{L_{PD}} - R^{L_{PE}}) = \$8 \times (R^{20} - R^{16}) = -\$0.64 \quad (7)$$

The negative sign indicates that this represents an expense to the highway agency. If the expected life of the pavement ($L_{PE}$) had been greater than the design life ($L_{PD}$), this value would have been positive, which represents a savings. In a similar manner, the equations for the adjustments resulting from the rescheduling of the next two overlays are written as follows:

$$A_2 = C_{02}(R^{L_{PD}+L_{OE}} - R^{L_{PE}+L_{OE}}) = \$7 \times (R^{30} - R^{26}) = -\$0.36 \quad (8)$$

$$A_3 = C_{02}(R^{L_{PD}+2L_{OE}} - R^{L_{PE}+2L_{OE}}) = \$7 \times (R^{40} - R^{36}) = -\$0.23 \quad (9)$$

By inspection of Equations 7 through 9, it is now possible to write the equation for the infinite series that gives the sum of all the individual pay adjustments:

$$\Sigma A_i = C_{01}(R^{L_{PD}} - R^{L_{PE}}) + C_{02}[(R^{L_{PD}+L_{OE}} - R^{L_{PE}+L_{OE}}) + (R^{L_{PD}+2L_{OE}} - R^{L_{PE}+2L_{OE}}) + (R^{L_{PD}+3L_{OE}} - R^{L_{PE}+3L_{OE}}) + \ldots] \quad (10)$$

By factoring and combining terms, this can be simplified as follows:

$$\Sigma A_i = (R^{L_{PD}} - R^{L_{PE}})[C_{01} + C_{02}(R^{L_{OE}} + R^{2L_{OE}} + R^{3L_{OE}} + \ldots)] \quad (11)$$

In this form, the last expression in the parentheses on the right is recognizable as a geometric progression (6, pp. 5-11), which, since R is less than unity, sums to $R^{L_{OE}}/(1 - R^{L_{OE}})$. Substituting this result into Equation 11 yields

$$\Sigma A_i = (R^{L_{PD}} - R^{L_{PE}})[C_{01} + C_{02}R^{L_{OE}}/(1 - R^{L_{OE}})] \quad (12)$$

as the sum of all pay adjustments. The final step is to combine this with the initial cost of the pavement ($C_P$) to write the equation for the appropriate pay factor:

$$F = \{C_P + (R^{L_{PD}} - R^{L_{PE}})[C_{01} + C_{02}R^{L_{OE}}/(1 - R^{L_{OE}})]\}/C_P \quad (13)$$

As a check, the values used in the numerical example will be substituted into this equation. With R = 1.10/1.15 = 0.9565, this yields

$$F = \{30 + (R^{20} - R^{16})[8 + 7R^{10}/(1 - R^{10})]\}/30 = 0.945 \quad (14)$$

which checks exactly with the result obtained earlier in Equation 6.

TYPICAL RESULTS

Before discussion of some additional factors that are involved, it will be of interest to use Equation 13 with the input parameters from the numerical example to compute the appropriate pay factor for various levels of the ratio of expected life to design life ($L_{PE}/L_{PD}$) as follows:

| $L_{PE}/L_{PD}$ | Appropriate Pay Factor |
|---|---|
| 0.0 | 0.597 |
| 0.2 | 0.709 |
| 0.4 | 0.802 |
| 0.6 | 0.880 |
| 0.8 | 0.945 |
| 1.0 | 1.000 |
| 1.2 | 1.046 |
| 1.4 | 1.084 |
| 1.6 | 1.116 |
| 1.8 | 1.143 |
| 2.0 | 1.165 |

There are two very interesting observations to be made from the values in this table. First, a pavement of such poor quality that its expected life is zero warrants a relatively high pay factor of about 60 percent. Second, a pavement of such exceptional quality that its expected life is double the design life warrants a bonus pay factor of approximately 116 percent, less than might have been anticipated.

