Development of Multicharacteristic Acceptance Procedures for Rigid Pavement

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The manner in which the design method of the American Association of State Highway and Transportation Officials (AASHTO) can be used to develop multi-characteristic acceptance procedures for rigid pavement is outlined. The AASHTO equation is used to compute both the expected load-bearing capacity based on the as-built characteristics of the pavement and the desired load-bearing capacity based on the design parameters. The ratio of these two values is then used to determine the appropriate pay adjustment, which may be either positive or negative. Sensitivity tests are performed to verify the reliability of this approach and computer simulation is used to demonstrate the effectiveness of several different acceptance procedures of this type. A secondary study is conducted to determine how the procedure based on the AASHTO equation compares with several other methods of treating multiple pay factors to obtain a single overall pay factor. Under the assumption that the AASHTO method is the fundamentally correct approach, the method of multiplying individual pay factors together is shown to be among the best of the other methods that were tested.

Statistical end-result specifications with adjusted pay schedules are now used by many highway agencies and are often based on more than a single quality characteristic. A statistical specification for rigid pavement, for example, may have separate acceptance procedures for compressive strength and thickness or a combination of these and other parameters. A pay factor is computed for each characteristic, and in order to arrive at an overall pay factor, a variety of methods have been used. Perhaps the most common method defines the overall pay factor as the product of the individual pay factors. Another approach is to use the smallest of the individual pay factors. Still another method is a cumulative one in which the individual pay adjustments are summed to obtain the total adjustment. The average of the individual pay factors has also been used.

Although all of these methods may be effective from a practical standpoint, the present state of the art is such that none of them has been conclusively demonstrated to be correct. Almost certainly, there is no single method that would be appropriate for all types of construction situations since this would be a function of the true quality-performance relationship and the degree of association among the various quality characteristics. However, in the case of rigid pavement, this problem can be avoided by working directly with the design equation of the American Association of State Highway and Transportation Officials (AASHTO) (1), which gives the number of equivalent 18-kip load applications that can be sustained as a function of several common quality characteristics. In this manner, the multiple effects of the individual characteristics can be reduced to a single fundamental effect--the resultant load-bearing capacity computed from the as-built characteristics of the pavement. By comparing this with the desired load-bearing capacity computed from the design parameters, a ratio is obtained that can be used to form the basis for a rational adjusted pay schedule.

It is the objective of this paper to outline how these concepts can be used to develop acceptance procedures based on the AASHTO design equation. Included are discussions of several factors that must be taken into consideration when using the AASHTO equation in this manner, sensitivity tests to confirm the soundness of certain assumptions, examples of several variations of this approach, and a series of computer simulation tests to demonstrate the effectiveness of the resultant acceptance procedures. In essence, this work constitutes a feasibility study that will serve as a useful guide for any highway agency desiring to develop a specification of this type.

AVERAGES VERSUS DISTRIBUTIONS

Many existing statistical specifications are based on the estimate of percent defective (2) of some construction parameter. These specifications recognize that the characteristic of interest is stochastically distributed and typically allow a small percent of the population to fall outside some critical limit (or pair of limits). Lot sizes are usually defined to be small enough so that, within each lot, the quality level is believed to be fairly uniform. To assure that a reasonable degree of uniformity has been achieved, a secondary requirement is often imposed that specifies additional limits within which the individual test values must fall.

The existence of a small amount of variability within a lot is not necessarily detrimental. In the case of a pavement lot the average quality of which is exactly at the desired quality level, this means that approximately half the pavement will require a little more than the normal amount of routine maintenance throughout its design life, whereas the other half will require a little less. These two conditions will tend to balance out provided the poorer-quality half is not so defective that it requires something more than routine maintenance. An additional limit on individual test values protects against this possibility.

An acceptance procedure based on the AASHTO design equation can be developed that will perform in much the same way. Each sample provides an independent estimate of the load-bearing capacity of the pavement lot and these can be combined to determine an average load ratio from which the appropriate pay factor is determined. As a safeguard against isolated sections of poor quality within a lot, a lower limit on the individual estimates can be imposed. Whenever an individual value falls below the lower limit, coring or other procedures can be employed to more precisely determine the quality of that particular section.

AASHTO EQUATION FOR RIGID PAVEMENT

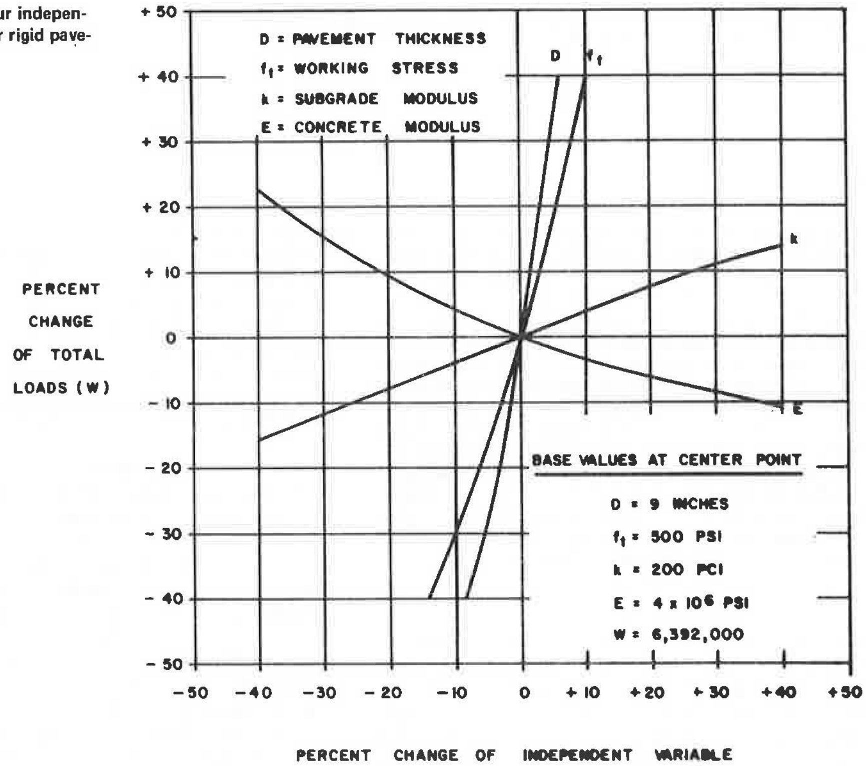
The equation for rigid pavement is presented in the AASHTO design guide (1) as follows:

$$\log W = 7.35 \log(D + 1) - 0.06 - 0.1761/[1 + 1.624 \times 10^7/(D + 1)^{8.46}] + 3.42 \log \{(f_t/690)(D^{0.75} - 1.132)/[D^{0.75} - 18.42/(E/k)^{0.25}]\} \quad (1)$$

where

W = number of equivalent 18-kip load applications,
D = pavement thickness (in),
f_t = working stress in concrete (psi),
E = concrete modulus of elasticity (psi), and
k = modulus of subgrade reaction (pci).

Figure 1. Sensitivity analysis of the four independent variables in AASHTO equation for rigid pavement.



