

13. R.E. Whiteside, T.Y. Chu, J.C. Cosby, R.L. Whitaker, and R. Winfrey. Changes in Legal Vehicle Weights and Dimensions: Some Economic Effects on Highways. NCHRP, Rept. 141, 1973.
14. B.C. Butler, Jr. Economic Analysis of Roadway Occupancy for Freeway Pavement Performance and

- Rehabilitation. FHWA, Rept. FHWA-RD-76-14, 1974.
15. Catalog of U.P.A. Prices for Roads and Bridges. IDOH, Indianapolis, 1973-1979.

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Incremental Cost-Allocation Analysis of Bridge Structures

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Methodologies pertaining to the allocation of costs for bridge superstructure by the incremental design method are developed. Generalized design relations are defined as a function of vehicle classes and are applied to three typical bridge structures. Three alternative allocation methodologies, which depend on the bridge functions, are also defined and applied to determine the cost functions for an entire state building and maintenance program taken over a six-year period. The results from these three methods are then compared for accuracy and amount of work required to implement them into a cost-allocation project.

Cost-allocation studies have traditionally been used to provide a systematic and logical basis for relating highway tax structures to highway program costs. There is no single accepted highway cost-allocation methodology, and the results of these studies often vary widely, depending on the method used. This is because much controversy currently exists as to whether roadway-related construction costs are design or damage related. Regardless of these difficulties, there is no doubt that the proper allocation of costs is an extremely important function that can significantly influence the amount of monies available for a highway program.

The proper execution of a cost-allocation project involves the occasioning of costs to numerous elements contained within any building or maintenance program. Considered in this paper is the methodology for the incremental design and subsequent allocation of costs to the superstructure elements of highway bridges. Although the total cost of such elements is often low as compared with that of other elements of the typical highway program (such as highway reconstruction and drainage), these elements may compose a high percentage of the allocatable costs within the program.

Finally, it is felt that the allocation of costs to bridge structures should potentially be one of the more accurate of any of the highway-related allocation methodologies in that the design process for bridges is well defined and well understood. If inaccuracies do appear in the allocation process for bridge structures, they are attributable to factors aside from the design function. Such factors can include

1. Lack of time to perform a detailed incremental design over the full range of vehicles,
2. Allocation of costs based on a single bridge that is not representative, and
3. Allocation of costs by methods not related to design.

Defined below are those methodologies that have been used to occasion the costs for bridge superstructure elements for an arbitrary set of highway loadings. These methods are applied to the actual

highway program in which the results of each are compared.

VEHICULAR LOADINGS

Bridge structures are designed to a standard set of vehicular loadings defined by the American Association of State Highway and Transportation Officials (AASHTO) (1). The loads specified are designated with an H prefix followed by a number that indicates the total weight of the truck in tons for two-axle trucks or with an HS prefix followed by a number that indicates the weight of the tractor in tons for tractor-trailer combinations. These H and HS truck loadings are placed on the spans to simulate the actual vehicles most encountered on the highway system along with the H and HS lane loadings to simulate a series of vehicles. Both the truck and lane loadings are placed on the bridge to produce maximum effects throughout the structure.

The three parameters that influence the level of stress on longitudinal members that compose the bridge superstructure are the gross vehicle weight (GVW), the axle loads, and the spacing between axles. AASHTO (1) specifies a fixed spacing between axles of 14 ft for the H truck and variable limits from 14 to 30 ft for the HS truck. These trucks are to be positioned on the span so as to give maximum stresses and deflections along with the associated lane loadings.

Vehicular Classification

The vehicles that use the Maryland (2) highway system are categorized into seven basic classifications, which can then be broken down by GVW group. A summary of such a classification is given in Table 1 where 59 GVW groups are distributed among the seven basic classes. As can be noted from the table, each GVW group is identified by its design axle loading and spacing.

Hand HS-Truck Correlation

It was first necessary to determine the relationship between the AASHTO Hand HS-truck loadings. This was done by placing each loading type on a series of simple span bridges that ranged from 42 to 400 ft in length, equating the maximum moments at the centerline, and performing the correlations by means of a straight-line least-squares fit.

AASHTO Truck and GVW Correlation

The correlation of the AASHTO truck types with the state GVW system requires that the effect of each of

Table 1. Correlation between state and AASHTO vehicle classifications.

Range No.	AASHTO Classification		State Vehicle Type (GVW in kips)									
	H	HS	Automobiles	Pickups and Vans	Buses	Two-Axle, Four-Tire	Two-Axle, Six-Tire	Three-Axle	Dump Truck	2S1	2S2	3S2
1	1.9	1.3				0-4						
2	3.9	2.6	X	X		4-8	4-8					
3	6.0	4.0				8-12	8-12					
4	7.9	5.3				12-16	12-16					
5	9.8	6.6				16-20	16-20	16-20			28-32	
6	11.9	8.0				20-24	20-24	20-24			32-36	
7	13.9	9.4				24-28	24-28	24-28		28-32	36-40	
8	15.8	10.7			X	28-32	28-32	28-32		40-44	44-48	
9	17.7	11.9				32-36	32-36	32-36		48-52	52-56	36-40
10	17.8	12.0								56-60	60-64	40-44
11	18.1	12.2								64-68	64-68	
12	18.8	12.7								48-44	68-72	44-48
13	19.6	13.2									52-56	
14	19.9	13.4										
15	20.0	13.5						40-44		36-40		
16	20.5	13.8								48-52		
17	21.5	14.5										56-60
18	21.7	14.6								52-56		
19	22.1	14.9						44-48				
20	23.0	15.5									60-64	
21	23.6	15.9						48-52				
22	24.1	16.3						52-56				
23	24.8	16.7									64-68	
24	26.7	18.0									68-72	
25	28.2	19.0								72-76	72-76	
26	29.7	20.0									76-80	
27	30.8	20.8								60-64 ^a		

