Suggested Improved Methodology for Relating Objective Profile Measurement with Subjective User Evaluation

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Considerable effort has been directed at relating subjective user evaluation with pavement profile characteristics. Improvements in the methodology for conducting subjective evaluations have recently been made and are cited. Profile characteristics that describe subjective evaluation have previously included statistical properties of elevation and slope. This paper questions the theoretical validity of correlating profile elevation and slope with subjective evaluation. Profile curvature is suggested as a theoretically sound profile measurement that can be related to subjective evaluation. Recommendations are made for verifying a relation between profile curvature and subjective evaluation, and the potential is outlined for its application in highway practice.

Since the conclusion of the American Association of State Highway Officials (AASHO) road test, much effort has been directed at relating subjective human response with the physical properties of a pavement profile. The methodology used for obtaining this subjective evaluation of pavement section, known as the present serviceability index (PSI) and outlined by Carey and Irick (1), has been generally accepted by the pavement evaluation community as a state-of-the-art method for determining pavement serviceability.

This subjective measure is currently popular; however, this method has important shortcomings in terms of the development of the rating scale and its application. Shortly after completion of the AASHO road test, Hutchinson (2) suggested that the principles of psychophysics, widely used in experimental psychology and marketing research, would be more appropriate than PSI for establishing a subjective rating scale for pavement evaluation. When this method of scale construction is used these systematic errors are removed:

1. The error of leniency, which refers to the constant tendency of a rater to rate too high or too low for whatever reasons;
2. The halo effect, which refers to the tendency of raters to force the rating of a particular attribute in the direction of the overall impression of the object rated; and
3. The error of central tendency, which refers to the fact that raters hesitate to give extreme judgments of stimuli and tend to displace individual ratings toward the mean of the group.

Hutchinson showed that these errors are inherent in the AASHO road test development of PSI.

The development and verification of a rational subjective scale for pavement evaluation that incorporates the methods of psychophysics has only recently been accomplished. Weaver and Clark (3) and Weaver (4) showed that the principles of psychophysics yield a representative rating scale for subjective pavement evaluation. The serviceability scale values obtained are similar to those of the AASHO road test PSI but cannot be related directly because of the inadequacies of the PSI in terms of subjective scale construction and data-gathering techniques.

The subjective human evaluation of pavements depends on the magnitude of the various acceleration components that occur as a vehicle traverses a pavement section. Shahin and Darter (5) cite numerous references that indicate that subjective human tolerance levels, in addition to acceleration magnitude, depend on factors such as duration of exposure and frequency of acceleration. Goldman (6) found that subjective human response to vibration is relatively constant in terms of average peak acceleration in the frequency range of 0-50 Hz. This is well within the range of frequencies encountered in typical pavement and vehicle response functions. Also, the duration of exposure encountered in most subjective pavement evaluations is similar in length and is relatively short. Based on Goldman's work, the U.S. Air Force (7) currently uses a nominal 0.4-g level for determining rough areas by using a known input profile in a computer code (TAXI) that simulates the vertical acceleration response of a given aircraft.

Although it is generally accepted that subjective human response is related to acceleration (the vertical direction being the main component), there is no consensus on the physical pavement profile parameter directly related to, and responsible for, this acceleration and resultant human response. Profile roughness numerics currently in use and outlined by Gillespie and others (8) are (a) spectral densities of elevation or slope and (b) root mean square (RMS) statistics of elevation or slope. Other numerics outlined include those obtained directly from the various response-type road roughness measuring systems (RTRRMS) currently in use, particularly the accumulated displacement value of the axle-body motion of a particular RTRRMS described as an inches-per-mile (I/M) statistic. Gillespie (8) suggested that this I/M statistic does not account for the velocity of the RTRRMS and should be transformed to obtain an average rectified velocity (ARV) to account for various RTRRMS operating speeds.

The problem with the various statistics outlined above is their apparent inability to predict the subjective human response of the traveling public. Although much of this apparent lack of correlation may be attributed to the past use of the AASHO road test PSI as the subjective measurement, one might also suspect the profile statistics currently being used to predict serviceability. Shahin and Darter (5) cite previous work that indicates the shortcomings of spectral density methods; Holbrook and Darlington (9) indicate that conventional multiple regression techniques are not applicable because of the high intercorrelation between frequency bands of pavement profile spectra. Holbrook and Darlington put forth a correlation method that eliminates this interdependency of frequency bands.