The first observation can readily be explained. Although many of the early attempts to establish appropriate pay factors tended to relate payment directly to performance, the justification for such an approach is rather dubious. Unless unusually drastic repairs are required, a pavement capable of providing essentially zero performance still has considerable value as the subsystem on which the first generation of overlay will be placed. In the sense of liquidated damages, the highway agency has been damaged only to the extent of the present worth of the cost to restore the serviceability of the pavement throughout its intended design life. This is the basis for the pay factors computed by Equation 13.

The second observation was more of a surprise. Apparently, based on the typical input values that were used, the highway agency benefits only marginally from an extended service life.

## UNQUANTIFIED FACTORS

In actual practice, there are additional factors that must be taken into consideration:

1. There will be administrative costs involved in preparing for the premature repair of poor-quality pavement;
2. There will be costs to the motoring public for the earlier disruption of traffic to make the necessary repairs;
3. For practical reasons, a small section of poor-quality pavement may make it necessary to overlay a larger section of pavement; and
4. Premature failures, if many should occur, could severely restrict the priority-setting capabilities of a highway agency.

Because these factors are extremely difficult to quantify, they will be dealt with only in a qualitative manner. Since all four represent additional expenses that may occur when the quality is substandard, they provide a valid argument for a lowering of pay factors that are less than F = 1.0. Conversely, it can be argued that these same factors will result in a saving to the highway agency when the quality is superior. This would justify an increase in pay factors greater than F = 1.0. The net effect of these unquantified factors, therefore, would be a slight broadening of the range of possible pay factors. In the previous example, a minimum pay factor somewhat lower than F = 0.597 might be appropriate and a maximum pay factor slightly greater than F = 1.165 might be justified. This is a decision that would have to be made by each highway agency based on its assessment of the importance of the unquantified factors. One way in which this might be handled will be discussed in the section on the development of pay schedules.

## SENSITIVITY TESTS

When Equation 13 is applied, some of the input variables will be well determined whereas others will be known with less certainty. Of particular concern are the interest and inflation rates ($R_{INT}$, $R_{INF}$), since these values must be projected many years into the future. Another important variable is the design life ($L_{PD}$) of the original pavement. Although it might seem that this variable would be known exactly, it is strongly dependent on the accuracy of the forecast of traffic volume. The design loading of the pavement may be reached several years ahead of schedule if the traffic volume is substantially underestimated. The remaining variables--the present cost of the original pavement ($C_P$), the present costs of the bituminous overlays ($C_{O1}$, $C_{O2}$), and the expected life of an overlay ($L_{OE}$)--would be known quite accurately and will not be treated as variables in this first test. The pay factors in the following table were computed with Equation 13 by using $C_P$ = 30, $C_{O1}$ = 8, $C_{O2}$ = 7, and $L_{OE}$ = 10:

| | Appropriate Pay Factor | | | |
|---|---|---|---|---|
| $L_{PE}/L_{PD}$ | $L_{PD}$=20 $R_{INT}$=15 $R_{INF}$=10 | $L_{PD}$= 20 $R_{INT}$=12 $R_{INF}$=7 | $L_{PD}$=20 $R_{INT}$=13 $R_{INF}$=10 | $L_{PD}$=16 $R_{INT}$=15 $R_{INF}$=10 |
| 0.0 | 0.597 | 0.599 | 0.575 | 0.652 |
| 0.2 | 0.709 | 0.711 | 0.679 | 0.743 |
| 0.4 | 0.802 | 0.804 | 0.773 | 0.821 |
| 0.6 | 0.880 | 0.882 | 0.857 | 0.890 |
| 0.8 | 0.945 | 0.946 | 0.932 | 0.949 |
| 1.0 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.2 | 1.046 | 1.045 | 1.061 | 1.045 |
| 1.4 | 1.084 | 1.082 | 1.116 | 1.083 |
| 1.6 | 1.116 | 1.113 | 1.165 | 1.117 |
| 1.8 | 1.143 | 1.139 | 1.209 | 1.146 |
| 2.0 | 1.165 | 1.161 | 1.248 | 1.171 |

It can be seen from the pay factors in this table that Equation 13 is quite stable over a wide range of input values. By comparing the second and third columns, it is observed that changing the interest and inflation rates from $R_{INT}$ = 15 and $R_{INF}$ = 10 to $R_{INT}$ = 12 and $R_{INF}$ = 7 produces virtually no change in the pay factors that are obtained. Therefore, the method is not sensitive to the actual values of interest and inflation but to their difference, a parameter that is somewhat easier for a highway agency to predict.