^a Designed in range 26.

these loadings be equated for their effect on bridge structures. Specifically, each GVW weight group within each class is placed on a series of simple spans that range from 40 to 400 ft in length increments of 5 ft. From this, the maximum H or HS loading encountered in the entire range is taken as the equivalent loading.

Range Number

A convenience adopted here to identify the smallest increment of the index for the H and HS vehicles is the range number. The smallest increment for the index is used to ensure that the minimum overlap will exist between the AASHTO and GVW groupings. The resulting correlations, which relate the AASHTO truck types to the state vehicular classification system, are shown in Table 1 for the 27 vehicular ranges selected.

Finally, it should be noted that no vehicular loadings that exceed those used to design the actual structure should be used in the incremental design process even though significant numbers of higher loads are traveling on the system either by permit or otherwise. All designs to be used in determining the allocation of responsibilities should follow the actual design criteria used by the state as closely as possible. Thus, if a state chooses not to design to permit vehicles or illegal overloads, they should not be included in the allocation process either. If this were not done, the costs arising from the incremental design would exceed the actual costs of the structures.

The above correlations allow for the proper interfacing between the vehicular classification system used by highway design engineers and that required for the design of bridge structures. Other methods do exist that perform the same task (the GVW

basis, for example), but they are believed to be less accurate in the correlations they yield than the method proposed here. Once the correlation process is complete, the AASHTO truck loadings can be used as the live loads as required for design.

INCREMENTAL DESIGN OF BRIDGE STRUCTURES

The incremental design of highway structures is based on the difference in design costs that results when various classes of vehicles are applied as loadings. The total cost (C_I) of any structural element I is given by the following expression:

$$C_I = \sum_{j=1}^N Q_{IJ} U_j \cdot I = I, M \tag{1}$$

where M is the number of elements that make up a structure (e.g., deck, stringer, pier), and N is the materials used to construct the elements. From this, Q_{IJ} is the quantity of each Jth material for the Ith structural element and U_J is the unit cost for that material. Here, the quantity of material, say the volume of steel in a bridge girder, will vary with the classes of vehicles that act as loadings on the structure. When this quantity is multiplied by the unit cost, the total cost of the element will result. Then as the vehicular classes are applied incrementally, the result is those incremental costs that are attributable to any respective class that caused the cost difference. Thus, Q_{IJ} represents a quantity function dependent on the class of loading that is applied to a structural element.

This paper will deal only with bridge structures, which are composed of steel stringers that act either compositely or noncompositely with a reinforced-concrete deck. For this type of structure, the index I in Equation 1 is defined as follows: 1

= superstructure reinforced-concrete deck, and 2 = steel stringer. Accompanying these elements are the material definitions for the index J in Equation 1: 1 = reinforced-concrete in place, 2 = structural steel in place. The definition of the incremental costs is given as follows for these elements and materials.

Bridge Decks

The bridge-deck elements consist of the reinforced-concrete deck, which acts either compositely or noncompositely with the longitudinal steel beam elements and that nonstructural part of the deck that acts as a wearing surface. The cost of the deck element can be written by using Equation 1:

$$C_1 = Q_{11} U_1 \quad (2)$$

Here, Q_{11} represents the quantity of reinforced concrete ($Q_{12} \equiv 0$) and U_1 represents the unit cost of reinforced concrete in place, including the cost of steel reinforcement. The cost and quantity of the actual slab are given by the following:

$$(C_1)_D = (Q_{11})_D U_1 = D_D T_D U_1 \quad (3)$$

Here, $(Q_{11})_D = D_D T_D$, where D_D and T_D are the area (in plan) and the thickness of the slab for the actual bridge, respectively.

The quantity of reinforced concrete for the theoretical structure under the design loading is given by

$$(Q_{11})_D = k_1 d_D t_D \quad (4)$$

where d_D and t_D are the area (in plan) and the thickness of the slab for the theoretical slab, respectively. If this is equated to the quantity of the actual bridge in Equation 3, the constant k_1 can be defined as follows:

$$k_1 = D_D T_D / d_D t_D \quad (5)$$

This represents a form factor to account for differences between the actual and the theoretical slab designs.

