Procedure for Estimating Freeway Trip Tables

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A method for estimating freeway trip tables with volume data by using a gravity-based model is presented. The 1969 San Francisco Bay Area origin-destination survey is used to test the estimates. The results of using the gravity-based model are compared with the estimates obtained by using the SYNODM approach developed at the University of California, Berkeley. The gravity-based approach achieves a closer fit to the actual trip tables than does the current version of SYNODM.

This paper describes a method for estimating freeway trip tables by using ramp volume data and compares the resulting estimates with those obtained by using the SYNODM approach (an estimation procedure currently used by traffic planners). Below is a description of both the general problem of estimating trip tables by using volume data and the freeway-restricted problem. Also included is a summary of currently proposed estimation procedures and their limitations.

GENERAL PROBLEM

For the past few years there has been a surge of interest in developing a technique for estimating a trip matrix or origin-destination (O-D) matrix for urban areas by using street volumes as the primary source of knowledge. This is because the collection of trip origin-destination data is costly, time consuming, and less accurate than the more easily collected traffic volume data. Volume data have been collected in most cities on a regular basis for a number of years through the use of automatic traffic counters. However, current O-D data, which require extensive travel surveys of the urban population, are not available to today's transportation planner and are not likely to be available in the future for the majority of our urban areas.

The state of the art in the general problem area is summarized by Willumsen (1), who divides the estimation methods into three broad groups of models. These include gravity-based models, network-equilibrium models, and entropy-maximizing models. Gravity-based models assume that trips follow a gravity pattern. In the approaches considered so far this leads to linear or nonlinear regression solutions. The linear models (2-7), where tested, are used to forecast link volumes. They give acceptable estimates of link flows (errors of 20 percent or less). The nonlinear approaches (8-10) yield slightly better results. However, accuracy is still based on observed flows. Since a variety of O-D matrices can produce the same pattern of link volumes in a network, this is not a sufficient test of the accuracy of the trip tables.

The second group of general models, network-equilibrium models, is based on Wardrop's first principle. Such models (11-13) yield solutions that depend on the initial solution assumed. The solutions are, therefore, not unique and have not been verified adequately.

The last group of models uses an entropy-maximizing approach to find the most likely trip matrix compatible with observed flows. In one such model the solution depends on an a priori estimation of the O-D matrix. Other approaches (1, 15-17) circumvent this problem but do not consider the impact of distance or travel time on tripmaking behavior. All three approaches to the general problem have not been tested against actual O-D data, although some have been checked by using synthetic networks.

FREEWAY OR CORRIDOR PROBLEM

Unlike the general problem, the freeway or corridor problem lends itself to model verification. Also, in dealing with a restricted network of corridor flows, we avoid some of the problems created by the ubiquitous nature of vehicle travel. The natural constraints on freeway flows reduce the relative number of unknowns, although the problem is still underdetermined.

Since an O-D matrix of the freeway portion of vehicle trips is required for certain traffic-planning models such as the FREQ6PE simulation model (18), these data are of immediate interest to traffic and transportation planners. Yet the methods used to collect these data, although easier than comprehensive travel surveys, are still costly and time consuming. The most widely used method is the license plate survey, where observers are positioned at every ramp for a particular freeway segment and license plates of passing vehicles are recorded. These are then traced to determine points of vehicle entry and exit. Although the surveys are inconvenient, they have been conducted in most major cities and therefore provide a basis for verification of models that attempt to estimate the freeway trip matrix by using ramp volumes as a basis. If sufficiently accurate models can be developed for this problem, they can be used not only to help monitor existing traffic congestion problems but also to illuminate the general problem. Also, since volume data are available for different time intervals (e.g., hours of the day), accurate models could reproduce the changes in trip patterns over time that would be of benefit to transit and traffic planners.

Although some work on generating freeway O-D tables (ramp-to-ramp trip tables) from link volumes has been reported, the results in this area are inconclusive and the theoretical basis of the proposed models is weak. The primary activity in this area comes from the developers of FREQ6PE, a combination traffic simulation and ramp control optimization model. This model requires a ramp-to-ramp trip table for every time interval (e.g., 15-min intervals) for the period of study (e.g., peak-hour period). A computer model called SYNODM (19) has been developed to synthesize the required trip tables. It is a simple proportionality scheme that distributes off-ramp traffic to upstream on-ramps. Specifically, if we let

M = set of all freeway entrances upstream of exit j,
V_i^* = total trips originating at i that have not yet been assigned a destination and are upstream from j,
D_j = total trips exiting at destination j, and
T_{ij} = total trips originating at i and exiting at j.