By comparing the second and fourth columns, it can be seen that a substantial decrease in this difference from $R_{INT} - R_{INF}$ = 15 - 10 = 5.0 to $R_{INT} - R_{INF}$ = 13 - 10 = 3.0 has very little effect on the pay factors below F = 1.0 but an increasingly noticeable effect on the pay factors above F = 1.0. The minimum pay factor is reduced from F = 0.597 to F = 0.575, whereas the maximum pay factor is increased from F = 1.165 to F = 1.248.

Finally, by comparing the second and fifth columns, a decrease in design life from $L_{PD}$ = 20 to $L_{PD}$ = 16 is observed to have a moderate effect. The minimum pay factor is raised from F = 0.597 to F = 0.652, whereas all other pay factors are affected to a lesser degree.

Another set of computations will be of interest. For a variety of reasons, the unit cost of the first resurfacing may occasionally be greater than the value of $C_{O1}$ = \$8/yd$^2$ assumed in the examples thus far. To test the effect of this parameter, the pay factors in the following table were computed with Equation 13 by using $C_P$ = 30, $C_{O2}$ = 7, $L_{PD}$ = 20, $L_{OE}$ = 10, $R_{INT}$ = 15, and $R_{INF}$ = 10:

| | Appropriate Pay Factor | | | |
|---|---|---|---|---|
| $L_{PE}/L_{PD}$ | $C_{O1}$ = 8 | $C_{O1}$ = 9 | $C_{O1}$ = 10 | $C_{O1}$ = 11 |
| 0.0 | 0.597 | 0.578 | 0.558 | 0.539 |
| 0.2 | 0.709 | 0.695 | 0.680 | 0.666 |
| 0.4 | 0.802 | 0.792 | 0.783 | 0.773 |
| 0.6 | 0.880 | 0.874 | 0.868 | 0.862 |
| 0.8 | 0.945 | 0.943 | 0.940 | 0.937 |
| 1.0 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.2 | 1.046 | 1.048 | 1.050 | 1.052 |
| 1.4 | 1.084 | 1.088 | 1.092 | 1.096 |
| 1.6 | 1.116 | 1.122 | 1.127 | 1.133 |
| 1.8 | 1.143 | 1.150 | 1.157 | 1.164 |
| 2.0 | 1.165 | 1.174 | 1.182 | 1.190 |

It can be seen from the values in this table that substantial changes in the cost of the first resurfacing have a noticeable but moderate effect. As this variable increases from $C_{O1}$ = 8 to $C_{O1}$ = 11, the minimum pay factor decreases from F = 0.597 to F = 0.539 and the maximum pay factor increases from F = 1.165 to F = 1.190.

In all the preceding tests, the value of $C_{O1}$ has been assumed to remain constant for all values of the ratio $L_{PE}/L_{PD}$. If the as-constructed quality of the pavement were so poor that immediate repair would be necessary, the first overlay might have to be thicker than usual. If so, it would be justifiable to increase the value of $C_{01}$ when computing the appropriate pay factors at or near the zero performance level. However, the only effect of such a change would be a slight decrease at the extreme lower end of the pay schedule. It will be discussed in the section on the development of pay schedules why this refinement is believed to be unnecessary.