The cost of any slab element $(C_1)_k$ is derived from the loading of the kth vehicle class and is given by

$$(C_1)_k = (Q_{11})_k U_1 = (D_D T_D / d_D t_D) U_1 d_k t_k = \alpha_1 d_k t_k \quad (6)$$

where d_k and t_k represent the area (in plan) and the thickness of slab under the kth design vehicular loading, respectively. The term α_1 represents a constant for each bridge and is represented by the following relation:

$$\alpha_1 = (D_D T_D / d_D t_D) U_1 \quad (7)$$

In order to determine the difference in the cost of successive slab designs for the kth and (k+1)th vehicle classifications, the following relation may be written:

$$(\Delta_1)_{k+1} = \alpha_1 (d_{k+1} t_{k+1} - d_k t_k) \quad (8)$$

It must be emphasized that the parameters d_k and t_k are functions of the kth vehicle loading since the plan area of deck is dependent on the length and width of the bridge, which in turn is dependent on the vehicle class (see Table 3). Further, the thickness is directly dependent on the axle shear loadings and the bending moment generated by the kth loading. However, for most slab designs,

the resulting plan area and thickness of the theoretical bridge proportioned for the design vehicle coincide with those of the actual bridge. Thus, $d_D = d_D$ and $t_D = t_D$, which yields $\alpha_1 = U_1$ and allows Equation 8 to take the following simplified form:

$$(\Delta_1)_{k+1} = U_1 (d_{k+1} t_{k+1} - d_k t_k) \quad (9)$$

The basic design parameter for the deck element is the slab thickness, since the volume of concrete depends directly on this dimension. The procedures used in the determination of thickness follow the AASHTO (1) specifications for the design of composite or noncomposite steel or concrete bridge structures. Specifically, two criteria are followed, each of which yields a required slab thickness. These, along with other conditions that define the load-geometry relationship, are given as follows:

Design Functions

Summarized in Table 2 are a total of 12 functions (2) used to define the basic geometry of design limits. These reflect those policies that could be practiced if bridge structures were to be designed for the full range of vehicular classes defined in Table 1.

Bending-Moment Criterion

One criterion used to determine the thickness of slab is that which satisfies the bending moment if it is assumed that the slab is continuously supported over three or more stringers.

Shear Criterion

The second criterion that must be satisfied is that which provides for a slab thickness adequate to sustain the punching shear due to a wheel load.

Deck Design

The width, length, and thickness of the deck slab are determined from the maximum thicknesses obtained by using the bending-moment and shear criteria for the geometry and loading associated with the various classes of vehicles considered.

In order to illustrate the method and the results obtained from the incremental analysis, an example was selected that is considered to be representative of deck-replacement projects. The results from the analysis are shown graphically in Figure 1, where the slab thickness is given as a function of the vehicle range number. Here it can be noted that abrupt changes occur between ranges 2 and 3, where the 2-in wearing surface is applied.

Longitudinal Elements

Longitudinal elements are bending members that are assumed to consist of rolled standard W sections or plate girders that act compositely with the reinforced-concrete deck. The total cost of this element can be written as follows from Equation 1:

$$C_2 = Q_{22} U_2 \quad (10)$$

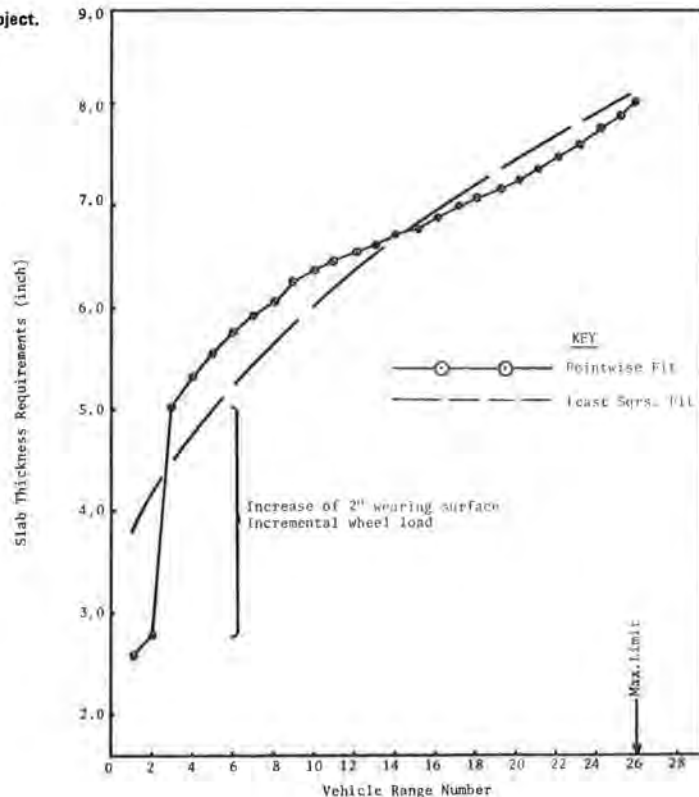
where $Q_{22} = 0$ (since no concrete exists in a steel stringer) and U_2 represents the unit cost of steel in place. The cost and the quantity of the actual structure are given by the following formula:

$$(C_2)_D = (Q_{22})_D U_2 = \left(\sum_{i=1}^P A_i S_i \right) DF(NS)_D U_2 \quad (11)$$

Table 2. Specification of bridge design functions.