Then,

$$T_{ij} = D_j \left(V_i^* / \sum_{\ell=1}^{M} V_\ell^* \right) \quad (1)$$

The trips are assigned beginning with the first upstream off-ramp and continuing to successive downstream off-ramps.

There are several problems with such a simplistic approach. The most noticeable is the implicit assumption that trip distance or travel time is not a factor in travel behavior. Thus, if there are 100 still unassigned vehicles that enter at on-ramp 1 (10 miles upstream from off-ramp j) and 100 vehicles enter at on-ramp 10 (1 mile upstream from the nearest off-ramp j), the number of vehicles from 1 to j and from 10 to j would be equal. Yet, intuitively one would expect that vehicles that enter at 10 would have a relatively low probability of getting off at the next exit. By the same token, vehicles that have already traveled 10 miles would have a relatively high probability of exiting at the next stop. Therefore, one could expect significant errors here in the form of overpredicting the number of very short trips and very long trips. The authors admit that the proportionality assumption is a crude approximation. In assessing the accuracy of their model they state that it "...does tend to distribute correctly 70-80 percent of the traffic in most cases, and in the absence of an O-D study that is probably a reasonable approximation" (20).

The level of error in those cases that are not correctly assigned is not discussed. If, as our intuition indicates, these errors could be substantial, the resulting O-D table is not valid. The developers are currently investigating other methods and have recently revised SYNODM to include known interchanges as inputs to improve accuracy (21,22). The gravity-based method proposed below also has this capability and includes an impedance factor as well.

RESEARCH DESIGN

A description of the proposed estimation procedure, the data base used for testing the trip estimates, and the error measurements used in comparing this procedure with the estimates obtained by using SYNODM is presented in this section.

Estimation Procedure

The procedure assumes a gravity-based model to be applied along a particular section of freeway. Three inverse impedance functions are calculated based on average trip distance along the section. (A separate curve was used for internal-internal or ramp-to-ramp trips, external-internal or mainstream-to-ramp trips, and internal-external or ramp-to-mainstream trips.) Since Voorhees (25) has shown that a gamma function is most appropriate for total trip patterns, a gamma function was assumed with adjustments made for external trip ends. Also assumed was that one could obtain a reasonably good estimate of through trips either by collection that used overpasses or from knowledge of previous O-D percentages.

The gravity-based model has the formulation shown below:

$$T_{ij} = O_i X_j f(d_{ij}) / \sum_j X_j f(d_{ij}) \quad (2)$$

$$\sum_i T_{ij} = D_j \quad (3)$$

$$f(d_{ij}) = [\beta^\alpha / \Gamma(\alpha)]\, d_{ij}^{(\alpha-1)} \exp(-\beta d_{ij}) \quad (4)$$

where

T_{ij} = trips from origin ramp i to destination ramp j,
O_i = trips that originate at ramp i,
D_j = trips that exit at ramp j,
d_{ij} = impedance of travel between i and j (e.g., distance or travel time),
f = inverse impedance function (i.e., travel propensity function), and
X_j = normalization factor for destination point j.

Given the impedance function, an iterative procedure is used to solve for X_j. This procedure has been shown to always converge to a unique solution (24,25).

Although several forms of inverse impedance functions have been tested in the past, Voorhees (25) showed rather conclusively that the gamma function gave the best fit when calibrating models to generate total travel matrices. This would appear to also apply to freeway trip tables because one expects a unimodal function to discourage both very short and very long trips. The shape parameter (α) was found by Voorhees to be approximately 1.5 for total travel for most cities. Since this was obtained for a total inverse impedance function, and since freeway travel can be expected to tolerate longer distances than can other types of trips, one would expect that a distribution less skewed to the left would be appropriate. A preliminary value for α of approximately 3 is suggested. (Note, for a sample experiment described later in this paper, values of the shape parameter were varied from 2 to 4 with minor differences in the resulting trip table.)

The size parameter (β) is equal to $\alpha/\bar{d}$ where $\bar{d}$ is the average impedance for the network. Since we know the total number of trips and the link volumes and impedances, this can be calculated as

$$\bar{d} = \sum_{k=1}^{K} v_k d_k / T \quad (5)$$

where

T = total trips (i.e., total number of origins or destinations),
v_k = volume on freeway subsection ℓ,
K = number of freeway subsections in study section, and
d_k = length of freeway subsection ℓ.