Because this equation is somewhat awkward to work with, the various tests described in this paper were performed with the aid of a computer. The following FORTRAN subroutine was written to execute the computations indicated in Equation 1:

```
SUBROUTINE AASHTO(THICK,WRKSTR,SUBMOD,CONMOD,LOADS)
TERM1=7.35*ALOG10(THICK+1.)-0.06
TERM2=0.1761/(1.+1.624E7/(THICK+1.)**8.46)
ROOT1=THICK**0.75
ROOT2=(CONMOD/SUBMOD)**0.25
TERM3=3.42*ALOG10((WRKSTR/690.)*(ROOT1-1.132)/(ROOT1-
  18.42/ROOT2))
LOADS=IFIX(10.**(TERM1-TERM2+TERM3)+0.5)
RETURN
END
```

where

THICK = pavement thickness (in),
WRKSTR = working stress in concrete (psi),
SUBMOD = modulus of subgrade reaction (pci),
CONMOD = concrete modulus of elasticity (psi), and
LOADS = number of equivalent 18-kip load applications computed and returned by subroutine.

The AASHTO equation gives the number of loads a pavement can sustain as a function of thickness, working stress, subgrade modulus, and concrete modulus. The relative importance of these variables can be judged from the steepness of the curves in Figure 1. Thickness (D) and working stress (f_t) are clearly the more important variables since a small change in either one produces a large change in the load-bearing capacity of the pavement. The subgrade modulus (k) and the concrete modulus (E), on the other hand, are less important since their curves are less steeply inclined. For this reason, plus the fact that compaction deficiencies can be corrected before the pavement is placed, the acceptance procedures developed in this paper are based on the two primary variables, thickness and working stress. Once the method has been established, a similar approach can be used to develop acceptance procedures based on all (or any subset) of the four AASHTO variables, if desired.

Although the acceptance procedure could be based on flexure tests (ASTM C78-75) that relate directly to the working stress, it will be more desirable from the standpoint of most highway agencies to base the procedure on standard compression tests (ASTM C39-80). The Portland Cement Association has published the following relationship (3, p. 57):

$$MR = K(f_c')^{1/2} \quad (2)$$

where

MR = modulus of rupture (psi),
f_c' = compressive strength of concrete (psi), and
K = a constant, usually between 8 and 10.

Then, since the working stress is defined by AASHTO as 75 percent of the modulus of rupture, Equation 2 can be rewritten as

$$f_t = 0.75K(f_c')^{1/2} \quad (3)$$

in which f_t is the working stress to be entered into Equation 1. It will be demonstrated in the next section that for purposes of the acceptance procedure, the value of K is not critical. If a midrange value of K = 9 is assumed for the working-stress constant, Equation 3 becomes

$$f_t = 6.75(f_c')^{1/2} \quad (4)$$

which makes it possible to base the AASHTO equation on concrete compressive strength.

ADDITIONAL SENSITIVITY TESTS

In the acceptance procedure to be developed, Equations 1 and 4 will be used to determine both the expected number of loads computed from the as-built measurements of thickness and compressive strength of the pavement and the desired number of loads computed from the design values for these same parameters. The ratio of these two values will then be used to determine the appropriate pay factor. It is the purpose of this section to show that this ratio is essentially independent of the nominal values assumed for subgrade modulus, concrete modulus, and the working-stress constant.

The first of these three variables to be tested is the subgrade modulus. The load ratios in the following table were computed by using a design average thickness of D = 9 in, a design average compressive strength of f_c' = 4000 psi, and nominal values for the concrete modulus and working-stress constant of $E = 4 \times 10^6$ psi and K = 9, respectively.

As-Built Values		Load Ratio		
D (in)	f_c' (psi)	k = 100 pci	k = 200 pci	k = 300 pci
8.5	3000	0.428	0.433	0.438
8.5	4000	0.699	0.709	0.716
8.5	5000	1.024	1.038	1.049
9.0	3000	0.611	0.611	0.611
9.0	4000	1.000	1.000	1.000
9.0	5000	1.465	1.465	1.465
9.5	3000	0.863	0.852	0.845
9.5	4000	1.411	1.394	1.381
9.5	5000	2.066	2.041	2.023

There are two interesting observations to be made from the values in this table. First, increasing the subgrade modulus by a factor of 3 from k = 100 pci to k = 300 pci produces only small changes in the resultant load ratios. Since a highway agency would know the nominal value of the subgrade modulus much more precisely than this, this variable will have essentially no effect on the load ratio that is computed as part of the acceptance procedure. The second observation to be made is that this type of acceptance procedure recognizes that, within reasonable limits, an excess in one quality characteristic can offset a deficiency in another. For example, although the assumed design values are D = 9 in and f_c' = 4000 psi, the third line in the table indicates that a pavement constructed with D = 8.5 in and f_c' = 5000 psi will actually have a slightly greater load-bearing capacity since the load ratios are greater than unity.

The next variable to be tested is the concrete modulus. The load ratios in the following table were computed by using the same design parameters of D = 9 in and f_c' = 4000 psi along with nominal values for the subgrade modulus and working-stress constant of k = 200 pci and K = 9, respectively:

As-Built Values		Load Ratio		
D (in)	f_c' (psi)	$E = 3 \times 10^6$ psi	$E = 4 \times 10^6$ psi	$E = 5 \times 10^6$ psi
8.5	3000	0.437	0.433	0.431
8.5	4000	0.714	0.709	0.705
8.5	5000	1.046	1.038	1.033
9.0	3000	0.611	0.611	0.611
9.0	4000	1.000	1.000	1.000
9.0	5000	1.465	1.465	1.465
9.5	3000	0.847	0.852	0.856
9.5	4000	1.385	1.394	1.400
9.5	5000	2.029	2.041	2.050

Precisely the same effects are observed in this table. The load ratios are very stable in spite of the large changes in concrete modulus, and the two as-built quality characteristics have the same offsetting property already noted. Like the subgrade modulus, the concrete modulus would be known sufficiently precisely that it would have no appreciable effect on the acceptance procedure.

The last variable to be tested is the working-stress constant. The load ratios in the following table were again computed by using the same design values of D = 9 in and f_c' = 4000 psi and nominal values for subgrade modulus and concrete modulus of k = 200 pci and $E = 4 \times 10^6$ psi, respectively:

As-Built Values		Load Ratio		
D (in)	f_c' (psi)	K = 8	K = 9	K = 10
8.5	3000	0.433	0.433	0.433
8.5	4000	0.709	0.709	0.709
8.5	5000	1.038	1.038	1.038
9.0	3000	0.611	0.611	0.611
9.0	4000	1.000	1.000	1.000
9.0	5000	1.465	1.465	1.465
9.5	3000	0.852	0.852	0.852
9.5	4000	1.394	1.394	1.394
9.5	5000	2.041	2.041	2.041

These results are even more consistent than those in the previous two tables. The working-stress constant has no effect on the load ratios that are computed.

Taken together, these tests indicate that it is not necessary to have an exact knowledge of the subgrade modulus, concrete modulus, or the working-stress constant for this application. With the substitution of nominal values for these parameters, the AASHTO equation can be used to obtain reliable estimates of the load ratio of the pavement based on the as-built measurements of thickness and compressive strength. This clears the way for the development of the remainder of the acceptance procedure.

