Computer Modeling of the Cross Canyon Culvert

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The CANDE finite element computer program was used to determine the effectiveness of three different soil models in predicting the behavior of an underdesigned reinforced concrete culvert installed under a deep fill. The analysis consisted of comparing the predicted behavior with the actual behavior of a dummy culvert installed at Cross Canyon, California, by using several different types of soil models. The actual behavior was obtained from information published by the California Department of Transportation. Comparisons included prefailure and postfailure behavior of the pipe culvert. An overburden-dependent soils model most accurately predicted the behavior of the culvert.

The California Department of Transportation (Caltrans) has conducted many research projects involving monitoring of culverts installed in deep embankments. In one of these projects a dummy reinforced concrete culvert was placed in the same embankment at Cross Canyon, California (1, Section I, Volumes 1 and 2; Section III; Section IV, Volume 1; and Section V, Volumes 7 and 9), with a functioning, 243.8-cm (96-in) diameter, prestressed concrete culvert. The dummy culvert had a 213.3-cm (84-in) inside diameter and was grossly underdesigned for the 55-m (180-ft) overfill. Backfill parameters and pipe strength were varied in ten different zones throughout the dummy pipe test section. Dummy sections were installed 3 m (10 ft) above and to the side of the functioning culvert. The reinforced concrete pipe was instrumented with displacement measuring devices, strain gauges, and soil stressmeters. This paper reports comparisons of finite element analyses performed by using the CANDE computer program (2) and the published behavior of one instrumented section (zone 6).

FIELD INSTALLATION

The dummy reinforced concrete had a wall thickness of 20.3 cm (8 in). The area of reinforcing steel was 275 cm^2/m (1.30 in^2/ft) for the inner cage and 184 cm^2/m (0.87 in^2/ft) for the outer cage. The culvert had a D-load rating of 2500 D. The load rating is the load in pounds per unit length of pipe divided by the internal diameter in feet that develops a 0.254-mm (0.01-in) wide crack 30.5 cm (12 in) long when force is applied by using a standard three edge bearing test. Development of the 0.254-mm crack is the accepted strength criterion. Based on the traditional Spangler design procedure (3), this pipe could be safely installed in a trench situation under overfills of 6-9 m (19-29 ft), depending on bedding conditions.

At zone 6, the reinforced concrete pipe was placed in a shallow trench made in previously placed fill material. Well-compacted structure backfill was installed adjacent to the pipe to provide increased lateral support. A layer of fine aggregate was placed beneath the pipe. Figure 1 shows bedding and backfill conditions for zone 6.

The structure backfill consisted of a well-graded gravelly sand with approximately 22 percent of material by weight finer than 0.074 mm (0.003 in). The largest particles were less than approximately 50.8 mm (2 in) in diameter. The wet unit weight of the structure backfill was 2113 kg/m^3 (131 pcf).

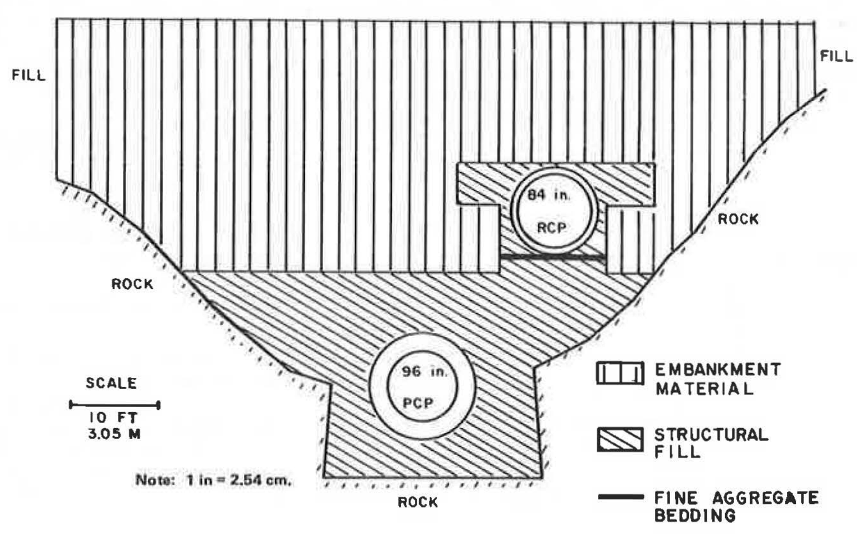
Only one gradation curve was available for embankment material; this curve shows that the material was a well-graded, gravelly, silty sand. The largest particle size was 64 mm (2.5 in), and 22 percent passed the 0.074-mm sieve. Liquid limit was 23, and unit weight was 2169 kg/m^3 (134.5 pcf).