In determining the impact of origins farther upstream or downstream from the freeway section, a simple constant that is equal to the largest possible value for that inverse impedance function was chosen. Thus, for example, for a travel function with α = 3 and $\bar{d}$ = 5.0, a gamma function would be generated as shown in Figure 1. If one end of the trip originated at the mainstream-on point or ended at the mainstream-off point this constant (f*) would be used for short trips (trips that have study section length d* or less). Thus, the external-internal and internal-external functions would resemble the solid line in Figure 1 and the internal-internal function would be a strict gamma function.

Figure 1. Example of inverse impedance curve.

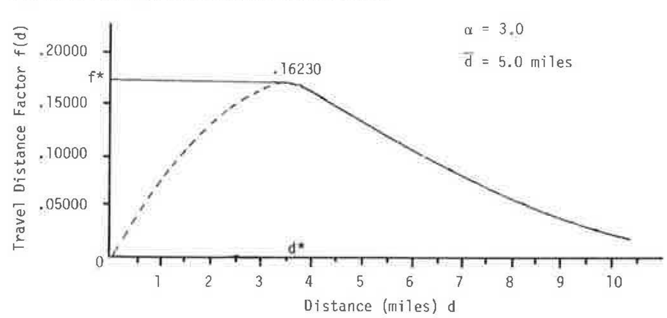

Table 1. Freeway subsection characteristics.

Sub-section No.	No. of Lanes	Length (ft)	Input	Subsection Description: Entry Point	Exit Point
1	5	1630	OD	Mainline origin	Powell off
2	5	1960		Powell off	Powell on
3	5	1550	OD	Powell on	Ashby off
4	4	1960		Ashby off	Ashby on
5	4	500	O	Ashby on	500-ft point
6	4	4790	D	500-ft point	University off
7	4	3030		University off	University on
8	4	2160	OD	University on	Gilman off
9	4	2030		Gilman off	Gilman on
10	5	1250	OD	Gilman on	Buchanan off
11	4	900	D	1 Buchanan off	Hoffman off, left
12	3	1320	D	Hoffman off	Pierce off
13	3	720		Pierce off	Pierce on
14	3	2610	OD	Pierce on	Central off
15	3	1660		Central off	Central on
16	3	1890	OD	Central on	Carlson off
17	3	2310		Carlson off	Carlson on
18	3	1460	OD	Carlson on	Potrero off
19	3	3800		Potrero off	Cutting on
20	4	1100	O	Cutting on	Grade change point
21	4	660	D	Grade change point	Macdonald off
22	4	1480	D	O Macdonald off	San Pablo off
23	3	1480		San Pablo off	San Pablo on
24	4	800	OD	San Pablo on	Solano off
25	3	4690	D	Solano off	San Pablo Dam off
26	3	2190		Dam Road off	Dam Road on
27	3	2320	OD	Dam Road on	Road 20 off
28	3	830		Road 20 off	Grade change point
29	3	1180		Grade change point	Road 20 on
30	3	2560	OD	Road 20 on	Mainline destination

Note: O = origin, D = destination.

In developing the above functions, an initial value of $\beta = \alpha/\bar{d}$ was chosen for all three curves. After each run of the model, new parameters were calculated for the external-internal (β_1), internal-internal (β_2), and internal-external (β_3) functions based on the average distances of these trips. Thus, for a freeway section that has N points of entry or exit,

$$\bar{d}_1 = \sum_{j\neq N} T_{1j}d_{1j}/O_1 \quad (6)$$

$$\bar{d}_2 = \sum_{i\neq 1}\sum_{j\neq N} T_{ij}d_{ij}\Big/\sum_{i\neq 1}\sum_{j\neq N} T_{ij} \quad (7)$$

$$\bar{d}_3 = \sum_{i\neq 1} T_{iN}d_{iN}/D_N \quad (8)$$

where point 1 represents the mainstream-on-node and point N the mainstream-off node.

Assuming that T_{1N} is given, the estimation procedure is as follows:

1. Calculate $\bar{d}$;
2. Remove through trips (e.g., subtract T_{1N} from O_1 and D_N);
3. Letting $\alpha = 3$ and $\Gamma(\alpha) = 2$, calculate initial values of $\beta_1 = \beta_2 = \beta_3 = \alpha/\bar{d}$, where β_1, β_2, and β_3 are the size parameters for the three inverse impedance curves;
4. Run the gravity model;
5. For the run obtained in step 4, calculate $\bar{d}_1$, $\bar{d}_2$, and $\bar{d}_3$ and use these new estimates to calculate β_1, β_2, and β_3; and
6. Repeat steps 4-5 until the $\bar{d}_i$s used in the travel functions agree with the $\bar{d}_i$s calculated (usually one or two iterations).

Step 2 was suggested by Willis of the University of California, Berkeley.