Still another set of computations will be useful. Although the present cost of the original pavement ($C_P$) and the present costs of the bituminous overlays ($C_{01}$, $C_{02}$) would be reasonably well known, there is no question that these prices will escalate and fluctuate with time. Furthermore, the experience of some highway agencies may suggest that the expected life of an overlay is different from the value of $L_{OE}$ = 10 years used in the preceding examples. To test the effect of these variables, the pay factors in the following table were computed with Equation 13 by using $L_{PD}$ = 20, $R_{INT}$ = 15, and $R_{INF}$ = 10:

| | Appropriate Pay Factor | | | |
|---|---|---|---|---|
| $L_{PE}/L_{PD}$ | $C_P$ = 30, $C_{01}$ = 8, $C_{02}$ = 7, $L_{OE}$ = 10 | $C_P$ = 60, $C_{01}$ = 16, $C_{02}$ = 14, $L_{OE}$ = 10 | $C_P$ = 27, $C_{01}$ = 8.8, $C_{02}$ = 7.7, $L_{OE}$ = 10 | $C_P$ = 30, $C_{01}$ = 8, $C_{02}$ = 7, $L_{OE}$ = 9 |
| 0.0 | 0.597 | 0.597 | 0.508 | 0.564 |
| 0.2 | 0.709 | 0.709 | 0.644 | 0.684 |
| 0.4 | 0.802 | 0.802 | 0.758 | 0.785 |
| 0.6 | 0.880 | 0.880 | 0.853 | 0.870 |
| 0.8 | 0.945 | 0.945 | 0.933 | 0.941 |
| 1.0 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1.2 | 1.046 | 1.046 | 1.056 | 1.050 |
| 1.4 | 1.084 | 1.084 | 1.103 | 1.091 |
| 1.6 | 1.116 | 1.116 | 1.142 | 1.126 |
| 1.8 | 1.143 | 1.143 | 1.175 | 1.155 |
| 2.0 | 1.165 | 1.165 | 1.202 | 1.179 |

The effects of both parallel and opposite movements of pavement and overlay costs can be judged from the second, third, and fourth columns in this table. The values in the second and third columns demonstrate that a uniform escalation of all prices produces no change in the pay factors that are computed. The values in the second and fourth columns show that an opposite movement of prices does have a noticeable effect. A decrease in pavement cost of 10 percent coupled with a 10 percent increase in overlay costs reduces the lowest pay factor from F = 0.597 to F = 0.508 while raising the largest pay factor from F = 1.165 to F = 1.202. Although this effect is not drastic, it suggests that the costs entered into Equation 13 should be composite values, averaged over a period of time, to minimize the influence of short-term price fluctuations.

The values in the second and fifth columns illustrate the effect of a 10 percent decrease in expected overlay life from $L_{OE}$ = 10 to $L_{OE}$ = 9. The lowest pay factor is reduced from F = 0.597 to F = 0.564 and the largest pay factor is increased from F = 1.165 to F = 1.179. Since a highway agency would have a reasonably accurate knowledge of the average life of an overlay, any error in the determination of this variable will not have a great effect on the resultant pay schedule.

Because the establishment of the minimum pay factor is an important step in developing any pay schedule, it will be of value to have a graph illustrating how this critical value is affected by uncertainty in the independent variables of Equation 13. The relative importance of each variable can be judged from the steepness of the curves in Figure 1. Of particular interest are the extremely shallow slopes and opposite inclination of the curves for interest and inflation rates, which indicates that a substantial degree of uncertainty in these variables can be tolerated. At the other extreme, the variable that has the steepest slope in Figure 1 is $C_P$, the cost of the original pavement. Since this variable has a strong influence on the resulting pay relationship, it is especially important that it be well determined. Fortunately, this can easily be accomplished by averaging the bid prices from several recent contracts.

Figure 1 can also be used as a guide to perform an especially severe test of the reliability of Equation 13. Although it would be very unlikely for any errors in the independent variables to all act in the same direction, the values used in the next test will be chosen to demonstrate the effect of such an improbable event. All variables will be incremented by 10 percent from their nominal values and, to produce the maximum effect, the variables $C_P$, $L_{OE}$, and $R_{INT}$ will move in a direction opposite the variables $C_{01}$, $C_{02}$, $L_{PD}$, and $R_{INF}$. The results are presented in the following table:

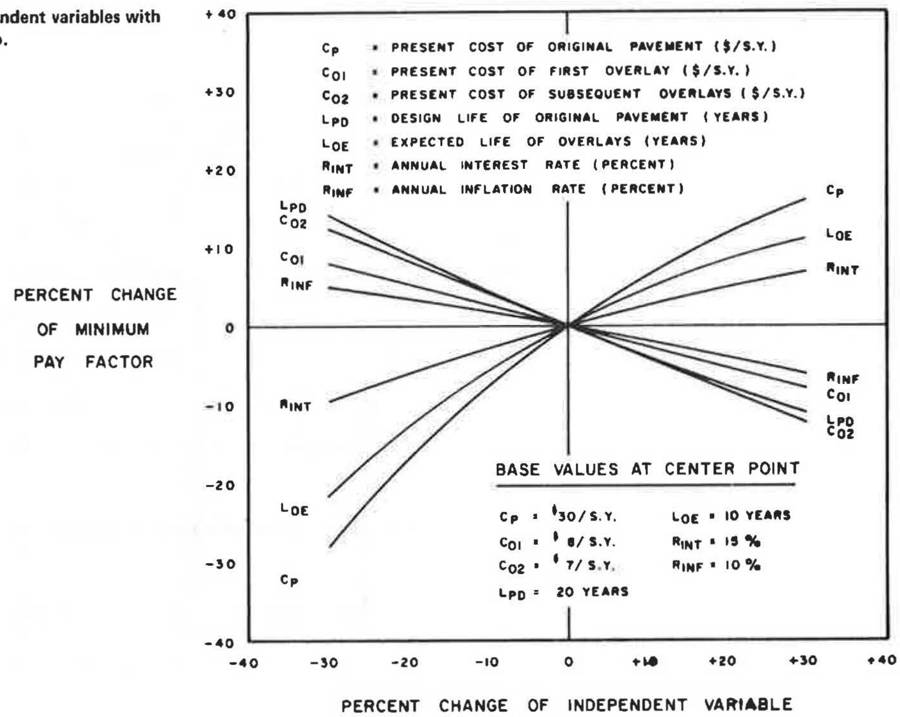
| | Appropriate Pay Factor | | |
|---|---|---|---|
| $L_{PE}/L_{PD}$ | $C_P$ = 30, $C_{01}$ = 8, $C_{02}$ = 7, $L_{PD}$ = 20, $L_{OE}$ = 10, $R_{INT}$ = 15, $R_{INF}$ = 10 | $C_P$ = 27, $C_{01}$ = 8.8, $C_{02}$ = 7.7, $L_{PD}$ = 22, $L_{OE}$ = 9, $R_{INT}$ = 13.5, $R_{INF}$ = 11 | $C_P$ = 33, $C_{01}$ = 7.2, $C_{02}$ = 6.3, $L_{PD}$ = 18, $L_{OE}$ = 11, $R_{INT}$ = 16.5, $R_{INF}$ = 9 |
| 0.0 | 0.597 | 0.376 | 0.724 |
| 0.2 | 0.709 | 0.526 | 0.808 |
| 0.4 | 0.802 | 0.663 | 0.875 |
| 0.6 | 0.880 | 0.786 | 0.927 |
| 0.8 | 0.945 | 0.898 | 0.968 |
| 1.0 | 1.000 | 1.000 | 1.000 |
| 1.2 | 1.046 | 1.092 | 1.025 |
| 1.4 | 1.084 | 1.176 | 1.045 |
| 1.6 | 1.116 | 1.251 | 1.061 |
| 1.8 | 1.143 | 1.320 | 1.074 |
| 2.0 | 1.165 | 1.382 | 1.083 |

As expected with the extremely unfavorable conditions assumed for this test, the resultant pay schedules are substantially affected. The greatest effect occurs at the minimum pay factor, which moves from F = 0.597 down to F = 0.376 with one combination of independent variables and up to F = 0.724 with the other. As with all of the tests, the pay factors closer to F = 1.0 are affected to a lesser degree.

Although this test was designed to produce a worst-case result, it emphasizes that the accuracy of the resultant pay schedule is dependent on the accuracy with which the independent variables have been determined. This suggests that pay schedules developed by use of Equation 13 should be reviewed periodically to verify that they are still appropriate, particularly if an unexpected change in any of the independent variables has occurred. However, as long as the input variables are average values that tend to be quite stable, a modification of the pay schedule should seldom be necessary in actual practice.