Results of laboratory triaxial tests on embankment and structure backfill materials were also published. Parameters have been derived for implementation of the Kondner-Duncan soil model in finite element computer codes. This model is the result of work by Kondner (4), Duncan and Chang (5), and Kulhawy, Duncan, and Seed (6). A cursory review of triaxial test data and a statistical analysis of the Kondner-Duncan model parameters indicated that, due to the number of samples and the standard deviation of the results, there was no significant statistical difference between embankment and structure backfill materials.

INSTRUMENTATION

Dummy test sections were instrumented to determine soil pressures acting on the external surface of the

Figure 1. Bedding and backfill conditions.



pipe, strains in the pipe wall, and displacements of the internal wall surface. Vertical settlements and soil stresses in the embankment were not obtained at zone 6.

Strain readings were taken for concrete and reinforcing steel. Weldable SR-4 gauges were placed on the inner and outer surfaces of inner and outer reinforcing bar cages. Concrete strain meters were placed at the midplane of the wall. Typically, eight points at 45° intervals around a section were instrumented with strain gauges.

Carlson and Cambridge contact stressmeters were installed at the outer surface of the concrete pipe. The Carlson meters provided normal pressures and the Cambridge meters normal and tangential pressures.

For measurement of interior wall displacements, steel balls were fastened at eight equally spaced points about the perimeter. An extensometer was used to determine 14 chord measurements, from which horizontal and vertical displacements could be calculated.

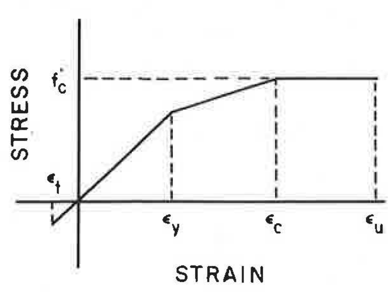
COMPUTER PROGRAM

The CANDE finite element computer program was used to perform all calculations described in this paper. CANDE is a FORTRAN program specifically made for the analysis and design of buried culverts. The program operates on three levels: (a) a modified elasticity solution developed by Burns and Richards (7), (b) a finite element procedure with automatic mesh generation, and (c) a finite element procedure in which the mesh is as defined by the user. The program has an executive routine that provides for the selection of common types of pipe culverts and five soil models. The common pipe types include steel, aluminum, concrete, plastic, and nonstandard. The soils models are linear elastic, orthotropic linear elastic, overburden dependent, extended Hardin, and an extended Hardin model for use with triaxial soil test data.

In this project, only the concrete pipe routine was used. The dummy pipe was assumed to have the nonlinear stress-strain relation shown in Figure 2. Cracking is handled by allowing only a small tensile strain (ϵ_t), usually taken as zero. For compressive loading, a trilinear curve is used. ϵ_y represents the strain at the elastic limit ($1/2\ f'_c/E_1$), where f'_c is the unconfined compressive strength in pounds per square inch and E_1 is the linear Young's modulus as defined by $E_1 = 33 \times \delta_w^{1.5}\ f'_c$, where δ_w is unit weight in pounds per cubic foot. ϵ_c represents the concrete strain at the ultimate strength, usually 0.002, and ϵ_u is the strain at crushing.

The concrete pipe was modeled in the finite element computer program by beam-column elements. To account for tension cracking and crushing of the concrete and yielding of the reinforcing steel, sectional properties of the beam column (area, centroid location, and moment of inertia) were changed to satisfy equilibrium and compatibility with the assumed stress-strain law for the concrete. Traditional assumptions of concrete analyses were maintained: (a) Circumferential strains varied in a linear fashion through the pipe wall section, (b) shear deformation was not included, and (c) longitudinal stresses and strains were neglected.