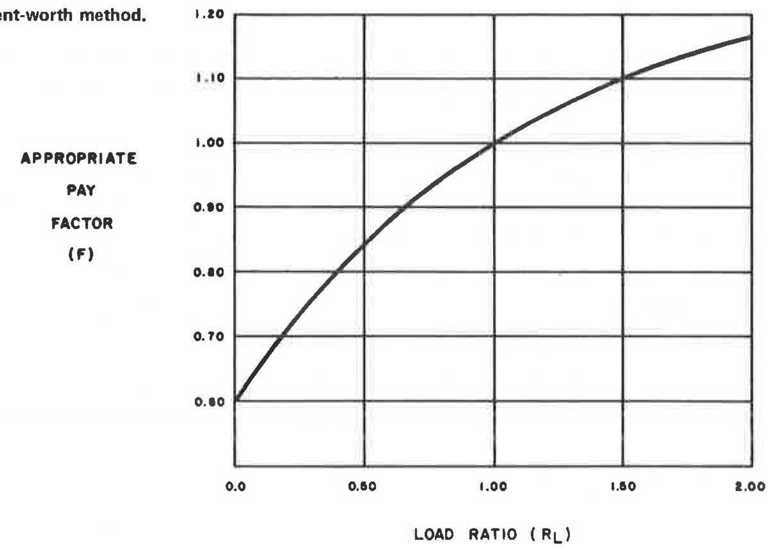
DEVELOPMENT OF PAY SCHEDULES

The derivation of pay schedules based on the concept of load ratio is developed in a companion paper by Weed in this Record and will be summarized only briefly here. If, due to construction deficiencies, the pavement is not capable of withstanding the design loading, it will fail prematurely. The necessity of repairing this pavement at an earlier date will result in an additional expense to the highway agency. Conversely, a pavement of superior quality that lasts longer than the intended design life will result in a savings. The appropriate pay adjustment is considered to be the present worth of any expense or savings expected to occur in the future as the result of a departure from the specified level of quality and may be positive or negative. In essence, a pay schedule based on this premise constitutes both a liquidated-damages clause and a bonus provision.

If a highway agency elects not to apply a bonus provision, the pay factors are limited to a maximum of 100 percent. However, this restriction tends to bias the higher pay factors downward and, in certain cases, this can create serious problems for both the contractor and the highway agency. This is discussed further in Example 7 in this paper and was explained in detail in an earlier paper (4).

Figure 2 illustrates a typical pay function that might result when bonus payments are permitted. The appropriate pay factor (F) is expressed as a decimal and is plotted as a function of the load ratio (R_L). For load ratios between R_L = 0.0 and R_L

Figure 2. Typical pay function derived by present-worth method.



= 2.0, the curve is concave downward and is seen to rise smoothly from a minimum of about F = 0.60 to a maximum of approximately F = 1.16.

Although it would be possible to develop a pay schedule that follows this curve, this turns out to be unnecessary. In actual practice, most pay schedules have various arbitrary conditions imposed on them, such as a minimum pay factor, a maximum pay factor, a requirement for retesting, and so forth, and these conditions invariably cause the operating-characteristic curve to depart from whatever pay function is used. This fact can be used to advantage as long as it is recognized and taken into account. The objective, of course, is to make the operating-characteristic curve, not the pay schedule itself, conform to the desired pay function.

In general, pay schedules may be of two types, stepped or continuous. Stepped pay schedules define discrete intervals for the quality characteristic and assign a specific pay factor for each interval. Continuous pay schedules employ an equation to compute the appropriate pay factor for any given quality level. The companion paper by Weed in this Record presents two pay schedules for use with the pay function shown in Figure 2. The first is a stepped pay schedule as follows:

Load Ratio	Pay Factor
<0.50	0.60
0.50-0.69	0.90
0.70-0.89	0.95
0.90-1.09	1.00
1.10-1.29	1.05
1.30-1.49	1.10
≥1.50	1.12

The second is given by the equation

$$F = 0.75 + 0.25R_L \quad (5)$$

with the added constraints that the maximum allowable pay factor is F = 1.12 and if the load ratio is less than R_L = 0.50, the pay factor is set at the minimum value of F = 0.60. Although this is a linear equation, it will be shown in a later section that its operating-characteristic curve conforms closely to the desired pay function within the primary region of interest.

Although stepped pay schedules are still more prevalent, continuous pay schedules are rapidly gaining acceptance and the reasons are quite obvious. Not only are they concise and easy to apply, they provide a more precise determination of the appropriate pay factor, a result beneficial to all parties. Because of these desirable features, plus the fact that the two types are essentially equivalent in the long run, equation-type pay schedules will be used for the remaining developments in this paper.

RETESTING PROVISIONS

Since it is common practice to require retesting when the first test indicates an unusually low level of quality, it will be worthwhile to consider the manner in which such a provision might be applied. There are two distinctly different ways in which the retest values can be processed and there are advantages and disadvantages associated with each. The first method combines the retest values with the original values and reevaluates the lot or sublot on the basis of the enlarged sample. The second method discards the original sample and evaluates the lot or sublot on the basis of the second sample only.

An advantage of the first method is that it uses all the available information. Advocates of this method argue that there is a cost associated with each sample and that it is wasteful to discard any valid information. An opposing viewpoint would question whether the original sample is truly valid. If the low quality level is the result of some malfunction of the testing process, then it would be more appropriate to discard the contaminated data.

This is a question of philosophy that each highway agency must answer for itself. However, there is another theoretical argument that can be offered in favor of the second method. When the retest values are combined with the original values, the probabilities of passing both the first test and the retest are correlated to some unknown degree. This lack of independence precludes the direct computation of the overall probability of acceptance. As a result, the operating-characteristic curve for the procedure must be determined somewhat imprecisely by a boundary method (5,6) or else obtained empirically by computer simulation. Consequently, if the test results are relatively inexpensive and easy to obtain, it may be more practical to use the second method and discard the original sample. Both methods will be illustrated in the examples that follow.

DEVELOPMENT OF ACCEPTANCE PROCEDURES

There are many ways in which the concepts outlined thus far can be applied, and it will be necessary to investigate several of them to understand the effects of minor procedural differences. For example, since the desired load-bearing capacity of a pavement is computed from the design average thickness and design average compressive strength, it might be thought necessary to compute the as-built average thickness and average compressive strength before entering these values into the AASHTO design equation to compute the as-built load capacity. Although this may turn out to be desirable, other procedures may be equally effective. One possible alternative would be to use the measured thickness and compressive strength from each sublot to determine the load ratios for all sublots. These would then be averaged to obtain the load ratio for the lot. An advantage of this approach is that the individual load ratios can be used to guard against isolated sections of poor quality within a single lot. As will be demonstrated in the following examples, these and still other variations can all be made to be extremely effective.

Example 1

For this example, it is desired to determine how well the pay schedule given by Equation 5 fits the desired pay function shown in Figure 2. The constraints imposed on this equation are a maximum pay factor of F = 1.12 and the stipulation that, for load ratios less than R_L = 0.50, the pay factor will be set at the minimum value of F = 0.60. There are no provisions for retesting in this case and the lot load ratio will be computed by using Equations 4 and 1 after first computing the average as-built thickness and compressive strength for each lot. A stratified random sampling plan is assumed with a single thickness and compressive strength determination made from each of a total of N = 5 sublots.

The only practical means to test this pay schedule is by computer simulation with the use of basic techniques described in a recent publication (7). For each of many different combinations of pavement thickness and compressive strength, a large number of random values of these parameters were generated and then processed in accordance with the requirements of the acceptance procedure. Figures 3 and 4 show the input and output stages of a typical computer run used to obtain points on the operating-characteristic curve for this plan.