Collectively, the tests in this section demonstrate that the values computed by Equation 13 are relatively insensitive to minor fluctuations of the independent variables. This indicates that this equation can be relied on to establish appropriate pay factors provided the input values are reasonably accurate.

**Figure 1. Sensitivity analysis of all independent variables with expected pavement life ($L_{PE}$) fixed at zero.**




DEVELOPMENT OF PAY SCHEDULES

There are three ways in which Equation 13 can be used to establish appropriate pay schedules. First, by defining discrete intervals for the ratio of expected life to design life (or its equivalent, the ratio of expected load-bearing capacity to the design loading) and then computing the pay factor associated with the midpoint of each interval, it is possible to construct a stepped pay schedule such as the following:

| Load Ratio | Pay Factor |
|---|---|
| <0.50 | 0.60 |
| 0.50-0.69 | 0.88 |
| 0.70-0.89 | 0.94 |
| 0.90-1.09 | 1.00 |
| 1.10-1.29 | 1.04 |
| 1.30-1.49 | 1.08 |
| 1.50-1.69 | 1.12 |
| 1.70-1.89 | 1.14 |
| ≥1.90 | 1.16 |

The pay factors in this table have been computed with Equation 13 by using the input parameters from the numerical example. The decision has been made in this particular case that below a load ratio of 0.50, the unquantified factors previously discussed take precedence and the pay factor is arbitrarily set at the minimum level of F = 0.60.

However, when the operating-characteristic curve for a pay schedule such as this is checked, it will usually be found that the provisions for minimum and maximum pay factors have biased it away from the desired curve. This effect is more pronounced with small sample sizes, and if it is considered large enough to require correction, either the sample size must be increased or some of the pay factors in the pay schedule must be raised. A typical refinement of this pay schedule, designed to achieve a close match with the desired curve between load ratios of 0.50 and 1.50, might be as follows:

| Load Ratio | Pay Factor |
|---|---|
| <0.50 | 0.60 |
| 0.50-0.69 | 0.90 |
| 0.70-0.89 | 0.95 |
| 0.90-1.09 | 1.00 |
| 1.10-1.29 | 1.05 |
| 1.30-1.49 | 1.10 |
| ≥1.50 | 1.12 |

Although stepped pay schedules are in common use, they do have a minor disadvantage. Unless the discrete intervals are quite small, the difference in pay between two successive steps may be fairly substantial. Whenever the true population quality happens to fall close to one of the boundaries in a stepped pay schedule, it is almost entirely a matter of chance whether the larger or smaller pay factor will be assigned. Although this tends to balance out in the long run, it can work to the disadvantage of either the highway agency or the contractor on a project with relatively few lots.

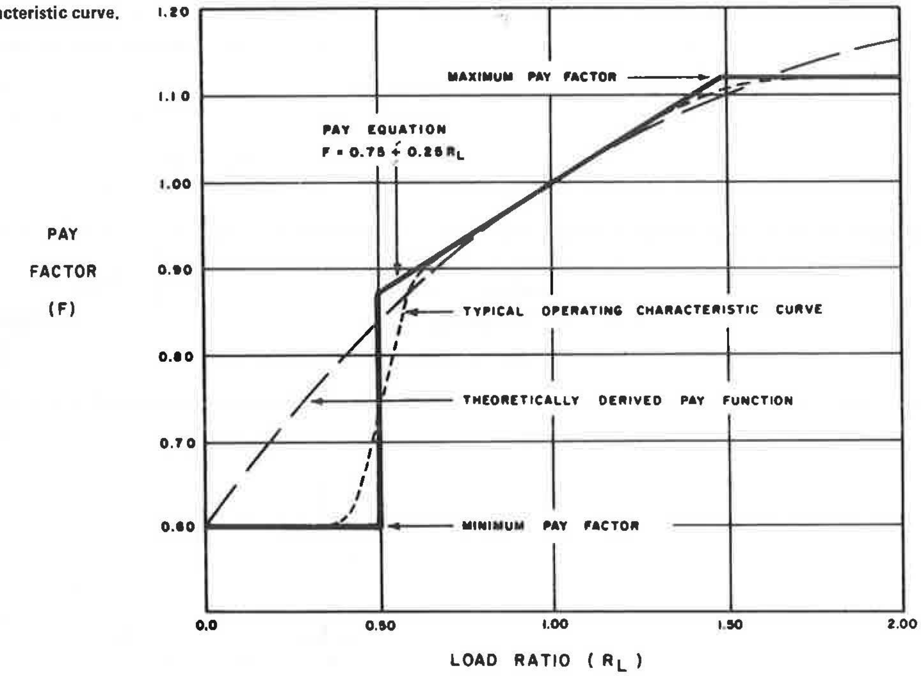
A second approach avoids this problem by expressing the pay schedule in the form of a continuous equation. When the pay factors computed previously are plotted as a function of the load ratio, they are seen to lie on a gentle curve as shown in Figure 2. Although it is possible to derive an equation that fits this curve, this turns out to be unnecessary. A simple linear pay equation can be found that will produce an operating-characteristic curve that closely approximates the desired curve.

