Abridgment

Optimal Speed Limit: A New Approach

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The private optimal speed for a driver and the optimal speed limit imposed by a government are two different things. An individual can determine his or her optimal driving speed by comparing the costs of increased speed (greater gasoline consumption and greater probability of an accident) with the benefits (reduced travel time). A driver's private optimum speed does not, however, take into account any damage that his or her extra speed may do to others. This is an external cost of speed, which provides a rationale for government regulation of highway speed. In this paper we present a method for calculating the optimal speed limit. It starts with the privately chosen speed and then adjusts it to account for external costs. One advantage of this method is that it is based on the driver's judgment about the value of his or her life rather than on an externally imposed estimate. After deriving a formula for the optimal speed limit, we use it in a simple numerical example that provides (a) an estimate of the optimal speed limit; (b) an estimate of the cost per life saved of a suboptimal speed limit, which can be compared with the costs of other ways of saving lives; and (c) an understanding of the types of information needed to improve estimates of the optimum.

In this paper we describe one rationale for government regulation of driving speed--externalities; that is, the costs imposed by one motorist on others. We also demonstrate that acceptance of this rationale leads to a method for calculating the optimum speed limit. As do all such calculations, our calculations depend on two uncertain parameters, the values of time and life. We show how to calculate the optimal speed limit without imposing outside judgments about these parameters. Someone's judgment must be used; we use that of the individual driver, which is inferred from the drivers' behavior. To demonstrate the methodology we make an illustrative calculation of the optimal speed limit for one type of road, uncongested freeways.

EXTERNALITY

The method is based on a crucial distinction: between the optimal speed limit (imposed by a government in the social interest) and the optimal speed for an individual driver.

An individual determines his or her optimal driving speed based on the private or internal costs (increased fuel use and increased possibility of an accident) and benefits (reduced travel time) of driving faster. The driver takes no account of the damage his or her extra speed may do to others (the external cost of speed). Society, however, must consider all costs, internal plus external, which is the major rationale for a speed limit.

ASSUMPTIONS

The formula for the optimal speed limit is derived from a simple economic model of driver behavior. [A more detailed analysis is given elsewhere (1)]. The model uses the following major assumptions:

1. A driver is rational; that is, given the necessary information, he or she would drive at the optimal speed;
2. The driver possesses the knowledge necessary to determine his or her private optimum speed or, at least, his or her knowledge is as good as the government's; and
3. There is a single representative driver, which implies that there is no variation in optimum speeds among drivers.

Deriving the Private Optimum Speed

Figure 1 shows how the private and social optimum speeds are determined. The marginal benefit curve shows the benefit of increased speed. The equation for this curve is derived from the total benefit of trip time (which is negative because trip time is a cost):

$$\text{Dollar benefit of trip time} = -V_T(D/S) \qquad (1)$$

where

V_T = value of time (\$/h),
D = trip distance, and
S = speed.

The marginal benefit of speed is the derivative of benefit with respect to S:

$$\text{Marginal benefit of speed} = V_T D/S^2 \qquad (2)$$

The curve that shows private marginal cost depends on the cost of gasoline and the increased probability of an accident. The equation is given by

$$\text{Private marginal cost} = [V_L b + P_G c]\, D \qquad (3)$$

where

V_L = the amount necessary to compensate a driver for an increase in the probability of a fatal accident;
b = the increased probability , per mile, of a fatal accident as speed increases 1 mph;
P_G = the price of gasoline; and
c = the increase in gasoline use, per mile, as speed increases 1 mph.

The private optimum is determined by the intersection of marginal benefit and marginal cost. Mathematically, this involves the equating of marginal benefit and cost and the solving of Equations 1 and 2 for S. S_p, the optimum speed for an individual driver, is the solution value of S:

$$S_p = V_T^{0.5}(P_G c + V_L b)^{-0.5} \qquad (4)$$

The private optimum depends directly on the value of time and inversely on the value of life and gasoline price, as well as on the increased gasoline use and fatality rate per mile of speed.

Deriving the Social Optimum Speed

To obtain a formula for the social optimum requires adjusting the earlier formula for marginal cost to account for externalities. The total effect of an increase in a driver's speed on the fatality rate is b + b', where b' is the effect on other people and b is the effect on the driver. Then (shown as an upward shift in marginal cost in Figure 1) marginal cost increases to $P_G Dc + V_L D(b + b')$, and the optimal speed is reduced to

$$S_s = V_T^{0.5}[P_G c + V_L(b + b')]^{-0.5} \qquad (5)$$

Figure 1. Social and private optimum speed limits.