From the fifth column of the computer output shown in Figure 4, the average load ratios obtained by simulation are seen to be in very close agreement with the population load ratios given in the third column. The average pay factors in the last column are used to plot the operating-characteristic curve for this plan in Figure 5. It can be observed from Figure 5 that, between load ratios of R_L = 0.60 and R_L = 1.60, the operating-characteristic curve matches the desired pay function quite closely. Although it begins to fall below the desired curve above R_L = 1.60, this is not considered to be a serious drawback because a pavement would seldom exceed this level of quality in actual practice. Below R_L = 0.60, the operating-characteristic curve drops rapidly to the minimum pay factor of F = 0.60. As explained in more detail in the companion paper in this Record, this is believed justifiable for such seriously defective pavement.

Although the operating-characteristic curve for this plan fits the desired pay function reasonably well, this is by no means the only pay schedule that could have been developed. Depending on the sample size, the critical load ratio below which the minimum pay factor is assigned, and the region within which a close fit is desired, a pay equation with a somewhat different intercept and slope might be appropriate. If an extremely close fit were required, a second-degree pay equation could be used although it is doubtful that the slightly better fit would justify the added complexity.

Example 2

The previous example contains no requirement on individual test results to guard against isolated sections of poor quality within a lot. For this next example, the same sample size of N = 5 will be used, the lot load ratio will be computed as before by first averaging the thickness and compressive strength results before entering them into the AASHTO equation, but an additional requirement will be imposed on each sublot. If the load ratio computed from the single values of thickness and compressive strength from a sublot is less than R_L =