One such equation is

$$F = 0.75 + 0.25R_L \quad (15)$$

in which $R_L$ is the load ratio. Two constraints that must be imposed on this equation are a maximum allowable pay factor of F = 1.12 and the restriction that if $R_L$ is less than 0.50, the pay factor will be set equal to the minimum value of F = 0.60. With these modifications, the equation-type pay schedule is essentially equivalent to the stepped pay schedule previously discussed.

Figure 2. Typical pay equation and operating-characteristic curve.




As can be seen in Figure 2, the operating-characteristic curve can be made to match the desired curve very closely between load ratios of $R_L$ = 0.60 and $R_L$ = 1.60. Although it begins to fall below the desired curve above $R_L$ = 1.60, this is not considered to be a serious drawback because it is believed that a pavement would seldom exceed this level of quality in actual practice. Below $R_L$ = 0.60, the operating-characteristic curve drops rapidly to the minimum pay factor of F = 0.60, a result considered justifiable because of the unquantified factors previously cited. It is because of this rapid drop, which provides considerable incentive to avoid extremely low levels of quality, that it is believed unnecessary to account for a possible increase in cost for the first resurfacing ($C_{01}$) at the zero performance level.

The third manner in which Equation 13 can be used to develop a pay schedule is less precise but still useful. If the acceptance procedure were based on some quality characteristic other than load ratio (such as the percent defective of some construction parameter), Equation 13 would not be directly applicable. However, it can be used to determine what the minimum and maximum pay factors should be. This would establish two extreme points, and an additional known point would be a pay factor of F = 1.0 at the acceptable quality level. Either a stepped or a continuous pay schedule could then be developed that would produce an operating-characteristic curve that passes through these points.

SUMMARY AND CONCLUSIONS

The same technology used to design a rigid pavement to have a specified service life can be used to estimate the expected life of a pavement whose as-built characteristics differ from the design values. Then, by the use of basic engineering economics methods, it is possible to compute the expense or savings that result from the rescheduling of the successive generations of overlays that eventually must be installed. On the assumption that it is justifiable to adjust the contract price by the amount of this expense or savings, an equation was developed to compute the appropriate pay factor for any quality level of rigid pavement as a function of input information readily available within most highway agencies. Additional factors were cited that should be taken into consideration and sensitivity tests were performed to show that the procedure is reliable provided the input values are determined with reasonable accuracy. Various methods of using the equation to establish pay schedules were then discussed.

The nominal input values used in the examples in this paper were obtained from recent construction cost records and other sources representative of a relatively urbanized area. Use of these values in Equation 13 plus consideration of the effect of several unquantified factors resulted in a pay schedule with pay factors that range from a minimum of F = 0.60 to a maximum of F = 1.12. The real importance of Equation 13, however, is that it provides a reliable and extremely easy method to develop pay schedules by using whatever input values a highway agency considers appropriate.

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