Item	Description or Definition	Unit
1. Parapet function	Description: Relationship between GVW and weight per running foot (W/p) of parapet, railing, and median. Assumption: Constant for all vehicle weights. Note: It is expected that any change in unit weight of parapet will be negligible.	Pounds per square foot
2. Vertical-clearance function	Description: Relationship between GVW and vertical clearance (H _v) for vehicle passage under a grade separation. Assumption: For all bridges, use a clearance of 8 ft for all vehicles equal to or less than H3.6 and 16 ft 9 in for all vehicles greater than H3.6.	Feet
3. Wearing surface	Description: Relationship between GVW and the unit weight of wearing surface (W _{ws}). Assumption: For all roads, H3 vehicle: W _{ws} = 0. For all roads, vehicles over H3 to HS20: W _s = 25 lb/ft ² (equivalent to a 2-in depth concrete).	Pounds per square foot
4. Lane-width function	Description: Relationship between GVW and the lane width (H _L). Assumption: Lanes will be identical with those given for the highway. For all highways and GVW < 10 000 lb, lane width = 11 ft; for GVW ≥ 10 000 lb, lane width = 12 ft.	Feet
5. Shoulder-width function	Description: Relationship between GVW and the shoulder width (W _s). Assumption: Shoulder width will be identical to that given for highway in cases where total deck width of hypothetical bridge is equal to or less than that of actual bridge. Here, for bridges on secondary roads, W _s = 6 ft for GVW < 10 000 lb and W _s = 8 ft for GVW > 10 000 lb. For bridges on primary and Interstate roads, W = 8 ft for GVW < 10 000 lb and 10 ft for GVW > 10 000 lb. For cases where hypothetical bridge deck width is greater than actual bridge deck width by using the lane-width function, lane-width function becomes W _s = 0.5 [actual deck width - (no. lanes) x (lane width)].	Feet
6. Slab-thickness function	Description: Relationship between GVW and slab thickness (t _s). Assumption: Given by current AASHTO specifications for design of reinforced-concrete deck slabs.	Inches
7. Stringer-spacing function	Description: Relationship between GVW and stringer spacing (S). Assumption: Stringer spacing nearest that given for existing bridge is used in design.	Inches
8. Length-to-depth ratio	Description: Ratio of bridge span length to depth of beam used in design process. Assumption: Maximum values of length-to-span ratio was held constant at 35 for all structures. Note: Value recommended by AASHTO is 25.	
9. Detail factor	Description: That factor (F _D) which when multiplied by computer dead load (consisting of the deck, stringer, parapet, railing, and wearing surface) will yield actual dead load of superstructure. This factor would typically account for connections, rebars, studs, etc. Assumption: Use 0.05 (5 percent) for rolled beams.	
10. Typical embankment slope	Description: Slope of embankment for a grade separation. Assumption: Use 2:1 or H ₁ = 2 horizontal, H ₂ = 1 vertical.	H ₁ :H ₂
11. Cover plate	Description: Policy of whether cover plates will be used on rolled sections or plate girder bridges. Assumption: No cover plates were used per se in design of longitudinal beam elements. The steel material volumes for hypothetical structure were adjusted to reflect volumes of actual structure, which may include cover plates.	
12. Overhang function	Description: Relationship between GVW and overhang distance (H ₀). Assumption: For vehicles from H 3 to HS 20 design overhang. For HS 20 vehicles, maximum of 3 ft 6 in.	Feet

Figure 1. Slab thickness requirements: deck-replacement sample project.



In Equation 11, the actual quantities of all steel are represented by the following:

$$(Q_{22})_D = \left(\sum_{i=1}^P A_i S_i \right) DF(NS)_D$$

where

- A_i and S_i = area and length of i th steel section, respectively, which compose the steel element;
 P = number of sections;
 DF = detail factor of bridge (to account for attachments, connections, etc.); and
 $(NS)_D$ = number of stringers of actual structure (which may differ from that for any theoretical structure).

The quantity of the stringer material $[(Q_{22})_D]$ for the theoretical structure under the actual bridge design loading is given by the following:

$$(Q_{22})_D = k_2 S_D a_D DF(NS)_D \quad (12)$$

where S_D and a_D are the length and the average cross-sectional area, respectively, of the stringer for the actual bridge design loading. If this is equated to the actual steel quantity given by Equation 11, the constant k_2 can be found:

$$k_2 = \left(\sum_{i=1}^P A_i S_i \right) S_D a_D \quad (13)$$

which represents a form factor to account for differences between the actual and idealized stringer designs.

The cost of any stringer element $[(C_2)_k]$ derived from the loading of the k th vehicle class is given by

$$\begin{aligned} (C_2)_k &= (Q_{22})_k U_2 \\ &= \sum_{i=1}^P A_i S_i [(DF) U_2] S_k a_k (NS)_k / S_D a_D \\ &= \alpha_2 S_k a_k (NS)_k \end{aligned} \quad (14)$$

Here, S_k , a_k , and $(NS)_k$ represent the length, cross-sectional area, and the number of stringers under the k th design vehicle loading, respectively. The term α_2 represents a constant for each bridge and is given by

$$\alpha_2 = \left(\sum_{i=1}^P A_i S_i / S_D a_D \right) (DF) (U_2) \quad (15)$$

In order to determine the difference in the cost of successive stringer designs between the k th and $(K+1)$ th vehicle classifications, the following cost difference may be written:

$$(\Delta_2)_{k+1} = \alpha_2 [S_{k+1} a_{k+1} (NS)_{k+1} - S_k a_k (NS)_k] \quad (16)$$

It must be emphasized here that the parameters S_k and $(NS)_k$ are functions of the k th vehicle. Here, the clearance and embankment slope both affect the length of the bridge. The roadway width affects the number of stringers as defined by item 7 in Table 2. These, along with the weight of the k th vehicle, greatly influence the resulting design area (a_k) of the steel section. The beam components are selected in accordance with the AASHTO specifications (1), where the moments of inertia determine the beam section. Specific design procedures are given as follows:

Design Function

Summarized in Table 2 are a total of the 12 functions (2) used to define the basic geometry and design limits for beam elements.