Figure 3. Input for typical computer simulation run.

```
run aashto13
EXECUTION BEGINS...

ENTER DESIGN VALUES FOR THICKNESS (INCHES) AND COMPRESSIVE STRENGTH (PSI)

9 4000

ENTER A AND B OF F = A + B(LOAD RATIO) AND MAXIMUM PAY FACTOR

0.75 0.25 1.12

ENTER LOWER LIMITING LOAD RATIO AND MINIMUM PAY FACTOR

0.50 0.60

ENTER RETEST LOAD RATIO AND NUMBER OF ADDITIONAL SAMPLES

0 0

ENTER STANDARD DEVIATIONS FOR THICKNESS AND COMPRESSIVE STRENGTH

0.25 500

ENTER MINIMUM, MAXIMUM, AND STEP SIZE FOR THICKNESS

8.50 9.50 0.25

ENTER MINIMUM, MAXIMUM, AND STEP SIZE FOR COMPRESSIVE STRENGTH

2500 5000 500

ENTER NUMBER OF LOTS PER RUN, SAMPLE SIZE, AND SEED NUMBER

500 5 1234567

ADVANCE PAPER TO NEW PAGE, DEPRESS SPACE BAR, AND RETURN CARRIAGE
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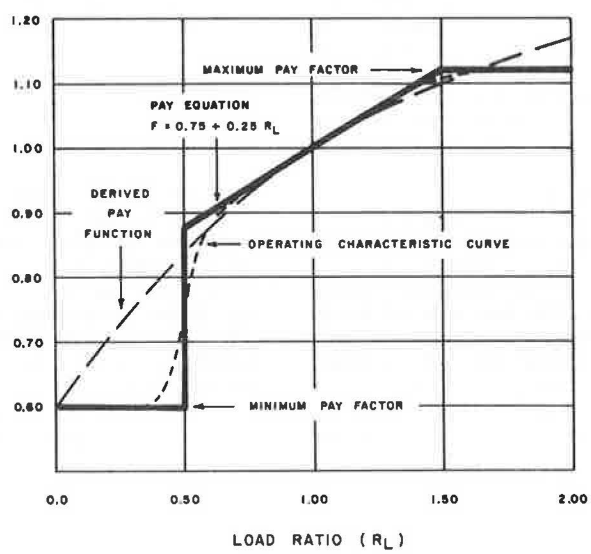
Figure 4. Output for typical computer simulation run.

POPULATION MEANS		LOAD RATIO COMPUTED FROM POPULATION MEANS	SIMULATION RESULTS		
THICKNESS	COMPRESSIVE STRENGTH		RETEST FREQUENCY	AVERAGE LOAD RATIO	AVERAGE PAY FACTOR
8.50	2500	0.317	0.0	0.318	0.600
8.50	3000	0.433	0.0	0.435	0.645
8.50	3500	0.564	0.0	0.571	0.850
8.50	4000	0.709	0.0	0.718	0.929
8.50	4500	0.867	0.0	0.873	0.968
8.50	5000	1.038	0.0	1.040	1.010
8.75	2500	0.377	0.0	0.379	0.607
8.75	3000	0.516	0.0	0.522	0.778
8.75	3500	0.671	0.0	0.672	0.910
8.75	4000	0.843	0.0	0.839	0.960
8.75	4500	1.031	0.0	1.037	1.009
8.75	5000	1.235	0.0	1.251	1.062
9.00	2500	0.448	0.0	0.454	0.682
9.00	3000	0.611	0.0	0.613	0.878
9.00	3500	0.796	0.0	0.793	0.948
9.00	4000	1.000	0.0	1.007	1.002
9.00	4500	1.223	0.0	1.232	1.057
9.00	5000	1.465	0.0	1.457	1.102
9.25	2500	0.529	0.0	0.529	0.781
9.25	3000	0.723	0.0	0.723	0.928
9.25	3500	0.941	0.0	0.942	0.985
9.25	4000	1.182	0.0	1.192	1.047
9.25	4500	1.446	0.0	1.459	1.101
9.25	5000	1.732	0.0	1.743	1.119
9.50	2500	0.624	0.0	0.622	0.872
9.50	3000	0.852	0.0	0.858	0.964
9.50	3500	1.109	0.0	1.117	1.029
9.50	4000	1.394	0.0	1.405	1.091
9.50	4500	1.705	0.0	1.719	1.118
9.50	5000	2.041	0.0	2.041	1.120

Figure 5. Simulation results for Example 1.



0.75, two additional cores will be taken from the sublot. The test results from these cores will be averaged with the original values to determine the thickness and compressive strength associated with that particular sublot. The pay factor for the lot will then be computed in the usual manner except that if the load ratio for any sublot is less than the minimum required value of $R_L = 0.50$, the sublot will be treated as a separate lot and assigned the minimum pay factor of F = 0.60. In this event, the pay factor for the remainder of the lot will be computed by using the remaining test results.

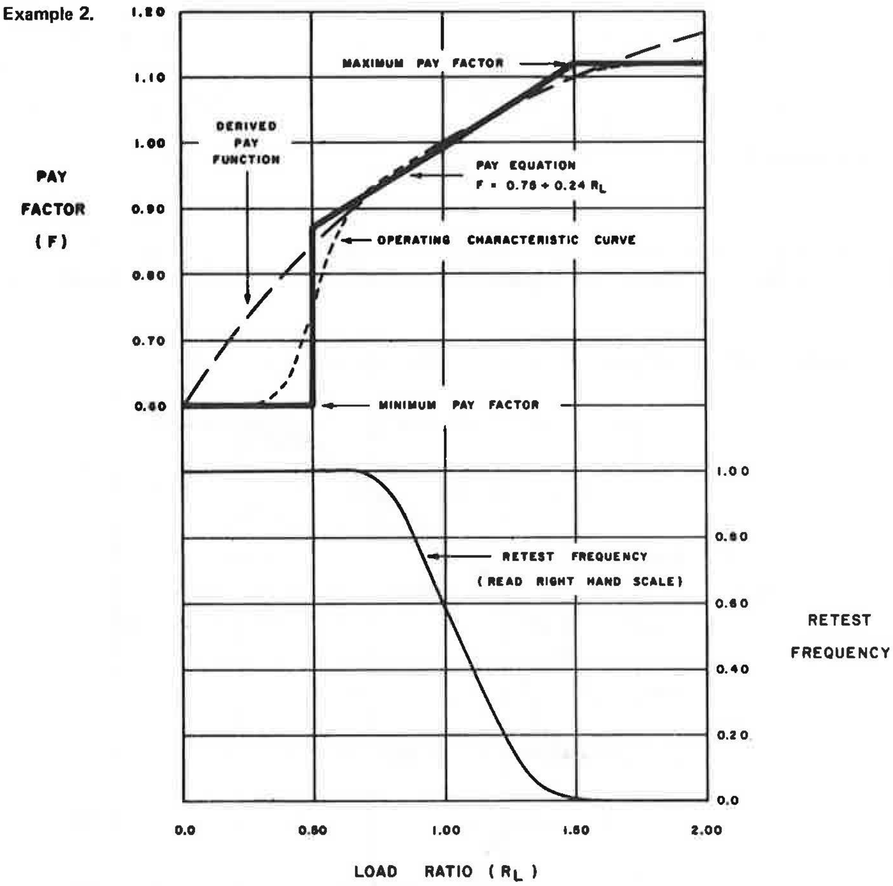
Because this acceptance procedure is different from that used in the first example, a slightly different pay equation is required. By trial and error, Equation 6 was found to be suitable for this application. The maximum pay factor of F = 1.12 continues to be satisfactory and the minimum pay factor remains unchanged at F = 0.60 for load ratios less than $R_L = 0.50$:

$$F = 0.75 + 0.24R_L \qquad (6)$$

This acceptance procedure was also tested by computer simulation and the results are plotted in Figure 6. Although the pay equation produces F = 0.99 at $R_L = 1.00$, the retest provision bows the operating-characteristic curve upward sufficiently to pass through the point at which $R_l = 1.00$ and F = 1.00, as it should. This effect is countered at the lower and upper ends by the minimum and maximum pay factors, which results in a good fit throughout the region between load ratios of $R_L = 0.60$ and $R_L = 1.60$. As in Example 1, the downward bias at the lower end is considered appropriate for such poor-quality pavement.

Although the operating-characteristic curve is quite satisfactory, this acceptance plan does have one serious drawback. As seen in Figure 6, the retest frequency is sufficiently high for normal construction that the plan would probably be con-

Figure 6. Simulation results for Example 2.



sidered impractical. For pavement that had a load ratio close to the desired value of R_L = 1.00, nearly 60 percent of the lots would require retesting. To correct this condition, the critical level of load ratio below which a retest is required must be set at a value substantially lower than R_L = 0.75. This modification will be made in the next example.

Example 3

This example is identical to the previous one except that the retest requirement is changed. First, a retest will not be required unless the load ratio for a single sublot is less than R_L = 0.50. Second, any sublot requiring a retest will be treated as a separate lot and N = 5 additional cores will be taken. Finally, the evaluation of the sublot will be based only on the new tests for strength and thickness; the original values will be discarded.

As in the previous example, the different acceptance procedure requires a different pay equation. In this case, Equation 7 was found to be appropriate. A maximum pay factor of F = 1.10 and the usual minimum pay factor of F = 0.60 for load ratios below R_L = 0.50 will be used with this equation.