There are limitations and constraints in conducting a statistical analysis of pavement profile parameters; however, the real issue may be that elevation or slope are not the true correlates of subjective user response.

PROFILE CURVATURE AS A THEORETICALLY SOUND CORRELATE

Seemann and Nielsen (10) related airfield pavement profile statistics with the computer-simulated response of aircraft. This computer simulation (TAXI) of an aircraft traversing a runway yields vertical acceleration information of the aircraft for the discrete profile data (usually 6 in or 2 ft). The acceleration data are then analyzed statistically to

Figure 1. Profiles of constant slope.

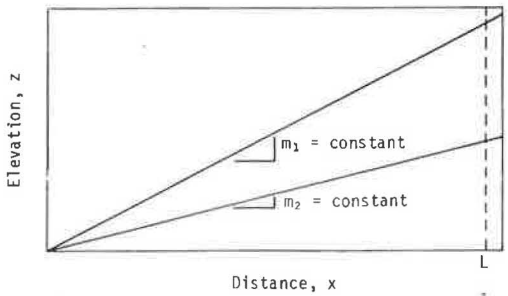


Figure 2. RMS acceleration versus RMS slope.

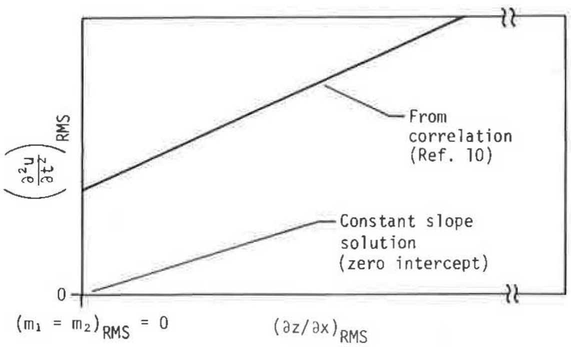
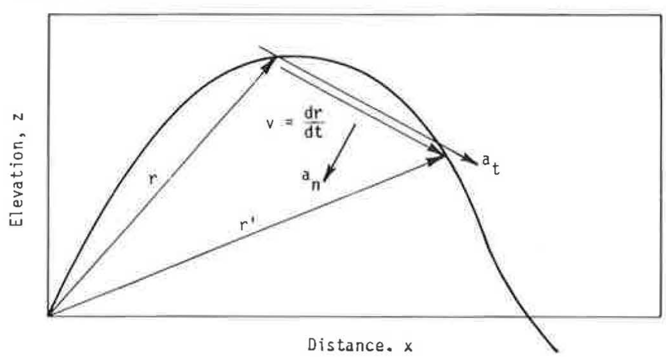


Figure 3. Curvilinear motion in vertical plane.



determine a RMS level of acceleration for an entire runway profile.

Their first attempt was to correlate RMS acceleration levels with elevation levels, or

$$(Z)_{RMS} \sim (\partial^2 u/\partial t^2)_{RMS} \quad (1)$$

which states that the RMS elevation level is proportional to aircraft vertical RMS acceleration for the entire runway profile. The result of this correlation was very poor. A second attempt was to correlate RMS change in elevation (or slope since the profile sample spacing is constant) between discrete elevation points with RMS acceleration values, or

$$(\Delta Z)_{RMS} \sim (\partial^2 u/\partial t^2)_{RMS} \quad (2)$$

This relation yielded an excellent correlation ($r = 0.96$) and further efforts to relate acceleration and profile data ceased because of the correlation obtained.

This correlation, although good, may not be theoretically sound or the best obtainable. Consider two different profiles of constant slope, $m_1 > m_2$, that otherwise are perfectly smooth (Figure 1). Because these two runway profiles are perfectly smooth the aircraft will not have a vertical component of acceleration because the aircraft velocity is constant. The RMS slope values for perfectly smooth profiles of differing slope are zero. The corresponding RMS acceleration level is also zero, so a plot of RMS slope versus RMS acceleration would have an intercept of zero. However, this is contradictory to the correlation obtained by Seemann and Nielsen (10) where a level of RMS acceleration exists for a RMS slope level of zero (Figure 2). This would suggest (excluding a nonlinear solution of RMS acceleration versus RMS slope) that the correlation of RMS acceleration with RMS slope is invalid.