Structural Analysis

The AASHTO live loadings for the H and HS truck and lane loadings are used to obtain the maximum shear and moment envelopes along the bridge span. The distribution factors, moments of inertia, effective widths of deck slabs, dead loads, and modular ratios for $n = 27, 9,$ and infinity as prescribed by the AASHTO specifications (1) are used. The basic analysis assumes that all beams are simply supported. Thus, where continuity exists between spans, simple beams are assumed. It is felt that this assumption will not result in a great degree of error in the volume of steel required for continuous structures since the k_1 -factor at least partly compensates for the lack of continuity, fatigue details, and connections.

Member Selection

The members are selected on the basis of a required section modulus calculated by dividing the material allowable, taken as 55 percent of the yield point, into the maximum moment for $[DL + (LL + I)]$ found within the span. The ratio of length to depth is used to determine the depth of the member (see Table 2). The results given for the designs cited herein were obtained from a computer program that has been used to design bridge structures (3).

A series of three sample (2) structures is given to illustrate the methodology and to indicate the wide variability of results obtained. The sample bridge structures defined in Table 3 were selected to yield the maximum, minimum, and representative levels of allocation of costs above the base structure. These are identified in Table 3 as structures 1, 2, and 3, respectively, along with a summary of the results of the cost-allocation process.

An example of the variation of the stringer area requirements is shown in Figure 2 for structure 1 as a function of the vehicle range number. It can be noted that the area increases stepwise, which is due to the selection of economic rolled shapes that suffice over a number of vehicle loading ranges. A continuous parabolic curve determined by the least-squares criterion is fitted to the area function obtained and is superimposed over the actual stepwise area requirements.

The total percentage of increase for AASHTO truck types for stringer and slab elements is shown in Figure 3 for all three sample structures. The results of structure 1, which represents the maximum allocation above the base bridge, lie below the results of all other examples. Further, the results of structure 2, which represents the minimum allocation, lie above all other curves. Finally, the results for structure 3, which represents the average bridge, lie unexpectedly close to the results of structure 2.

Another unexpected result of the incremental design of the sample structures is that the cost functions for the stringer, deck, and the sum of the stringer and deck are nearly linear for vehicle loadings above that point at which the bridge geometry does not change. The linear cost functions for the stringer, slab, and combined stringer plus slab are shown in Figure 4. Again, structure 1 lies below all other results, and structure 2 lies above all other results and is close to the results of structure 3.

Table 3. Definition of superstructure sample projects.

Item	Sample Structure		
	1 (maximum allocation)	2 (minimum allocation)	3 (representative allocation)
Design vehicle	HS 20	HS 20	IIS 20
Structure type	Grade separation	Railroad type	River crossing
No. of lanes per bridge	2	2	2
Span length (span 2) (ft)	86.0	156.00	90.0
No. of stringers	6	6	6
Steel	A588 GR 50	A588	A588
Construction type	Composite	Composite	Composite
Slab thickness (in)	8.5	8.5	8.5
Deck width (ft)	40	43.2	30
Stringer spacing (ft)	7.33	7.63	5.75
Additional dead load (kips/ft)	0.5	0.5	0.5
Overhang width (ft)	3.27	3.00	2.23
Raised deck width (ft)	0.0	0.0	0.0
Haunch thickness (in)	2	2.0	2.0
Key	0.0	0.0	0.0
Fillet angle (degrees)	90	90	90
Sidewalk dead load	0.0	0.0	0.0
Sidewalk live load (lb/ft ²)	0.0	0.0	0.0
Detail factor	1.05	1.05	1.05
Unit cost (1980)			
Steel (\$/ft ³)	281	318	281
Concrete (\$/yd ³)	338	383	338
Stringer			
Original cost (\$)	154 000	766 000	2 217 250
Theoretical cost (\$)	113 152	845 756	1 573 176
Difference (%)	26.5	10.4	29
To first increment (%)	13.5	83	84.2
Allocatable to trucks (%)	86.5	17	15.8
Deck			
Original cost (\$)	117 610	350 710	
Theoretical cost (\$)	61 052	237 287	
Difference (%)	48.1	2.3	
To first increment (%)	19.9	33.8	
Allocatable to trucks (%)	80.1	66.2	
Total			
Original cost (\$)	271 610	1 116 710	2 230 750
Theoretical cost (\$)	174 204	1 083 043	2 019 949
Difference (%)	35.9	3	9.4
To first increment (%)	15.7	72.2	72.7
Allocatable to trucks (%)	84.3	27.8	27.3

ALLOCATION METHODS

The techniques that have been used to allocate the construction costs of superstructure elements to a generalized set of vehicular loadings fall under one of the following four basic methodologies:

1. Full-design method,
2. Representative-bridge method,
3. Semistatistical method, or
4. Heuristic methods.

All these methods can use, to varying degrees, the incremental structural design procedures outlined above. However, the results obtained from using any one method can differ considerably from those from another. The methods and results obtained for these methods are given in the following paragraphs.