$$F = 0.70 + 0.30R_L \quad (7)$$

The results of the simulation of this plan are shown in Figure 7. The operating-characteristic curve is very close to the desired pay function between load ratios of R_L = 0.55 and R_L = 1.50. However, unlike the previous example, the retest frequency is at an acceptably low level throughout the range within which most pavement would normally fall. At the design load ratio of exactly R_L = 1.00, for example, the retest frequency is about 5.0 percent.

Example 4

The remaining examples will all make use of individual sublot load ratios that will be averaged to obtain the load ratio for the lot. Each example will be constructed to be comparable with one of the previous examples to investigate the effect of the alternative method of computing the average load ratio.

This example is designed to be similar to Example 1. The maximum pay factor is F = 1.12, the minimum pay factor for load ratios below R_L = 0.50 is F = 0.60, and there is no retest provision. An appropriate pay schedule is given by Equation 8:

$$F = 0.745 + 0.25R_L \quad (8)$$

The simulation results are plotted in Figure 8. Although the pay equation produces F = 0.995 at R_L = 1.00, the operating-characteristic curve is bowed upward sufficiently to provide a good fit throughout the region between load ratios of R_L = 0.60 and R_L = 1.60.

Example 5

This example is designed to be comparable with Example 2 except that the critical value of load ratio below which a retest is required is set at a more practical level of R_L = 0.50. When a retest is required, two additional cores will be taken, which results in a total of three individual R_L values to be averaged together to obtain the load ratio for the sublot. If the average load ratio for any sublot is less than the minimum required level of $R_L = 0.50$, the sublot will be treated as a separate lot as described in Example 2. By using a maximum pay factor of F = 1.12 and a minimum pay factor of F = 0.60 for load ratios below R_L = 0.50, a suitable pay schedule is given by Equation 9:

$$F = 0.75 + 0.245R_L \quad (9)$$

The operating-characteristic curve and retest frequencies for this plan are shown in Figure 9. Although this plan is basically different from the one illustrated in Example 2, the resulting operating-characteristic curve is very nearly the same. However, unlike Example 2, the retest frequency is at a very tolerable level of about 5.0 percent for pavement with a load ratio close to the desired value of R_L = 1.00.

Example 6

This example is meant to be compared with Example 3. Whenever an individual load ratio for a sublot

Figure 7. Simulation results for Example 3.

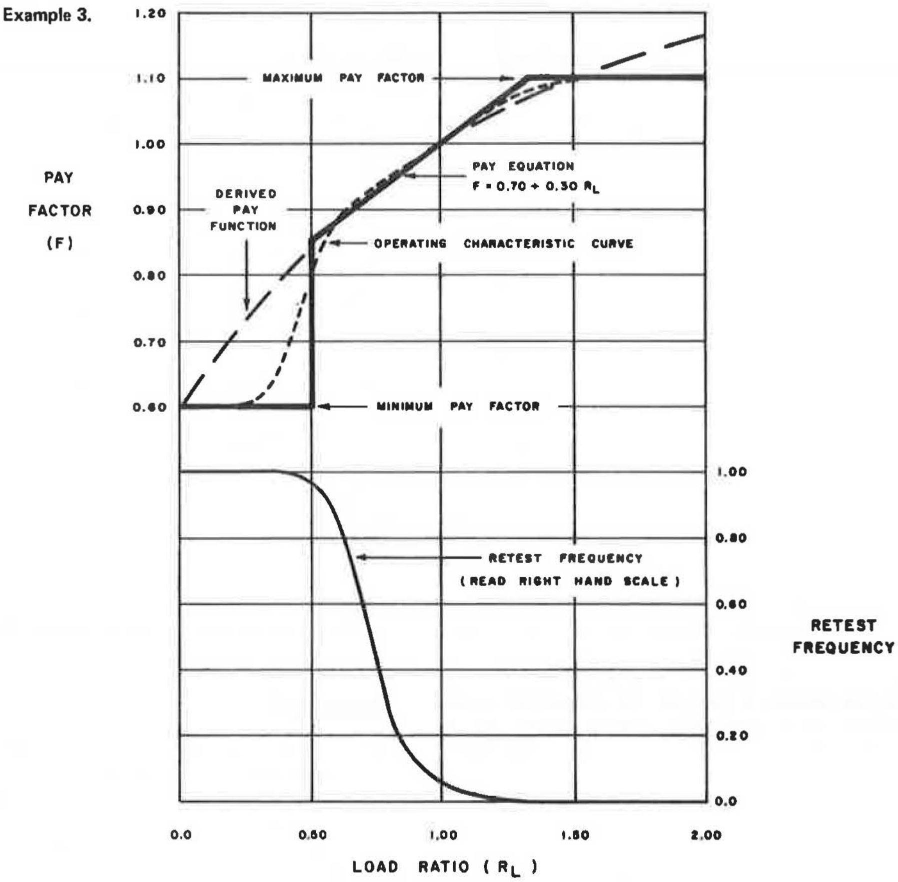


Figure 8. Simulation results for Example 4.

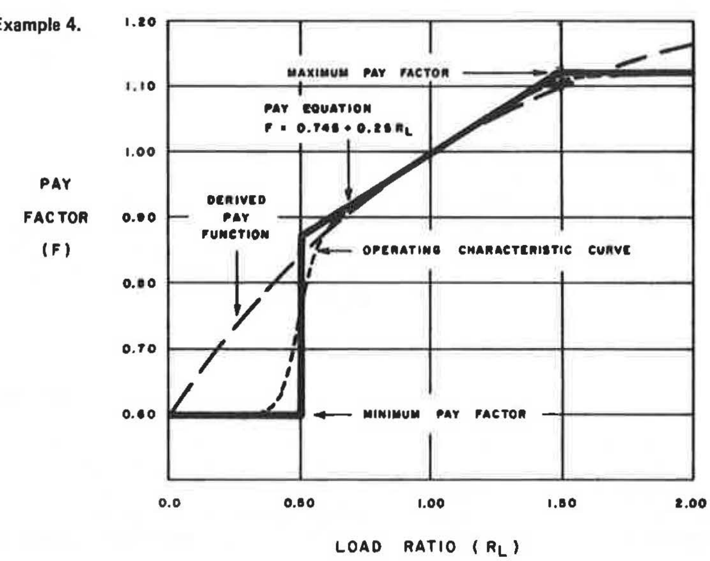


Figure 9. Simulation results for Example 5.

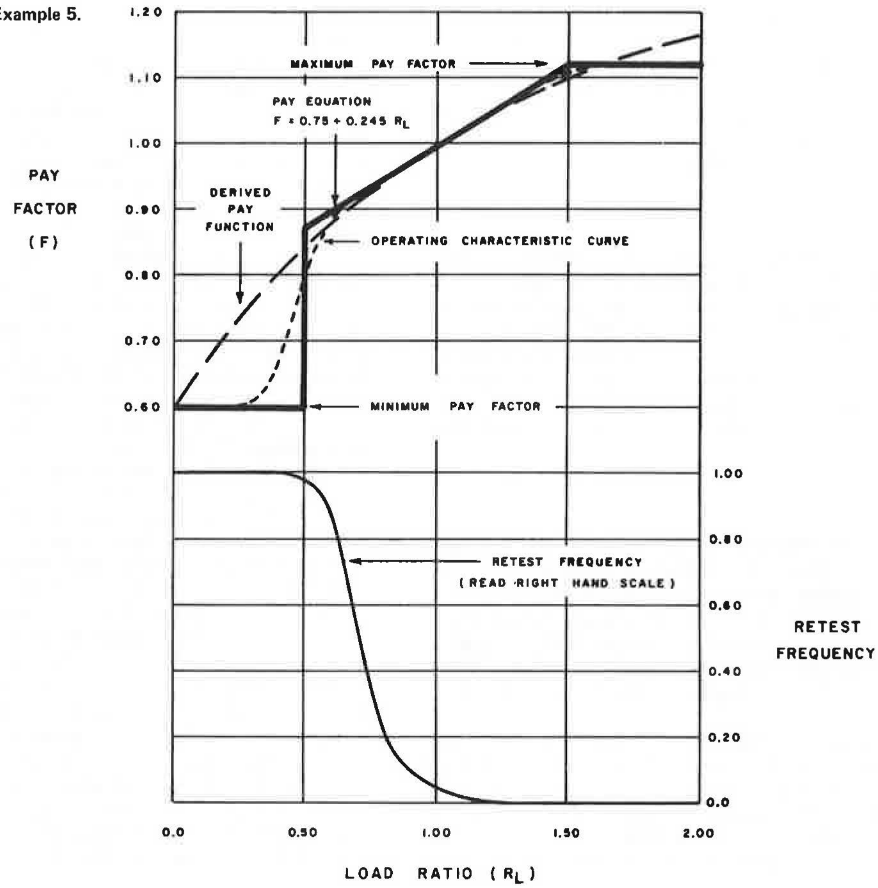
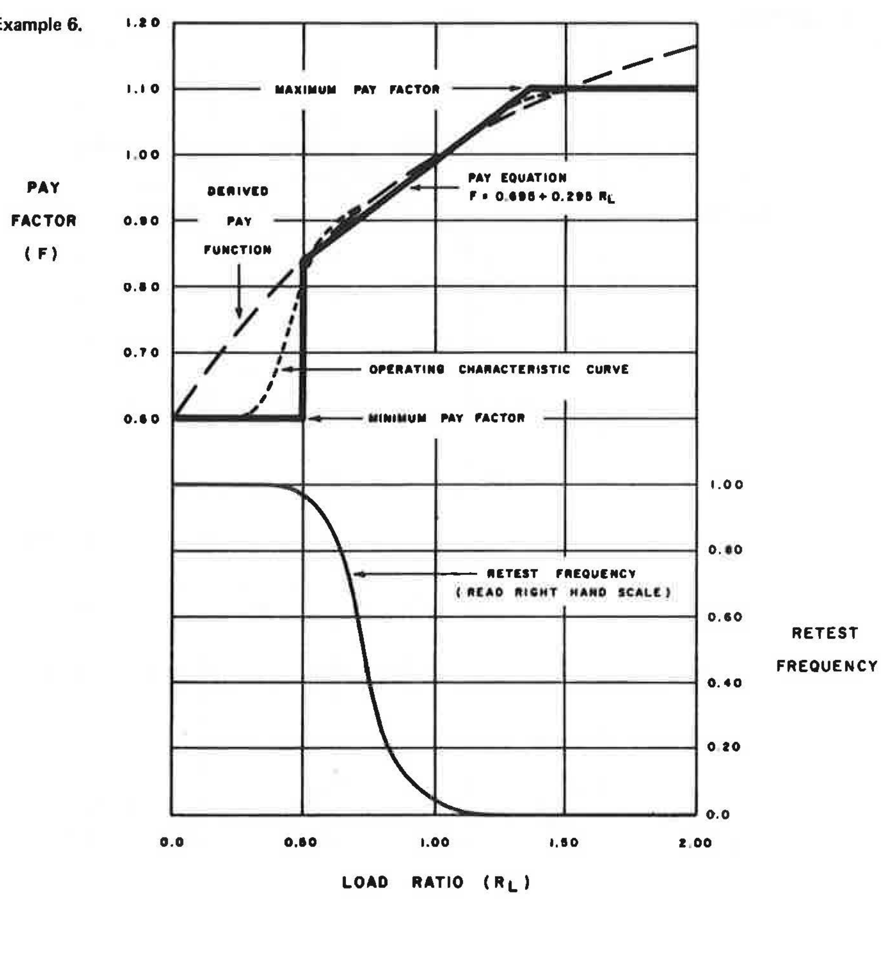


Figure 10. Simulation results for Example 6.



is below R_L = 0.50, the sublot will be treated as a separate lot, N = 5 additional cores will be taken, and the original test value for the sublot will be discarded. With a maximum pay factor of F = 1.10 and a minimum pay factor of F = 0.60 for load ratios below R_L = 0.50, Equation 10 was found to provide an appropriate pay schedule:

$$F = 0.695 + 0.295R_L \quad (10)$$

The simulation results for this example are plotted in Figure 10. The operating-characteristic curve matches the desired pay function just slightly better than the curve obtained in Example 3, although the results are so nearly the same that there may be no practical difference. Since the retest provisions are identical, so are the retest curves that indicate a normal retest frequency of about 5.0 percent.

Example 7

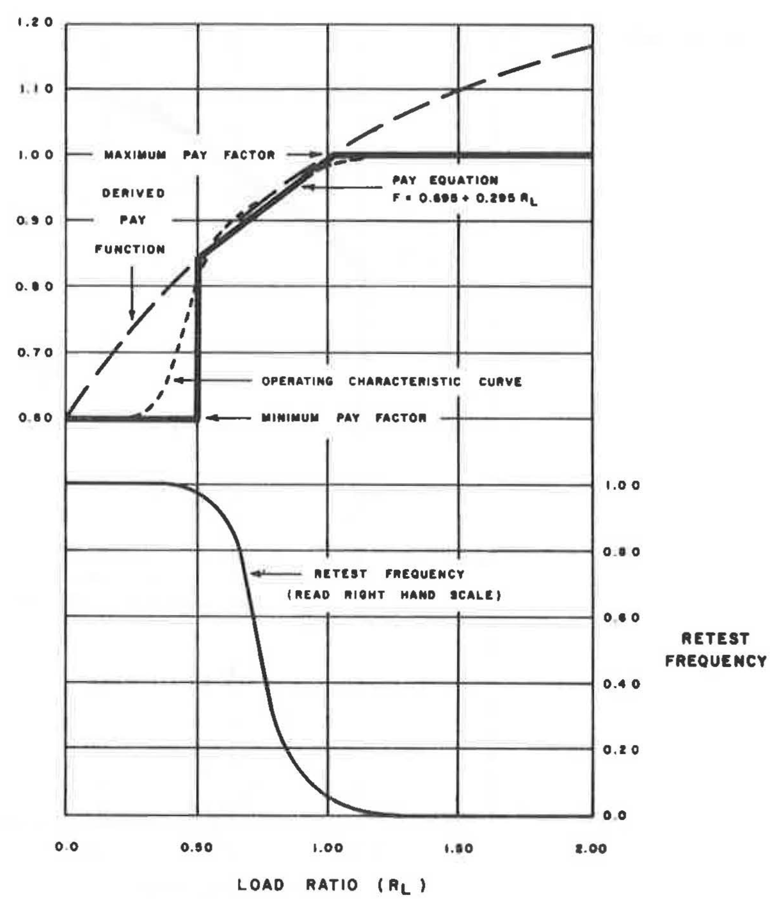
It was stated in an earlier section that if a highway agency did not elect to apply a bonus provision, the maximum pay factor could be limited at F = 1.00 (100 percent). To see what effect this would have on the operating-characteristic curve, this example duplicates the conditions of Example 6 except that the maximum pay factor is reduced from F = 1.10 to F = 1.00.

The results are plotted in Figure 11. As expected, the operating-characteristic curve matches the desired pay function very well except at the upper end, where it becomes increasingly biased downward. For a load ratio exactly at the design value of R_L = 1.00, the expected pay factor is approximately F = 0.985. The retest frequency curve, of course, remains unchanged.

Although the downward bias of 1.5 percent at the desired quality level seems relatively small, it was demonstrated in a recent paper (8) that with specifications based on the concept of percent defective, this can force contractors and producers to supply a level of quality substantially above that which is desired or economically justifiable from the highway agency's standpoint. For acceptance procedures based on the concept of load ratio, the effect is similar but somewhat less severe. In Figure 11, it is seen that a load ratio of about R_L = 1.20 or more is required to achieve an average pay factor of F = 1.00. By substituting typical values into the AASHTO equation, it is found that this would require either an increase in pavement thickness of about 0.3 in or an increase in compressive strength of approximately 500 psi.