Figure 2. Stress and strain curve for concrete.



The soil was modeled by using plane-strain quadrilateral and triangular elements. The quadrilateral element, a nonconforming element developed by Hermann (8), was two triangular elements, each a quadratic interpolation function. The quadrilateral element has four nodes with a vertical and horizontal degree of freedom at each node and is generated from the triangular elements by applying constraints to ensure compatibility and static condensation procedures.

Several types of soil models were used during the course of the project: (a) linear elastic, (b) overburden dependent, and (c) extended Hardin. In the linear elastic model, the matrix [C] is constant and is completely defined by two parameters: Young's modulus (E) and Poisson's ratio (ν). The [C] matrix relates stress to strain, where C_{11}, C_{12}, C_{21}, C_{22}, and C_{33} are material constants defined below for plane strain conditions:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma \end{bmatrix} \quad (1)$$

where

σ_x = vertical stress,
σ_y = horizontal stress,
τ = shearing stress,
ϵ_x = vertical strain,
ϵ_y = horizontal strain, and
γ = shearing strain.

$$C_{11} = C_{22} = [E(1-\nu)]/[(1+\nu)(1-2\nu)] \quad (2)$$

$$C_{21} = E\nu/[(1+\nu)(1-2\nu)] = C_{12} \quad (3)$$

$$C_{33} = E/[2(1+\nu)] \quad (4)$$

In the overburden-dependent model, the nonlinear behavior of soil is characterized by secant values of the soil stiffness modulus that vary as a function of overburden pressure. The two parameters needed to characterize the soil are the secant constrained modulus (M_s) and the coefficient of horizontal earth pressure (K_o). In confined compression conditions, lateral deformations are prevented and $K_o = \nu/(1-\nu)$. Poisson's ratio (ν) is generally constant under this type of loading condition. The coefficients of the plane strain constitutive matrix are $C_{11} = C_{22} = M_s$, $C_{12} = C_{21} = M_sK_o$, and $C_{33} = M_s(1-K_o)/2$. The secant confined modulus is related to secant Young's modulus (E_s) by

$$\left\{(1-\nu)/[(1+\nu)(1-2\nu)]\right\} E_s = M_s \quad (5)$$

Reasonable estimates of E_s as a function of overburden pressure are available from the CANDE engineering manual and from Duncan (9). Values from the CANDE manual are presented below for structure backfill (granular soil with good compaction) and embankment material (mixed soil with good compaction) (1 psi = 6.89 kPa):

Overburden Pressure (psi)	E_s (psi) Structure Backfill	Embankment Material
5	1130	600
10	1300	850
15	1500	1000
20	1650	1100
25	1800	1200
30	1900	1250
40	2100	1350
50	2250	1450

Poisson's ratio (ν) for the CANDE values is 0.3-0.35 for structure backfill and 0.35-0.40 for embankment material.

Values of E_s recommended by Duncan (9) for a compacted SW soil are as follows:

Overburden Pressure (psi)	E_s (psi)
5	833
10	1040
20	1260
30	1440
40	1570

Duncan's values are approximate averages of the values suggested by the CANDE manual. Chang, Espinoza, and Selig (10) have suggested revisions of values provided in the CANDE manual. Only the suggested values of E_s and ν given in the two tables above were used during this project.

The third type of soil model used in this study was the extended Hardin model. The model originally developed by Hardin (11) relates shear stress (τ) to shear strain (γ) by a secant shearing modulus (G_s), where $\tau = G_s\gamma$. The secant shearing modulus (G_s) is expressed in hyperbolic form as $G_s = G_{max}/(1 + \gamma_h)$, where γ_h is the hyperbolic shear strain. The advantage of the original model was that the coefficients (given below) were related to fundamental soil parameters (plasticity index, degree of saturation, void ratio, and soil type). The general relations of Hardin's work are as follows:

$$G_{max} = S_1 \sqrt{\sigma_m} \tag{6}$$

$$\gamma_h = \gamma/\gamma_r \left\{1 + [a/\exp(\gamma/\gamma_r)^{0.4}]\right\} \tag{7a}$$

$$\gamma_r = G_{max}/c_1 \tag{7b}$$

where

- γ = shear strain;
- σ_m = spherical stress = $(\sigma_{11} + \sigma_{22} + \sigma_{33})/3$;
- γ_r = reference strain;
- S_1 = soil parameter related to void ratio = 1230F;
- a = soil parameter related to soil type and degree of saturation = 3.2 for granular soil, 2.54(1 + 0.02S) for mixed soil, and 1.12(1 + 0.02S) for cohesive soil;
- c_1 = soil parameter related to void ratio, plasticity index, and degree of saturation = $(F^2R^2)/[0.6 - 0.25(PI)^{0.6}]$;
- F = (2.973 - e)/(1 + e);
- R = 1100 for granular soil and 1100 - 6S for mixed or cohesive soil;
- e = void ratio;
- S = degree of saturation ($0 \leq S \leq 100$); and
- PI = plasticity index ($0 \leq PI < 1.0$).

Hardin's law was extended in the CANDE engineering manual by adding a functional relation for Poisson's ratio. As in Hardin's work, a hyperbolic relation was developed for a secant Poisson's ratio (ν_s):

$$\nu_s = (\nu_{min} + \gamma_p \nu_{max})/(1 + \gamma_p) \tag{8a}$$

$$\gamma_p = q\,\gamma/\gamma_r \tag{8b}$$

where

- ν_{min} = minimum Poisson's ratio at zero shear strain,
- ν_{max} = Poisson's ratio at failure, and
- q = dimensionless parameter for curve shape.

The CANDE program can be used with recommended values for soil parameters, or the user may define them from triaxial tests.

ANALYSIS

The computer analysis of zone 6 of the Cross Canyon culvert was done in two phases. In the first phase, Lee (12) modeled the dummy pipe in the conventional fashion by assuming vertical symmetry and only represented the culvert as a series of semicircular connected beam elements. Quadrilateral elements, representing the soil, extended above, below, and to the right of the culvert. In the second analysis phase, the entire cross section of the canyon, including the 243.8-cm (96-in) functioning culvert, was included. This grid is shown in Figure 3.

Lee compared results of calculations by using the CANDE program with the measured performance of zone 6. He obtained the most appropriate overburden soil modulus by a trial-and-error procedure until he reduced the difference between the measured and computed performance of the culvert. Lee's recommended values for structure backfill and embankment are given below (1 psi = 6.89 kPa):

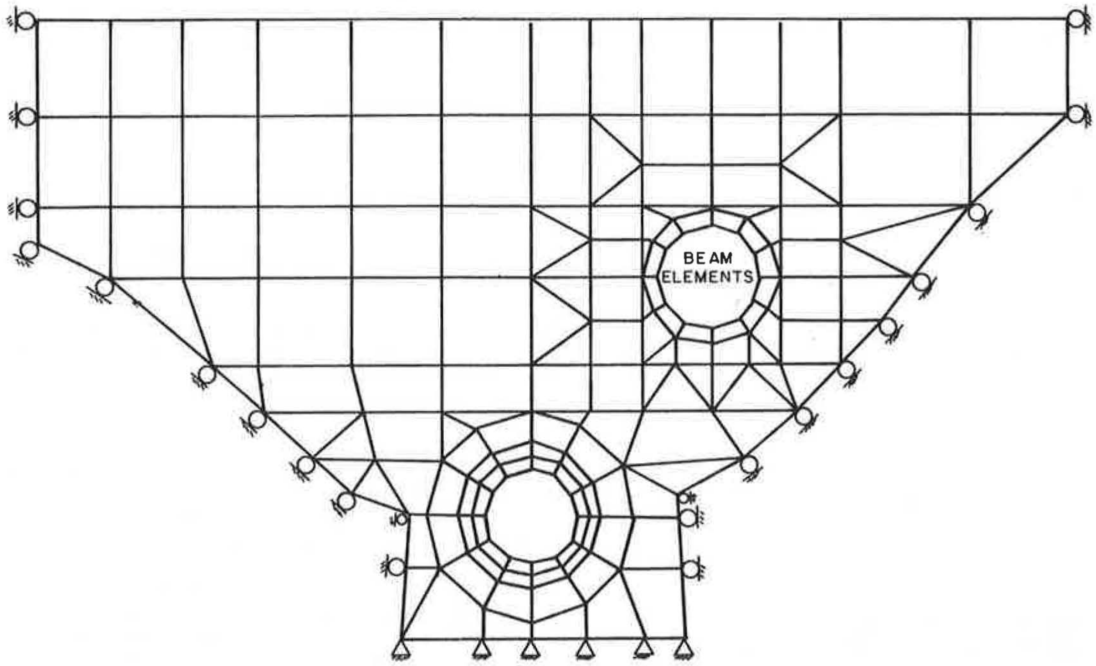
Overburden Pressure (psi)	E_s (psi) Structure Backfill	Embankment Material
5	6900	5100
10	7500	5500
15	7900	5900
20	8400	6200
25	8700	6500
30	9000	6800
40	9500	7200
50	9960	7600