Full-Design Method

This method uses all bridges designed within a representative time period to reach the cost-distribution function. Because so many bridges are generally involved, the design procedure often must be simplified. The methodology outlined above in the section on the incremental design of bridge structures was applied to all projects that involved the construction of bridge superstructures in Maryland (2) during a six-year base period (1978-1984). Here, all new construction and rehabilitation projects

were considered in the incremental design process. From the summary tabulated below, it can be noted that 105 spans were constructed during the period, which entailed 2730 discrete designs (since 26 load increments are required).

Item	Amount
No. of spans on Interstate system	20 (19 percent)
No. of spans on primary system	36 (34 percent)
No. of spans on secondary system	49 (47 percent)
Total no. of spans	105
No. of spans over rivers	73 (70 percent)
No. of spans over roads	32 (30 percent)
Total no. of spans	105
Total length of all spans (ft)	10 063
Avg span length (ft)	95.8
Total no. of contracts	10
Total cost (actual) (\$)	63 815 749
Total cost (computed) (\$)	22 038 633
Cost to base vehicle (%)	
Avg	33.0
Maximum	34.2
Minimum	31.3
Type of construction	All composite

The total cost of the superstructure elements during the period was \$63 815 749, which involved the construction of 10 063 linear feet of bridge roadway. The curve representing the distribution of all costs relative to the design and rehabilitation of superstructure over the six-year period is given in Figure 5.

It should be pointed out that the cost function given in Figure 5 represents the percentage of the superstructure cost that must be borne by any given AASHTO truck. The responsibility determined by the cost-allocation process involves forming differences between subsequent vehicle ranges and distributing them by means of some allocator (say, vehicle miles of travel) to those vehicles that fall into that AASHTO weight grouping.

Representative-Bridge Method

As the name implies, this method requires that a representative bridge structure be selected and subjected to a detailed incremental design in order to formulate the cost function. This is then used as the cost function for all bridges within the representative period. As was indicated previously, a considerable spread can result for the allocation function for different projects. The degree of this spread can be noted in Figure 3, where as much as a 50 percent difference can occur. Further, the selection and the incremental design of the representative sample structure thus resulted in an allocation function that was slightly below that which represented the minimum allocation for vehicles above the base vehicle.

The cost-allocation project conducted by the Federal Highway Administration (FHWA) (4,5) used the representative-bridge method where bridges were selected to represent construction types for both grade separations and river crossings. The allocation functions derived from the detailed design process resulted in a spread of about 10 percent between bridges. It must be noted that great care was taken in this study in the selection of the representative bridges. Further, the cost factors were altered to give a true representation of the type of project the bridge was to reflect.

The representative-bridge method is tempting in that the incremental design process is required only on one structure and, indeed, the method is the most popular one currently in use. However, unless great care is taken in the selection of the representative

Figure 2. Stringer area requirements for sample structure 1.

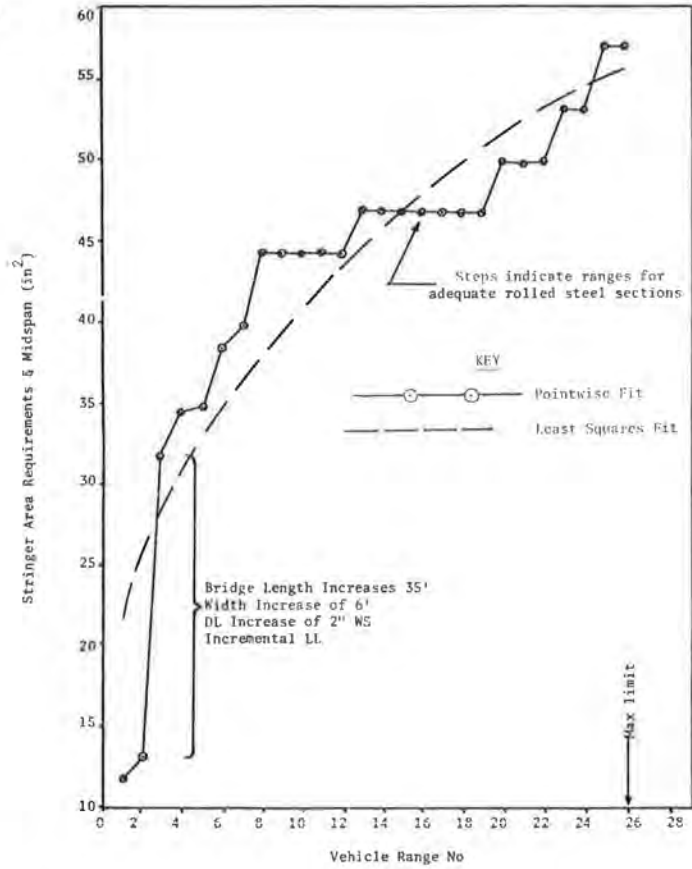


Figure 3. Added stringer and deck cost for all AASHTO loadings.