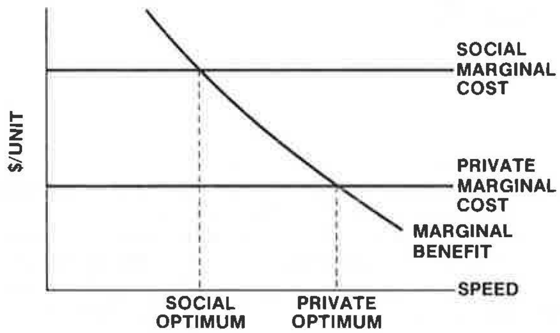

Table 1. Estimates of private and socially optimal speeds.

Value of Time ($/h)	Value of Life ($/life)	Speed (mph) Private Optimum	Speed (mph) Social Optimum
5	100 000	60.8	60.3
5	1 000 000	53.4	50.3
5	10 000 000	29.4	24.9
10	100 000	86.0	85.3
10	174 256	85.0	83.8
10	1 000 000	75.5	71.1
10	10 000 000	41.5	35.2
15	100 000	105.4	104.5
15	1 000 000	92.5	87.0
15	1 716 697	85.0	78.0
15	10 000 000	50.9	43.2
20	100 000	121.7	120.7
20	1 000 000	106.8	100.5
20	3 259 137	85.0	75.6
20	10 000 000	58.8	49.9

Numerically Calculating the Optimal Speed Limit

We now have formulas for the private optimal speed and the optimal speed limit. To make numerical calculations, we need values for the parameters. It is not too difficult to find data on gasoline prices or on the relations between driving speeds and fatality rates (2) and driving speed and gasoline mileage (3-5). The value of time and the value of life, however, are uncertain.

One strategy is to calculate the optimal private speed and the optimal social speed by using alternate values for life and time, which leads to the results in Table 1. The social speed limit varies greatly within the table, from 30 mph to more than 120 mph. To determine the optimal speed limit one would have to choose the most likely values for time and a life, a task that involves massive uncertainty.

We propose an alternative method, one that avoids imposing the value of a life from outside the driver's behavior. Our method uses the driver's private optimum (which we assume equals the average speed where there is no speed limit) and the other parameters to make inferences about the value drivers place on their own lives and to derive the optimal speed limit.

To illustrate our method, suppose the private optimum speed is 85 mph (roughly the actual, unrestricted speed on the German Autobahn). Our method involves restricting attention to the cases in Table 1 that are consistent with a private optimum of 85 (cases 10, 15, and 20). Note the drastic reduction in the range of socially optimal speeds. Rather than assuming a value of life and then solving for the private and social optimum speeds, we assume a value for the private optimum and then solve for the implied value of a life and social optimum.

Application to the 55-mph Speed Limit

This section applies our model to a current policy question, evaluation of the 55-mph speed limit. We can use our model not only to calculate the optimal speed limit, which does take account of highway deaths, but also to answer the question: What is the cost per life saved of imposing the 55-mph limit if this differs from the socially optimal speed limit?

We answer this question in the following way. First, we compute the lifesaving cost of a suboptimal speed limit: the costs less the benefits, where the benefits exclude the value of saving lives. We then divide this figure by the number of lives saved.

If the private optimum speed is 85 mph, the optimal speed limit is 78 mph, and the value of a life is $1.7 million (from Table 1), we find that the lifesaving cost of having a speed limit of 55 mph rather than 78 mph is $3.2 million per life saved. [For the calculations, see elsewhere (1).] In our case, this not only exceeds the value individuals place on their own lives ($1.7 million) but also proves to be a more expensive way of saving lives than many other proposed lifesaving policies, for example, the installation of smoke alarms.

SUMMARY

We have developed a method for calculating the optimal speed limit. The conclusions of our work can be stated as follows.

To determine the optimal speed limit, two parameters are of crucial importance: (a) the speed in the absence of a speed limit and (b) the ratio of external costs to internal costs. We conclude that data-collection efforts should be focused on these two parameters, which would involve a major redirection of federal highway research.

If our crude calculations are at all indicative, the optimal speed limit is above 55. Maintenance of the 55-mph limit is an expensive way to save lives.

In our method, the value of a life does not directly enter the calculation of the optimal speed limit. The value of time enters with less importance than in usual calculations. In general, the information needed for the calculation is changed as well as the relative importance of different pieces of information.

What we have presented is a highly simplified model, without precise parameter estimates. What is needed is a more sophisticated model with the same emphases--the focus on externalities and the ability to use the vast amount of information embodied in observed speeds. Also needed are the data (especially on externalities) to apply the model.

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