Poisson's ratio (ν) for Lee's values is 0.20 for structure backfill and 0.24 for embankment material.

Lee's values are much stiffer than those recommended by Duncan or the CANDE manual. Lee felt he must follow Chang's lead and develop a new overburden model because the existing recommended values produced poor comparisons between calculated and measured performance of the culvert. He also derived an overburden-dependent model from the published triaxial soil test data, but again calculated performance did not agree with measured performance. Lee came to these conclusions by using a conventional symmetrical finite element grid that modeled only culvert and soil and not a grid that incorporated the entire canyon cross section.

In the second phase of the analysis, the grid shown in Figure 3 was used. Beam elements were used to represent the 213.4-cm (84-in) dummy concrete culvert. Two layers of quadrilateral elements were used to represent the 243.8-cm (96-in) functioning prestressed culvert. The soil was represented by triangular and quadrilateral elements. The grid

Figure 3. Finite element grid.



extended to an elevation of 5.5 m (18 ft) above the top of the dummy culvert. Fill placed above this elevation was represented by a surcharge pressure applied to the top of the mesh.

Based on published soil test data, there was no statistically significant difference between structure backfill and embankment material. For the preliminary results presented in this paper, the soil was thus considered to be homogeneous. The concrete in the 213.4-cm pipe was modeled as a nonlinear material. The concrete in the 243.8-cm functioning culvert was modeled as linearly elastic. As previously mentioned, the three types of soil models were linear elastic, overburden dependent, and extended Hardin.

For the linear elastic model, a value of 96.46 MPa (14 000 psi) was selected as an average value for Young's modulus for the fill. This was the initial tangent Young's modulus determined from a triaxial compression test when the overburden pressure was one-half the final overfill. A Poisson's ratio of 0.20 was used.

Two overburden-dependent models used average values of Lee's results and overburden pressures suggested by Duncan (Duncan's values of Young's modulus for a well-graded sand compacted to 95 percent of its maximum dry density were given previously). A Poisson's ratio of 0.2 was used. The comparison of the two overburden-dependent models also provided a method of checking Lee's suggested values.

The following table defines the input parameters for the extended Hardin soils model:

Parameter	Value
Soil type	Mixed
Minimum void ratio	0.1
Maximum void ratio	0.49
Poisson parameter q	0.260
Degree of saturation	0.46
Plasticity index	0.020
Density (lb/ft^3)	131