Data Base

The experiment was performed for a subsection of the San Francisco Bay Area freeway network. The study corridor included the northbound portion of the Eastshore Freeway (I-80), beginning upstream from the Powell exit ramp and terminating downstream from the CA-20 entrance ramp. Table 1 describes this freeway section and its points of entry and exit. Volume data for five 15-min time slices starting at 3:45 p.m., October 1969, were used as the O-D inputs for the gravity-based model. An O-D survey performed during the same time periods was used for comparing the accuracy of gravity-based model outputs with those estimated by the SYNODM procedure.

Error Measures

Three types of error measures were considered. These were average absolute trip errors, average total percentage of trip errors, and average individual percentage of trip errors. These are defined below.

$$\text{Avg trip error} = \sum_{ij} |T^c_{ij} - T^o_{ij}|/N \quad (9)$$

$$\text{Avg total percentage trip error} = \sum_{ij}(|T^c_{ij} - T^o_{ij}|/T) \times 100 \quad (10)$$

$$\text{Avg individual percentage trip error} = \sum_{ij} [(|T^c_{ij} - T^o_{ij}|/T^o_{ij})/N] \times 100 \quad (11)$$

where

T^c_{ij} = trips from i to j calculated,
T^o_{ij} = trips from i to j observed,
N = number of error values, and
T = total trips in cells used.

Cells that have less than five observed trips were ignored in the above calculations to avoid unreasonably high individual percentage errors associated with very few trips.

RESULTS

Observed and calculated trip tables that represent the 15-min slices from 3:45 to 5:00 p.m. were compared. SYNODM trip tables were also calculated. Figure 2 shows an example comparison for time slice 3. As expected, the SYNODM trip tables were more likely to overpredict short trips than was the gravity-based model. A summary of the average trip distances given in Table 2 bears this out. The average distance for internal trips for the SYNODM estimate is consistently shorter than that observed; however, the gravity-based approach is very close in its estimates of all three types of trip lengths.

Figure 3 shows an example comparison of the trip-length distributions for the actual and gravity-based calculated trips. For all five time slices these frequency distributions are very close.

A final comparison of observed, estimated, and SYNODM estimated trips is given in Table 3. In all three error measures, the gravity-based approach is consistently better. However, as total trip volumes increase (time slices 4 and 5) the SYNODM approach becomes competitive. As expected, the individual percentage of error is somewhat high due to relatively low numbers of trips in some cells.

INTERPRETATION OF RESULTS

Although the results of this experiment are by no means conclusive, distance impedance should be considered in estimating freeway trip tables, even for relatively short freeway subsections. If through

Table 2. Average trip distances along freeway section.

Time Slice	Internal-Internal, $\bar{d}_2$ Actual	Internal-Internal, $\bar{d}_2$ Gravity-Based Estimated	Internal-Internal, $\bar{d}_2$ SYNODM	External-Internal, $\bar{d}_1$ Actual	External-Internal, $\bar{d}_1$ Gravity-Based Estimated	External-Internal, $\bar{d}_1$ SYNODM	Internal-External, $\bar{d}_3$ Actual	Internal-External, $\bar{d}_3$ Gravity-Based Estimated	Internal-External, $\bar{d}_3$ SYNODM	$\bar{d}$
1	3.33	3.61	2.51	3.67	3.65	3.85	4.74	4.48	4.16	4.29
2	3.48	3.65	2.70	3.51	3.56	3.84	5.40	5.10	4.88	4.57
3	3.39	3.56	2.53	3.88	3.94	3.95	5.04	5.04	4.44	4.51
4	3.36	3.49	2.80	3.35	3.32	3.93	5.35	5.24	5.19	4.86
5	3.03	3.21	2.53	3.78	3.66	3.81	4.96	5.08	4.67	4.31

Figure 2. Freeway trip tables for time slice 3.

Observed Trips

190	285	250	59	175	310	10	124	52	45	46	126	32	94	35	201
	2		8		13		8		13	5		3	8	8	29
			3		19		8	4	6	7	2	7	19	10	65
					47		4	6	21	2		8	21	11	70
				15	11		6	6	29	2		7	21	11	72
										2		1	5	3	19
								2	4	2	2	3	9	4	34
									2	4		5	15	7	47
												12	33	16	109
												12	35	17	116
														8	52
															40

Gravity-Based Estimated Trips

190	286	233	63	163	338	8	117	51	82	43	77	39	102	43	200
	1	10	3	8	17		6	3	4	2	4	2	5	2	30
		8	4	12	26	1	10	5	8	4	7	4	9	4	51
			1	6	16	1	11	6	12	7	13	8	20	8	82
				1	3		7	5	11	8	15	9	24	11	88
								1	2	1	3	2	5	2	15
									2	2	5	4	11	5	32
										2	5	5	16	8	44
										1	3	11	36	20	99
												7	33	24	116
														3	57
															40