It is now suggested that the second derivative of elevation with respect to distance be considered; i.e., the rate of change of slope [$\partial^2 z/\partial x^2$], or profile curvature. From an intuitive viewpoint this seems to make sense. Take for example a vertical curve. The response of a vehicle passing through a vertical curve can be reduced by reducing the difference in initial and final grades, by lengthening the vertical curve distance, or by a combination of both. These variables affect the rate of change of slope; i.e., for the above vertical curve the rate of change of slope is decreased.

The suggested numerical relation between vertical acceleration and profile curvature is

$$(\partial^2 u/\partial t^2)_{RMS} \sim (\partial^2 z/\partial x^2)_{RMS} \quad (3)$$

and if a constant of proportionality is included, then

$$(\partial^2 u/\partial t^2)_{RMS} = k(\partial^2 z/\partial x^2)_{RMS} \quad (4)$$

The above equation is similar to the classical wave equation

$$\partial^2 z/\partial t^2 = c^2 (\partial^2 z/\partial x^2) \quad (5)$$

where c is the wave speed.

If aircraft elevation at pilot station (u) and profile elevation (z) can be related, Equation 4 can be put into a form similar to Equation 5. From the following relation between u and z,

$$u = z + z_{ps} \quad (6)$$

where z_{ps} is the distance from the profile to the pilot station (point at which vertical acceleration is of interest), it is evident that

$$\partial^2 z/\partial t^2 = (\partial^2 u/\partial t^2) - (\partial^2 z_{ps}/\partial t^2) \quad (7)$$

If one assumes that z_{ps} is constant, then

$$\partial^2 z/\partial t^2 = \partial^2 u/\partial t^2 \quad (8)$$

and the wave equation is satisfied. This formulation assumes that the aircraft is a particle, with velocity (v), in continuous contact with the profile.

Furthermore, the theory of curvilinear particle dynamics (11, pp. 464-466) and elementary calculus (12, pp. 551-553) shows that the following relations exist. Take a typical profile (Figure 3) with position vectors r (time t) and r' (time t + Δt). The velocity of a particle that traverses this profile is shown to be

$$v = dr/dt \quad (9)$$

and for the case of an airplane (particle) that

Figure 4. RMS acceleration versus RMS curvature.

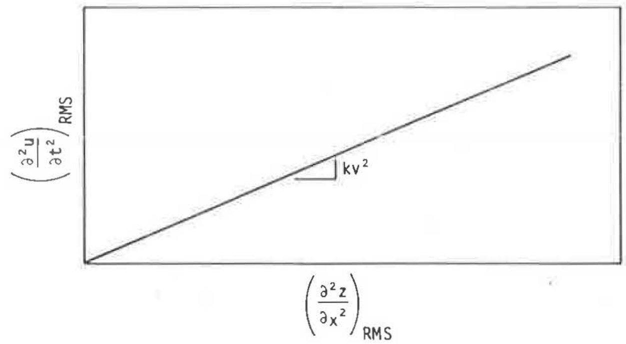


Figure 5. Idealized profile.

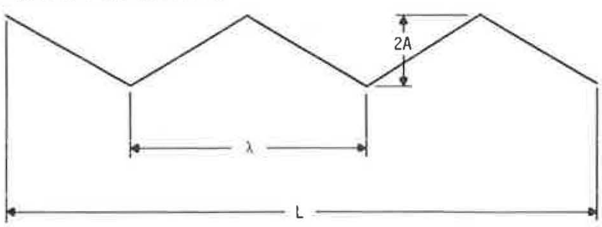
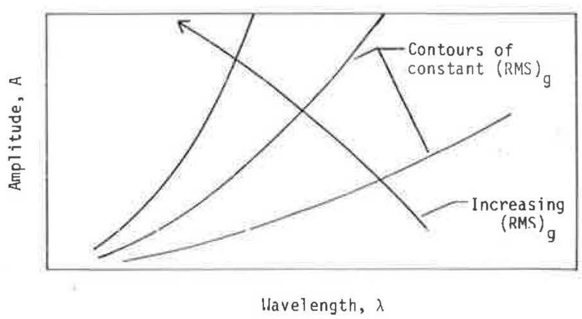