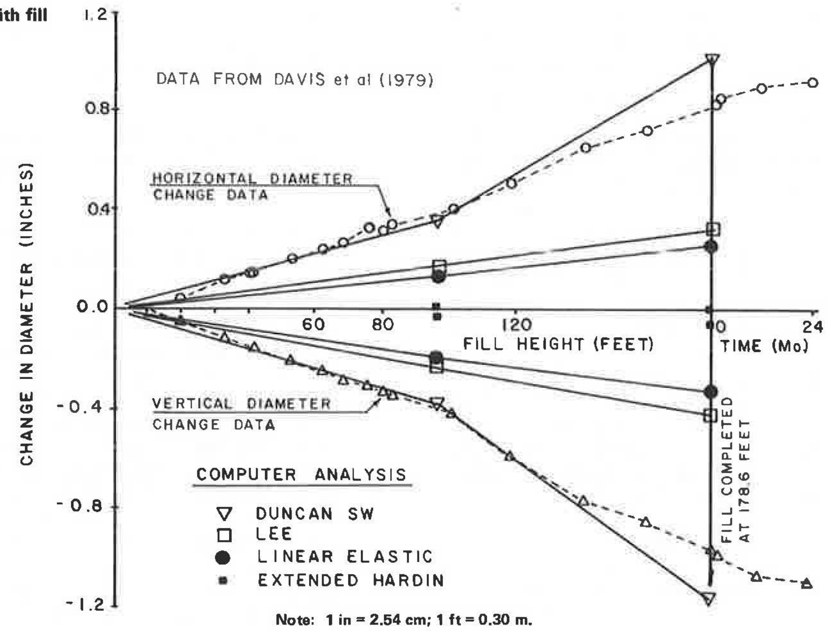
BEHAVIOR OF DUMMY CULVERT

Vertical and horizontal diameter changes of the 213.4-mm (84-in) reinforced concrete pipe installed at zone 6 at Cross Canyon are shown in Figure 4. The changes in vertical diameter increase in a linear fashion up to an overfill of 30.5 m (100 ft). For overfills greater than 30.5 m (100 ft), the vertical diameter changes in a nonlinear fashion. The change in horizontal diameter varies essentially in a linear fashion with increasing fill height.

Development of a 0.25-mm (0.01-in) crack 30.5 cm (12 in) long has long been an industrial strength criterion. Such a crack was not recorded until the overfill had reached 30.3 m (99.3 ft). Delamination was observed in different segments at an average overfill of 27.7 m (91 ft). Delamination, or "bowstringing", occurs when reinforcing steel with a minimal amount of embedment and subjected to tensile stresses on the concave section of the pipe straightens and forces the inner surface concrete to separate from the central concrete core. The yield strength of the reinforcing steel was reached at approximately 19.5 m (64 ft). The break of the slope of the vertical diameter change in Figure 4 was most likely due to delamination. A similar discontinuity, though less pronounced, is visible in the horizontal diameter change curve. The pipe was considered to have reached failure at an overfill of 30.3 m (99.3 ft).

Shown in Figure 4 are vertical diameter changes computed by using the three types of soil models. Only two data points were plotted for each soil model, one at 29.6 m (97 ft) and the other at 54.6 m (179 ft). At the 29.6-m overfill, the linear elastic model and overburden-dependent model, based on Duncan's recommended values, produce reasonable estimates of deflection. At an overfill of 54.6 m (179 ft), the Duncan overburden-dependent model overestimated the vertical diameter by 19.6 percent. The other two soil models grossly underesti-

Figure 4. Diameter changes with fill height.



Note: 1 in = 2.54 cm; 1 ft = 0.30 m.

mated the vertical diameter changes. The errors for the linear elastic model and for Lee's overburden model at an overfill of 54.5 m (178 ft) for the vertical diameter change were 66 and 57 percent, respectively. The extended Hardin model predicted only negligible diameter changes.

In summary, results were obtained by using only one construction increment. An appropriate pressure was applied to the top of the finite element grid to represent different overfills. The computer program did not take into account reinforcing steel movements due to delamination. Duncan's recommended values produced reasonable comparisons in the second phase of the analysis, when the boundary conditions included the canyon walls and the presence of the functioning culvert. The lack of these boundary conditions was most likely the reason for the unsatisfactory results achieved by using recommended overburden-dependent moduli in the first phase of the analysis. Hence, accurately representing boundary conditions is important in backfiguring or improving soil modulus values.

Duncan's overburden model was used to compare measured culvert behavior with observed behavior when the pipe was well within its allowable overfill. Deflections, bending moments, and external normal pressures were compared for an overfill of 12.5 m (41 ft). Computed vertical and horizontal diameter changes were -3.8 mm (-0.149 in) and 3.2 mm (0.126 in), respectively. Actual diameter changes were -3.86 mm (-0.152 in) and 3.68 mm (0.145 in). Average percentage error between the measured results and the calculated results was 14 percent.