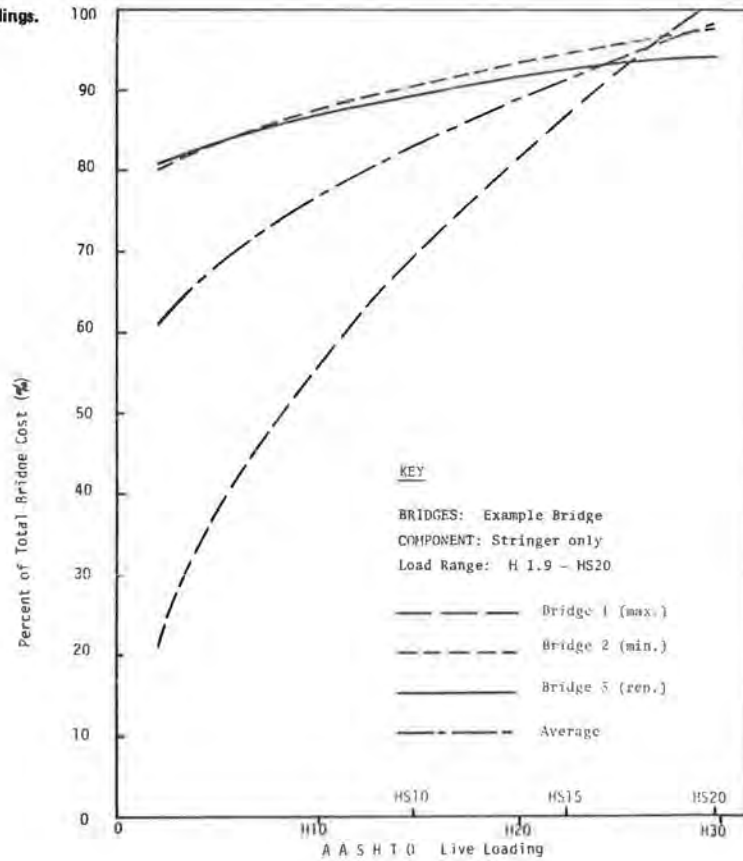


Figure 4. Added stringer cost over linear range AASHTO loadings.

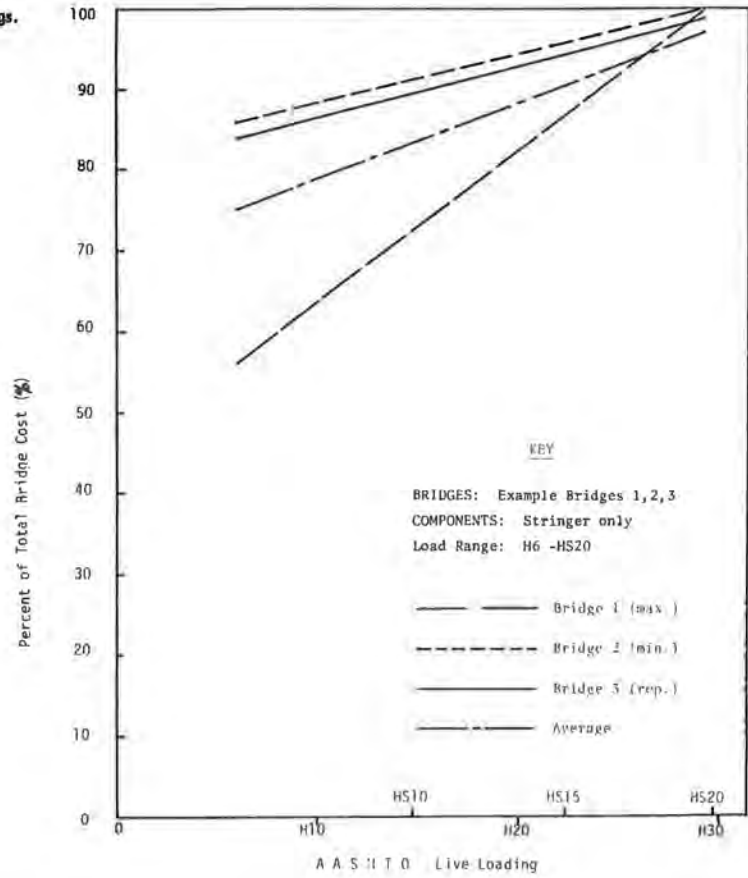


Figure 5. Comparison of cost distribution methodologies for narrow bridge.