Figure 6. Vertical acceleration versus amplitude and wavelength.



traverses the above profile, this velocity it taken to be constant. The acceleration for the above particle is the vectoral sum of tangential and normal components

$$a = a_t i_t + a_n i_n \quad (10)$$

where i_t and i_n are unit vectors in the tangential and normal directions, respectively. The acceleration components are

$$a_t = dv/dt \quad (11)$$

which is zero because v is constant, and

$$a_n = v^2/\rho \quad (12)$$

where ρ is the radius of curvature. The normal acceleration can be taken as

$$a_n = d^2z/dt^2 \quad (13)$$

and, from calculus, the curvature κ is

$$\kappa = 1/\rho = (d^2z/dx^2)/[1 + (dz/dx)^2]^{3/2} \quad (14)$$

but the $[dz/dx]^2$ term in the denominator can be assumed to be zero for virtually all pavement profiles (1.5 percent error for a 10 percent slope); therefore,

$$1/\rho = d^2z/dx^2 \quad (15)$$

and now substitution of Equations 13 and 15 into Equation 12 yields

$$d^2z/dt^2 = v^2 (d^2z/dx^2) \quad (16)$$

which is identical to the wave equation (Equation 5).

The constant k in Equation 4 might be interpreted as a function of the aircraft type and its associated dynamic properties as well as the velocity of the aircraft. This would suggest that Equation 4 might take the form of

$$(\partial^2 u/\partial t^2)_{RMS} = k\, v^2\, (\partial^2 z/\partial x^2)_{RMS} \quad (17)$$

where v is the aircraft velocity and k is a constant that accounts for the dynamic response of the aircraft. Assuming that u and z from Equation 8 are interchangeable, Equation 17 is virtually identical to Equations 5 and 16.

For a given aircraft and velocity, Equation 17 can be assumed as shown in Figure 4. Notice that the intercept is zero; this accounts for the case of a constant slope, perfectly smooth profile.

Typical profiles exhibit a characteristic wavelength (λ) and corresponding amplitude (2A) that exist for the most part over the entire profile length (L). This wavelength and amplitude are functions of the soil type and structural stiffness of the profile pavement. This can be represented pictorially as in Figure 5. The curvature can be expressed in finite difference form as

$$\partial^2 z/\partial x^2 = (z_i - 2z_{i-1} + z_{i-2})/(\Delta x)^2 \quad (18)$$

If one now looks at a characteristic wavelength and amplitude of a profile, Equation 18 can be expressed as

$$\partial^2 z/\partial x^2 = -16A/\lambda^2 \quad (19)$$

Substitution of Equation 19 into Equation 4 yields

$$\partial^2 u/\partial t^2 = -k\,(16A/\lambda^2) \quad (20)$$

and replacement of the partial derivative with the RMS acceleration yields

$$(RMS)_g = k'\,(A/\lambda^2) \quad (21)$$

where k' = 16 k, the negative sign being eliminated because of the use of RMS levels of acceleration and profile curvature. Equation 21 can be rearranged as

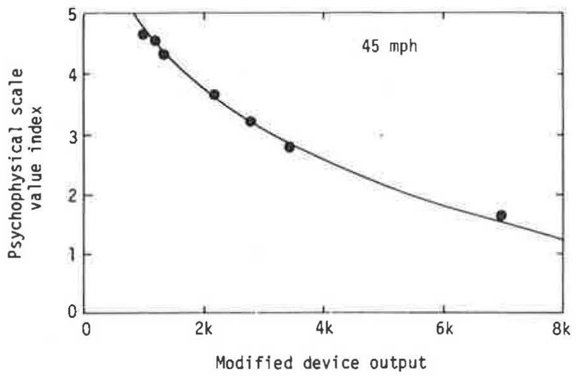
$$A = [(RMS)_g/k']\,\lambda^2 \quad (22)$$

McKeen (13) found from his work on expansive soils that A, λ, and $(RMS)_g$ are related graphically as in Figure 6. If A is plotted versus λ^2, one might expect a plot similar to that of Figure 6 [i.e., a family of straight lines with slope $(RMS)_g/k'$]. Mathematically this is identical to Equation 22. Thus, the theoretical basis of Equations 4 and 17 would seem substantiated based on this experimental work of McKeen.