Figure 5 shows computed and actual bending moments around the pipe culvert. Actual bending moments were computed from strain readings. As the figure shows, computed and actual bending moments compare favorably at the crown and invert but not at the springing line on the left side of the diagram.

Figure 6 shows a comparison between computed and measured normal pressures acting on the external surface of the dummy culvert. Measured normal pressures were determined from Carlson and Cambridge stressmeters. Field data showed a maximum pressure at the invert and another maximum pressure at 10 o'clock with respect to the crown. High pressure at the invert was probably due to the contact stress

Figure 5. Comparison of experimental and computed bending moments.

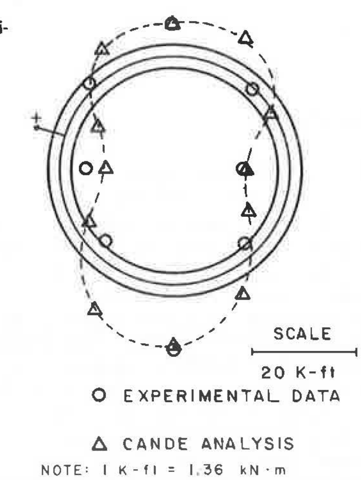
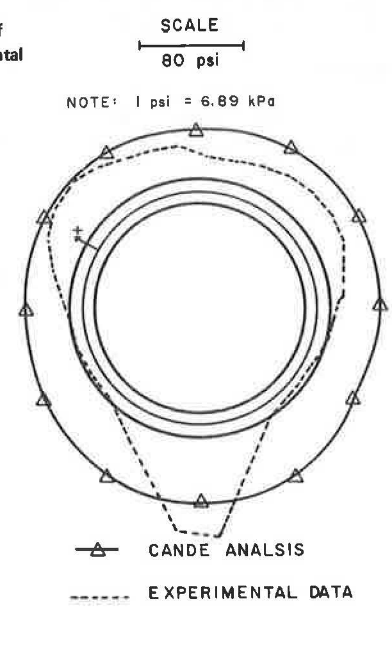


Figure 6. Comparison of computer and experimental normal pressures.



between the pipe and the aggregate bedding. Except at the invert, the computed normal pressures were greater than any other measured normal pressures. Computed pressures do not necessarily produce a conservative design since the horizontal pressures provide a restraint against lateral movement. A decrease in lateral pressure would increase lateral movement and alter the wall bending moments.

My research (13) has shown that finite element analysis of buried culverts performed by using a linear elastic soil model is very dependent on the proper selection of Poisson's ratio for the soil. To show the effects of Poisson's ratio on Duncan's model, additional calculations were made for an overfill of 27 and 54.6 m (90 and 178 ft) for Poisson's ratios of 0.1 and 0.3. Vertical and horizontal diameter changes were compared with field data. The results, summarized below, show that Poisson's ratio affects diameter changes but does not produce as great an error as is produced when another soil model is used (1 ft = 0.3 m):

		Error (%)	
Overfill (ft)	Poisson's Ratio	Vertical Diameter Change	Horizontal Diameter Change
90	0.1	29	0
	0.2	-1	17
	0.3	15	35
178	0.1	7	-16
	0.2	-21	-26
	0.3	31	40

CONCLUSIONS

Three different types of soil models were compared for their effectiveness in predicting the behavior of an underdesigned reinforced concrete culvert installed in a deep fill. The extended Hardin model, a linear elastic model, and two overburden-dependent models were used. The overburden-dependent model recommended by Duncan predicted the actual behavior with the least error. The analysis was performed by using the CANDE computer program. Only one construction increment was used in conjunction with a surcharge pressure applied to the top of a relatively shallow finite element grid to simulate additional overburden. The program reasonably predicted prefailure and postfailure behavior of the pipe culvert. The need to accurately represent the actual boundary conditions in backfiguring overburden-dependent values was also shown.

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