An acceptance procedure such as this is misleading at best. Unless the contractor knows the degree of overdesign required, even good-quality work may receive a sufficient number of pay reductions to substantially reduce the expected profit margin on the job. Fortunately, there are two ways in which this undesirable feature can be corrected; both require that the average pay factor be 100 percent when the work is exactly at the desired quality level. The first method is simply to permit bonus pay factors as was done in the first six examples. It can be observed in each of Figures 5 through 10 that the operating-characteristic curve passes through the point at which R_L = 1.00 and F = 1.00, as desired. The second method permits pay factors in excess of 100 percent to be averaged with lower pay factors but is not a true bonus provision be-

Figure 11. Simulation results for Example 7.



cause the overall pay factor for specific intervals of time is still limited to a maximum of 100 percent. This method has been described in detail in an earlier paper (4).

ALTERNATIVE METHODS OF COMBINING MULTIPLE PAY FACTORS

The computer programs developed for this work provided the capability to conduct an interesting secondary study. By using a typical pay equation similar to those developed in the seven examples, it was possible to compute appropriate pay factors for various combinations of thickness and compressive strength and then compare these with the pay factors that would result from the several other methods that have been used to combine multiple pay factors. The results obtained by using five alternative methods are listed in Table 1. The product, average, minimum, and maximum methods are self-explanatory. For the cumulative method, the individual pay adjustments are summed to determine the total adjustment.

Assuming that the method based on the AASHTO equation is the fundamentally correct approach, the

Table 1. Comparison of alternative methods of combining multiple pay factors.

THICKNESS			COMPRESSIVE STRENGTH			METHOD BASED ON AASHTO EQUATION		PAY FACTOR DERIVED FROM INDIVIDUAL PAY FACTORS				
VALUE	LOAD RATIO	PAY FACTOR	VALUE	LOAD RATIO	PAY FACTOR	LOAD RATIO	PAY FACTOR	PRODUCT	AVERAGE	MINIMUM	MAXIMUM	CUMULATIVE
8.50	0.71	0.93	3000	0.61	0.90	0.43	0.86	0.84	0.92	0.90	0.93	0.83
8.50	0.71	0.93	3500	0.80	0.95	0.56	0.89	0.88	0.94	0.93	0.95	0.88
8.50	0.71	0.93	4000	1.00	1.00	0.71	0.93	0.93	0.96	0.93	1.00	0.93
8.50	0.71	0.93	4500	1.22	1.06	0.87	0.97	0.98	0.99	0.93	1.06	0.98
8.50	0.71	0.93	5000	1.46	1.12	1.04	1.01	1.03	1.02	0.93	1.12	1.04
8.75	0.84	0.96	3000	0.61	0.90	0.52	0.88	0.87	0.93	0.90	0.96	0.86
8.75	0.84	0.96	3500	0.80	0.95	0.67	0.92	0.91	0.95	0.95	0.96	0.91
8.75	0.84	0.96	4000	1.00	1.00	0.84	0.96	0.96	0.98	0.96	1.00	0.96
8.75	0.84	0.96	4500	1.22	1.06	1.03	1.01	1.01	1.01	0.96	1.06	1.02
8.75	0.84	0.96	5000	1.46	1.12	1.23	1.06	1.07	1.04	0.96	1.12	1.08
9.00	1.00	1.00	3000	0.61	0.90	0.61	0.90	0.90	0.95	0.90	1.00	0.90
9.00	1.00	1.00	3500	0.80	0.95	0.80	0.95	0.95	0.97	0.95	1.00	0.95
9.00	1.00	1.00	4000	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
9.00	1.00	1.00	4500	1.22	1.06	1.22	1.06	1.06	1.03	1.00	1.06	1.06
9.00	1.00	1.00	5000	1.46	1.12	1.46	1.12	1.12	1.06	1.00	1.12	1.12
9.25	1.18	1.05	3000	0.61	0.90	0.72	0.93	0.94	0.97	0.90	1.05	0.95
9.25	1.18	1.05	3500	0.80	0.95	0.94	0.99	0.99	1.00	0.95	1.05	0.99
9.25	1.18	1.05	4000	1.00	1.00	1.18	1.05	1.05	1.02	1.00	1.05	1.05
9.25	1.18	1.05	4500	1.22	1.06	1.45	1.11	1.10	1.05	1.05	1.06	1.10
9.25	1.18	1.05	5000	1.46	1.12	1.73	1.18	1.17	1.08	1.05	1.12	1.16
9.50	1.39	1.10	3000	0.61	0.90	0.85	0.96	0.99	1.00	0.90	1.10	1.00
9.50	1.39	1.10	3500	0.80	0.95	1.11	1.03	1.04	1.02	0.95	1.10	1.05
9.50	1.39	1.10	4000	1.00	1.00	1.39	1.10	1.10	1.05	1.00	1.10	1.10
9.50	1.39	1.10	4500	1.22	1.06	1.70	1.18	1.16	1.08	1.06	1.10	1.15
9.50	1.39	1.10	5000	1.46	1.12	2.04	1.26	1.23	1.11	1.10	1.12	1.21

Table 2. Evaluation of alternative methods of combining multiple pay factors.

	DEVIATIONS FROM CORRECT PAY FACTOR FOR VARIOUS METHODS				
	PRODUCT	AVERAGE	MINIMUM	MAXIMUM	CUMULATIVE
	-0.02	0.06	0.04	0.07	-0.03
	-0.01	0.05	0.04	0.06	-0.01
	0.0	0.03	0.0	0.07	0.0
	0.01	0.02	-0.04	0.09	0.01
	0.02	0.01	-0.08	0.11	0.03
	-0.01	0.05	0.02	0.08	-0.02
	-0.01	0.03	0.03	0.04	-0.01
	0.0	0.02	0.0	0.04	0.0
	0.0	0.0	-0.05	0.05	0.01
	0.01	-0.02	-0.10	0.06	0.02
	0.0	0.05	0.0	0.10	0.0
	0.0	0.02	0.0	0.05	0.0
	0.0	0.0	0.0	0.0	0.0
	0.0	-0.03	-0.06	0.0	0.0
	0.0	-0.06	-0.12	0.0	0.0
	0.01	0.04	-0.03	0.12	0.02
	0.0	0.01	-0.04	0.06	0.0
	0.0	-0.03	-0.05	0.0	0.0
	-0.01	-0.06	-0.06	-0.05	-0.01
	-0.01	-0.10	-0.13	-0.06	-0.02
	0.03	0.04	-0.06	0.14	0.04
	0.01	-0.01	-0.08	0.07	0.02
	0.0	-0.05	-0.10	0.0	0.0
	-0.02	-0.10	-0.12	-0.08	-0.03
	-0.03	-0.15	-0.16	-0.14	-0.05