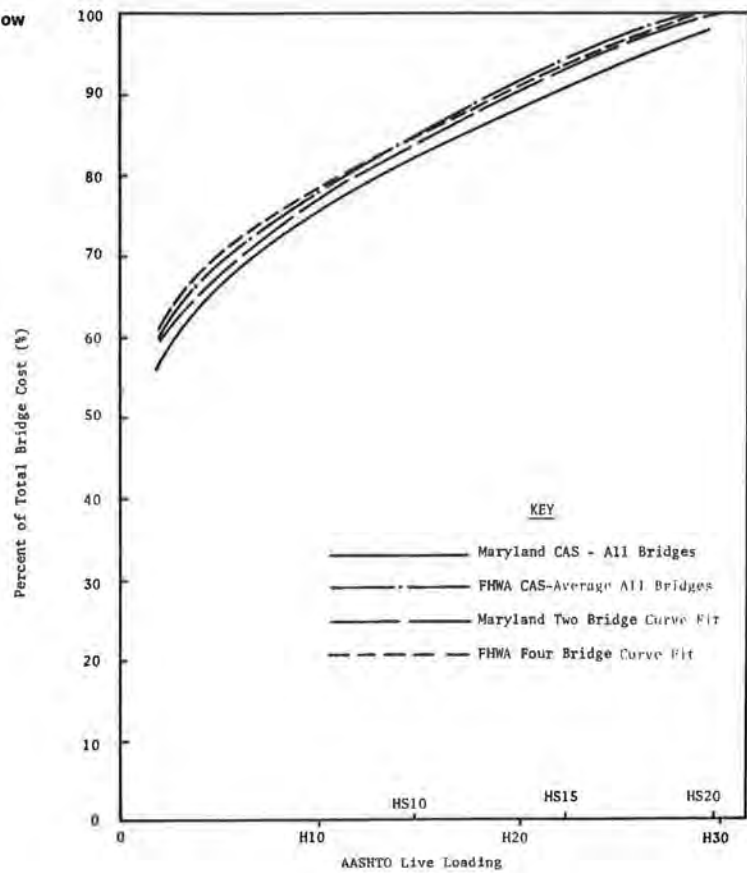


Table 4. Summary of methods.

Method	No. of Designs Required	Accuracy Expected	Design Mode
Full design	Many (3730)	Excellent	Some approximations used in design process
Representative bridge	Few (26)	Variable (10-50 percent)	Generally a detailed design process
Semistatistical	Few (3)	Excellent (4 percent)	Can be as detailed a design process as required
Heuristic	None	Unknown	Generally no design used

structure, the allocation function that results from the incremental design of the structure will differ significantly from that obtained from an analysis of all superstructure elements. If the structure is selected with care, results are good, as indicated by the relatively low spread obtained in the FHWA study (4,5).

Semistatistical Method

This method uses a combination of design and statistics to arrive at the allocation function. It attempts to minimize the possible spread by performing incremental designs on more than one representative structure. The steps are given as follows:

1. Select two or more structures that are considered representative of those bridges that the allocation function is to represent;

2. Design the representative structures for the minimum vehicle in order to determine the base facility by using all the geometrical functions as necessary;

3. Average the results obtained for the base facility as a percentage of the total costs expended for original construction; and

4. Fit a parabola through the percentage obtained in step 3 and 100 percent for the design vehicle by using the method of least squares. The parabola should be of the form $a + b(x)^{1/2}$, where a and b are constants obtained from the least-squares analysis and x represents the live loading (such as the range number, the AASHTO vehicular index, etc.). This method was used both for the sample structures representing Maryland bridges and for those used in the FHWA cost-allocation study. The results of this relatively simple method yield the curves given in Figure 5. Assuming that the solid curve representing all bridges in Maryland is the most correct, a curve fit that uses only sample structures 1 and 2 yields results within about 2 percent of those obtained for all bridges; the resulting curves for the FHWA bridge averages are all within about 4 percent above that given by Maryland study results. The allocation functions for any one of the Maryland or FHWA bridges all lie considerably outside these results.

Thus, the use of a parabolic curve positioned by the least-squares criterion through points obtained for a few different structures appears to be much more accurate and much less effort than performing a detailed incremental design over many steps for one representative bridge structure.

Heuristic Methods

These methods generally involve basing the allocation function on various relationships believed to be representative. Relationships such as the proportionalities between the cost and the maximum

moment in simple spans or the combination of dead load and live loading to the cost functions are typical. In defense of such practices, it can only be said that at least they are based on a consistent criterion arrived at by engineers generally knowledgeable in structural design methods rather than those conjured by legislators steeped only in the knowledge of law.

SUMMARY AND CONCLUSIONS

The procedures used for the allocation of the costs for the construction of bridge superstructures are highly variable and subject to wide variations in the results they yield. Identified here are four basic methods that traditionally have been used to determine the allocation function. These are summarized in Table 4 along with the benefits and drawbacks of each. As can be noted, the semistatistical method seems to be the most attractive; good accuracies are attained with relatively few designs.

It must be pointed out, however, that the cost-allocation process is not so much a science as an art. For this reason, it is extremely difficult to determine accuracy, since no one true answer exists. However, it is possible to examine the methods, as was done in this study, where the variances of the results are compared to some norm.

Finally, it must be concluded that greater standardization should be sought in the definition and specification of the design parameters that relate to the incremental design of bridge superstructure. If the state design agencies are to be the basic source of expertise in this area, they must be given better guidelines to follow from AASHTO. This will allow the results forthcoming from any state-generated cost-allocation analysis to better withstand the political pressures that seem to be inevitable.

REFERENCES

1. Standard Specifications for Highway Bridges, 12th ed. American Association of State Highway and Transportation Officials, Washington, DC, 1977.
2. D.R. Schelling, J.G. Saklas, and J. Banks. Maryland Cost Allocation Study. Univ. of Maryland, College Park, 1982.
3. Documentation for the Maryland Continuous Beam Design Rating and Routing System. Maryland Department of Transportation; Division of Bridge Development, State Highway Administration, Baltimore, 1980.
4. Final Report on the Federal Highway Cost Allocation Study, Vols. 1 and 2. FHWA, May 1982.
5. B.A. Sinclair and Associates, Inc. Incremental Analysis of Structural Construction Costs. FHWA, Rept. FHWA/PL/81/008, 1981.

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