PROFILE CURVATURE VERSUS SUBJECTIVE EVALUATION

Although the dynamic responses of aircraft and automobiles are quite different (the latter being less complicated), it should be evident that the above theoretical considerations are easily applicable to

Figure 7. Psychophysical rating versus RTRRMS output.



highway pavements and the associated problem of relating a pavement profile with subjective human response. The described wave equation applies to the curvilinear motion of particles and is not directly applicable to systems that have frequency response characteristics. Equation 17 is suggested as a modified wave equation that incorporates a factor (k) to account for the dynamic response of a system that traverses a pavement profile. This factor should be dependent on frequency and complex in nature.

The work of Weaver and Clark (3) would seem to support the above postulate. Their Figure 22 can be viewed as a semilogarithmic transform of the wave equation (reproduced in Figure 7). A psychophysical scale value of five (perfect pavement) corresponds to the threshold of perception of human response to vertical acceleration, which approaches zero, and a scale value of zero (impassable pavement) can be thought of as an acceleration that is very great and in theory approaches infinity. The modified device output is a measurement obtained from RTRRMS, which can be thought of as a relative measure of profile curvature.

According to the wave equation formulation, contours of increasing constant velocity would lie closer to the origin in Figure 7. Weaver has indicated that this is generally the case for most pavement systems. However, for rigid pavement systems the opposite condition has been known to occur. The presence of construction joints and expansion joints in rigid pavements permits the possibility of discontinuous slope and elevation of a pavement section, which will affect the dynamic response of a test vehicle. Characteristic wavelengths and amplitudes associated with these joints tend to be more pronounced in a rigid pavement system than in a flexible pavement system as well as being more uniform in nature. Because the factor k of Equation 17 is postulated to be dependent on frequency (and therefore on wavelength), the suggested wave equation would explain Weaver's observations.

Furthermore, McKeen (13) found from his work on expansive soil-pavement interaction that characteristic wavelengths and amplitudes depend on the type of clay and the pavement rigidity. It has been postulated that characteristic wavelengths and amplitudes may exist for all engineering pavements, these characteristics being dependent on soil and pavement type as well as structural rigidity.

The above discussion suggests that the wave equation formulation may be a feasible method for relating pavement profile and subjective human response. It is anticipated that a subjective evaluation can be related to a statistical property of the curvature. This statistical property might be a power spectral property or a RMS property of the curvature. The latter might be formulated in a fashion similar to Equation 17 as

$$\text{S.E.} = kv^2\,(d^2z/dx^2)_{RMS} \tag{23}$$

where S.E. is the subjective evaluation.

The power spectral formulation is developed in Equations 24-34. Equation 17 can be cast as

$$\ddot{z}(t) = v^2 z''(x) \tag{24}$$

where the double dot ($\ddot{o}$) and double prime (") denote second derivative operations of elevation with respect to t and x, respectively. The autocorrelation of the above is

$$R_{\ddot{z}\ddot{z}}(\tau) = v^4\,E\left\{z''[x(t)]\,z''[x(t+\tau)]\right\} \tag{25}$$

where E{ } denotes the expected value of. The above reduces to

$$R_{\ddot{z}\ddot{z}}(\tau) = v^4\,E\left\{z''(vt)z''[v(t+\tau)]\right\} \tag{26}$$

where v is the vehicle velocity. Further reduction yields the autocorrelation of acceleration in terms of the autocorrelation of curvature

$$R_{\ddot{z}\ddot{z}}(\tau) = v^4\,R_{z''z''}(v\tau) \tag{27}$$

The spectral density of Equation 27 is

$$S_{\ddot{z}\ddot{z}}(\omega) = v^4\int_{-\infty}^{\infty} R_{z''z''}(v\tau)e^{-i\omega\tau}d\tau \tag{28}$$