SYNODM Estimated Trips

190	273	218	54	133	281	7	103	46	73	37	69	41	112	57	356
	14	11	3	7	15		5	2	4	2	3	2	5	3	15
		21	5	13	27	1	10	4	7	3	6	4	10	5	31
			8	19	39	1	14	6	10	5	9	6	15	8	47
				19	39	1	14	6	10	5	9	5	15	7	46
							4	2	2	1	2	1	3	1	9
								4	6	3	5	3	8	4	25
									8	4	7	4	12	6	37
										10	18	11	29	15	91
												14	37	19	116
													14	7	42
															43

trips can be estimated or measured, the gravity-based approach looks reasonably accurate. If this were coupled with knowledge of one or two other interchange values, the results should approach observed values. For example, for some reason the number of observed vehicles getting on at Gilman and directly off at Buchanan is high in some time slices. If this were suspected in advance, trips from Gilman to Buchanan might be measured and used as an additional factor in calibrating the model (adjusting the travel distance factor for that particular interchange).

Most ramp interchanges that are closely spaced, however, do not exhibit this property. Thus, distance appears to be a factor and methods such as SYNODM, which do not take distance into account, will not do as well. The errors that result from this omission are higher than indicated by our error measures because cells that have four or fewer trips were dropped from the analysis. Consider, for example, the observed trip table for time slice three and the estimated tables shown in Figure 2. Because of the short time interval, several of the less frequented interchanges have no observed trips and were dropped before calculating the percentage error terms. Yet SYNODM in several cases estimates significant numbers of trips for these cells, thus the true error differences between the two procedures is even larger. In both cases, the predictions might improve if larger time intervals were used.

Note also that SYNODM overpredicts through trips. These represent very long trips, and this is another indication that distance is a factor. Even if through trips were assumed as known in the SYNODM procedure, other very long trips would probably be overpredicted.

Figure 3. Trip length distribution for time slice 3.

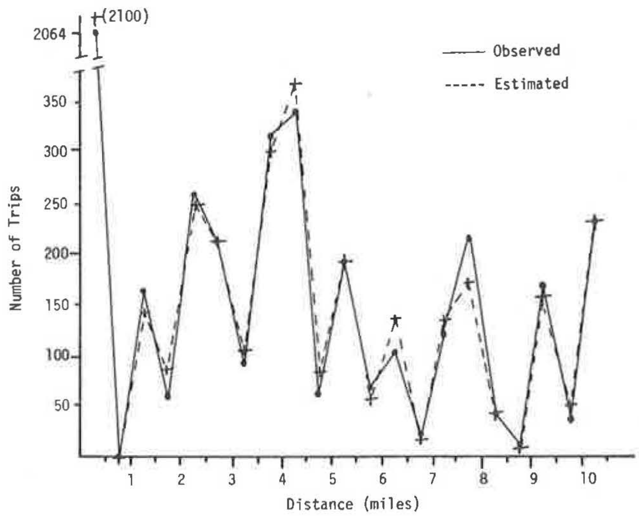

Table 3. Comparison of trip errors.

Time Slice	Avg No. of Trips in Error: Proposed Method	Avg No. of Trips in Error: SYNODM	Avg Total Trip Error (%): Proposed Method	Avg Total Trip Error (%): SYNODM	Avg Individual Trip Error (%): Proposed Method	Avg Individual Trip Error (%): SYNODM
1	5.9	9.1	12.6	15.4	21.1	21.7
2	8.1	10.0	17.0	22.2	33.1	34.9
3	6.6	12.8	12.1	23.9	18.4	31.5
4	11.4	11.5	20.7	21.5	39.2	40.2
5	8.3	8.5	17.1	17.4	27.5	27.6

SUMMARY AND CONCLUSIONS

Knowledge of the number of through trips on a freeway section appears sufficient to calibrate a reasonably accurate gravity-based trip-table estimator. Whether knowledge of the number of trips for any major O-D pair would work as well should be explored because overpasses may not always coincide with the freeway subsection under study. In an upcoming revision, SYNODM may also assume knowledge of through trips to improve its accuracy. However, without incorporation of an impedance factor, it may still have unreasonably high predictions for very short and very long trips.

In any event, further exploration of the accuracy of these techniques with other O-D data bases is needed to determine whether existing models are sufficiently accurate. The possibility of improving accuracy by obtaining data on one or more interchanges in lieu of a complete O-D survey should also be investigated.

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