AVERAGE:	-0.001	-0.007	-0.046	0.035	-0.001
STANDARD DEVIATION:	0.013	0.054	0.056	0.066	0.020
RANK FOR ACCURACY:	1 - 2	3	5	4	1 - 2
RANK FOR PRECISION:	1	3	4	5	2
OVERALL RANK:	1	3	4 - 5	4 - 5	2

values in the last five columns of Table 1 can be used to judge which of the other methods is most appropriate in this particular case. To accomplish this, the deviations from the values obtained by the fundamental method have been tabulated for each of the other methods in Table 2. The summary statistics for each column are printed at the foot of the table. For a method to be judged both accurate and precise, the average deviation must be close to zero and the standard deviation should be small. On this basis, the five methods have been ranked for accuracy and precision and the overall rank has been determined by weighting these two separate ranks equally.

What emerges from this rather cursory investigation is evidence that the method of multiplying individual pay factors together is equal or superior to any of the other methods that were tested, at least for this particular application. This is encouraging, not only because this approach is widely used, but also because it suggests a method by which additional quality characteristics not included in the AASHTO equation might be incorporated into acceptance procedures for rigid pavement.

SUMMARY AND CONCLUSIONS

A method has been outlined by which the AASHTO design equation can be used to develop acceptance procedures for rigid pavement. By computing the expected load-bearing capacity from the as-built characteristics of the pavement and comparing this with the design loading, a ratio is obtained that forms the basis for a rational pay schedule. Sensitivity tests were performed to confirm the reliability of this approach, and several different acceptance procedures were developed and tested by computer simulation. In all cases, it was possible to make the operating-characteristic curve conform closely to the desired pay function.

It was demonstrated that the limitation of pay factors to a maximum of 100 percent biases the operating-characteristic curve downward, which makes it difficult for contractors to know how to bid or perform under such a specification. This situation can be alleviated by allowing a bonus provision or by permitting pay factors greater than 100 percent to be used to offset other pay factors less than 100 percent.

Finally, a secondary study was conducted to compare various methods currently in use for combining multiple pay factors. Under the assumption that the method based on the AASHTO equation is fundamentally correct, it was demonstrated that the method of multiplying the individual pay factors together is among the best of the other methods that were tested.

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Interest on Capital Invested in Construction as Delay Damages

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The potential for contractors to recover extended financing costs that result from a construction delay is investigated. Legal case histories arising from the federal courts and boards of contract appeals are reviewed, and recent developments related to federal construction contract procedures are presented. Legal case studies are cited that indicate that delay damages can be recovered under the suspension-of-work clause even though no written directive is issued. Delay damages under the change clause are generally not recoverable, although the general conditions of construction contracts of the General Services Administration and the Department of Defense do permit recovery of cost of delays related to change orders. Legal precedents are reviewed that suggest that interest on borrowed funds that was necessitated by a delay can also be recovered. Regulations that prohibit recovery of interest on borrowed funds governed by most federal construction contracts are reviewed. These have been challenged and upheld in the U.S. Court of Claims. Since 1976, boards of contract appeals have awarded imputed-interest damages. These damages result when a contractor is required by a delay to increase the capital investment in a construction project. This increased investment represents a loss of profit because these funds could otherwise be invested in short-term securities and treasury notes. Cost Accounting Standard 417, effective December 1980, provides for the recovery of imputed-interest damages resulting from a delay. The calculation procedure presented in CAS 417 is illustrated with a construction example. It is shown that on a project that costs $2 380 750 and experiences a three-month suspension-of-work delay, the contractor is entitled to $29 702 in imputed-interest damages in addition to any other damages that may have been incurred.

Acceptable cash flow for a construction contractor is largely dependent on ability to achieve satisfactory progress with regard to the project schedule. Unanticipated delays in the construction process, regardless of the cause or responsible party, will likely result in additional direct and indirect