The change of variable $\theta = v\tau$ yields

$$S_{\ddot{z}\ddot{z}}(\omega) = v^4\int_{-\infty}^{\infty}(1/v)\,R_{z''z''}(\theta)e^{-i(\omega/v)\theta}\,d\theta \tag{29}$$

$$= v^3\int_{-\infty}^{\infty} R_{z''z''}(\theta)e^{-i(\omega/v)\theta}d\theta \tag{30}$$

or

$$S_{\ddot{z}\ddot{z}}(\omega) = v^3\,S_{z''z''}(\omega/v) \tag{31}$$

The above states that the spectral density of acceleration at the pavement vehicle interface is related to the spectral density of the pavement curvature.

The spectral density of the vehicle response is

$$S_{\ddot{u}\ddot{u}}(\omega) = |H(\omega)|^2 S_{\ddot{z}\ddot{z}}(\omega) \tag{32}$$

with substitution of Equation 31 to yield

$$S_{\ddot{u}\ddot{u}}(\omega) = v^3|H(\omega)|^2 S_{z''z''}(\omega/v) \tag{33}$$

where $|H(\omega)|^2$ is the frequency response of the vehicle system (the unsprung and sprung mass) as well as the human support system (car seat).

The power spectral formulation for subjective human response can now be assumed to be

$$\text{S.E.} = v^3|H(\omega)|^2 S_{z''z''}(\omega/v) \tag{34}$$

where $|H(\omega)|^2$ is as above but also includes the response of the human body.

Equations 23 and 34 both state that the output (S.E.) is proportional to the input (curvature), the coefficient of proportionality being a complex frequency-dependent transform.

The frequency response terms need not be known explicitly, but it may be determined by means of statistical regression between S.E. and the profile curvature. The curvature portion of the regression might consist of a linear or nonlinear combination of the area under the curvature power spectral function plus higher order statistical moments. The higher order statistical moments represent the mean and standard deviation and reflect quantitatively the distribution of the curvature spectral function. Note also that the frequency response term in Equation 34 is a statistical property that reflects the inability of performing a power spectra transformation on the S.E. term.

APPLICABILITY TO HIGHWAY PRACTICE

RTRRMS measurements can also be related to profile curvature. With a knowledge of the curvature properties of various test sections, RTRRMS should be easily calibrated to yield a relation between device output and curvature at various travel speeds and subsequently to yield a measure of S.E.

It was shown that Figure 7 is a transform of the wave equation for a given RTRRMS. Similar relations for various RTRRMS should be easily obtainable. Gillespie and others (8) suggested the use of an average rectified velocity (ARV) as a measure of a RTRRMS output. A similar statistic could be developed that would be compatible with the wave equation formulation; this statistic might be called an average rectified acceleration (ARA). This ARA value could then be related to curvature and in turn to subjective evaluation or serviceability.

CONCLUSIONS AND RECOMMENDATIONS

The previous discussion indicates that RMS elevation levels do not correlate with RMS vertical acceleration; however, RMS slope levels correlate with RMS vertical acceleration but do not yield a unique solution. A profile curvature statistic will more adequately relate to statistics of vertical vehicle response based on a wave equation formulation that is known to have a solid basis in theoretical mechanics. It is anticipated that a relation between profile curvature and vehicle response can be extended to describe subjective evaluation, as acceleration and human response are known to correlate.

Verification of the relation between curvature and vehicle-human response will require correlation and parametric studies of Equations 17, 23, and 34. Subsequently, calibration of the various RTRRMS could be accomplished based on the relations of Equations 23 and 34.

ACKNOWLEDGMENT

The above paper is a result of work conducted for the U.S. Air Force Engineering and Services Center by the New Mexico Engineering Research Institute (NMERI) at the University of New Mexico. Appreciation is extended to R. Gordon McKeen and John P. Nielsen of NMERI for their input and encouragement. Special thanks are extended to Thomas L. Paez of the University of New Mexico Civil Engineering Department for his input on random processes.

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Publication of this paper sponsored by Committee on Pavement Condition Evaluation.